ICTP Summer School on Cosmology 2022

4. "Dynamical Dark Energy"

Filippo Vernizzi, IPhT CEA/Saclay

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4.1 QUINTESSENCE

- Quintessence

True value of Λ is zero and cosmic acceleration is due to the potential of a scalar field (not really improving the C.C. problem but we must have an alternative to compare to the C.C.).

$$S = \int dx^4 \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right)$$

$$\Rightarrow \quad T_{\mu\nu}^{(\phi)} = \partial_\mu \phi \, \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} (\partial \phi)^2 + V(\phi) \right)$$

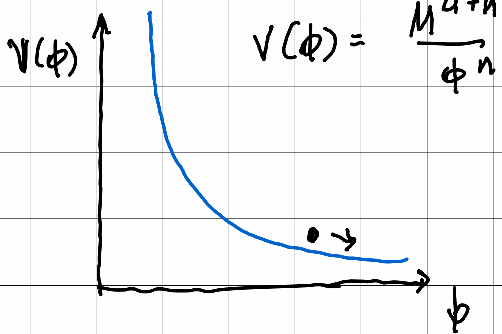
Homogeneous field $\phi = \phi(t)$

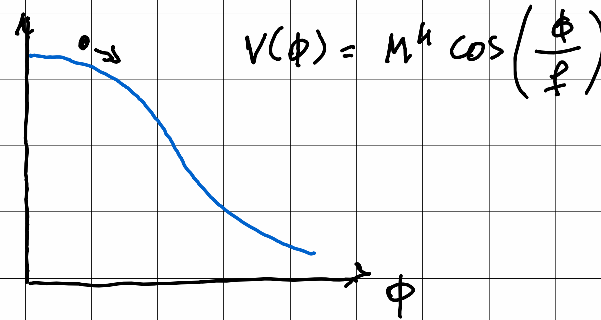
$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad \& \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

$$\Rightarrow \quad w_\phi = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} \gtrsim 1 \qquad \phi = \phi(t)$$

To have $w_\phi \simeq -1$ we need $\dot{\phi}^2 \ll V(\phi)$

- this can be realized in different ways





w_ϕ goes to -1 : "Freezing"

w_ϕ goes away from -1 : "Thawing"

- If slow-roll $H\dot{\phi} \sim V'$: $\Delta\phi \sim \dot{\phi} H^{-1} \sim \frac{V'}{H^2} \sim \frac{M_p^2}{\phi}$

where $M_p^2 \equiv \frac{1}{8\pi G}$

$V' \sim V/\phi$
$V \sim H^2 M_p^2$

$\frac{\Delta\phi}{\phi} \sim \frac{M_p^2}{\phi^2}$ super-Planckian field values for $\Delta\phi \ll \phi$

Difficult to realize in concrete examples : eg swampland conjecture $V' \gtrsim \frac{V}{M_p} \Rightarrow \phi \lesssim M_p$.

- Power-law potentials: eg.

$$m_\phi^2 \phi^2 \simeq H_0^2 M_p^2 \quad \Rightarrow \quad m_\phi \ll H_0 \sim 10^{-33}\,eV$$

$$\lambda \phi^4 \simeq H_0^2 M_p^2 \quad \Rightarrow \quad \lambda \ll 10^{-120}$$

Extremely tiny mass or quartic coupling: unnatural potential

- No fine-tuning if there is a symmetry protecting the potential

eg. PNGB $\quad V = M^4 \cos(\phi/f)$

However one needs $f \gg M_p$, hard to achieve

- Tracking field

Quintessence redshifts much slower than matter & radiation; one has to start with a very subdominant component. In some cases, quintessence tracks the dominant component and dominates at late times: it alleviates the coincidence problem!

$$V(\phi) = \frac{M^{n+4}}{n\,\phi^n} \quad n > 2 \quad \Rightarrow \quad \ddot{\phi} + \underbrace{\frac{3\alpha}{t}}_{3H}\dot{\phi} - \underbrace{\frac{M^{n+4}}{\phi^{n+1}}}_{V'(\phi)} = 0 \quad \text{for } a \propto t^{\alpha}$$

