ICTP Summer School on Cosmology 2022

5. "Modified Gravity"

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5.1 GRAVITATIONAL TESTS

- Cosmological effects

Jordan frame, $ds^2 = -(1+2\Phi)\,dt^2 + a^2(t)(1-2\Psi)\,d\vec{x}^2$

$$
\begin{tikzcd}
\Phi \arrow[rr, leftrightarrow, "\vec{v} = -\vec{\nabla}\Phi"] \arrow[d, leftrightarrow] & & \vec{v} \arrow[d, leftrightarrow] \\
\Psi \arrow[rr, leftrightarrow] & & \delta
\end{tikzcd}
$$

$\vec{v} = -\vec{\nabla}\Phi$

Euler eqn.

$\Psi = \dfrac{1-2\alpha^2}{1+2\alpha^2}\,\Phi$

Continuity eqn.

$\dot{\delta} + \dfrac{1}{a^2}\vec{\nabla}\vec{v} = 0$

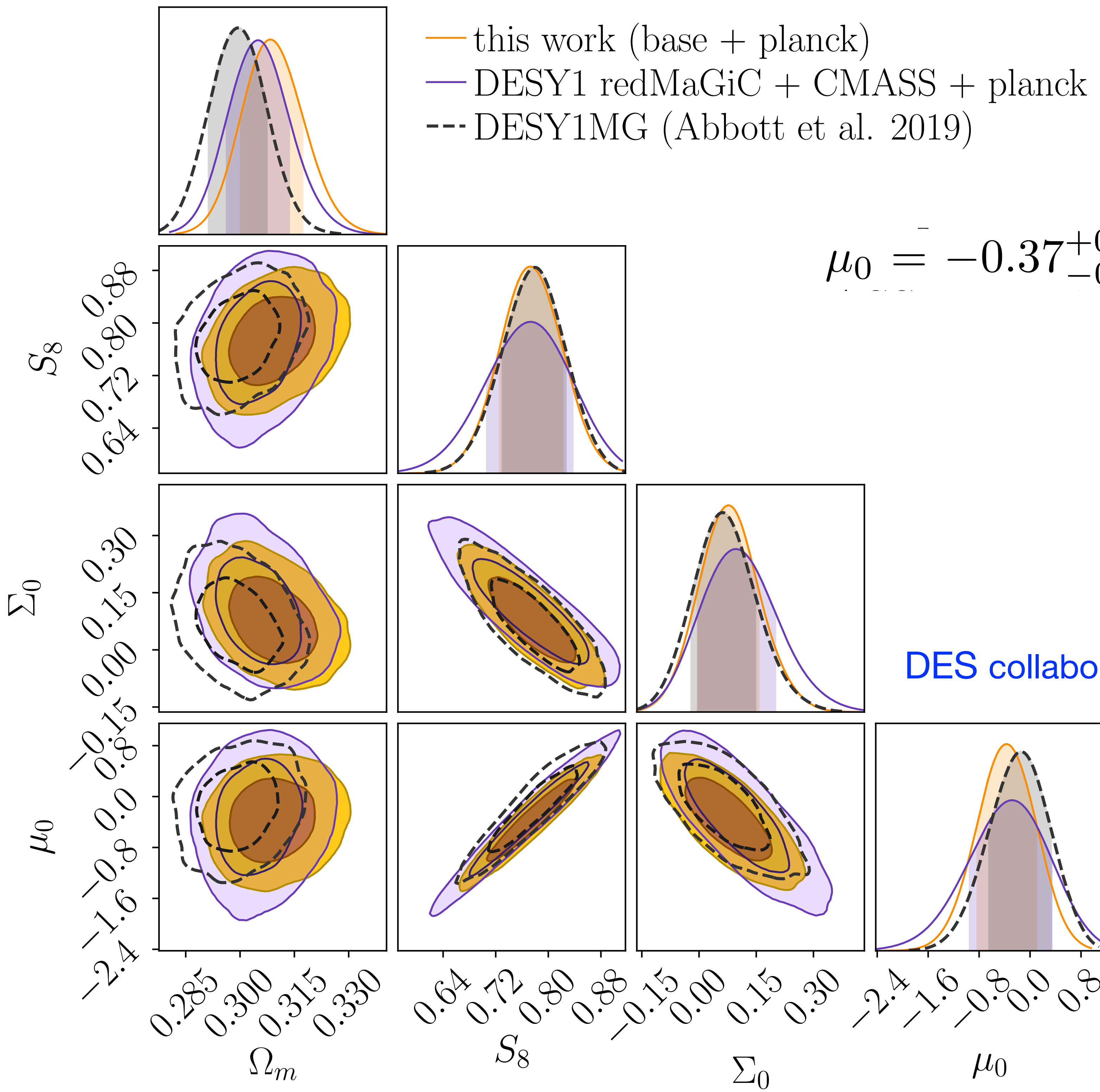
$\nabla^2\Psi = 4\pi G\,(1-2\alpha^2)\,a^2\delta$

"Poisson" eqn.

