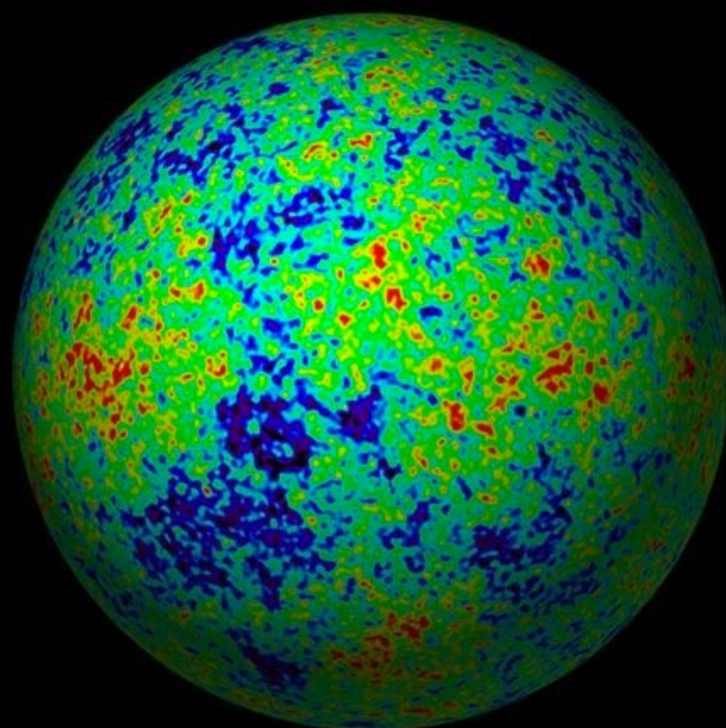
The Cosmic Microwave Background
Lecture 4: CMB tests of inflation; CMB polarization



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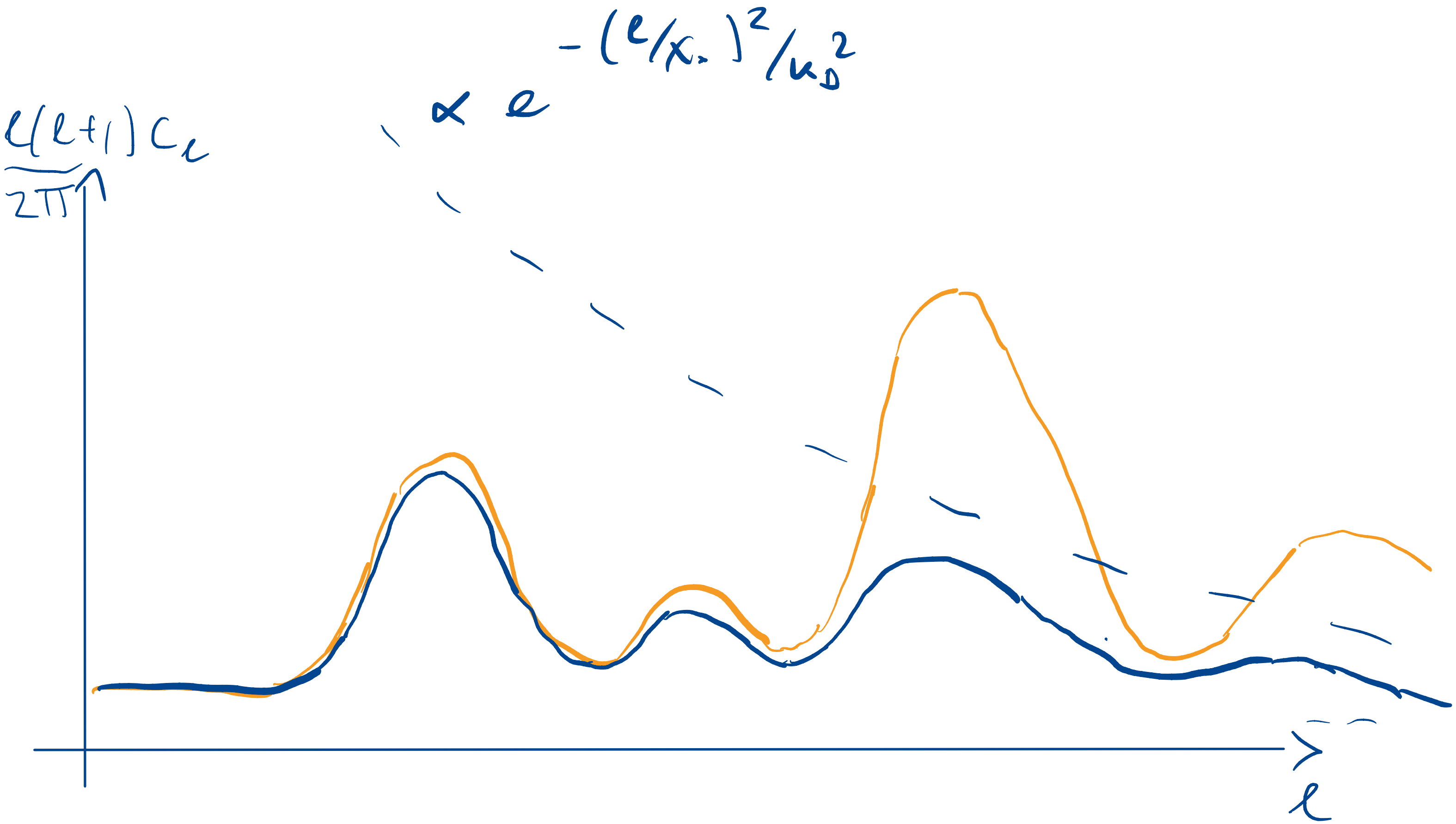
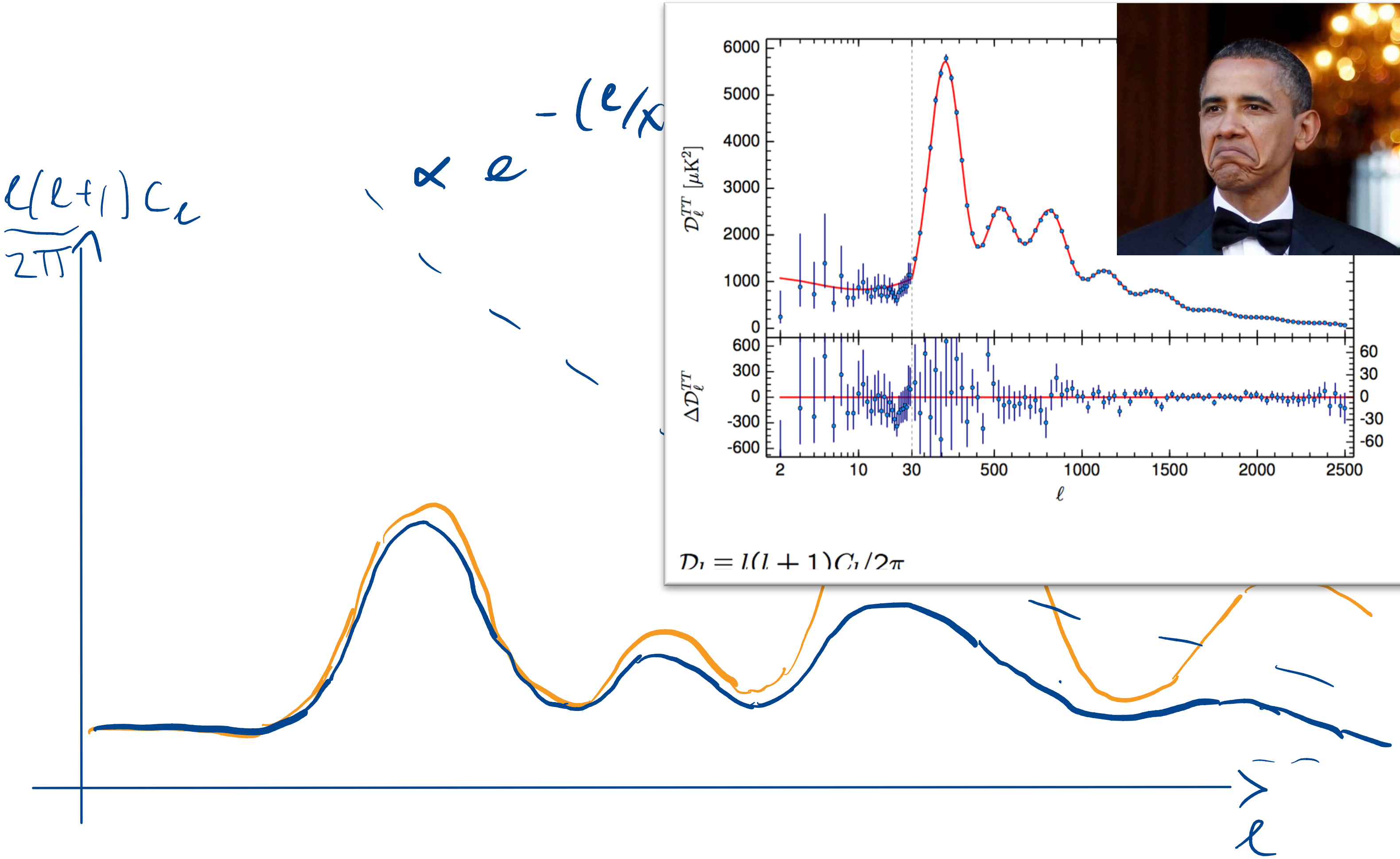
Reminder: Derived Form of Power Spectrum



- Ingredients: acoustic oscillations + baryons + doppler + damping + reionization

Reminder: Derived Form of Power Spectrum



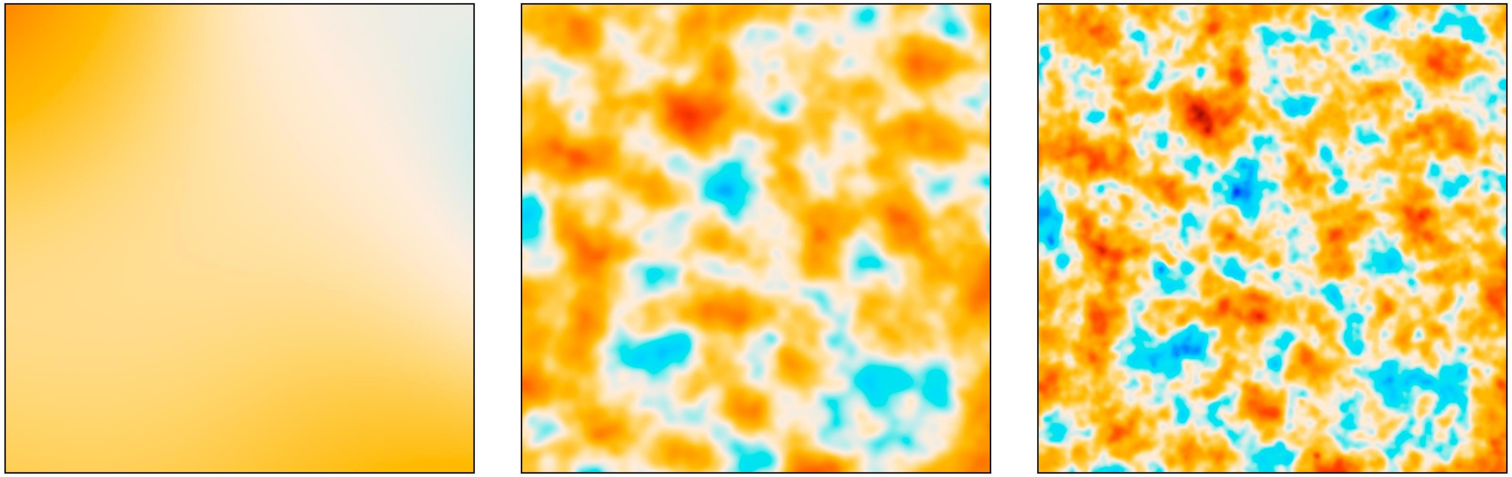
- Ingredients: acoustic oscillations + baryons + doppler + damping + reionization

Reminder: Understood how CMB can constrain parameters!

Parameter	Plik best fit	Plik [1]
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012
$100\theta_{MC}$	1.040909	1.04092 ± 0.00031
τ	0.0543	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.0448	3.044 ± 0.014
n_s	0.96605	0.9649 ± 0.0042
$\Omega_m h^2$	0.14314	0.1430 ± 0.0011
H_0 [km s^{-1}Mpc^{-1}]	67.32	67.36 ± 0.54
Ω_m	0.3158	0.3153 ± 0.0073
Age [Gyr]	13.7971	13.797 ± 0.023
σ_8	0.8120	0.8111 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.8331	0.832 ± 0.013
z_{re}	7.68	7.67 ± 0.73
$100\theta_*$	1.041085	1.04110 ± 0.00031
r_{drag} [Mpc]	147.049	147.09 ± 0.26

Outline

- Inflation: brief reminder and tests with the CMB temperature
- CMB polarization: basics of physics and spectra
- Testing inflation with CMB B-modes



Quick summary: inflation and initial conditions

- Reminder: inflation is a way to solve problems in cosmology to do with homogeneity and flatness. Typically done with slowly rolling scalar field.

During inflation: $H = \sqrt{\frac{V(\phi)}{3M_{\rm pl}^2}} \approx \text{constant}$ $\quad a \propto e^{Ht}$