$\left(\alpha = \frac{1}{2} \text{ in RD} ; \alpha = \frac{2}{3} \text{ MD}\right)$

$$\phi = C\,t^{\beta} \Rightarrow \frac{C\beta(\beta-1)}{t^2} + \frac{C\,3\alpha\beta}{t^2} - \frac{M^{n+4}}{C^{n+1}t^{\beta(n+1)}} = 0 \qquad \beta = \frac{2}{n+2}$$

$$C: \ \beta(\beta-1+3\alpha) = \frac{M^{n+4}}{C^{n+2}}$$

One can check that this solution is an attractor: $\phi = C t^{\beta} + \varphi$

$$\ddot{\varphi} + \frac{3\alpha}{t}\dot{\varphi} + \underbrace{\frac{(n+1)M^{n+4}}{\phi^{n+2}}}\varphi = 0$$

$$\frac{(n+1)\,M^{n+4}}{C^{n+2}\,t^2} = \frac{(n+1)\,\beta(\beta-1+3\alpha)}{t^2}$$

Solution $\varphi \propto t^{\gamma}$ with

$$\gamma = \frac{1}{2}\left(1 - 3\alpha \pm \sqrt{\text{negative}}\right)$$

decaying solution

How does ρ_ϕ redshift with expansion?

$$\dot\phi^2 = \beta^2 C^2 t^{2(\beta-1)} = \frac{\beta^2 C^2}{t^{\frac{2n}{2+n}}}$$

$$V(\phi) = \frac{M^{n+4}}{n C^n t^{n\beta}} = \frac{M^{n+4}}{n C^n t^{\frac{2n}{n+2}}}$$

$\beta = 2/(n+2)$

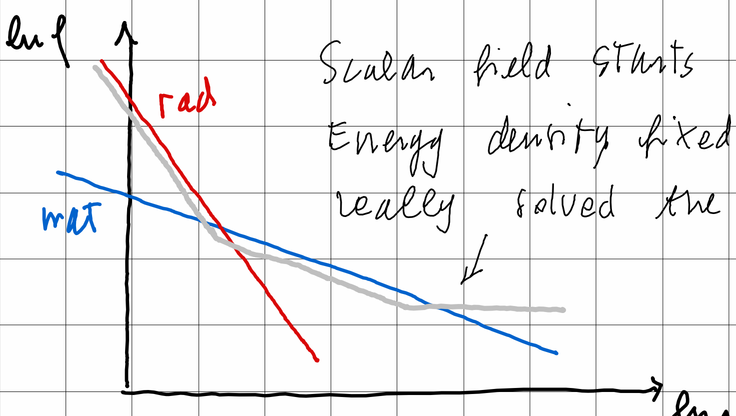
$$\left.\right\} \dot\phi^2 \propto V \propto \rho_\phi \propto t^{-\frac{2n}{n+2}}$$

$$\frac{1}{t^2} \propto H^2 \propto \rho_i$$

On the other hand, if a "i" component dominates I have $\rho_i \propto t^{-2}$

$$\Rightarrow \boxed{\frac{\rho_\phi}{\rho_i} \propto t^{\frac{4}{n+2}}} \longrightarrow \text{const for } n \gg 1$$

It always track the dominant component



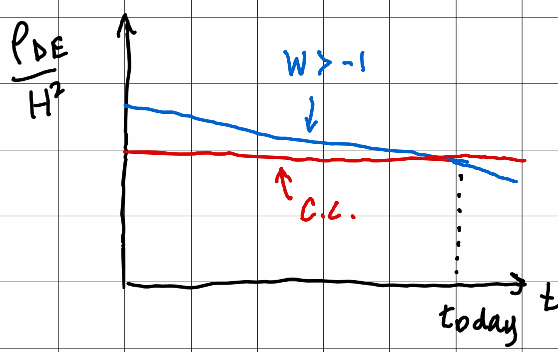
Scalar field starts entering slow-roll at $\phi \sim M_p$

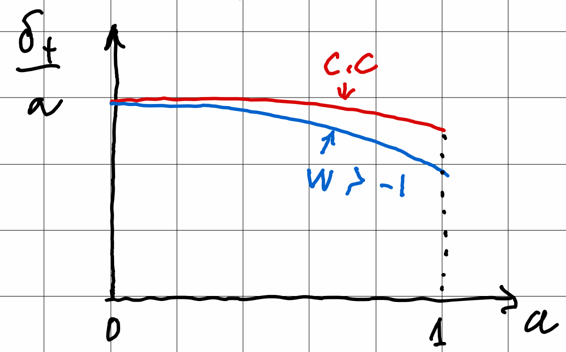
Energy density fixed by M, it doesn't really solved the "why now" problem.

