Approaching the Big Crunch in Cyclic/Bouncing Cosmologies

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General References

- *Bouncing Cosmology made simple*,
  A. Ijjas and P Steinhardt, CQG 35 (2018), 1803.01961

- *Beyond Standard Inflationary Cosmology*,
  R. Brandenberger, 1809.04926

- *A Critical Review of Classical Bouncing Cosmologies*,
  D. Battefeld and P. Patrick, Phys. Rept. 571 (2015)

- *Ekpyrotic and Cyclic Cosmology*,
  J. Lehners, Phys. Rept. 465 (2008), 0806.1245

- *Numerical approaches to space-time singularities*,
  B. Berger, Living Rev. Rel. (2002), grqc/0201056

More Technical References

- *Dynamical attractors in contracting spacetimes dominated by kinetically coupled scalar fields ,*
  A. Ijjas, FP, P. Steinhardt and D. Garfinkle, JCAP 12 (2021), 2109.09768

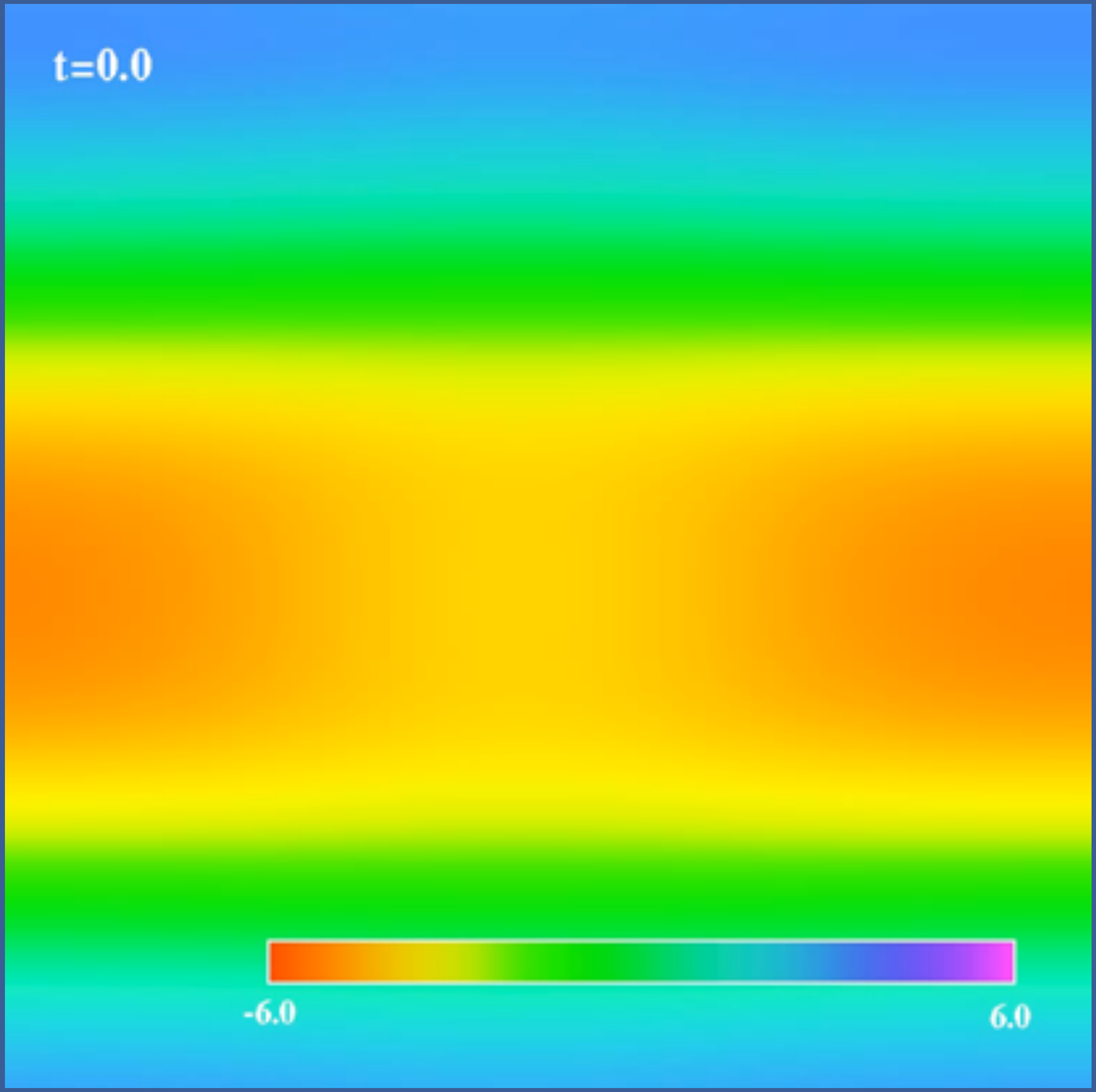
- *Ultralocality and slow contraction,*
  A. Ijjas, A.P. Sullivan, FP, P. Steinhardt and W.G. Cook, JCAP 06 (2021), 2103.00584