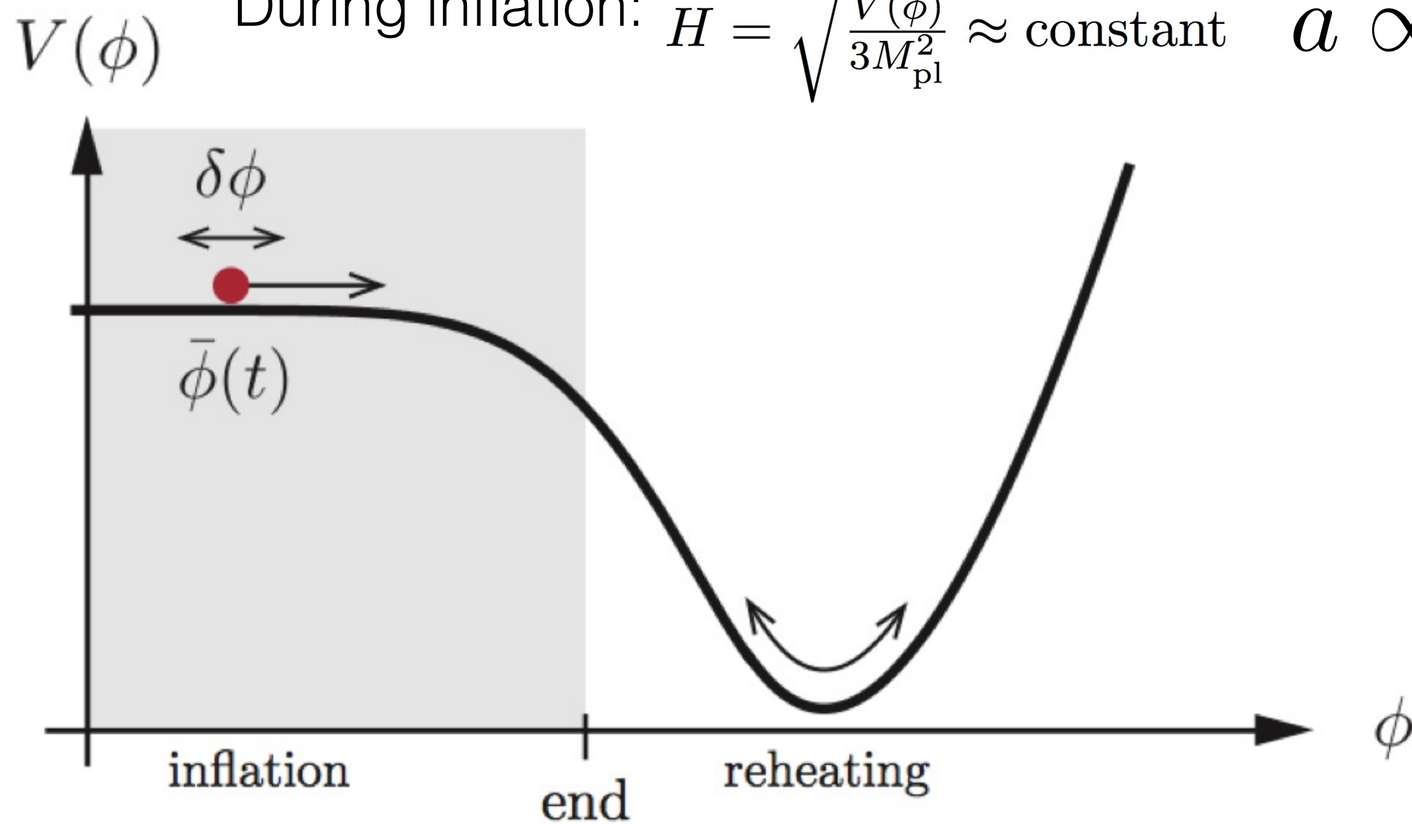
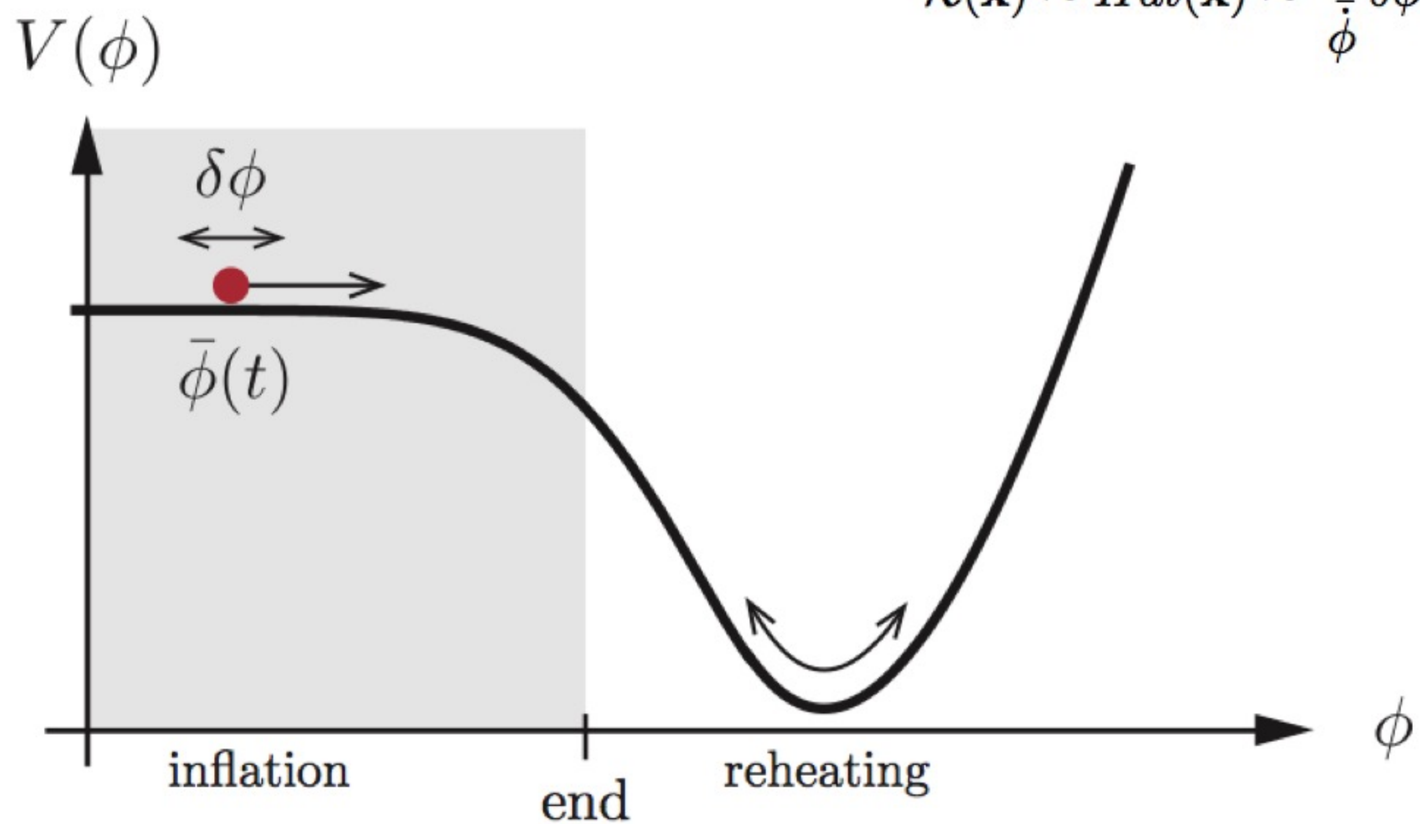


Image credit: D. Baumann

Quick summary: inflation and initial conditions

- But: quantum fluctuations change time to end of inflation! This changes the amount of expansion, giving:

$$\mathcal{R}(\mathbf{x}) \sim H dt(\mathbf{x}) \sim \frac{H}{\dot{\bar{\phi}}}\delta\phi(\mathbf{x})$$



Quick summary: inflation and initial conditions

- Write down action for scalar field

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

- Expand as $\phi = \bar{\phi} + \frac{f}{a}$ to obtain equation of motion:

$$f''_{\mathbf{k}} + (k^2 - \frac{a''}{a}) f_{\mathbf{k}} = 0$$

Inflation field fluctuation

Quick summary: inflation and initial conditions

$$f''_{\mathbf{k}} + (k^2 - \frac{a''}{a})f_{\mathbf{k}} = 0$$

- Important point: (on sub-horizon scales $k^2 >> a''/a$) this behaves just like a harmonic oscillator.

- We can quantize each k-mode of the field just like a harmonic oscillator! Introduce raising and lowering operators

$$\hat{f}_{\mathbf{k}} \sim f^*_{\mathbf{k}}(\eta)\hat{a}^\dagger_{\mathbf{k}} + f_{\mathbf{k}}(\eta)\hat{a}_{\mathbf{k}}$$

Quick summary: inflation and initial conditions

$$\hat{f}_{\mathbf{k}} \sim f^*_{\mathbf{k}}(\eta)\hat{a}^\dagger_{\mathbf{k}} + f_{\mathbf{k}}(\eta)\hat{a}_{\mathbf{k}}$$

- Now have non-zero expectation value for field value

$$\langle 0|\hat{f}^2_{\mathbf{k}}|0\rangle \neq 0$$

- and hence non-zero power spectrum of fluctuations

$\delta\phi = f/a$ and $\mathcal{R} \sim \frac{H}{\dot{\phi}}\delta\phi$

Quick summary: inflation and initial conditions

- and hence non-zero power spectrum of fluctuations

$\delta\phi = f/a$ and $\mathcal{R} \sim \frac{H}{\dot{\phi}}\delta\phi$. Result from calculation:

$$\frac{k^3}{2\pi^2} P_{\mathcal{R}} = \frac{1}{2\epsilon M_{pl}^2} \left(\frac{H}{2\pi} \right)^2_{k>aH}$$

- Since H and ϵ vary VERY slowly during inflation (V nearly constant), predict

Inflationary predictions I

- Inflation predicts nearly scale invariant spectrum of curvature perturbations:

$$\boxed{\frac{k^3}{2\pi^2}P_{\mathcal{R}} \propto k^{n_s-1} \approx \text{const.}}$$

- But not quite scale invariant because inflaton is slowly rolling down potential

Inflationary predictions II: gravitational waves

- Not just fluctuations in inflaton: fluctuations in metric

$$ds^2 = a^2(\eta)\left[d\eta^2 - (\delta_{ij} + 2\hat{E}_{ij})dx^i dx^j\right]$$

- Can write E as two gravitational wave polarizations

$$\frac{M_{\rm pl}}{2} a\hat{E}_{ij} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} f_+ & f_\times & 0 \\ f_\times & -f_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Inflationary predictions II: gravitational waves

- Insert in action

$$S = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g} R$$

- Get exact same equation of motion for f_+ and f_x as for inflaton fluctuation f! So power also a near scale invariant power spectrum of gravitational waves:

$$\frac{k^3}{2\pi^2} P_t \equiv 2 \frac{k^3}{2\pi^2} P_{\hat{E}} \approx \text{const.}$$

Testing inflation with CMB T: large scales

- Reminder: $$\frac{l(l+1)}{2\pi}C_l \approx T_S^2(\eta_*, k) \times \left[\frac{k^3}{2\pi^2}P_{\mathcal{R}}(k)\right]$$

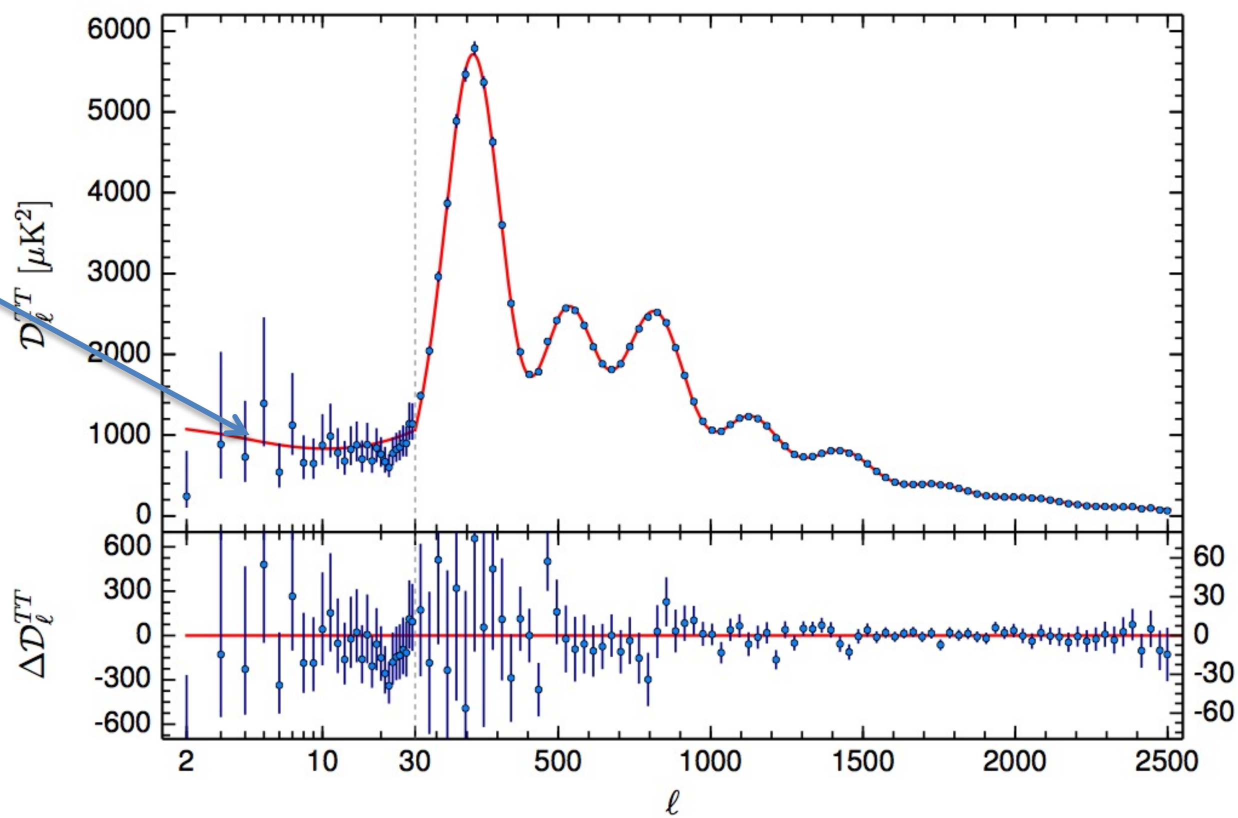
  Taking the large scale limit of $-\frac{1}{5}\mathcal{R}(\mathbf{k})[(1+3R)\cos(kr_s) - 3R]$
  we obtain: T_s=-1/5

- On superhorizon, large scales we are directly seeing the primordial fluctuations (superhorizon correlations not trivial!)

Testing inflation with CMB T: large scales

- Prediction: $$\frac{l(l+1)}{2\pi}C_l \approx \frac{1}{25}\left[\frac{k^3}{2\pi^2}P_{\mathcal{R}}\right] \approx \text{const.}$$

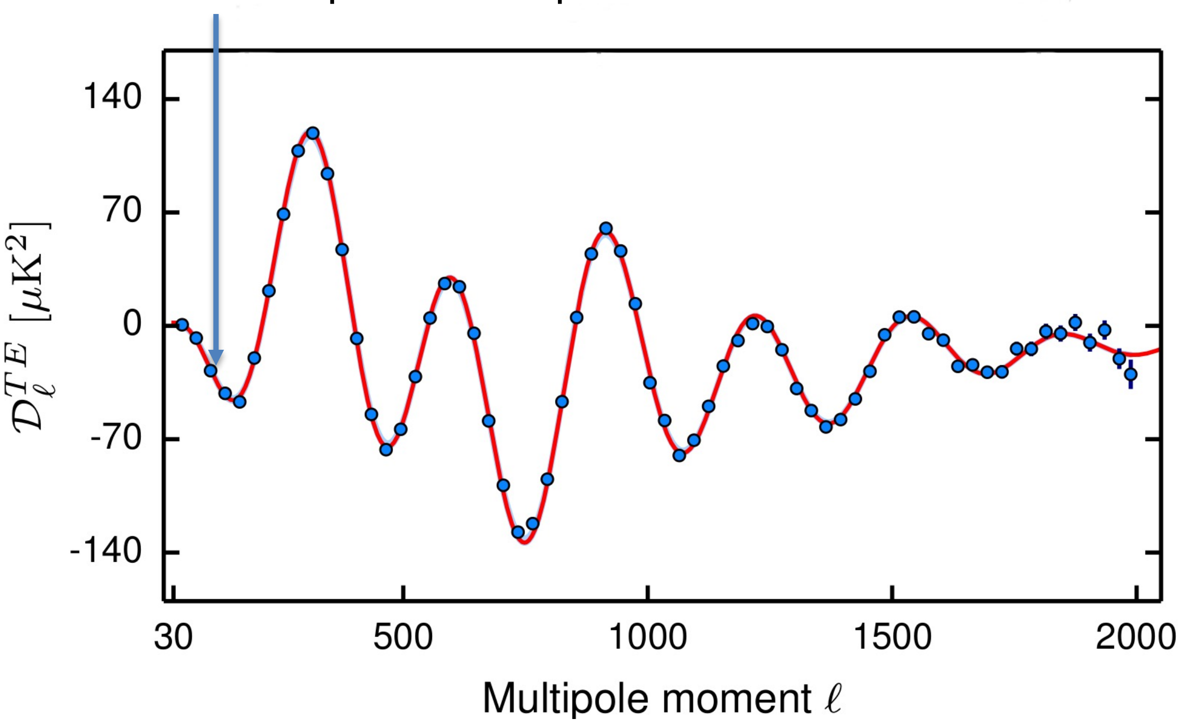
Observed!



$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$

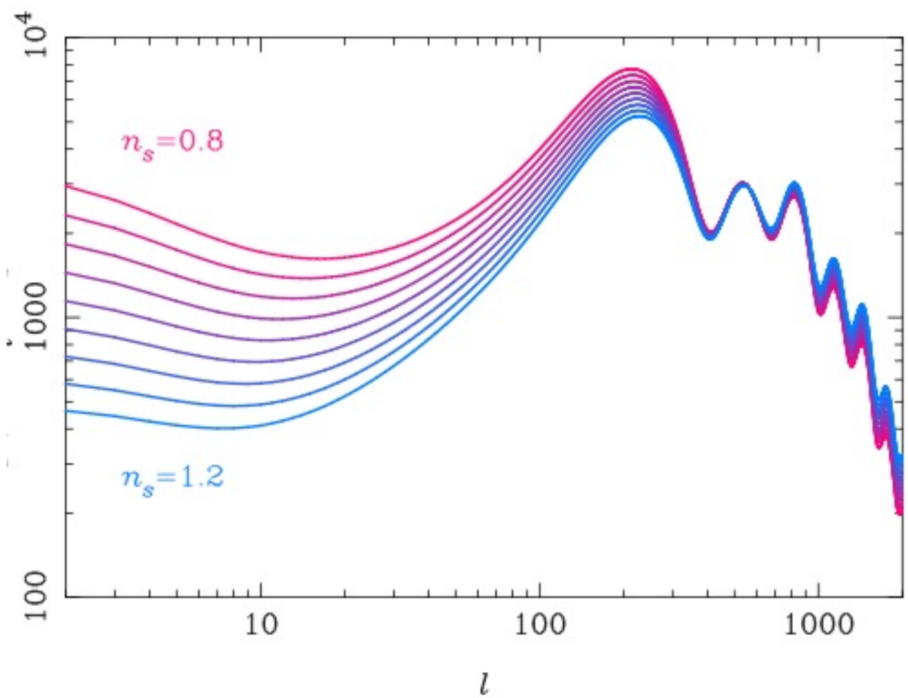
CMB T x E polarization (see later) – even better test!