• Quintessence perturbations

If quintessence has a relativistic speed of "sound", $c_s \simeq 1$, then it does not cluster: $\ddot{\delta\phi} + 3H\dot{\delta\phi} + c_s^2 k^2 \delta\phi$ = metric perts.

Scalar field fluctuations can travel large distances $a(t)\int_0^t \frac{c_s\, d\tilde{t}}{a(\tilde{t})}$ and homogenize. However, $w > -1$ affects the growth: (this eqn. is still valid. $\ddot{\delta} + 2H\dot{\delta} = 4\pi G \rho_m \delta$).





Quintessence is more relevant in the past, & suppresses more perturbations.

- Clustering quintessence

Quintessence can cluster $\qquad \ddot{\delta\phi} + 3H\dot{\delta\phi} + c_s^2 k^2 \delta\phi = \ldots$

$$S = \int dx^4 \sqrt{-g}\, P(\phi, X) \qquad X = g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

$$T_{\mu\nu} = 2P_{,X}\,\partial_\mu\phi\,\partial_\nu\phi + P\,g_{\mu\nu} \qquad \text{where } P_{,X} \equiv \frac{\partial P}{\partial X}$$

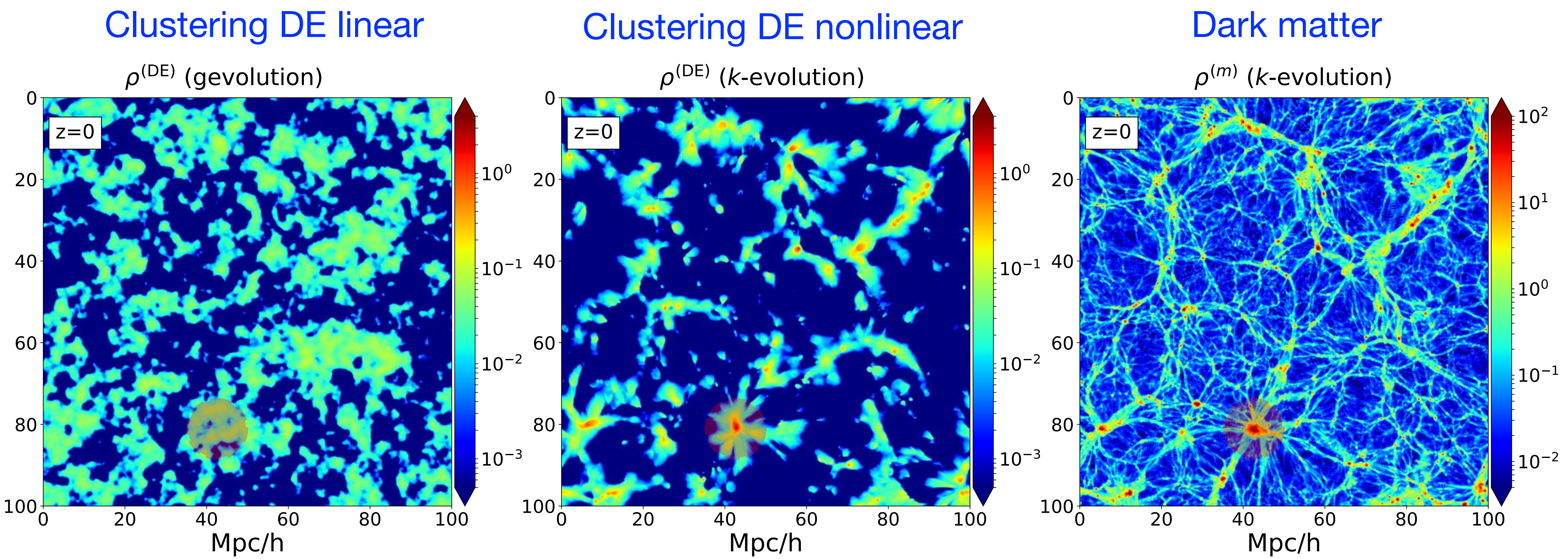
Perfect fluid form with $\rho = 2P_{,X}X - P$, $p = P$, $u_\mu = \dfrac{\partial_\mu\phi}{\sqrt{-X}}$