- Redshift-space distortions measure δ & $\vec{v}$
- Weak lensing measures $\Phi + \Psi$

Modified gravity constraints

Combination of weak-lensing measurements (DES) and redshift-space distortion measurements (CMASS - BOSS):



$$\mu_0 = -0.37^{+0.47}_{-0.45} \text{ and } \Sigma_0 = 0.078^{+0.078}_{-0.082}$$

$$\Psi = 4\pi G(1+\mu_0)\delta\rho_m$$

$$\Phi + \Psi = 8\pi G(1+\Sigma_0)\delta\rho_m$$

DES collaboration '21

• Gravitational tests on "small" scales

GR works very well. Many precise tests on many scales (see e.g. Will 1403.7377)

• **Tests of WEP** (inertial and gravitational mass are the same)

Eötvos type experiments : $\eta \lesssim 10^{-15}$

$$\eta(A,B) = 2\,\frac{\left(\frac{m_g}{m_i}\right)_A - \left(\frac{m_g}{m_i}\right)_B}{\left(\frac{m_g}{m_i}\right)_A + \left(\frac{m_g}{m_i}\right)_B} \Rightarrow \text{All objects fall in the same way.}$$

In GR, objects fall the same way irrespectively of their composition and their gravitational self energy: eg. a feather and a BH.

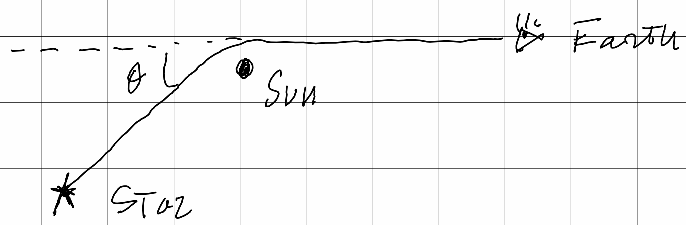
If you add a scalar field, it should be universally coupled to SM, However, objects with high gravitational self energy tend to violate the EP (Nordtvedt effect).

• PPN Parameters (Nordtvedt Phys Rev. 169 '68)

$$ds^2 = -(1+2\Phi-2\beta_{PPN}\Phi^2)dt^2 + (1-2\gamma_{PPN}\Phi)\,d\vec{x}^{\,2}$$

$\gamma_{PPN} = \dfrac{\Psi}{\Phi}$ constrained by measurements involving light: time delay & light bending

$$\Phi + \Psi = (1+\gamma_{PPN})\,\Phi$$



e.g. $\theta = 2(1+\gamma_{PPN})\dfrac{GM_\odot}{r}$

Shapiro time delay from Cassini $|\gamma_{PPN}-1| \lesssim 2\cdot 10^{-5}$

For the previous theories: $\gamma_{PPN}-1 = -\dfrac{4\alpha^2}{1+2\alpha^2} \Rightarrow \alpha \lesssim 5\cdot 10^{-3}$

β measures non-linearity of gravity $|\beta_{PPN}-1| \lesssim 10^{-4}$

- Variation of G_N

$\dot{G}_N/G_N \lesssim 4 \cdot 10^{-13}\ yr^{-1}$ from Lunar Laser Ranging

- Short-scale experiments

$\Phi = -\frac{GM}{r}\left(1 + 2\alpha^2 e^{-r/\lambda}\right)$ No evidence of $\alpha \simeq 1$ for $\lambda \gtrsim 56\,\mu m$

Fifth force and anomalous light bending

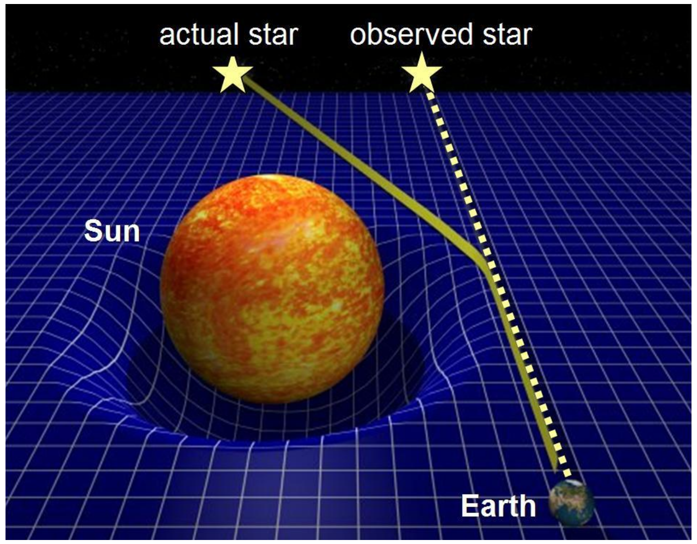
- **Fifth force**

$$\Phi = \Phi_E + \beta\phi = -(1+2\alpha^2)\frac{GM}{r}$$

$$\Psi = \Psi_E - \beta\phi = -(1-2\alpha^2)\frac{GM}{r}$$

Post-Newtonian Parametrization (PPN):

Slip parameter $\gamma = \Psi/\Phi$



- **Light bending**

$$\theta = 2(1+\gamma)\frac{GM_\odot}{r} = \frac{1+\gamma}{2}\theta_{\rm GR}$$

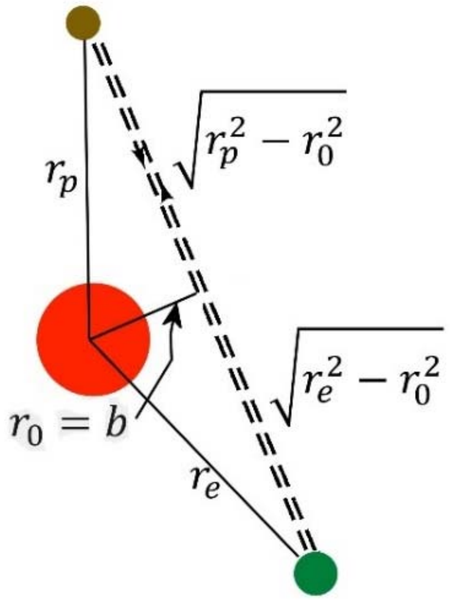
$$\gamma - 1 = (-1.7 \pm 4.5)\times 10^{-4}$$