- Inflation skeptic: could say +ve superhorizon power could just be random noise, not real correlations.
- Even stronger proof would be –ve correlation on superhorizon scales.
- Observed in TE power spectrum!



Testing inflation with CMB T: spectral index

- Prediction: $\frac{k^3}{2\pi^2} P_{\mathcal{R}} \propto k^{n_s - 1} \approx \text{const.}$
  where n_s is very close to 1 (but slightly different due to inflaton rolling down potential)



- Planck measurement: $n_s = 0.967 \pm 0.006$

Additional tests:

- Isocurvature modes: if perturbations are not adiabatic, oscillations are not pure cosine, some sine is generated. No evidence!

- Single field slow-roll inflation produces negligible non-Gaussianity. Field has been showed to be gaussian at level of few in 10^5!

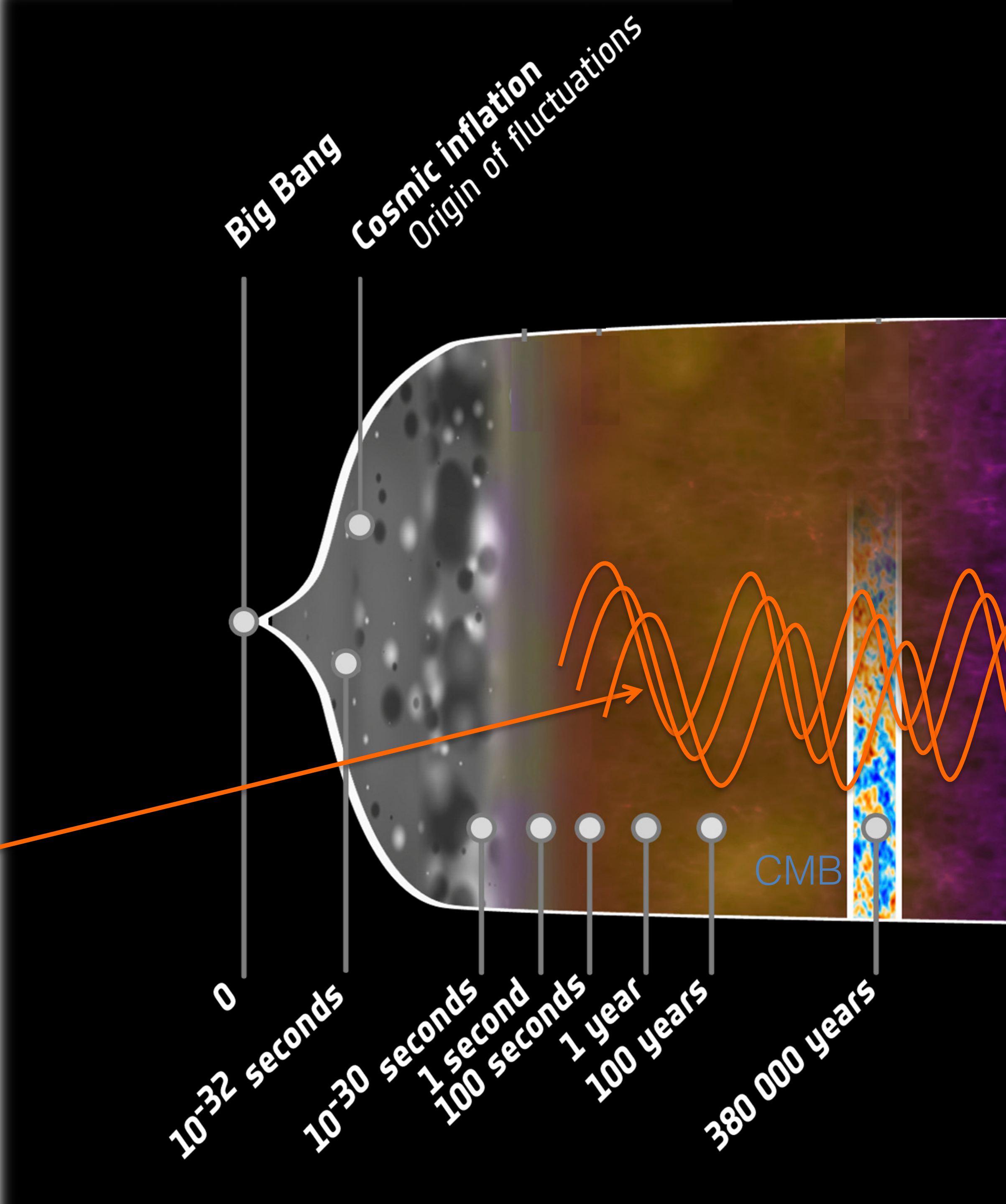
Gravitational waves from inflation?

- Would give physics at ultra-high energy (at the doorstep of the Planck scale)

$$V^{1/4} = 1.04 \times 10^{16}\mathrm{GeV} \left(\frac{r_*}{0.01}\right)^{1/4}$$

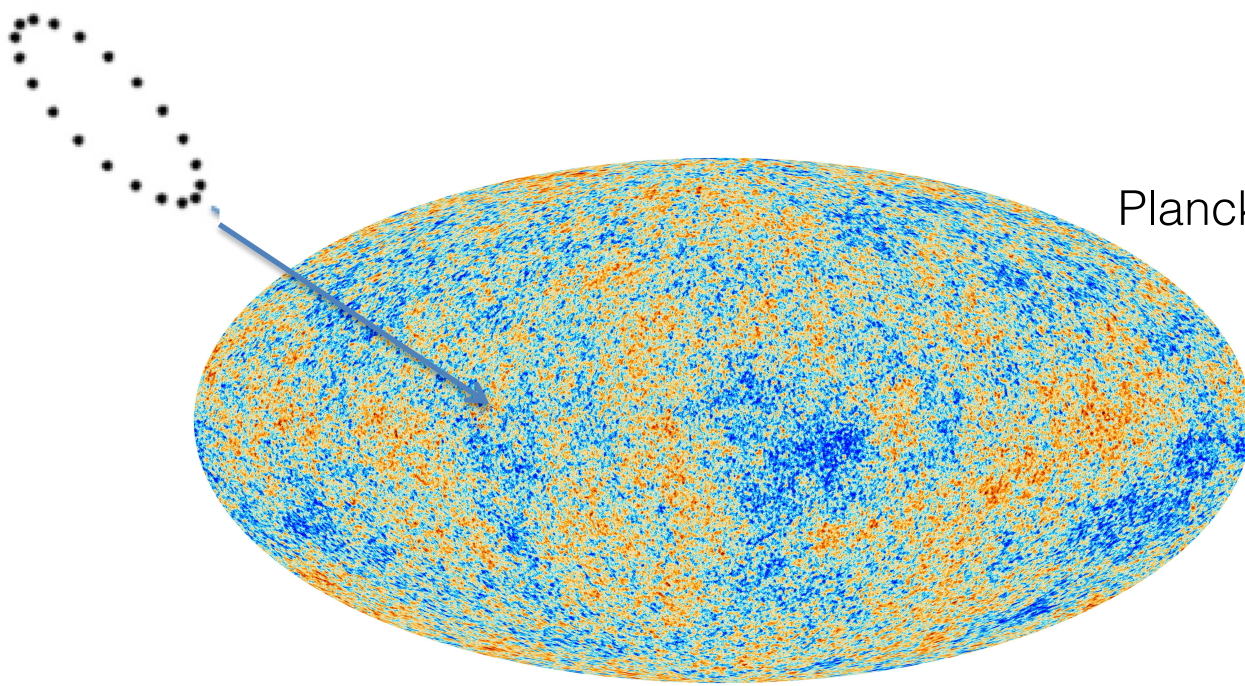
- The strength of the waves - tensor-to-scalar ratio r - tells us about the energy at which inflation happened!

N.B. Even improved upper limits interesting.



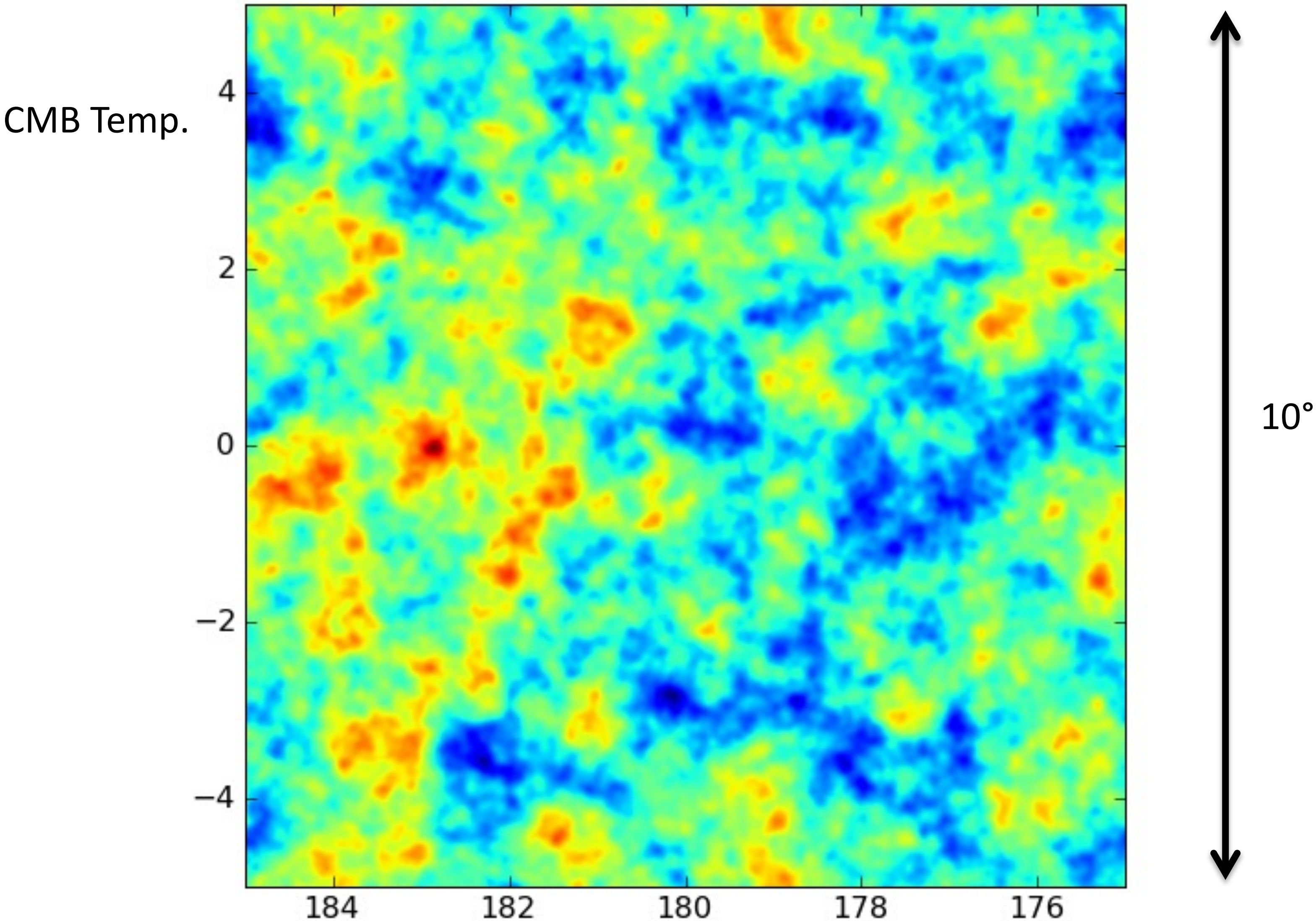
Testing Inflation with the CMB: Gravitational Waves

- Want to test inflationary prediction: gravitational waves
- Gravitational waves also red/blueshift photons and induce CMB patterns



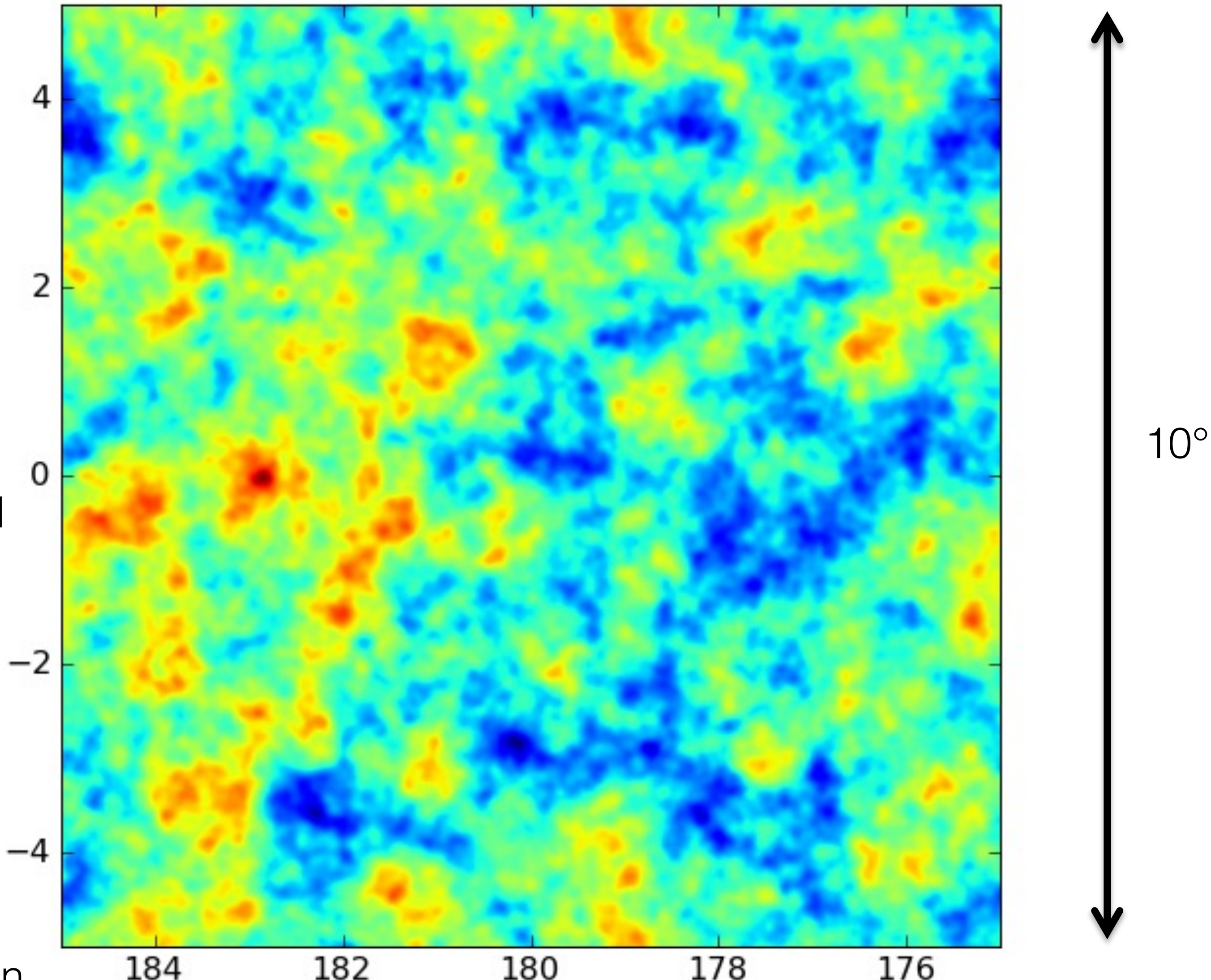
fluctuations in 2.73 K blackbody radiation

CMB temperature with no r



CMB temperature with very small r

CMB Temp. (cartoon picture)



difficult to find r due to confusion and cosmic variance from scalar density perturbations

Need to look in polarization!