Perturbing the Lagrangian using $\phi = \phi_0(t) + \delta\phi(t, \vec{x})$ $\quad (X_0 = -\dot{\phi}_0^2)$
up to second order:

$$\mathcal{L} = -P_{,X}\left[\dot{\delta\phi}^2 - (\vec{\nabla}\delta\phi)^2\right] - 2P_{,XX}X_0\,\dot{\delta\phi}^2 + \ldots \qquad c_s^2 = \frac{P_{,X}}{P_{,X} + 2X\,P_{,XX}}$$

Dark energy can cluster if $X P_{,XX} \gg P_{,X} \Rightarrow c_s^2 \ll 1$

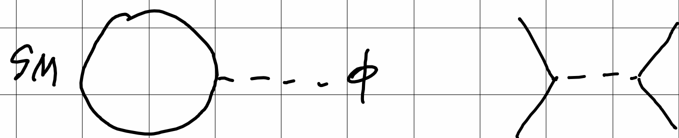
Clustering dark energy



4.2 TRADITIONAL SCALAR-TENSOR THEORIES

- Coupling to matter

Dark energy could be coupled to matter: maybe the acceleration is not due to Λ or $V(\phi)$ but to a genuine modification of GR.



fifth-force for ϕ light

The Equivalence P. is very well tested (see later). Better to have a universal coupling!

$\Rightarrow$ Scalar-Tensor gravity. traditionally:

$$S = \int d^4x \sqrt{-g} \left[f(\phi) R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_m [g_{\mu\nu}, \psi_m]$$

modified Einstein eqn.

$$\Rightarrow \quad f(\phi) G_{\mu\nu} - \nabla_\mu \nabla_\nu f(\phi) + g_{\mu\nu} \Box f(\phi) = \frac{1}{2} T^{(m)}_{\mu\nu} + \frac{1}{2} T^{(\phi)}_{\mu\nu} \qquad (\text{Ex } 4.1)$$

where $T^{(\phi)}_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \left(\frac{1}{2} (\partial\phi)^2 + V(\phi) \right)$; $\quad T^{(m)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$

When matter is coupled only to gravity we are in <u>Jordan-frame</u>.

Einstein frame :

Conformal transformation of the metric $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

$$\tilde{R} = \Omega^{-2} \left\{ R - 6 \nabla_\mu \nabla^\mu \log \Omega - 6 \nabla_\mu \log \Omega \, \nabla^\mu \log \Omega \right\}$$

(see Wald's GR book Appendix D)

$$\Rightarrow \int d^4x \sqrt{-\tilde{g}}\, f(\phi) \tilde{R} = \int d^4x \sqrt{-g}\, \underbrace{\Omega^2 f}_{\text{choose } \Omega^2 = f^{-1}} \left(R - 6 \nabla_\mu \nabla^\mu \log \Omega - 6 \nabla^\mu \log \Omega \, \nabla_\mu \log \Omega \right)$$

$g^{(E)}_{\mu\nu} = f(\phi)\, g^{(J)}_{\mu\nu}$ the kinetic term is modified but can be cast in standard form by a field red $\tilde{\phi}(\phi)$. $V(\phi)$ is rescaled.

$$S_m [g^{(J)}_{\mu\nu}, \psi_m] \longrightarrow S_m [f^{-1}(\phi)\, g^{(E)}_{\mu\nu}, \psi_m]$$

Jordan Frame $\qquad\qquad$ Einstein frame

"Frame" is a misnomer: The change of frame is just a "field redefinition" of the metric. Nothing to do with reference frame.