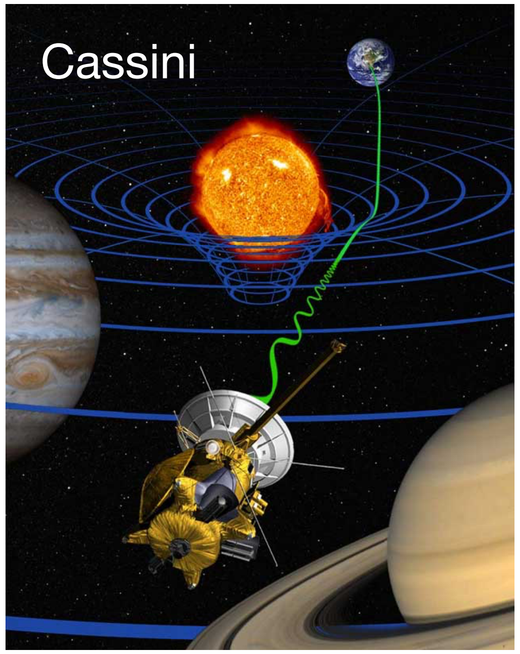
- **Shapiro time-delay**

$$\Delta t = 2(1+\gamma)GM_\odot\left[\ln\left(4\frac{r_p r_e}{r_0}\right)+1\right]$$

$$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5}$$

$$\gamma - 1 = -\frac{4\alpha^2}{1+2\alpha^2} \qquad |\alpha| \lesssim 5\cdot 10^{-2}$$





Fifth force and anomalous light bending

- **Lunar laser ranging**: accurate measurement of lunar period and mean distance

$$|\dot{G}/G| \lesssim 4 \pm 9 \times 10^{-13}\,\mathrm{yr}^{-1}$$

$$\frac{\dot{G}}{H_0 G} \lesssim 10^{-3} \qquad \Rightarrow \qquad \frac{\dot{G}}{H_0 G} = 2\alpha \frac{\dot{\phi}}{H_0 M_{\mathrm{Pl}}} \lesssim 10^{-3}$$

3.2 GALILEONS

• Vainshtein screening

Consider the action (in flat spacetime)

$$S = \int dx^4 \left[-\frac{c_2}{2}(\partial\phi)^2 - \frac{c_3}{\Lambda^3}(\partial\phi)^2 \Box\phi + \mathcal{L}_m\left(A^2(\phi) g_{\mu\nu}, \psi_m\right)\right]$$

EOM are invariant under $\phi \to \phi + c + b_\mu x^\mu$, c, b_μ constants, because they contain only two derivatives (also the action is invariant). Galileon symmetry (analogous to $\vec{x} \to \vec{x} + \vec{x}_0 + \vec{v}_0 t$ for $\ddot{\vec{x}} = \vec{F}(\vec{x} - \vec{y})$)

EOM: $$c_2 \Box\phi + \frac{2c_3}{\Lambda^3}\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] + \frac{\alpha}{M_p} T = 0$$

Non-linear, but no more than two derivatives: I do not have to specify more data than position & velocity to specify initial conditions: well defined Cauchy problem.

Consider a static point mass source (spherical symmetry):

$$\vec{\nabla}\cdot\left(c_2 \vec{\nabla}\phi + \hat{r}\,\frac{2c_3}{\Lambda^3}\,\frac{(\vec{\nabla}\phi)^2}{r}\right) = \frac{\alpha M}{M_p}\,\delta^{(3)}(\vec{x})$$

We can solve this eq. like we do in electrodynamics and we find

$$c_2\phi' + \frac{2c_3}{\Lambda^3}\,\frac{\phi'^2}{r} = \frac{\alpha M}{4\pi r^2 M_p} \qquad \text{or}$$

Vainshtein radius

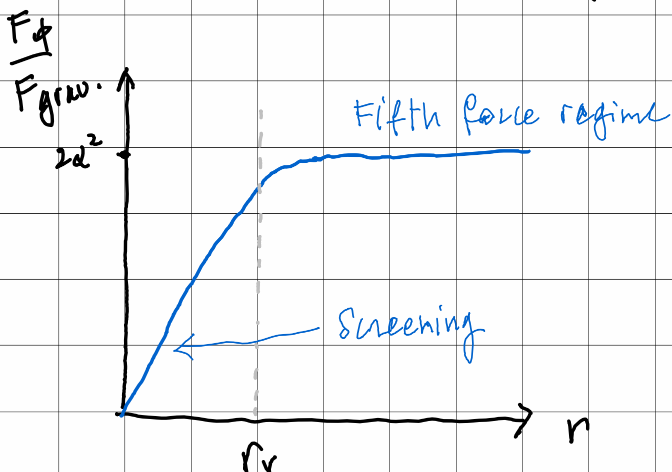
$$\boxed{\phi'(r) = -\frac{r\alpha}{2\pi r_V^3}\,\frac{M}{M_{pl}}\left(1 \pm \sqrt{1+\left(\frac{r_V}{r}\right)^3}\right)} \qquad \text{with} \qquad r_V \equiv \frac{1}{\Lambda}\left(\frac{4c_3\alpha}{c_2\pi}\,\frac{M}{M_p}\right)^{\frac{1}{3}}$$

solution giving $\phi \propto \frac{1}{r}$ at $r\to+\infty$

At the V. radius the linear approx breaks down similarly to what happens in GR at the Schwarzschild radius

• For $r \gg r_V$ $\quad \phi' \simeq \dfrac{\alpha}{4\pi r^2}\dfrac{M}{M_p} \Rightarrow \dfrac{F_\phi}{F_{grav.}} \simeq 2\alpha^2$

• For $r \ll r_V$ $\quad \phi' \simeq \left(\dfrac{r}{r_V}\right)^{3/2}\dfrac{\alpha}{2\pi r^2}\dfrac{M}{M_p} \Rightarrow \dfrac{F_\phi}{F_{grav.}} \simeq 4\alpha^2\left(\dfrac{r}{r_V}\right)^{3/2}$



Remember $\Phi = \Phi_E + \overset{\alpha/M_p}{\beta}\,\phi$

$$F_\phi \equiv \frac{\alpha}{M_p}\phi'$$

$$F_{grav} \equiv \frac{GM}{r^2} = \frac{1}{8\pi r^2}\frac{M}{M_p^2}$$

Size of the V. radius is set by Λ. What do we choose?