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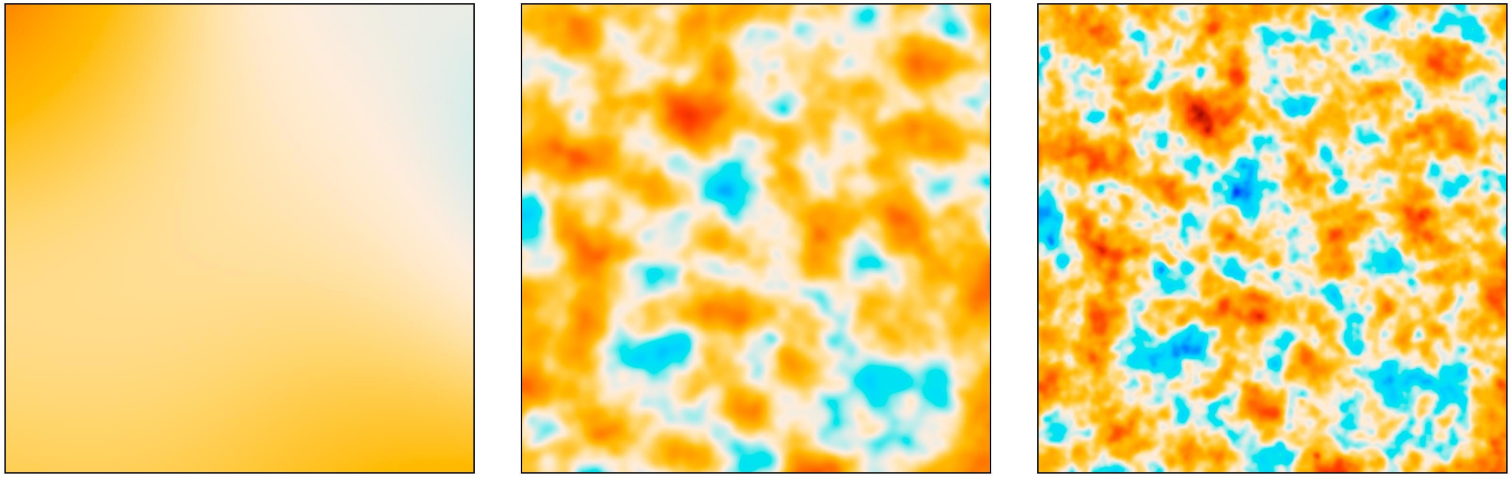
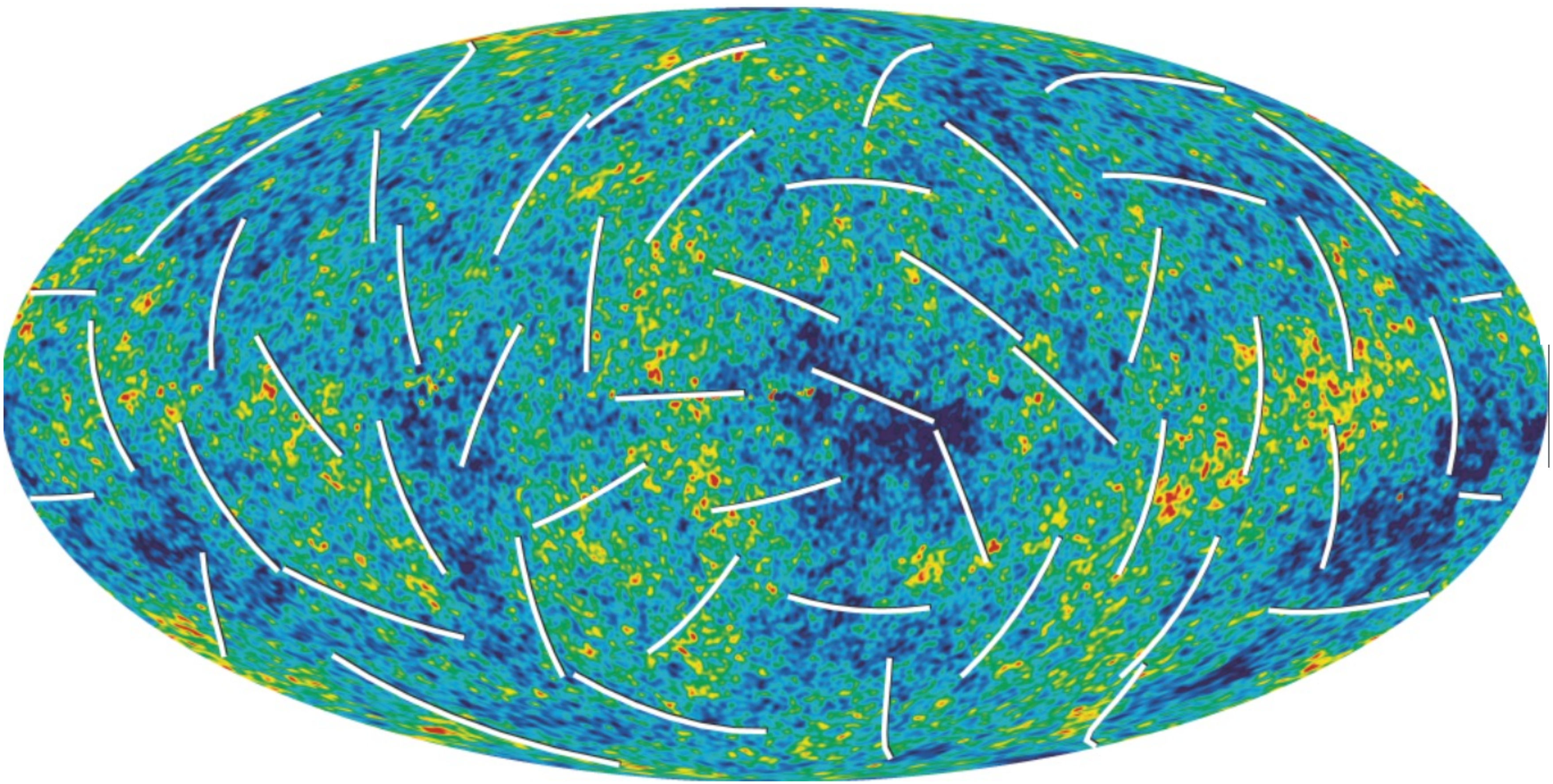


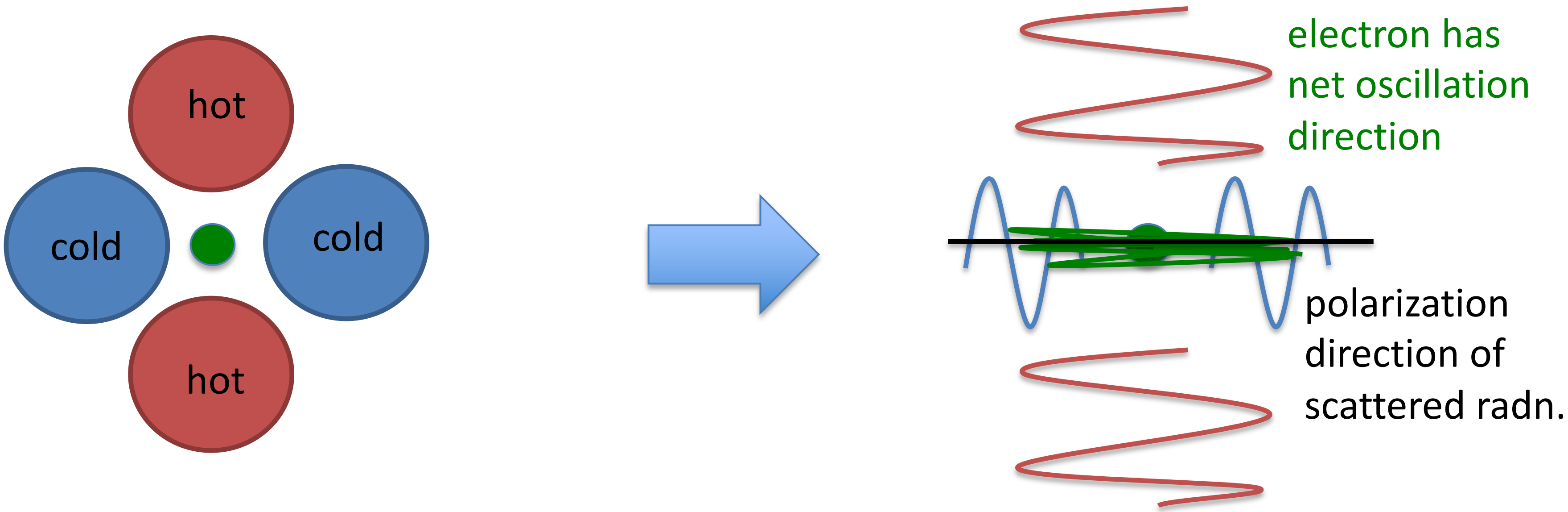
CMB Polarization

- The CMB is weakly polarized at the 10% level



The CMB Is Polarized by Anisotropic Scattering

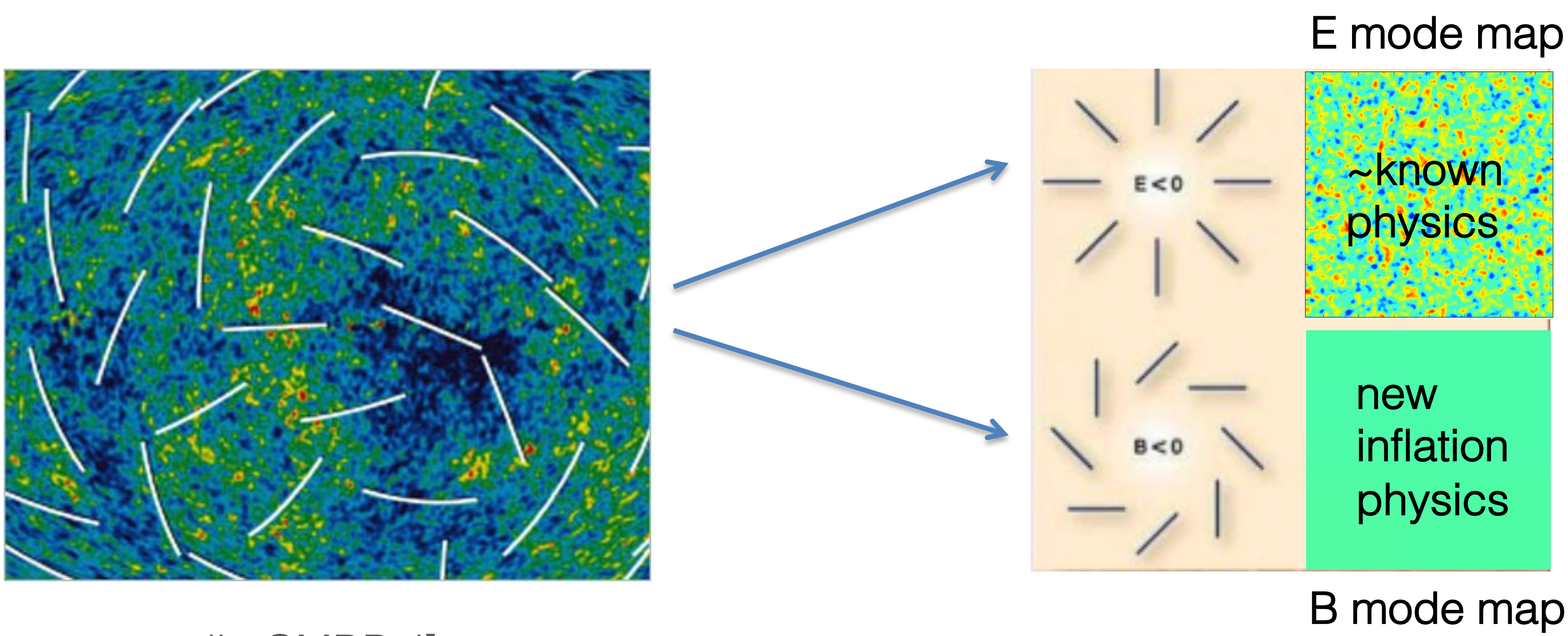
- Consider looking at an electron at the last scattering surface



- Direction in which electron is "seen" to oscillate gives direction of polarization of scattered radiation (perpendicular to hot direction)
- Hence, have net linear polarization if an electron sees a quadrupolar temperature variation.

CMB Polarization: Big Picture Overview

- Any polarization map can be decomposed into E and B mode fields
- <u>B-mode: contains signals from inflation, if there</u>



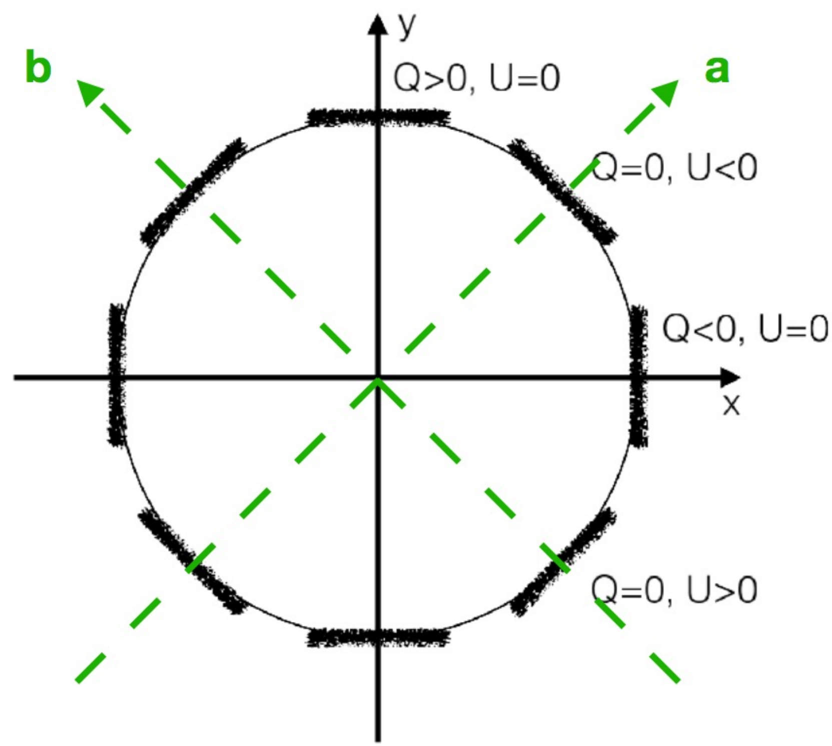
[Image credit: CMBPol]

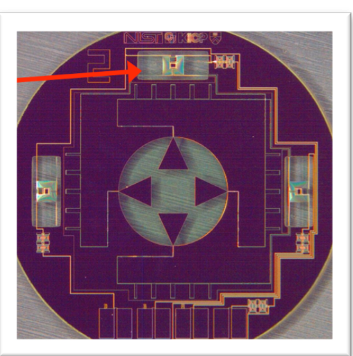
CMB Polarization Details

- Describe with Q and U fields defined via electric field:

$$Q \propto E_x^2 - E_y^2$$

$$U \propto E_a^2 - E_b^2$$

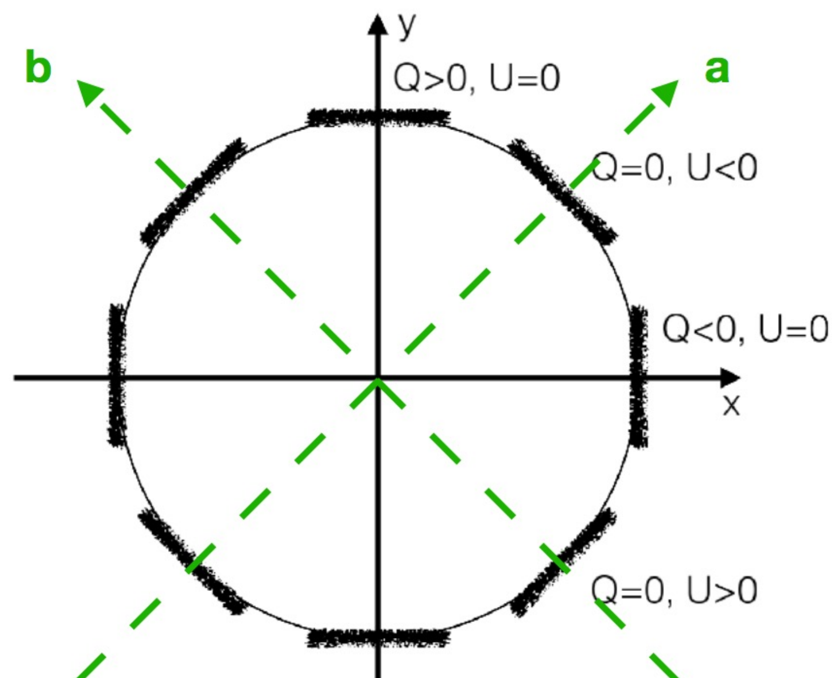




CMB Polarization Details

- Problem: coordinate dependent. If rotate coordinate system, Q/U transform: $$\begin{pmatrix} \tilde{Q} \\ \tilde{U} \end{pmatrix} = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ -\sin 2\varphi & \cos 2\varphi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}$$

$$Q \propto E_x^2 - E_y^2$$
$$U \propto E_a^2 - E_b^2$$



- Solution: define Q/U for each l relative to coords. along l

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- We can write Q+iU for convenience. Under rotation, now, equivalently:

$$\tilde{Q} + i\tilde{U} = \exp(-2i\varphi)(Q + iU)$$

CMB Polarization Details

- Need a coordinate-invariant expression. To do this, go to Fourier space and flat sky

$$Q(\mathbf{x}) + iU(\mathbf{x}) = \int \frac{d^2 l}{(2\pi)^2} a_{\mathbf{l}} e^{i\mathbf{l}\cdot\mathbf{x}}$$

$$\mathbf{l} = (l \cos\phi_{\mathbf{l}}, l \sin\phi_{\mathbf{l}})$$

angle of wavevector l relative to x axis

- If we rotate, a also changes, still. To avoid this, slightly modify the Fourier coefficient by setting

$$a_{\mathbf{l}} = -{}_{+2}a_{\mathbf{l}} e^{2i\phi_{\mathbf{l}}}$$

CMB Polarization Details

- If we rotate, a also changes, still. To avoid this, slightly modify the Fourier coefficient by setting

$$a_{\mathbf{l}} = -{}_{+2}a_{\mathbf{l}} e^{2i\phi_{\mathbf{l}}}$$

- Under rotation, angle of Fourier wavevector changes as

$$\phi_{\mathbf{l}} \to \phi_{\mathbf{l}} - \varphi$$

- Hence get same as factor on LHS so ${}_2a_l$ is unchanged:

$$Q(\mathbf{x}) + iU(\mathbf{x}) = -\int \frac{d^2 l}{(2\pi)^2} {}_{+2}a_{\mathbf{l}} e^{2i\phi_{\mathbf{l}}} e^{i\mathbf{l}\cdot\mathbf{x}}$$

$$\to e^{-2i\varphi}(Q(\mathbf{x}) + iU(\mathbf{x})) = -\int \frac{d^2 l}{(2\pi)^2} {}_{+2}a_{\mathbf{l}} e^{2i\phi_{\mathbf{l}}} e^{-2i\varphi} e^{i\mathbf{l}\cdot\mathbf{x}}$$