COMMENTS:

1. Jordan-frame may not be unique if different species couple to different metric: e.g. SM & DM.

2. In Einstein frame gravity is standard but the scalar field is coupled to the trace of the SE Tensor. Since it is light it mediates a long-range force: 5th force!

$$T_E^{\mu\nu} = \frac{2}{\sqrt{-g^E}} \frac{\delta S_m^E}{\delta g^E_{\mu\nu}} \qquad \nabla_\mu^E T_E{}^\mu{}_\nu = \frac{A'(\phi)}{A(\phi)} T_E \nabla_\nu^E \phi$$

Non-standard conservation

coupling to the Trace

$$\Box_E \phi - V_E'(\phi) = -\frac{A'(\phi)}{A(\phi)} T_E$$

The scalar is coupled to T_E

3. In Jordan frame matter is minimally coupled to $g_{\mu\nu}$.

Thus matter Test particles follow geodesics: $\nabla_\mu T^\mu{}_\nu = 0$

But Einstein and scalar field eqn are modified

$$\Box \phi - V'(\phi) + f' R = 0$$

(Einstein eqn was shown before)

Usually we interpret physical effects in this frame.
In Einstein frame masses of particles can vary ...

4. Three relevant effects:

- Fifth force

non-relativistic limit

J-frame: $\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0 \Rightarrow \ddot{x}^i + \Gamma^i_{00} = 0$

$g_{00} = -(1+2\Phi) \Rightarrow \boxed{\ddot{\vec{x}} = -\vec{\nabla}\Phi}$

but Φ does not satisfy the usual eqs.

$$\ddot{x}^i + \underbrace{\Gamma^i_{E00}}_{\nabla^i \Phi_E} = \underbrace{\Gamma^i_{E00} - \Gamma^i_{00}}_{-\frac{A'(\phi)}{A(\phi)}\nabla^i\phi = -\beta\nabla^i\phi}$$

$\Rightarrow \boxed{\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta\vec{\nabla}\phi}$ & $\boxed{\Phi = \Phi_E + \beta\phi}$

Consider a static body of mass M

$$\nabla^2 \Phi_E = 4\pi G \, M \, \delta^{(3)}(\vec{x}) \Rightarrow \Phi_E = -\frac{GM}{r}$$ Gravity is standard in E frame

One can neglect time derivatives for NR sources (quasi-static approx.)

$$(\nabla^2 - m^2)\phi = \beta M \delta^{(3)}(\vec{x}) \Rightarrow \phi = -\frac{\beta M}{4\pi r} e^{-m_\phi r}$$

attractive force $\propto \beta^2$ Yukawa supp.

Using $\Phi = \Phi_E + \beta\phi \Rightarrow$ $\boxed{\Phi = -(1+2\alpha^2)\frac{GM}{r}}$ $m_\phi \lesssim H$

$\alpha \equiv \frac{\beta}{\sqrt{8\pi G}}$ the "Newtonian" potential is enhanced

A mass $m_\phi \gtrsim H$ will induce modifications <u>scale (i.e. k) dependent</u>

- $\Phi \neq \Psi$

$$g_{\mu\nu} = A^2(\phi)\, g^E_{\mu\nu} \,;$$

$$\delta g_{\mu\nu} = 2AA'\bar{g}^E_{\mu\nu}\,\phi + A^2\,\delta g^E_{\mu\nu}$$

"00" $\quad \Phi = \Phi_E + \beta\phi \,;$

"ij" $\quad \Psi = \Psi_E - \beta\phi \quad$ with $\quad g_{ij} = (1 - 2\Psi)\,\delta_{ij}$

Using $\Phi_E = \Psi_E$ gravity is standard in E frame $\Rightarrow$ $\boxed{\Psi = -\left(1 - 2\alpha^2\right)\frac{GM}{r}}$

Light couples to $\Phi + \Psi$ therefore ϕ cancels, as it should, given that ϕ couples to T_m (trace).