- Self-acceleration

Let us switch on gravity and consider the action

$$S = \int dx^4 \left[\frac{M_p^2 R}{2} - \frac{c_2}{2}(\partial\phi)^2 - \frac{c_3}{\Lambda^3}(\partial\phi)^2 \Box\phi \right]$$

I'm interested in the homogeneous evolution : $\begin{cases} \phi = \phi_0(t) \\ ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 \end{cases}$

"Friedmann" eqn.

$$3M_p^2 H^2 = \frac{c_2}{2}\dot{\phi}_0^2 - \frac{6Hc_3}{\Lambda^3}\dot{\phi}_0^3$$

scalar field EOM

$$\partial_t \left[a^3 \left(c_2 \dot{\phi}_0 - 3H\dot{\phi}_0^2 \frac{2c_3}{\Lambda^3} \right) \right] = 0$$

Remarkably $H = H_0$ & $\dot{\phi}_0 = b H_0 M_p$ is a solution : de Sitter

$\Rightarrow$ Self-acceleration! Acceleration without a C.C.

$$\frac{\Lambda^3}{c_3} \simeq \frac{6b}{c_2} H_0^2 M_p \quad \Rightarrow \quad \Lambda \sim (H_0^2 M_p)^{1/3} \sim 10^{-13}\,\text{eV} \sim (1000\,\text{km})^{-1}$$

Very low scale

$$c_2 = -6/b^2$$

This model is ruled out by the ISW-gal. correlation (see Renk et al. '17). But it can be generalized to

$$S = \int dx^4 \left[\frac{M_P^2 R}{2} - G_2(\phi, X) - G_3(\phi, X) \frac{\Box \phi}{\Lambda^3} \right]$$

where $$G_i(\phi, X) = X \sum_{n=0} c_n^{(i)}(\phi) \left(\frac{X}{H_0^2 M_p^2} \right)^n$$

NOTE: How big is the Vainshtein radius with $\Lambda \sim (1000 \text{ km})^{-1}$?

$$r_V \sim 100 \left(\frac{M}{M_\odot} \right)^{1/3} \text{pc} \Rightarrow$$

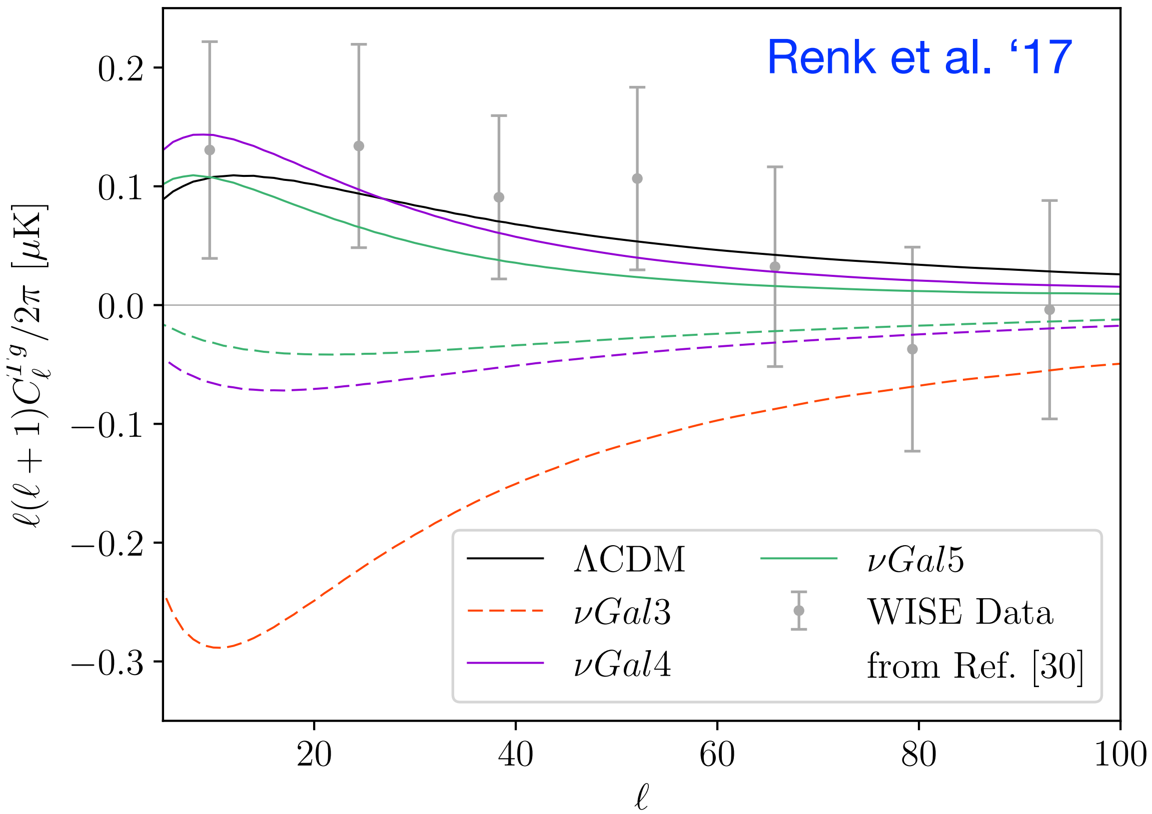
$r_V^{\text{Earth}} \sim 0.1$ pc

$r_V^{\text{Sun}} \sim 100$ pc

$r_V^{\text{Milky Way}} \sim 1$ Mpc

Cubic galileon

Model ruled out by integrated Sachs-Wolfe/ galaxy correlation (Wide-field Infrared Survey Explorer). The gravitational potentials deepen instead of getting swallower as in LCDM