CMB Polarization Details

- This gives

$$Q(\mathbf{x}) \pm iU(\mathbf{x}) = -\int \frac{d^2l}{(2\pi)^2} {}_{\pm 2}a_{\mathbf{l}} e^{\pm 2i\phi_{\mathbf{l}}} e^{i\mathbf{l}\cdot\mathbf{x}}$$

- And now defining ${}_{\pm 2}a_{\mathbf{l}} \equiv -(E_{\mathbf{l}} \pm iB_{\mathbf{l}})$
  we obtain:

$$Q(\mathbf{x}) \pm iU(\mathbf{x}) = \int \frac{d^2l}{(2\pi)^2} (E_{\mathbf{l}} \pm iB_{\mathbf{l}}) e^{\pm 2i\phi_{\mathbf{l}}} e^{i\mathbf{l}\cdot\mathbf{x}}$$

CMB Polarization Details

- Equivalently:

$$Q(\mathbf{x}) = \int \frac{d^2l}{(2\pi)^2}(E_{\mathbf{l}} \cos 2\phi_{\mathbf{l}} - B_{\mathbf{l}} \sin 2\phi_{\mathbf{l}})e^{i\mathbf{l}\cdot\mathbf{x}}$$

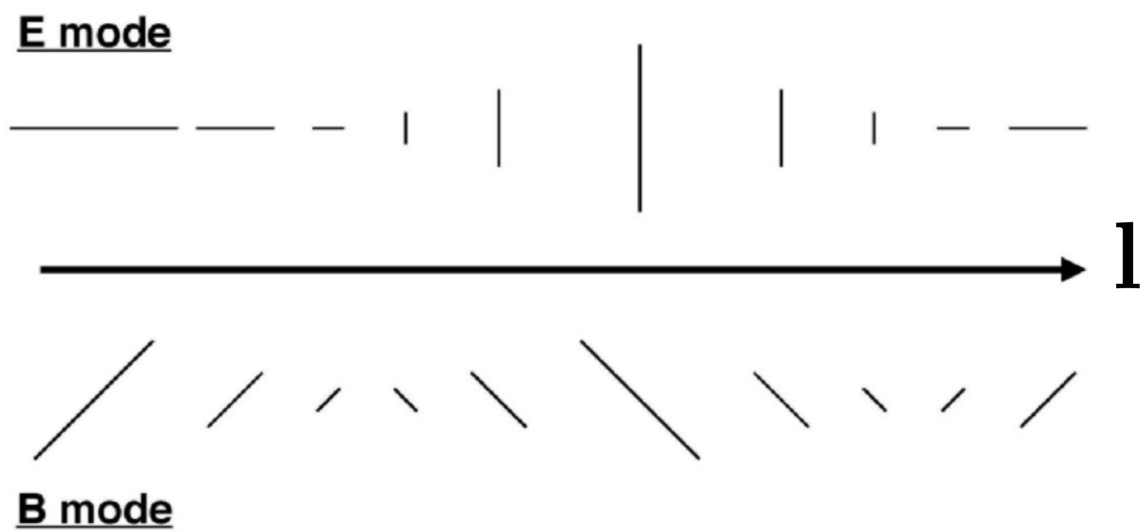
$$U(\mathbf{x}) = \int \frac{d^2l}{(2\pi)^2}(E_{\mathbf{l}} \sin 2\phi_{\mathbf{l}} + B_{\mathbf{l}} \cos 2\phi_{\mathbf{l}})e^{i\mathbf{l}\cdot\mathbf{x}}$$

- Where we have defined CMB E and B-modes. Can easily solve for E/B. These are now frame-independent!
- Interpretation: Have rotated frame describing Q / U to be aligned with wavevector l -> E / B

Interpretation

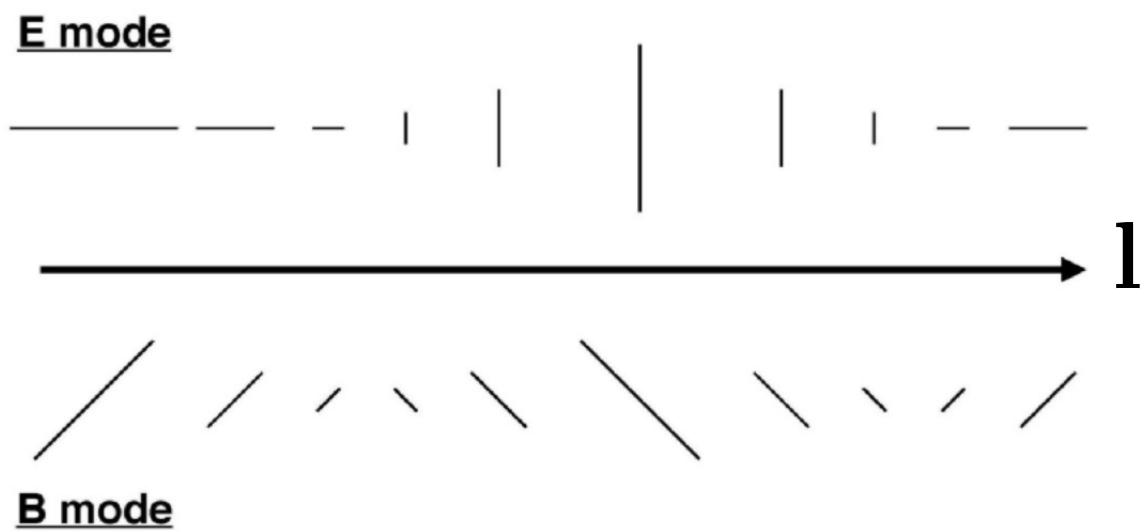
- Consider only one mode aligned with x axis. Now:

$$Q(\mathbf{x}) = E_{\mathbf{l}} e^{i\mathbf{l}\cdot\mathbf{x}}$$
$$U(\mathbf{x}) = B_{\mathbf{l}} e^{i\mathbf{l}\cdot\mathbf{x}}$$



Interpretation

- E is: i) Q in frame oriented with wavevector ii) parallel and perpendicular to wave vector; iii) parity even
- B is i) U relative to wavevector; ii) at 45 degrees to wavevector; iii) parity odd.



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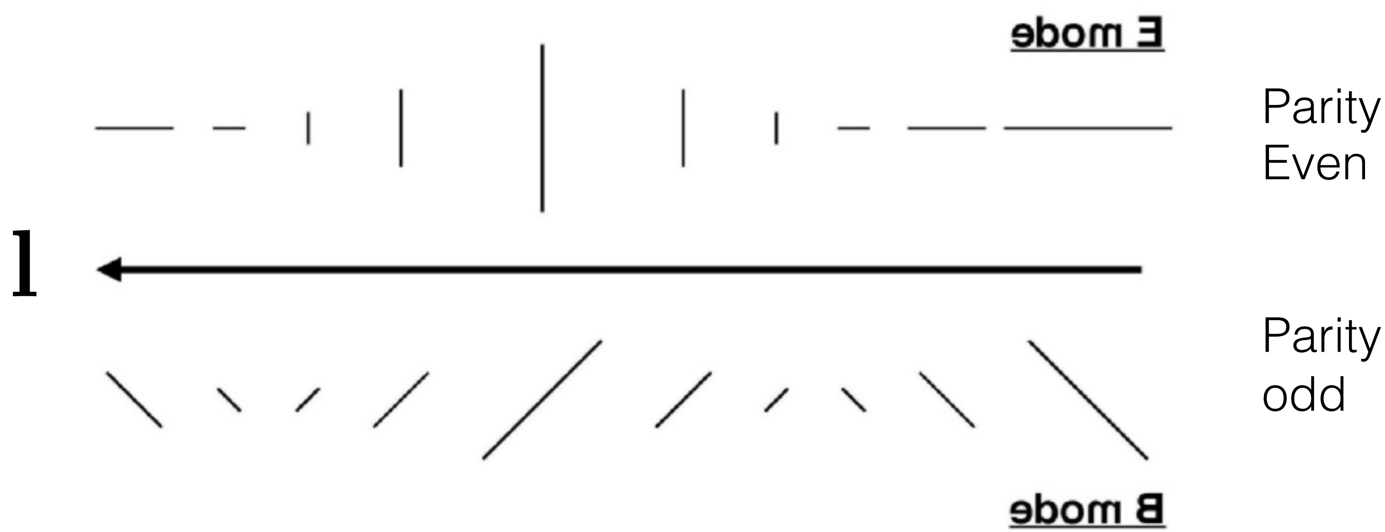
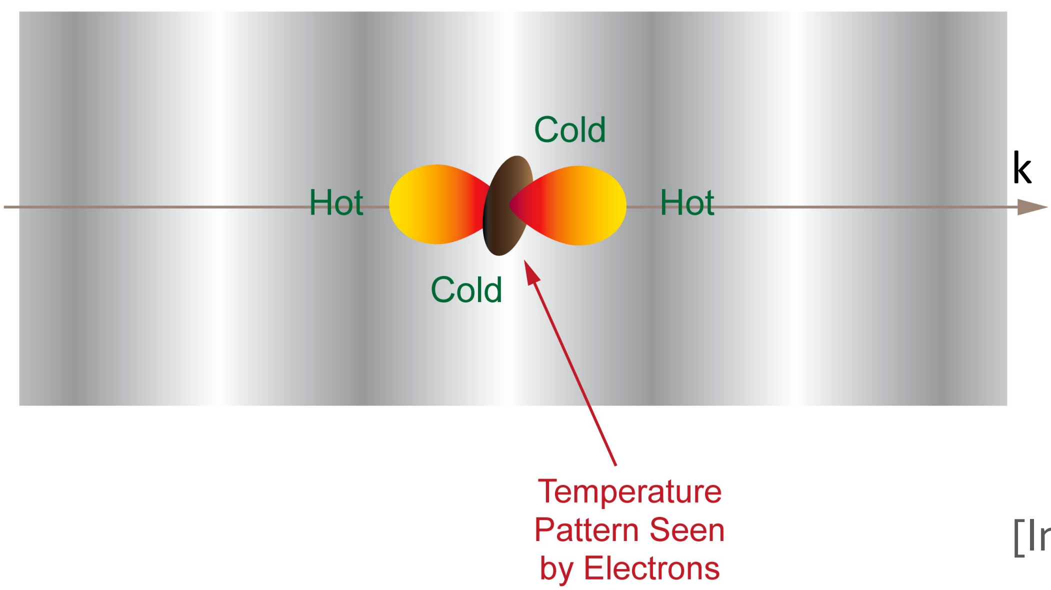


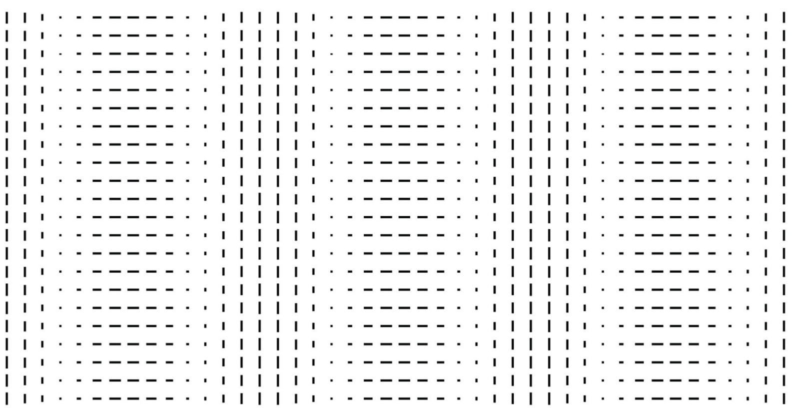
Scalar Perturbations: Only E, **no B!**

- Scalar density fluctuations can cause quadrupolar brightness fluctuations at last scatter -> polarization
- But: only E modes generated! To see this, consider one density wave on the last scattering surface
- By symmetry, see polarization must be parallel or perpendicular to k – characteristic of E

Density Wave



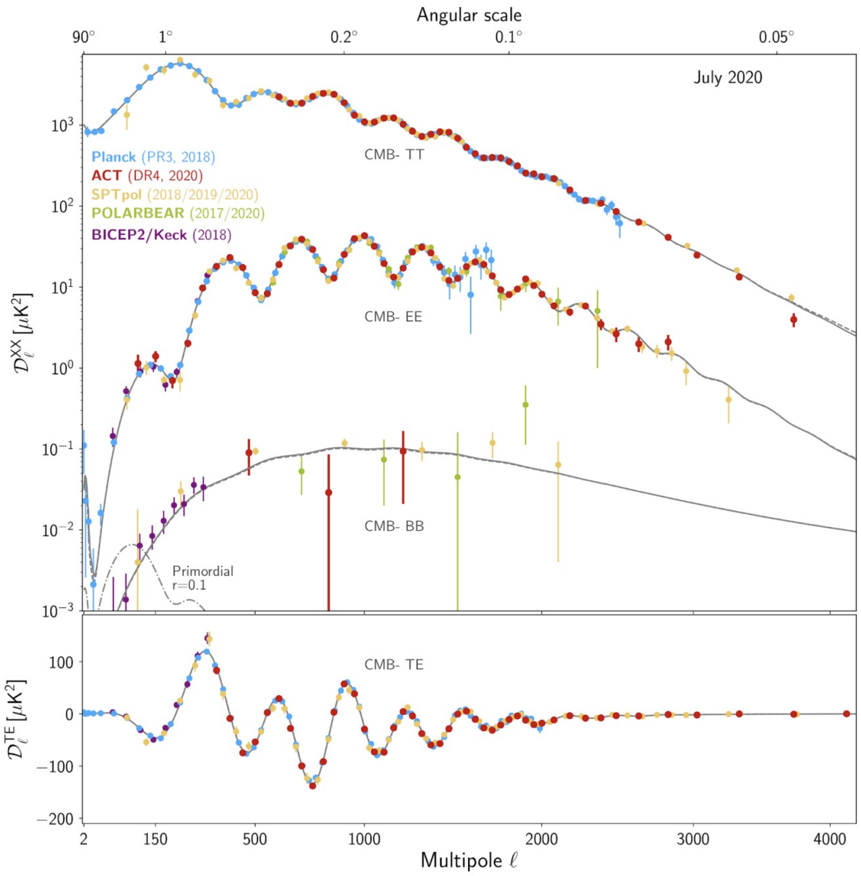
E-Mode Polarization Pattern



[Image credit: BICEP-2 collaboration]