- Variation of G_N

$$\frac{\dot{G}_N}{G_N} = 2\,\frac{A'}{A}\,\dot{\phi} = 2\beta\dot{\phi}$$

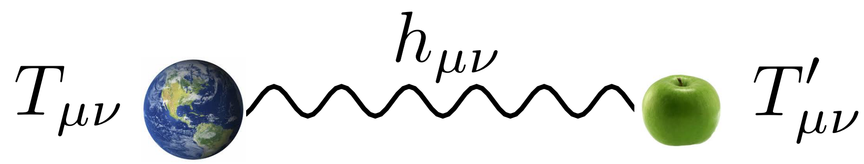
If ϕ evolves cosmologically, so does G_N

Jordan/Einstein frame

Jordan frame:

$$\mathscr{L} = -(\partial h)^2 - (\partial\phi)^2 - \beta\partial h\partial\phi - \frac{1}{2M_{\rm Pl}}h_{\mu\nu}T^{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



Solution of the modified (due to coupling between gravity and scalar field) Poisson equation

$$\Phi = -\frac{GM}{r}\left(1 + 2\alpha^2\right)$$ Fifth force

$$\alpha = M_{\rm Pl}\beta$$

Matter is conserved

$$\nabla_\mu T^\mu_\nu = 0$$

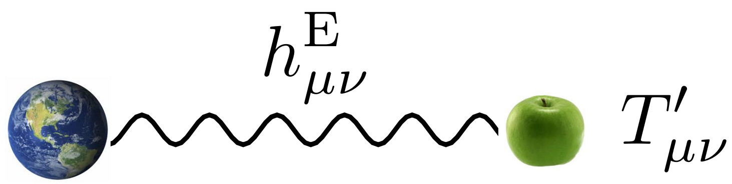
Particles follow geodesics

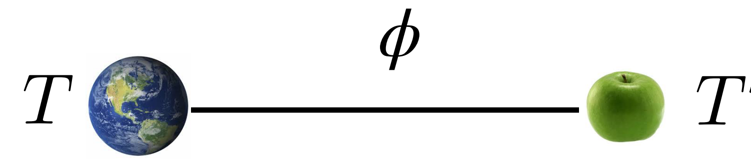
$$\ddot{\vec{x}} = -\vec{\nabla}\Phi$$

$$h_{00} = -2\Phi$$

Jordan/Einstein frame

Einstein frame: $$\mathscr{L} = -(\partial h_E)^2 - (\partial \phi)^2 - \frac{1}{2M_{\rm Pl}} h^E_{\mu\nu} T^{\mu\nu}_E - \frac{\beta}{M_{\rm Pl}} \phi T_E$$





$$\Phi_{\rm E} = -\frac{GM}{r}$$

$$\phi = -\beta \frac{GM}{r}$$

Matter is not conserved

$$\nabla^{\rm E}_\mu T^{\mu\nu}_{\rm E} = \frac{\alpha}{M_{\rm Pl}} T_{\rm E} \nabla^{\rm E}_\nu \phi$$

Particles do not follow geodesics: fifth force!

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta \vec{\nabla}\phi = -\vec{\nabla}\Phi$$

$$h^{\rm E}_{00} = -2\Phi_{\rm E}$$

$$h_{00} = -2\Phi$$

$$\Phi = \Phi_E + \beta\phi = -(1 + 2\alpha^2)\frac{GM}{r}$$

$$h_{ij} = -2\Psi\delta_{ij}$$

$$\Psi = \Psi_E - \beta\phi = -(1 - 2\alpha^2)\frac{GM}{r}$$

5. F(R) gravity

Consider the action $S = \frac{1}{16\pi G}\int dx^4 \sqrt{-g}\,\left(R + F(R)\right)$ (Ex. 4.2)

We can recast this action as

$$S = \frac{1}{16\pi G}\int dx^4\sqrt{-g}\,\left(R + F(\phi) + \frac{dF}{d\phi}(R-\phi)\right)$$

the field ϕ is non dynamical and its EOM, assuming $F'' \neq 0$, is

$\cancel{F'} + F''(R-\phi) - \cancel{F'} = 0 \Rightarrow \phi = R$ so we get the first action

this is like the action of scalar-tensor gravity but without kinetic term & $f(\phi) = 1 + \frac{dF}{d\phi}(\phi)$ & $V(\phi) = \phi - F(\phi)$

Conclusions Lecture 4

- Dark energy can be **dynamical**: a scalar field rolling down a potential. Generically **w> -1** and **this slows down** dark matter **clustering**. But dark energy could even cluster itself if its Lagrangian contains derivative couplings. These models suffer similar **fine tuning problems** that the CC.

- Can the accelerated expansion be due to a **genuine modification of gravity** (CC set to zero)?