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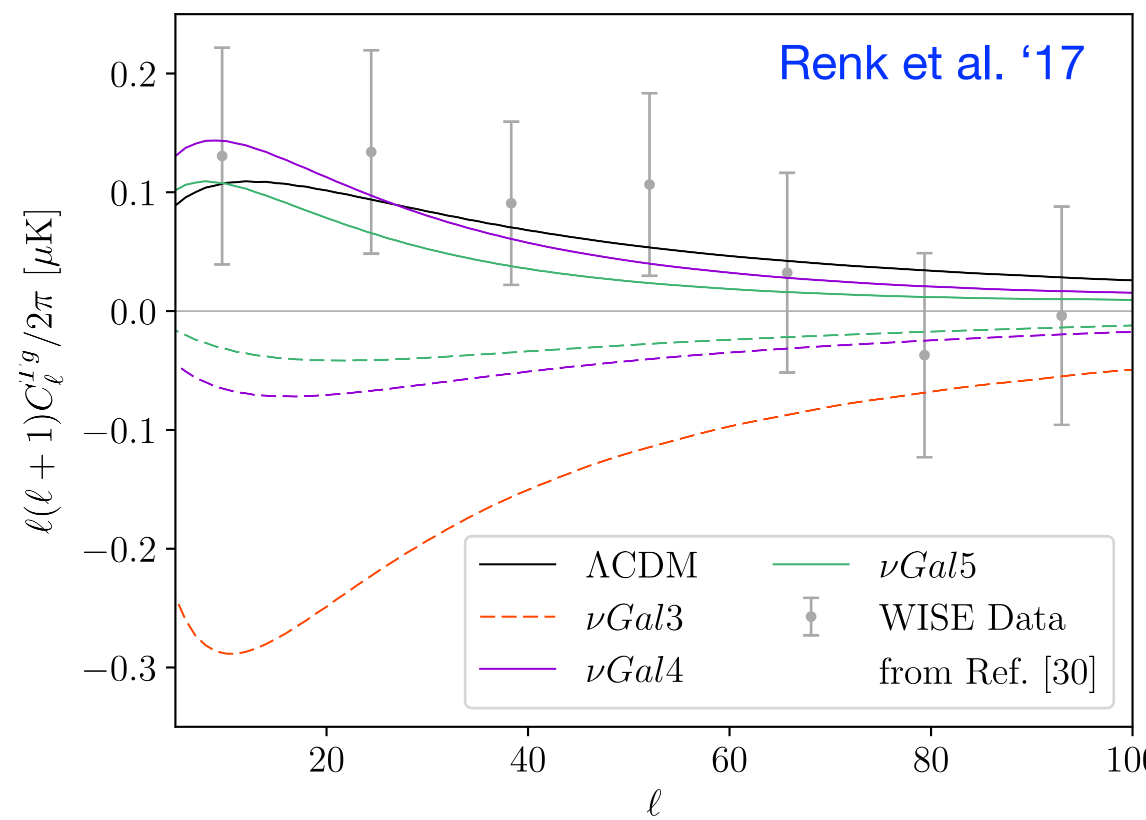


We can easily generalize the model

$$S = \int d^4x \left[\frac{M_{\rm Pl}^2 R}{2} - \frac{c_2}{2}(\partial\phi)^2 - \frac{c_3}{\Lambda_3^3}(\partial\phi)^2\Box\phi \right]$$

$$G_2(\phi, X) \qquad G_3(\phi, X) \qquad X = \nabla_\mu\phi\nabla^\mu\phi$$

$$G_i(\phi, X) = X \sum_{n=0} c_n^{(i)}(\phi) \left(\frac{X}{H_0^2 M_{\rm Pl}^2} \right)^n$$

In principle, self-acceleration and screening. Much wider range of parameter space: more difficult to constrain

- Galileons

Galileons are scalar fields invariant under

$$\phi \to \phi + c + b_\mu x^\mu \qquad \text{generalized shift symmetry}$$

For instance terms constructed from $\partial_\mu \partial_\nu \phi$ (we are in flat space):

e.g.: $(\partial^2 \phi)^2$, $(\partial^2 \phi)^n$

There are operators with minimal number of derivatives; they give second-order EOM:

$$\mathcal{L}_2 = (\partial\phi)^2$$

$$\mathcal{L}_3 = (\partial\phi)^2 \, \partial^2\phi$$

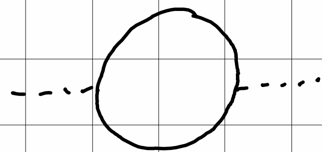
$$\mathcal{L}_4 = (\partial\phi)^2 \left[(\partial^2\phi)^2 - (\partial_\mu\partial_\nu\phi)^2 \right]$$

$$\mathcal{L}_5 = (\partial\phi)^2 \left[(\partial^2\phi)^2 + 2(\partial_\mu\partial_\nu\phi)^3 - 3\,\partial^2\phi\,(\partial_\mu\partial_\nu\phi)^2 \right]$$

Second-order EOM: no extra propagating modes!