E-mode polarization power spectra

- Expect E ~ velocity ~ sine oscillations. Powerful consistency test of standard model, and even more constraining power on cosmological parameters



More details on polarization generation

- Consider scalar perturbations. Polarization depends on local quadrupole generated by photon diffusion, with last scattering l_p away. Flux for electron at origin:

$$\Theta(\mathbf{e}) = S(0) + e_i l_p \partial_i S(0) + \frac{1}{2} l_p^2 e_i e_j \partial_i \partial_j S(0) + e_i v_i(0) + e_i e_j l_p \partial_j v_j(0)$$

- Quadrupole (from trace-free part of $e_i e_j$ components):

$$\frac{1}{2} l_p^2 \partial_i \partial_j S(0) + l_p \partial_j v_j(0) \sim \frac{1}{2} k^2 l_p l_p S + k l_p v_j$$

- Assuming we are considering large scales kl_p<<1 we can assume that the polarization is ~ $kl_p v$ ~ $k l_p \sin(k r_s)$

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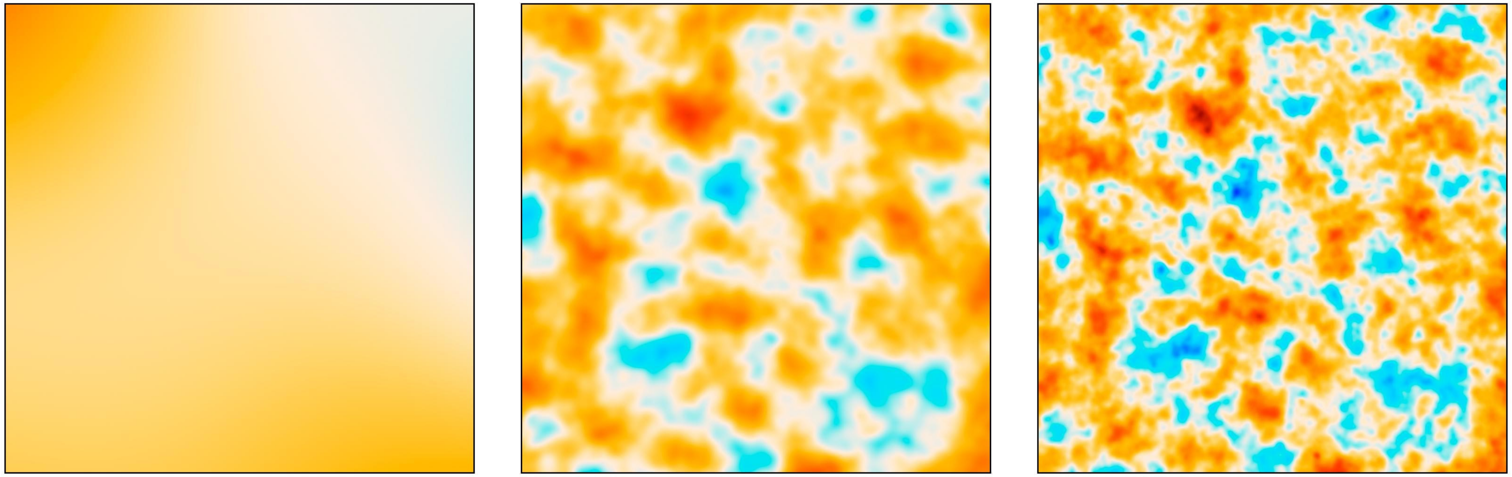
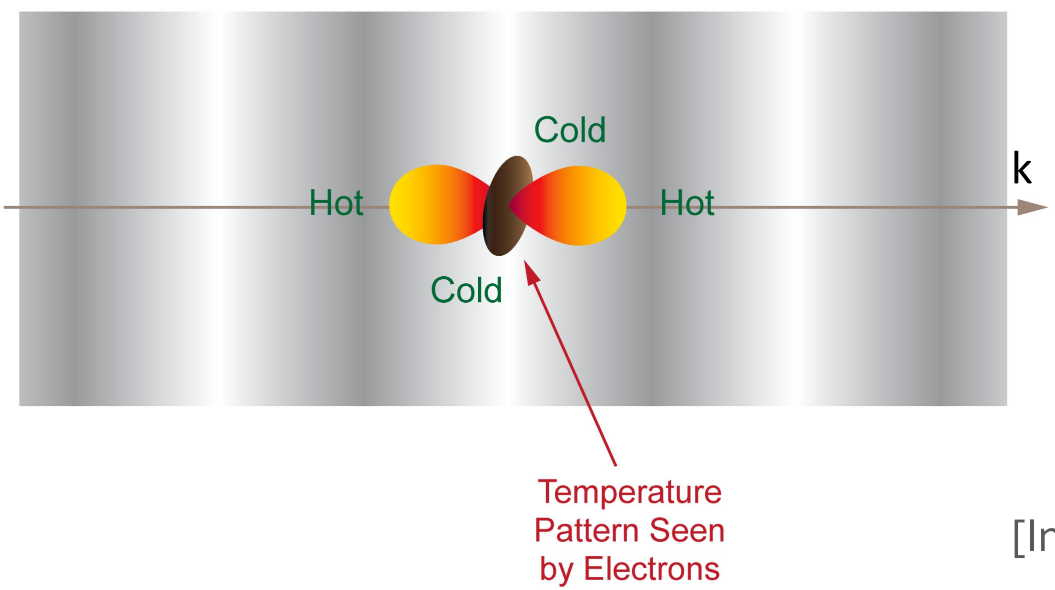


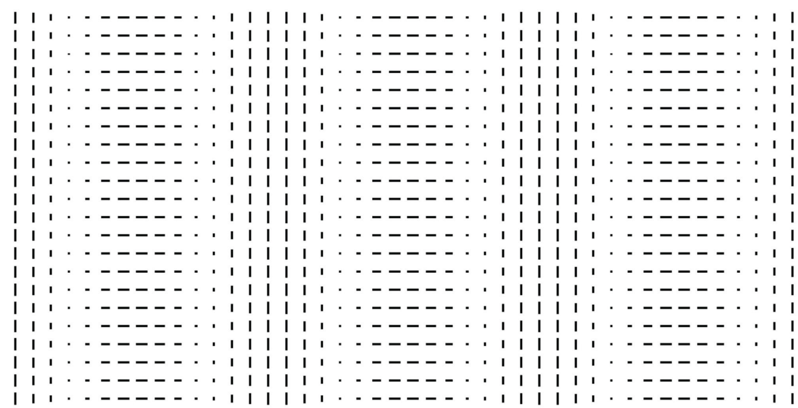
Gravitational Waves: Also B!

- By symmetry, scalar polarization must be parallel or perpendicular to k – characteristic of E [n.b. spectra interesting in their own right!]
- Gravitational waves have extra degree of freedom: polarization direction
- Not restricted to just make E, also makes B!

Density Wave



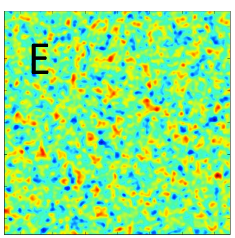
E-Mode Polarization Pattern

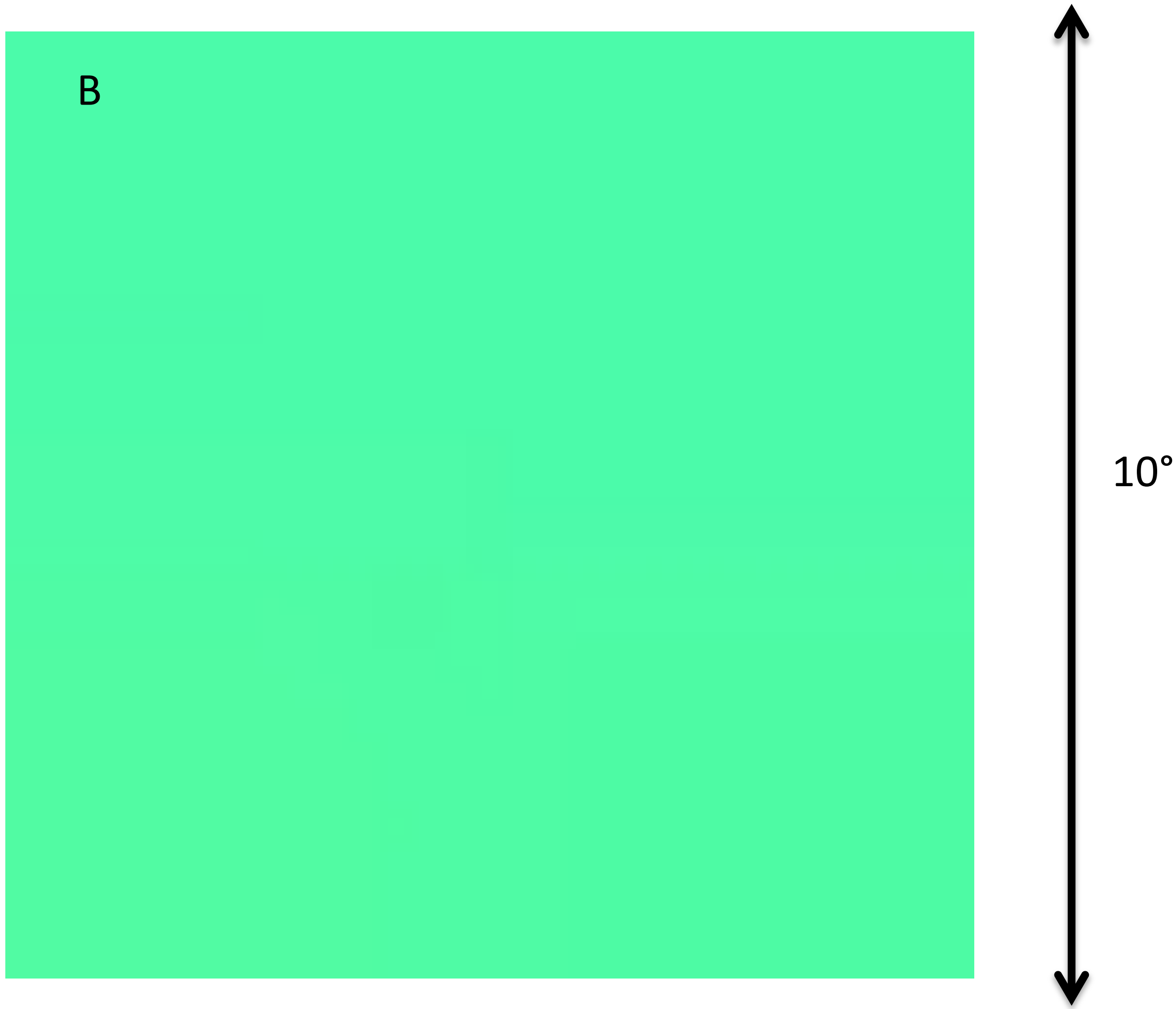


[Image credit: BICEP-2 collaboration]

CMB B polarization* with r = 0

Construct B-mode polarization: no leading order signal from scalar density perturbations!

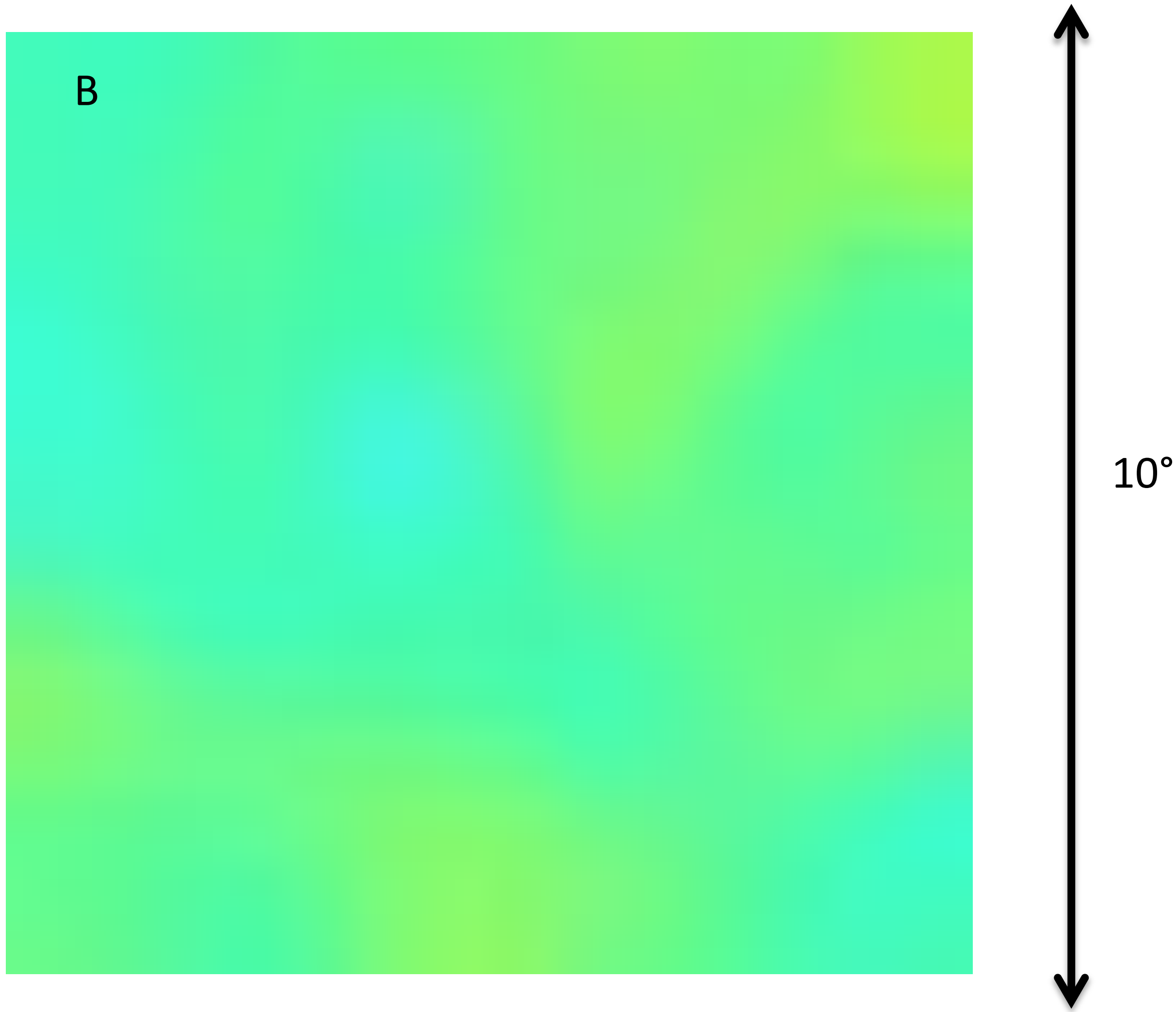




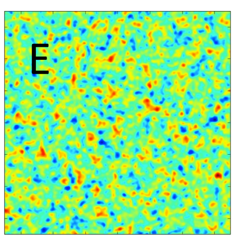
*ignoring lensing for now

CMB B polarization* with r>0

See r clearly as there is no background variance from normal (scalar) density perturbations



B-modes are a "null channel"

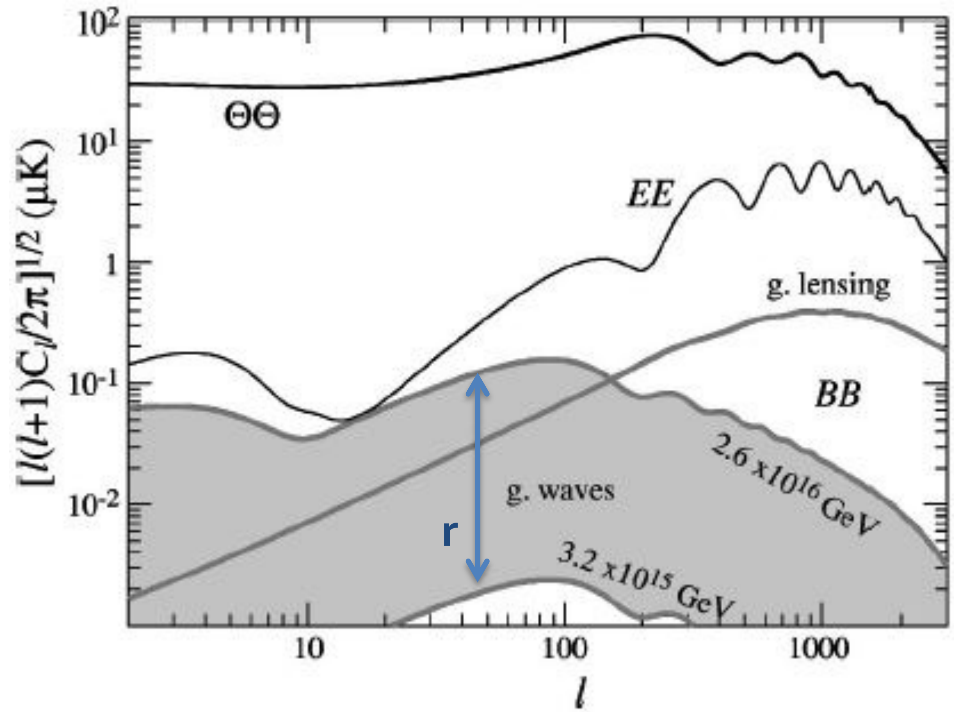


*ignoring lensing for now

The B-mode Power Spectrum

- Temperature and E-mode polarization are also sourced by large scalar density fluctuations – hard to disentangle r