- We have studied **traditional scalar-tensor** theories. Description can be done in **Jordan** and **Einstein "frame"**. In Jordan frame test particles follow geodesics of the Jordan frame metric but this metric does not satisfy the usual gravitational equations. In Einstein frame gravity is standard but matter is coupled to the scalar field. The physics is the same.

- Independently on the frame, there are three main effects. There is a **fifth force** that enhances the standard gravitational force. The two gravitational potentials are not the same (**anomalous light bending**). The **Newton constant** can **depend on time**.

EXERCISES

4.1 Derive the equations of motion from the action

$$S = \int dx^4 \sqrt{-g}\left[f(\phi) R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] + S_m[g_{\mu\nu}, \psi_m]$$

(Use $\sqrt{-g}\, g^{\mu\nu}\delta R_{\mu\nu} = \nabla_\alpha u^\alpha$ with $u^\alpha = \sqrt{-g}\left(g_{\mu\nu}\nabla^\alpha \delta g^{\mu\nu} - \nabla_\mu \delta g^{\alpha\mu}\right)$)

4.2 Consider the action $S = \int dx^4 \sqrt{-g}\, F(R)$

a) Show that the EOM obtained by $\delta S / \delta g^{\mu\nu} = 0$ are

$$F' R_{\mu\nu} - \frac{1}{2} F g_{\mu\nu} - \nabla_\mu \nabla_\nu F' + g_{\mu\nu} \Box F' = 0$$

b) Consider $F = \frac{1}{16\pi G}\left(R - \frac{3\mu^4}{R}\right)$ where μ is some energy scale.

Verify that $R = 3\mu^2$ & $R_{\mu\nu} = \frac{3}{4}\mu^2 g_{\mu\nu}$ is a solution of the EOM above and that it corresponds to de Sitter expansion (i.e. self-acceleration). Find H in terms of μ.

4.3 Show that starting from

$$S = \int dx^4 \sqrt{-g} \left[f(\phi) R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

with the following field redefinitions:

$$\chi = f(\phi) \qquad \partial_\mu \chi = f' \partial_\mu \phi \qquad f'(\phi)^2 \equiv \frac{\chi}{\omega}$$

one obtains the Jordan - Brans - Dicke action.

$$S = \int dx^4 \sqrt{-g} \left[\chi R - \frac{\omega}{2\chi} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right]$$

4.4 Consider the JBD theory above with matter

$$S = \int dx^4 \sqrt{-g} \left\{ \left[\chi R - \frac{\omega}{2\chi} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right] + \mathcal{L}_m [g_{\mu\nu}, \psi_m] \right\}$$

a) Find that the field eqs read:

$$\chi G_{\mu\nu} - \nabla_\mu \nabla_\nu \chi + g_{\mu\nu} \Box \chi - \frac{\omega}{\chi} \left(\nabla_\mu \chi \nabla_\nu \chi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \chi \nabla^\alpha \chi \right) = 8\pi G T_{\mu\nu}$$

$$\Box \chi = \frac{8\pi G}{3+2\omega} T$$

b) In the non-relativistic limit we have

$$G_{00} = 2\vec{\nabla}\Psi$$

$$\left(ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)d\vec{x}^2\right)$$

$$G_{ij} = (\partial_i\partial_j - \delta_{ij}\nabla^2)(\Psi - \Phi)$$

Consider a matter source $T_{00} = m\,\delta^{(3)}(\vec{x})$; $T_{0i} = T_{ij} = 0$

Show that:

$$2\nabla^2\Psi - \nabla^2\varphi = 8\pi G\, m\, \delta^{(3)}(\vec{x})$$

$$(\partial_i\partial_j - \delta_{ij}\nabla^2)(\Psi - \Phi - \varphi) = 0$$

$$\nabla^2\varphi = -\frac{8\pi G}{3+2\omega}\, m\, \delta^{(3)}(\vec{x})$$

c) Find that $G_* = \dfrac{4+2\omega}{3+2\omega}G$ & $\gamma = \dfrac{1+\omega}{2+\omega}$

for $\Phi = -\dfrac{G_* m}{r}$ & $\Psi = -\gamma\dfrac{G_* m}{r}$

d) What are the constraints on ω from the Cassini time-delay constraints?