Terms with higher derivatives must be there: loops generate them

$\frac{1}{\Lambda^3}(\partial\phi)^2\,\partial^2\phi \longrightarrow$



$\frac{(\partial^2\phi)^2}{\Lambda^2}$

suppressed by $\frac{\partial}{\Lambda} \ll 1$: $\frac{(\partial^2\phi)^2}{\Lambda^2} \ll (\partial\phi)^2$

- Low energy expansion dominated by $\mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4, \mathcal{L}_5$ with 2nd order EOM

- Higher derivative terms are generated; however, they are suppressed by $\frac{\partial}{\Lambda}$ w.r.t. $\mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4, \mathcal{L}_5$ so I can treat them perturbatively. If treated perturbatively, higher derivatives do not generate new modes.

Example: $\ddot{x} + k^2 x + \frac{1}{2\Lambda^2} \ddddot{x} = 0$

- $-\omega^2 + k^2 + \frac{\omega^4}{2\Lambda^2} = 0 \Rightarrow \omega = \pm \Lambda \sqrt{1 \pm \sqrt{1 - 2\frac{k^2}{\Lambda^2}}}$

  4 modes, 2 propagating d.o.f.

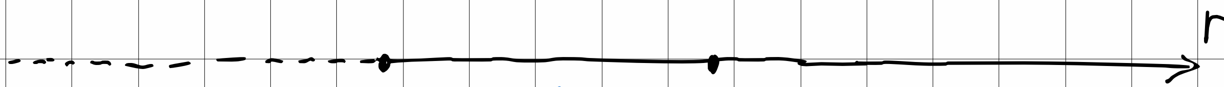
- Perturbatively $\frac{\omega^2}{\Lambda^2} \ll 1$

  At lowest order $\omega_0 = \pm k$

  $-\omega^2 + k^2 + \frac{\omega_0^4}{2\Lambda^2} = 0 \Rightarrow \omega = \pm k \left(1 + \frac{1}{4}\frac{k^2}{\Lambda^2}\right)$

  2 modes, 1 propagating d.o.f.

• Higher derivative corrections ≠ non-linearities :



both quantum & classical non-l. are important — R_q — classical nonlinearities are important but quantum small — R_{cl} — neither quantum not classical nl. are important

$$\text{GR} : \quad R_q = \ell_{pl} \; ; \quad R_{cl} = R_{sch}$$

$$\text{galileon} : \quad R_q = \Lambda^{-1} \; ; \quad R_{cl} = R_{\text{vainshtein}}$$

• Galileons are non-renormalized by quantum corrections
Effective action can only generate higher derivatives, e.g.

$\frac{(\partial^2\phi)^2}{\Lambda^2}$ but not $(\partial\phi)^2$ or $\frac{(\partial^2\phi)^4}{\Lambda^8}$ but not $\frac{(\partial\phi)^2\Box\phi}{\Lambda^3}$

$$\mathcal{L}_3[\phi+\phi_0] \supset \partial^\mu\phi_0\partial_\mu\phi\,\Box\phi = \partial^\mu\phi_0\partial^\nu\left[\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}(\partial\phi)^2\right] + \text{b.t.}$$

$$e^{i\Gamma^{1\text{-loop}}[\phi_0]} = \int \mathcal{D}\phi\; e^{-\frac{i}{2}\int dx^4\left(-2\,\partial_\mu\partial_\nu\phi_0\,[\;''\;]\right)}$$

always two derivatives on the external legs

Low-energy solutions are protected against quantum corrections.

◦ Massive gravity & DGP

Massive gravity : massive spin-2 field has 5 d.o.f. : $j = 2s+1$, $s=2$

$$h_{\mu\nu} = H_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\partial_\mu\partial_\nu\phi$$

15 components but diff. invariance removes 10 : $15-4-4-1-1=5$

$$H_{\mu\nu} \to H_{\mu\nu} + \partial_\mu \chi_\nu + \partial_\nu \chi_\mu \quad ; \quad A_\mu \to A_\mu - \chi_\mu$$

$$A_\mu \to A_\mu + \partial_\mu \psi \quad ; \quad \phi \to \phi - \psi$$

5.3 GENERALISED SCALAR-TENSOR THEORIES

Horndeski theories

Covariantization: Most general Lorentz-invariant scalar-tensor theories with 2nd-order equations of motion: 1 scalar + 2 tensor polarisations

$$\mathcal{L}_{\mathrm{H}}^{(2)} = G_2(\phi, X) \qquad\qquad X = \nabla_\mu\phi\nabla^\mu\phi$$