- B-mode polarization is a “null channel”: **only* see inflationary gravitational waves with little confusion**

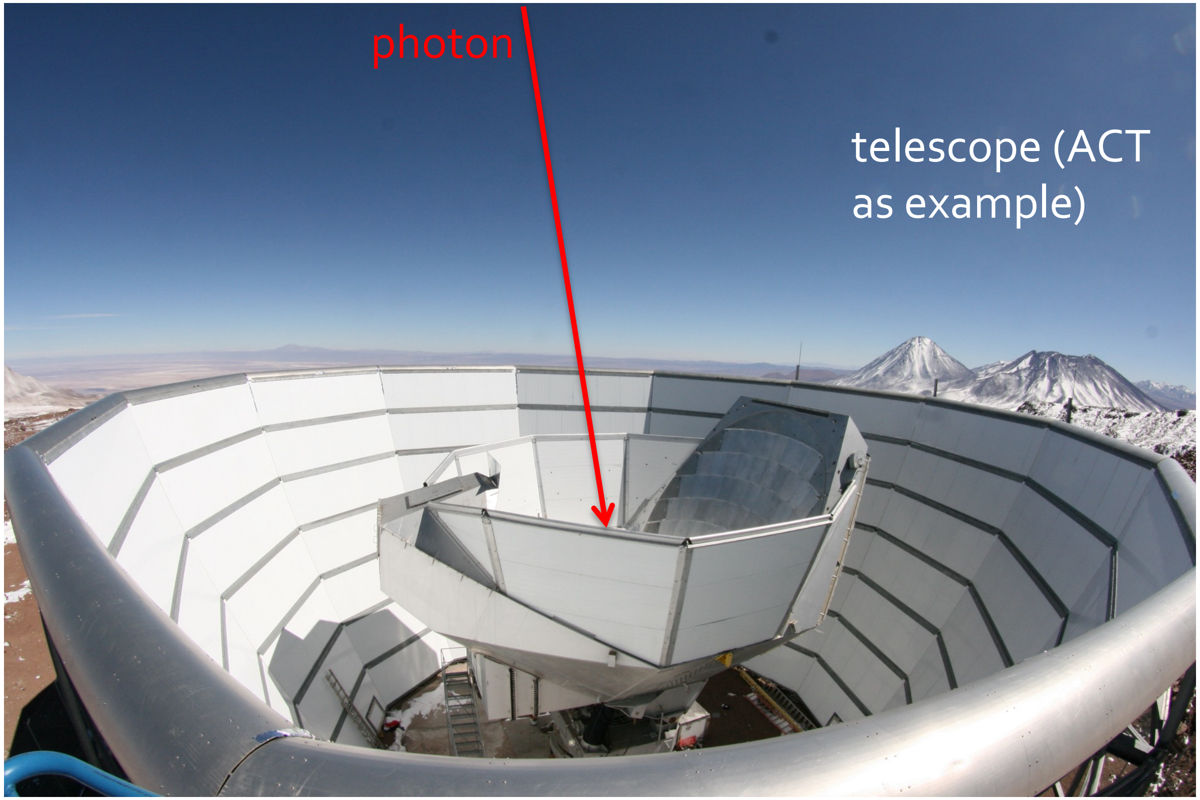


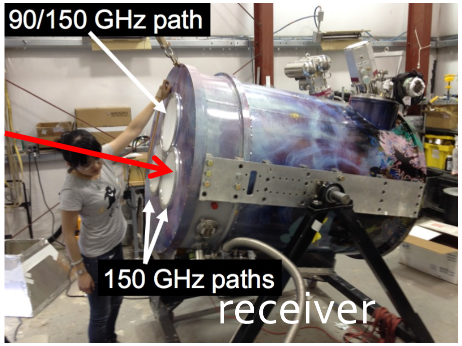
$$r \equiv \frac{P_t}{P_{\mathcal{R}}}$$

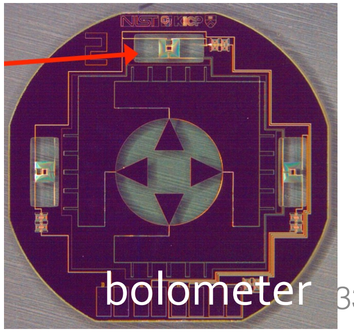
Want to measure B-mode power and tensor-scalar ratio r!

Measuring Polarization: CMB Telescopes

- Polarization-sensitive receivers containing large number of TES bolometers
- Located at high-and-dry sites







Atacama: Now, Stage-III

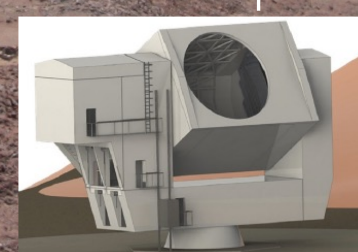
ACT (ACTPol/AdvACT)

6m, 6000 detectors
@ 28/41/90/150/230 GHz

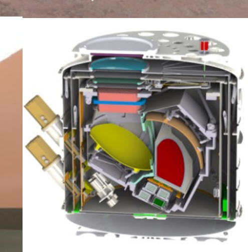
POLARBEAR2/SimonsArray

2.5m, 20000 detectors
@ 90/150/220/280 GHz

+CCAT-p



+QUBIC

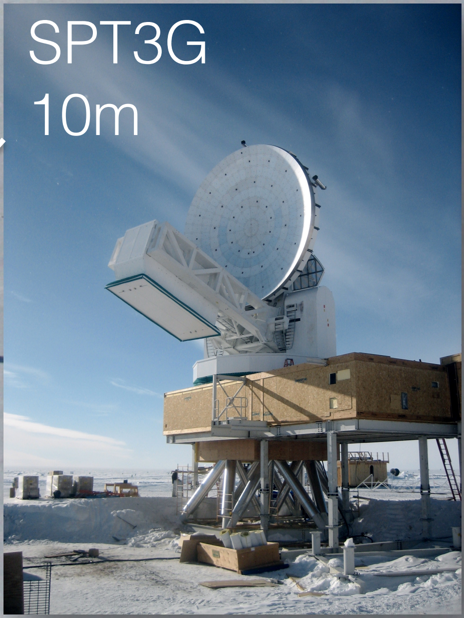


Pole: Now, Stage-III

BICEP3

3000 detectors
@ 95 GHz

SPT3G
10m

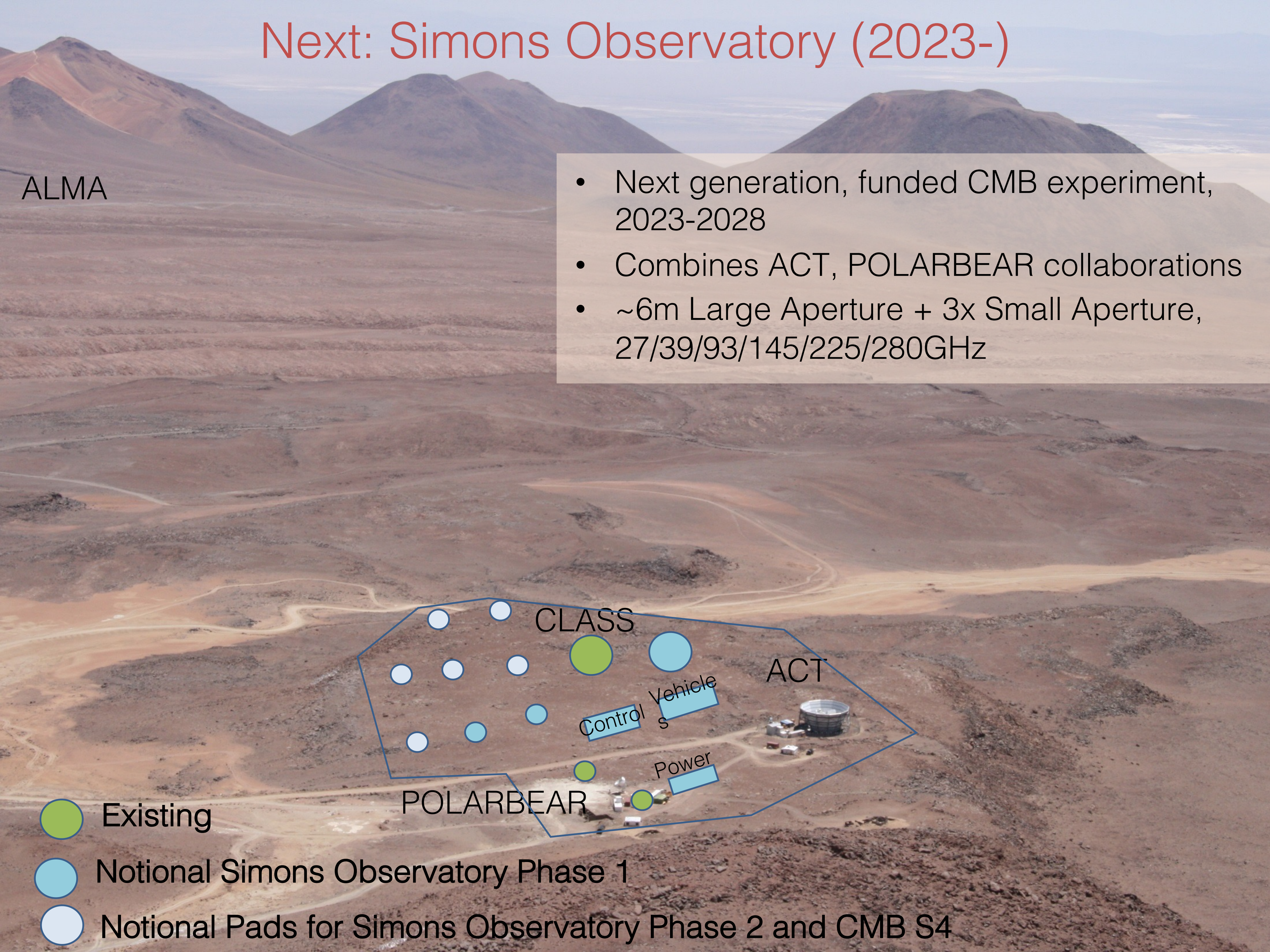


16000 detectors
@ 90/150/220 GHz

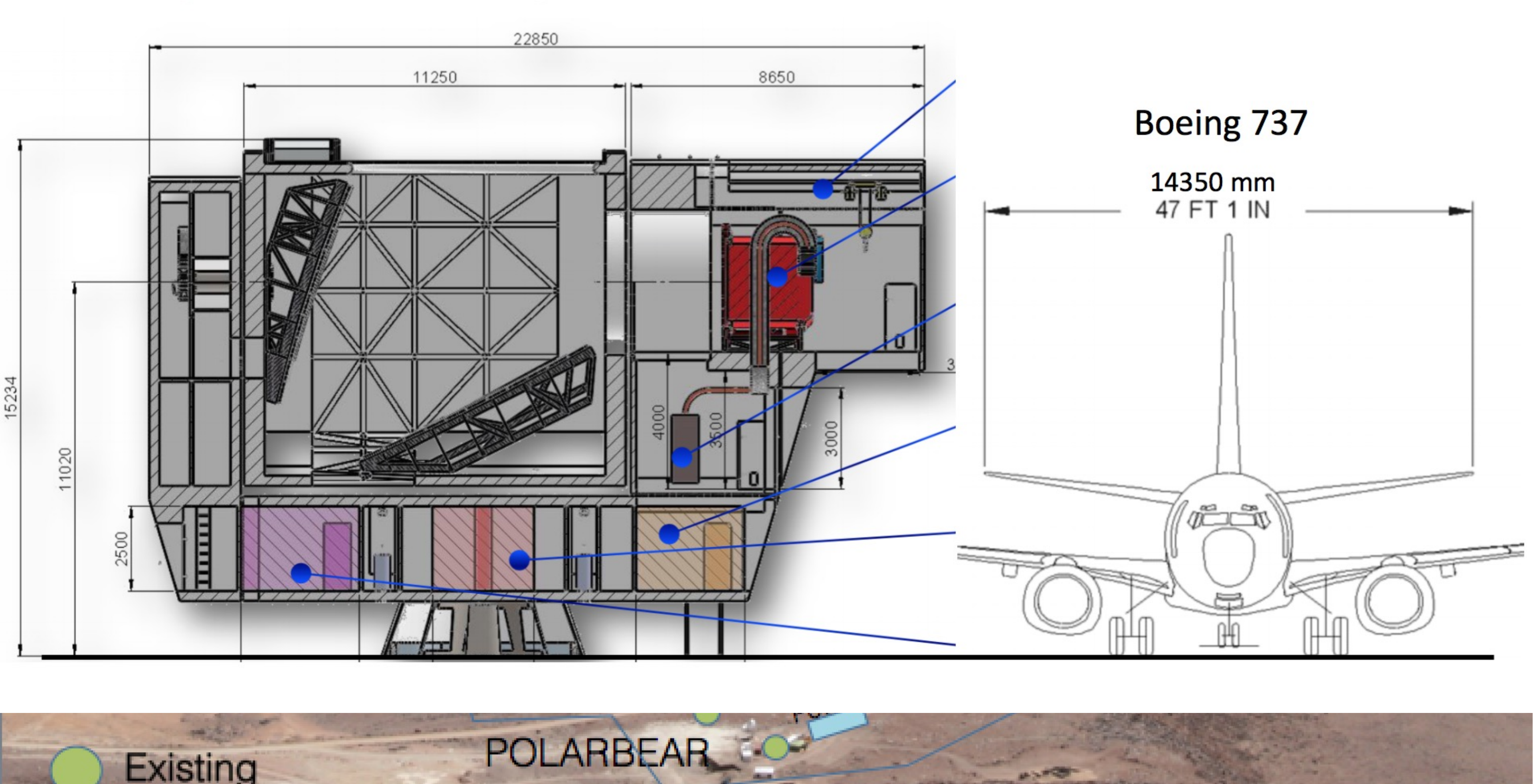
Cardiff experimental involvement

Next: Simons Observatory (2023-)

- Next generation, funded CMB experiment, 2023-2028
- Combines ACT, POLARBEAR collaborations
- ~6m Large Aperture + 3x Small Aperture, 27/39/93/145/225/280GHz