$$\mathcal{L}_{\mathrm{H}}^{(3)} = G_3(\phi, X)\Box\phi$$

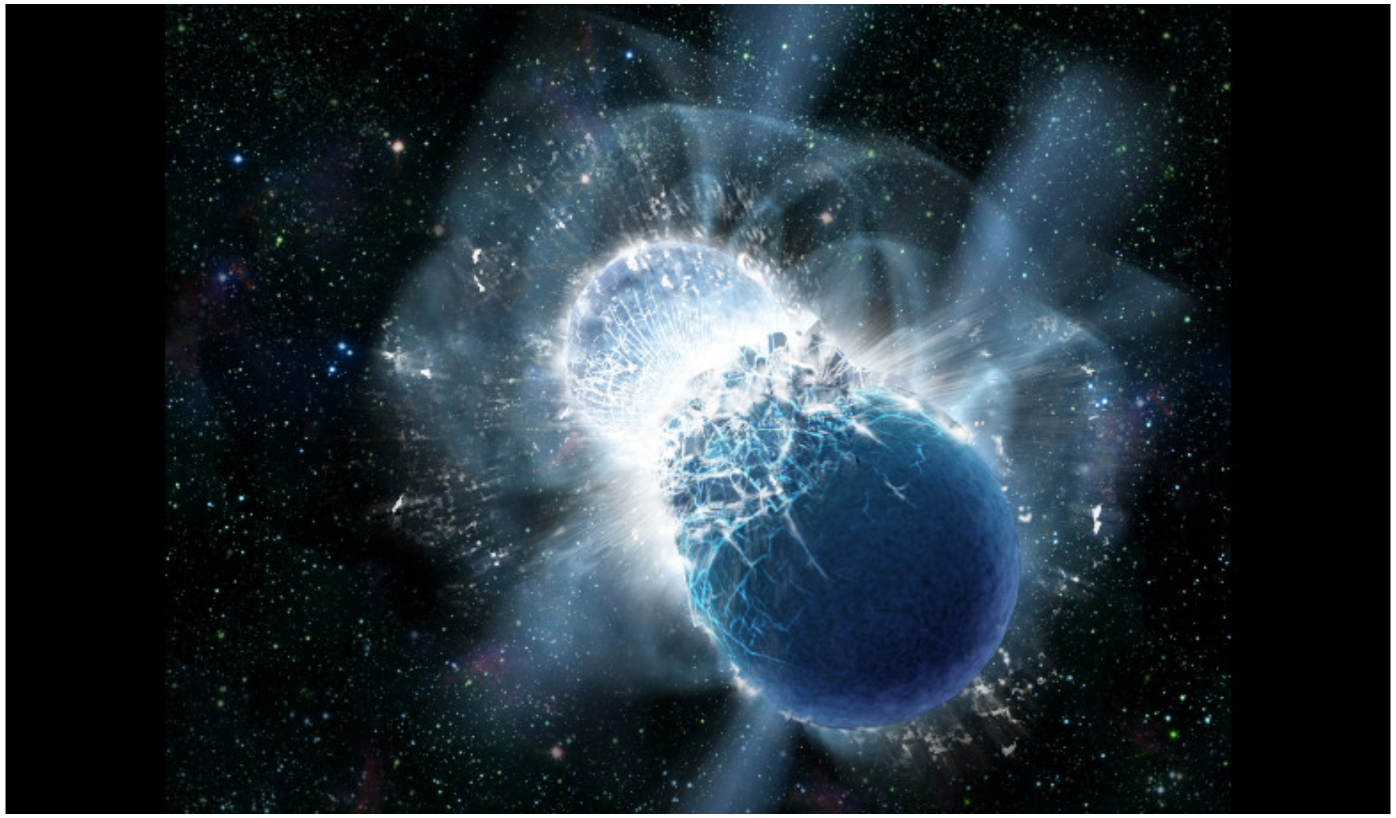
$$\mathcal{L}_{\mathrm{H}}^{(4)} = G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

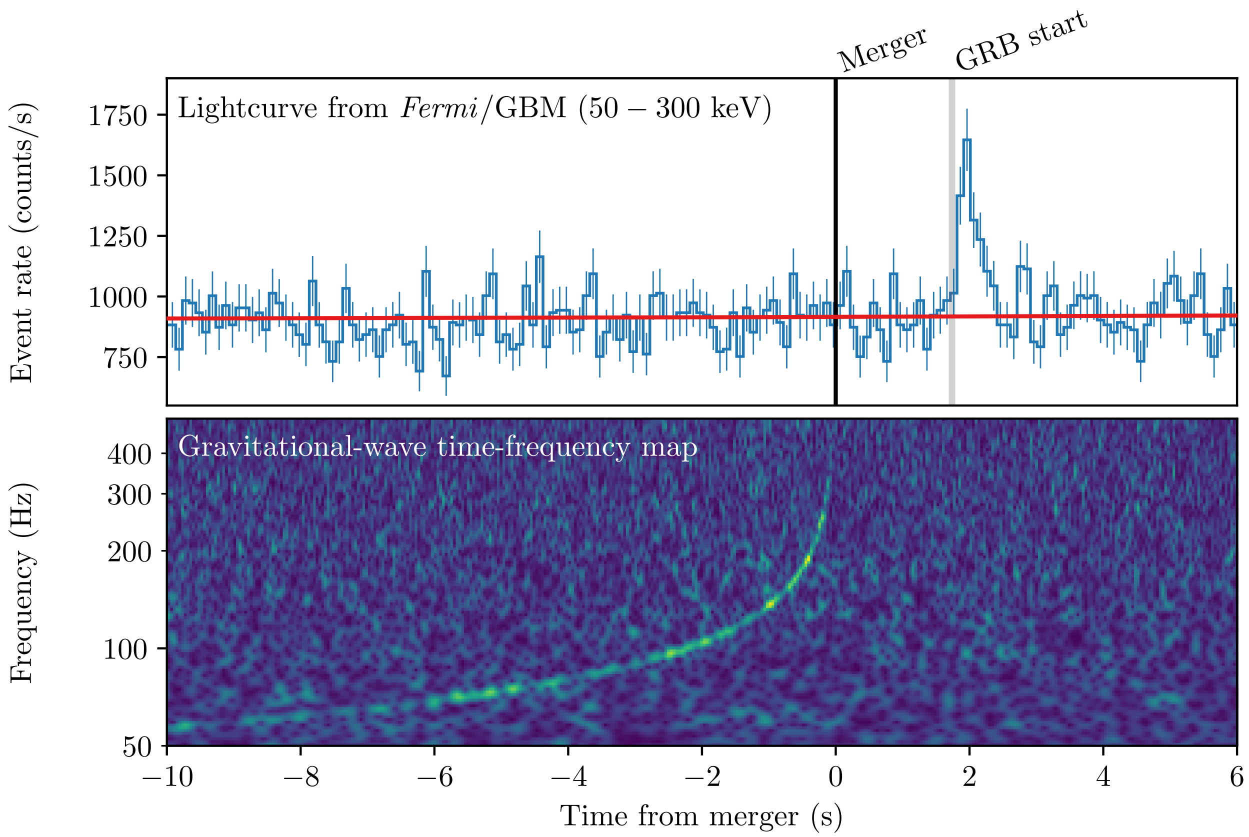
$$\mathcal{L}_{\mathrm{H}}^{(5)} = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$$G_i(\phi, X) = X\sum_{n=0} c_n^{(i)}(\phi)\left(\frac{X}{H_0^2 M_{\mathrm{Pl}}^2}\right)^n$$

Without gravity, $M_{Pl} \to \infty$ (and Λ_3 const.), Horndeski theories reduce to Galileons. Galileons are the **skeletons of modified gravity**

Gravitational waves





$$-3 \times 10^{-15} \leq \frac{c_T - c}{c} \leq 7 \times 10^{-16}$$

Gravitational waves

Lorentz invariance is spontaneously broken. Gravitational waves acquire non trivial speed of propagation. (Think of light travelling in a material.)

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + c_T^2 k^2 \gamma_{ij} = 0$$

$$\mathcal{L}_{\rm H}^{(2)} = G_2(\phi, X)$$

$$\mathcal{L}_{\rm H}^{(3)} = G_3(\phi, X)\Box\phi$$

$$\dot{\gamma}_{ij}^2 - (\partial_k \gamma_{ij})^2$$

$$\dot{\gamma}_{ij}^2$$

$$\mathcal{L}_{\rm H}^{(4)} = G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

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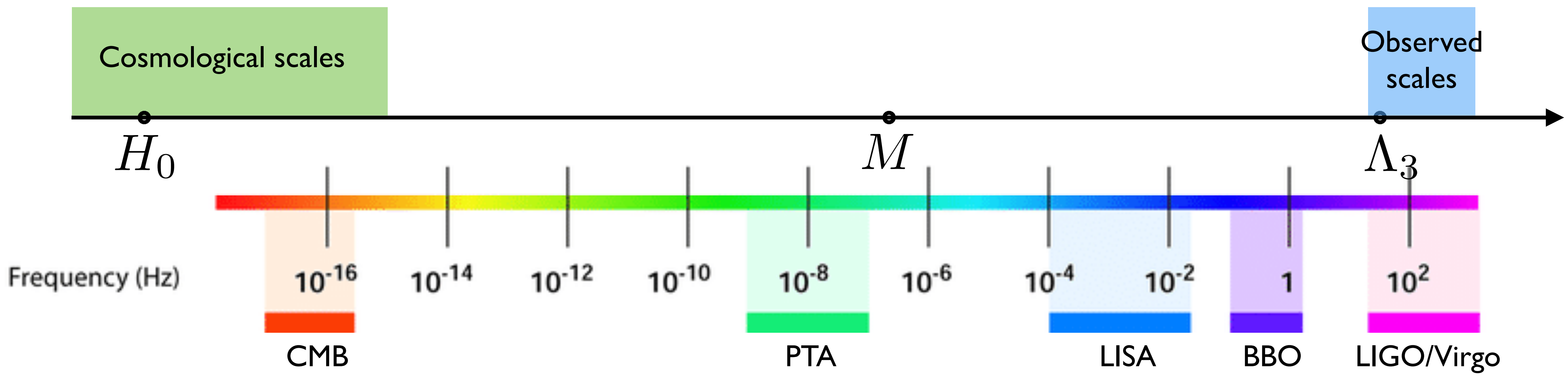
$$\mathcal{L}_{\rm H}^{(4)} = G_4(\phi, \cancel{X})R \cancel{- 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]}$$

$$\mathcal{L}_{\rm H}^{(5)} = \cancel{G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]}$$

Frequency dependence?



EFT of cosmological scales may not apply to LIGO-Virgo scales [de Rham, Melville '18]

Theory may break down and tensor speed may go back to luminal on short scales

$$n(\omega) = 1 + \frac{2\pi N e^2}{m_e} \frac{f}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

Can we use GW observations to constrain these theories?

Conclusions Lecture 5

- Modifications of gravity can be probed by cosmological observations: **redshift-space distortion, weak lensing**. However, they are tightly constrained by **Solar System tests**. How can we modify gravity on cosmological scales and simultaneously pass SS tests?

- We have studied the **screening mechanism** displayed by a cubic Galileon. Nonlinearities of the scalar field induced by overdensities (like the Earth or the Sun) **suppress the fifth force**.

- A covariant Galileon naturally displays **self-acceleration**: observed acceleration explained by modified gravity, **without a CC**. Ruled out by ISW-gal correlation, but one can enlarge the parameter space.

- **Galileons**: set of Lagrangian with the same characteristics: important nonlinearities that contribute to **screening** and **self-acceleration** but second-order EOM (only one propagating dof). **Non-renormalization theorem**: quantum corrections are small at low energy.

- **Horndeski theories**: covariantized version of the Galileons: most general scalar-tensor theories with second-order EOM. Used as a **testbeds** for modified gravity in cosmology. A large set of them possibly **ruled out by GW170817**.