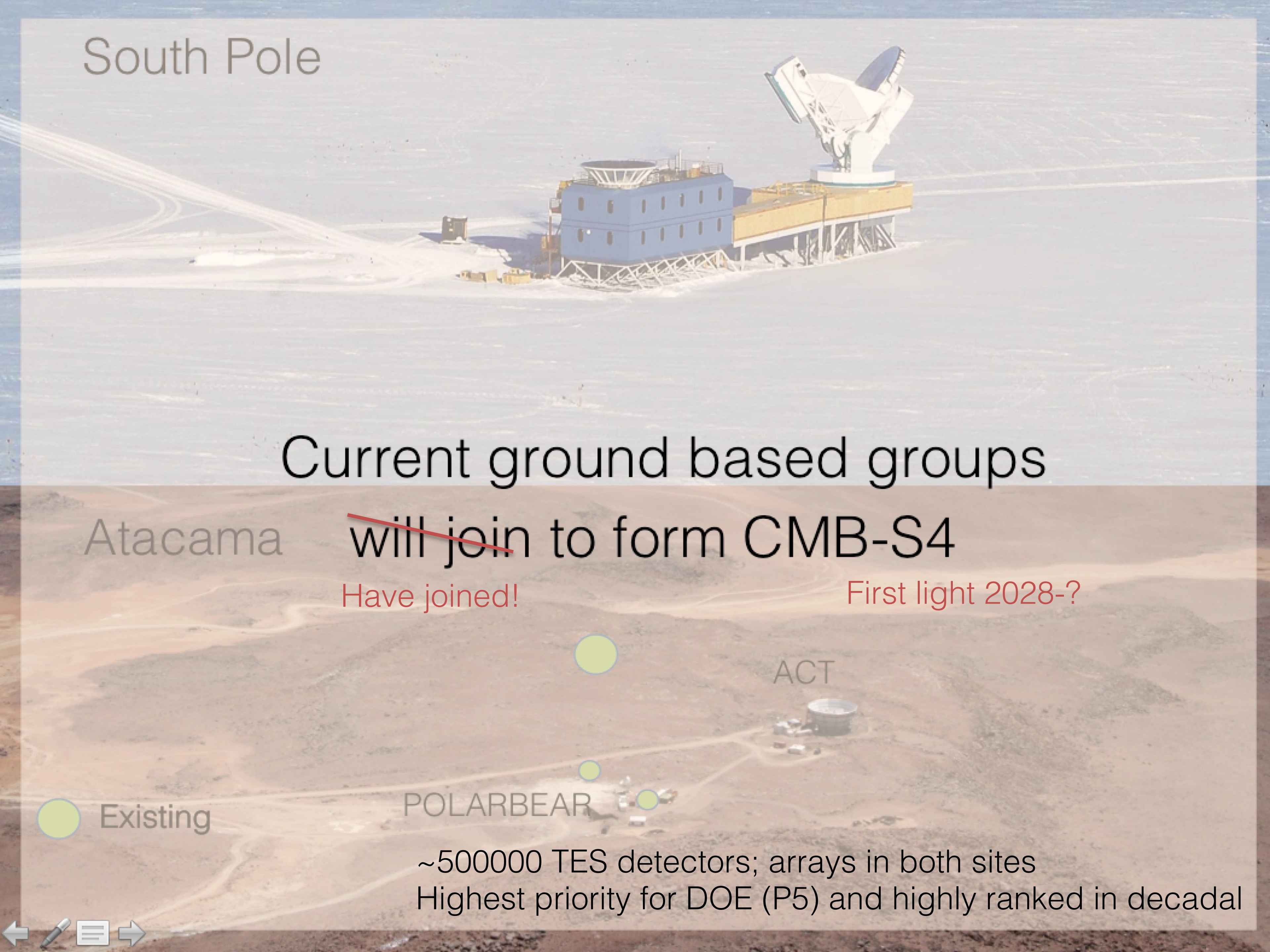
Simons Observatory



Existing

Notional Simons Observatory Phase 1

Notional Pads for Simons Observatory Phase 2 and CMB S4



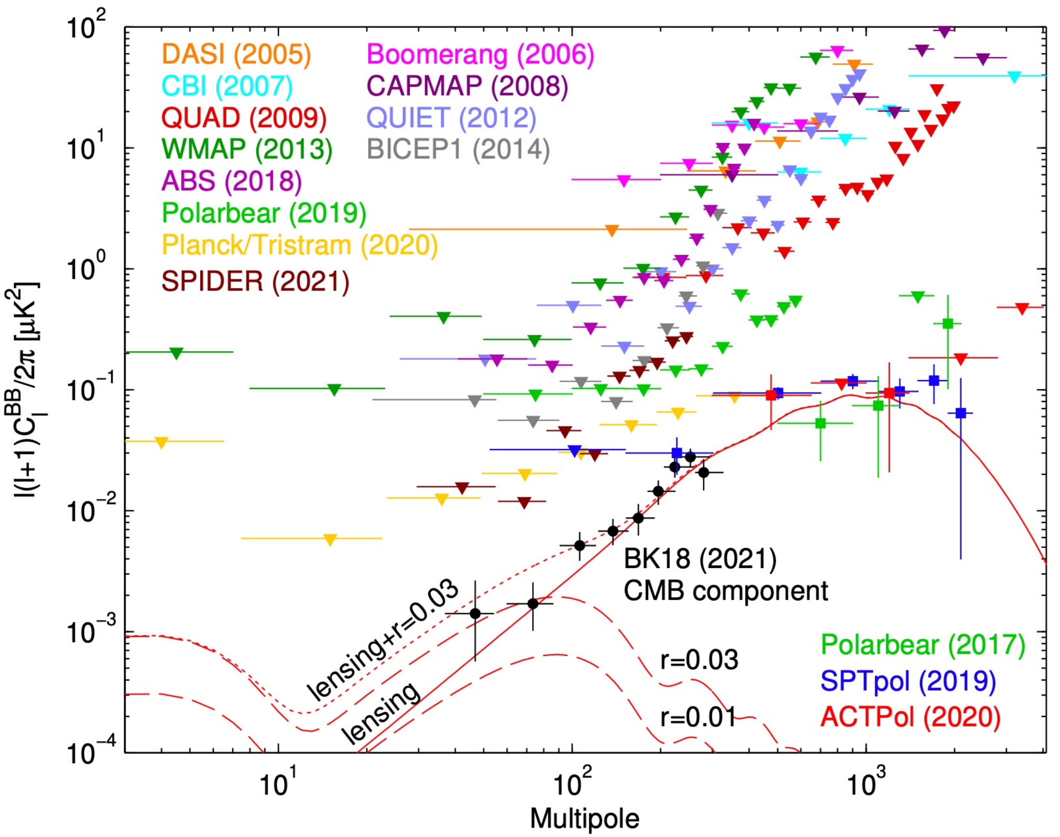
Current ground based groups ~~will join~~ to form CMB-S4

Have joined!

First light 2028-?

~500000 TES detectors; arrays in both sites
Highest priority for DOE (P5) and highly ranked in decadal

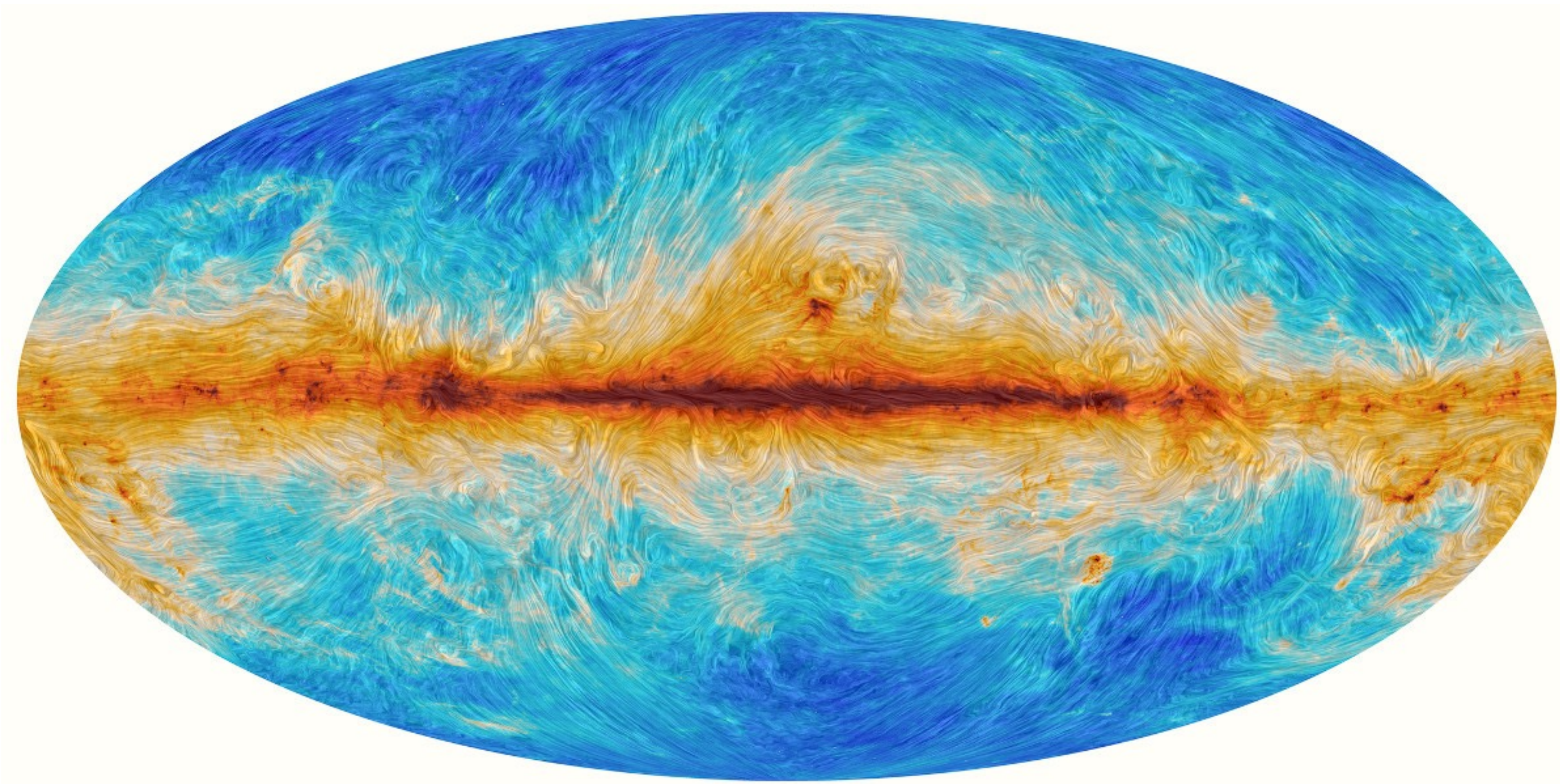
Current State of the Field



- Upper limit: **r<0.036 at 95% confidence** – BICEP/Keck 2022
- Want to progress by ~2 orders of magnitude to rule out r~0.001 – **challenges?**

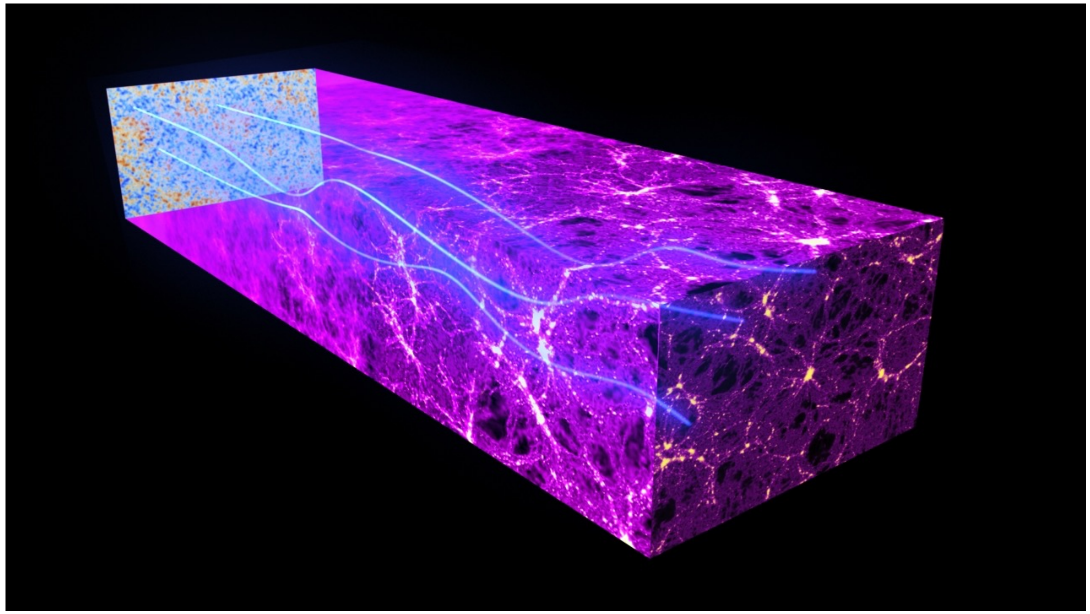
Challenge I - Foregrounds

- Galactic emission also sources B-mode polarization.



Dust Polarization (Planck)

Challenge II - CMB Lensing



- Cosmic Microwave Background (CMB) photons are gravitationally lensed by large scale mass distribution
- Many small deflections remap the CMB, define deflection d
- Converts some E-modes into small scale lensing B-modes! $B(\mathbf{l}) \sim \int d\mathbf{l}' f(\mathbf{l}') E(\mathbf{l}') \mathbf{d}(\mathbf{l}-\mathbf{l}')$

Confirming – and learning about – inflation models

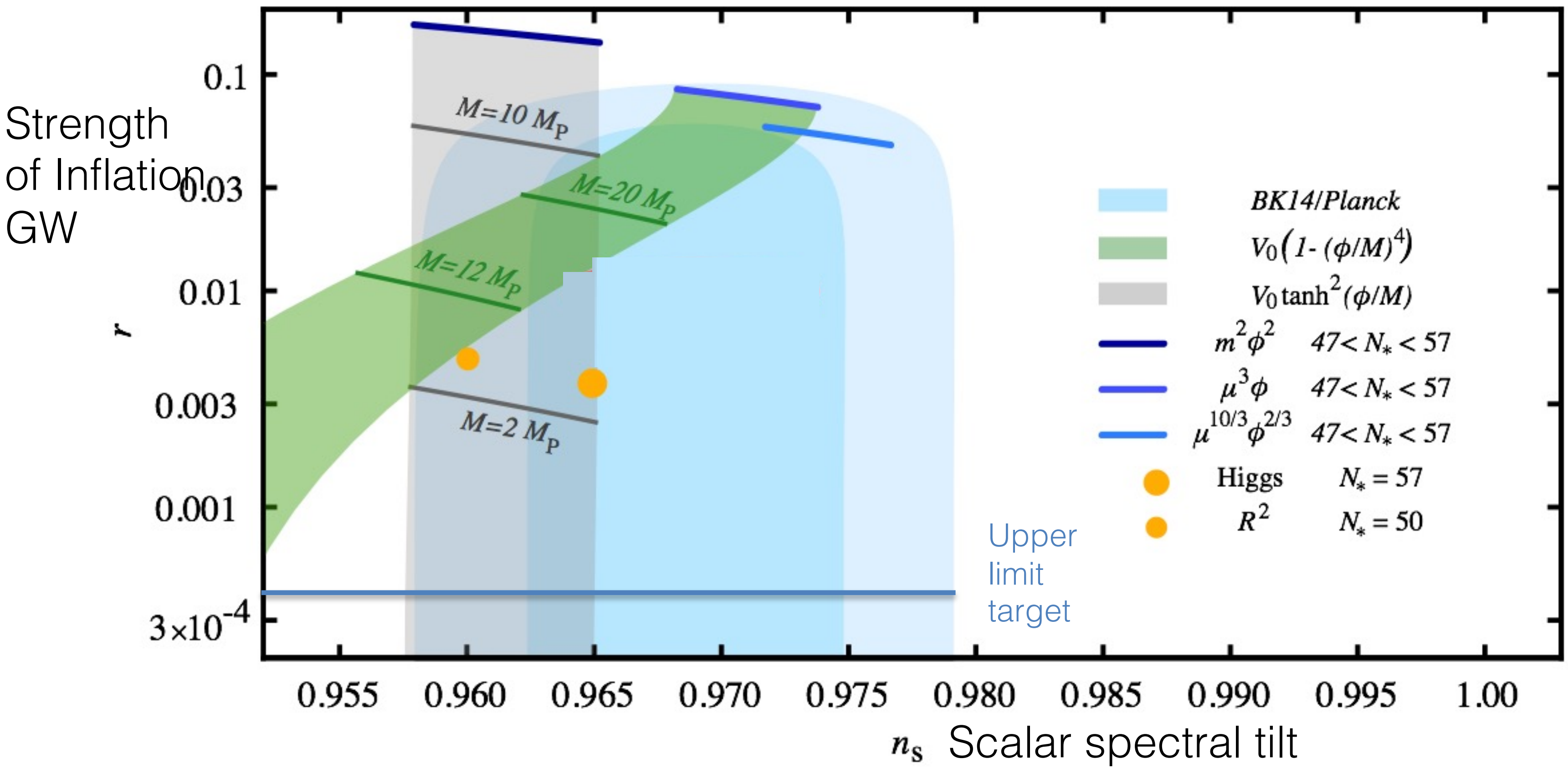
- If we have a measurement of r, and a measurement of n_s, we can use that

$$n_s - 1 = -2\epsilon - \eta \qquad\qquad r = 16\epsilon$$

$$\frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 = \epsilon \qquad\qquad M_{\rm pl}^2\left(\frac{V_{,\phi\phi}}{V}\right) = 2\epsilon - \frac{\eta}{2}$$

- Hence for each inflation potential, predictions for r, n_s can be obtained

Confirming – and learning about – inflation models



- Exciting prospects for ruling out broad class of inflation models over next 5-10 years
  – or confirming one of them!