- *Spike behavior in the approach to spacetime singularities,*
  D. Garfinkle and FP, PRD 102 (2020), 2010.01399

- *Evolution to a smooth universe in an ekpyrotic contracting phase with $w > 1$,*
  D. Gafinkle, W.C. Lim, FP, P. Steinhardt, PRD 78 (2008), 0808.0542

Shear^3 in a contracting spacetime

- Vacuum
- Periodic in (x,y), planar symmetry in z
- Overall blow-up of shear factored out
- A time coordinate chosen so that t→ ∞ is the singularity



Smoothing Example

- The next few slides are from a similar case, except planar symmetry along two spatial dimensions (so an effect 1+1D evolution), and now with a scalar field with an ekpyrotic-like potential

$$V(\phi) = -V_0 e^{-k\phi}$$

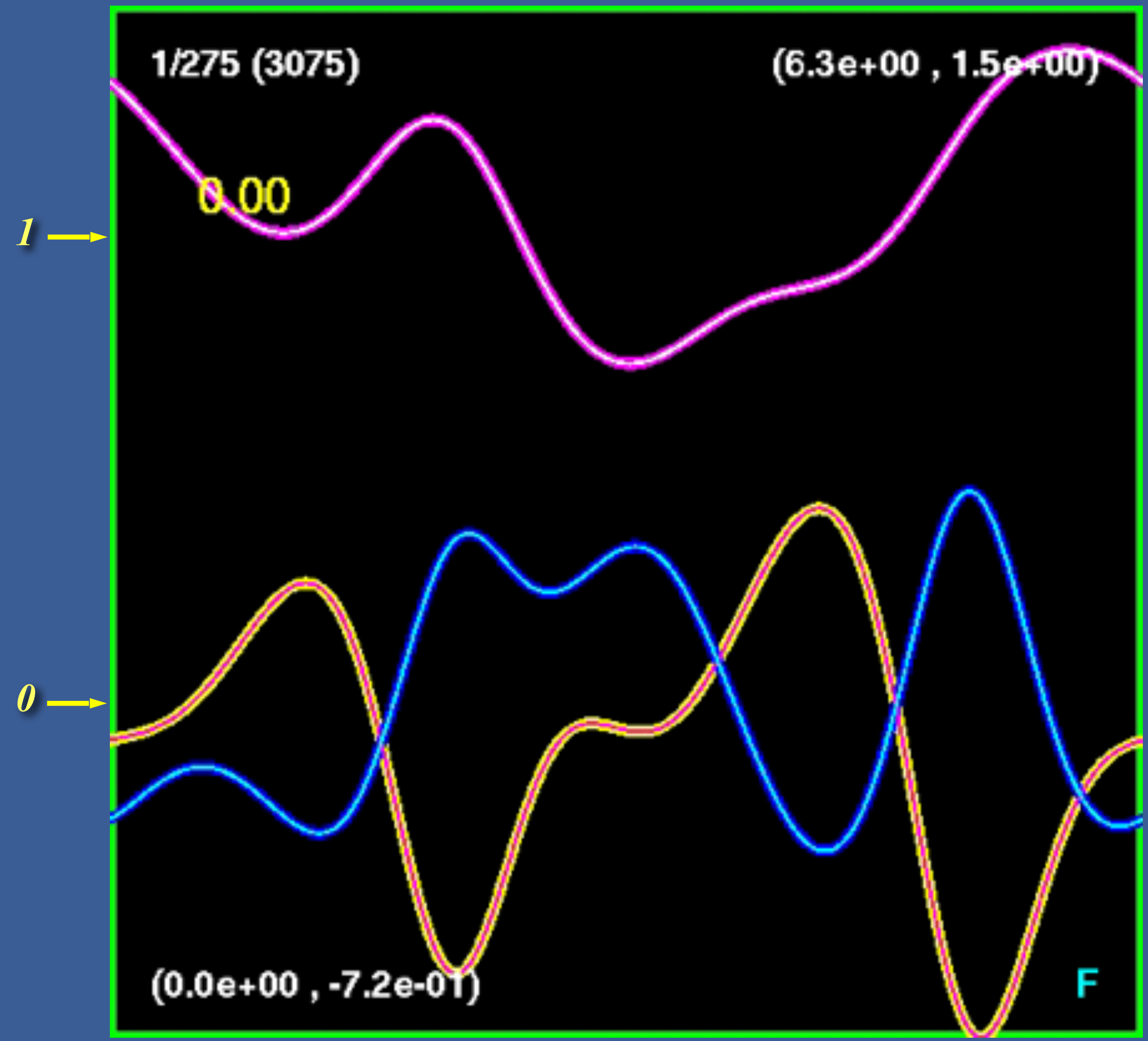
  with k=10

- Starting with initial data that is very far from FRLW, as quantified via Ω's, the normalized contributions to the energy density:

$$\begin{aligned}
\Omega_m &\equiv \tfrac{1}{6}W^2 + \tfrac{1}{6}S^\alpha S_\alpha + \tfrac{1}{3}\overline{V} \\
\Omega_s &\equiv \tfrac{1}{6}\Sigma^{\alpha\beta}\Sigma_{\alpha\beta} \\
\Omega_k &\equiv \tfrac{1}{6}N^{\alpha\beta}N_{\alpha\beta} - \tfrac{1}{12}(N_\gamma{}^\gamma)^2 + A^\alpha A_\alpha - \tfrac{2}{3}E_\alpha{}^i \partial_i A^\alpha
\end{aligned}$$

$$\Omega_m + \Omega_k + \Omega_s = 1$$

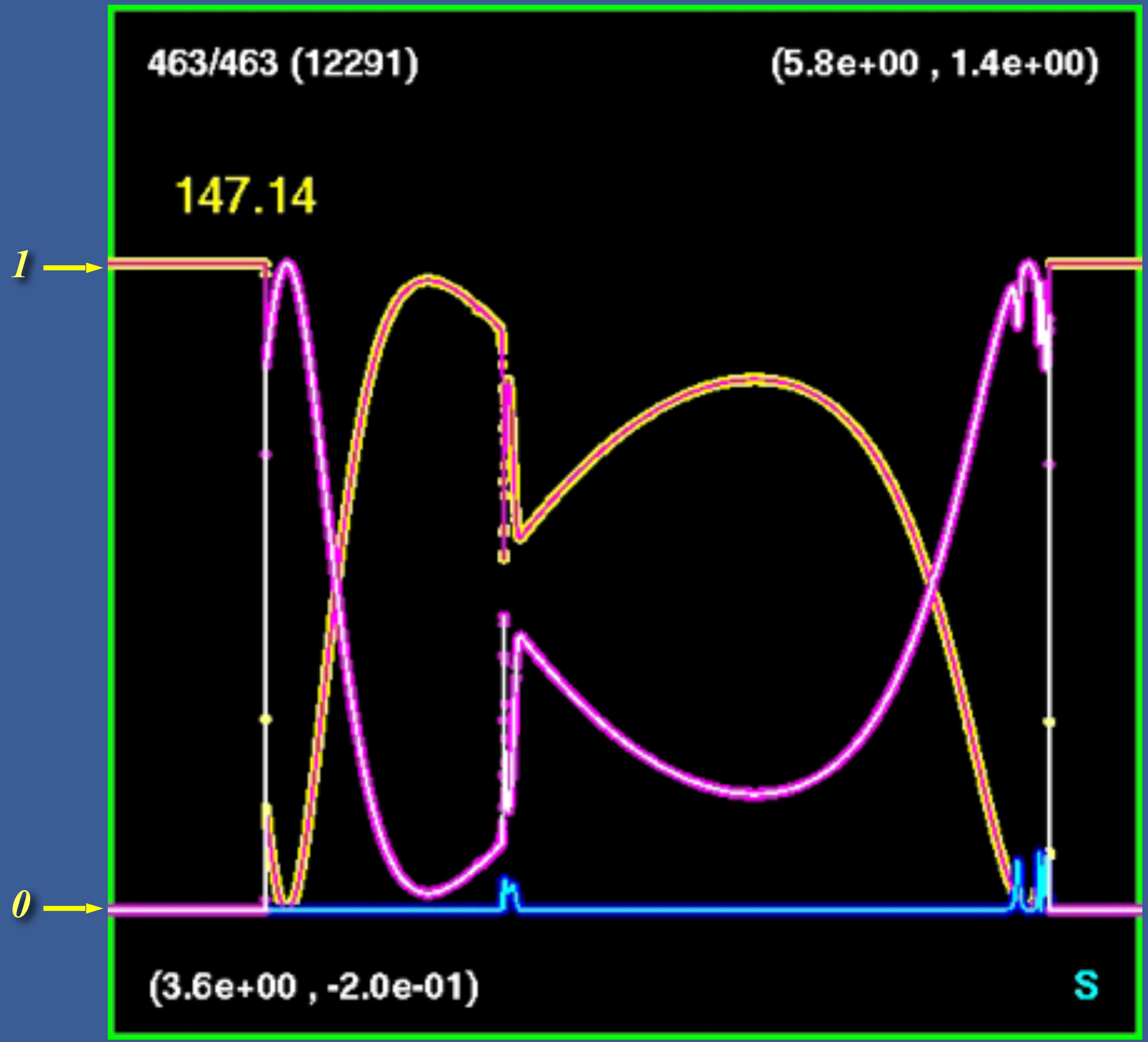
Ω at early times



- Yellow --- Ω_m
- Blue --- Ω_k
- Pink --- Ω_s

In these coordinates the singularity occurs at $t=\infty$

Zoom-in of Ω at late times

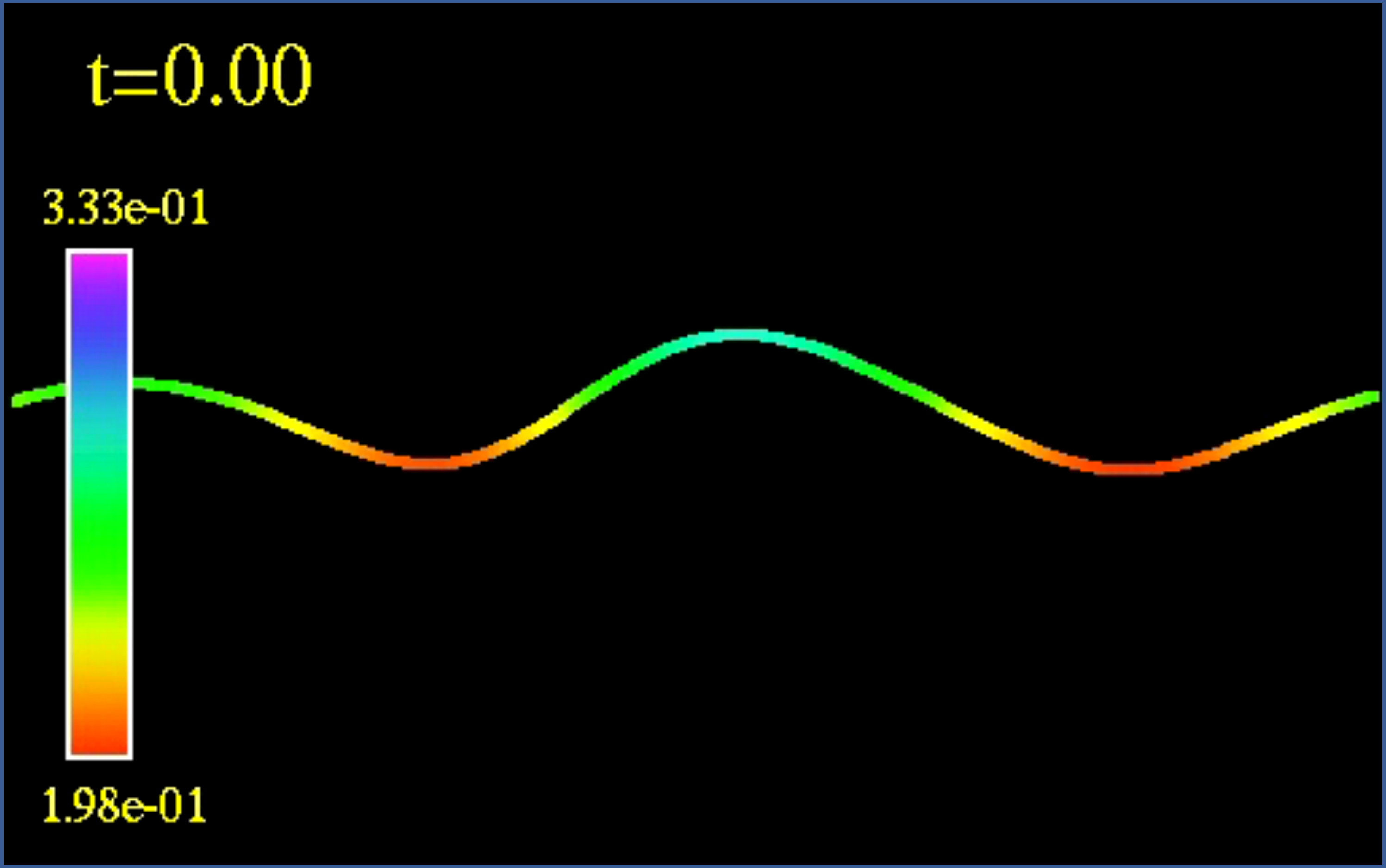


- Yellow --- Ω_m
- Blue --- Ω_k
- Pink --- Ω_s

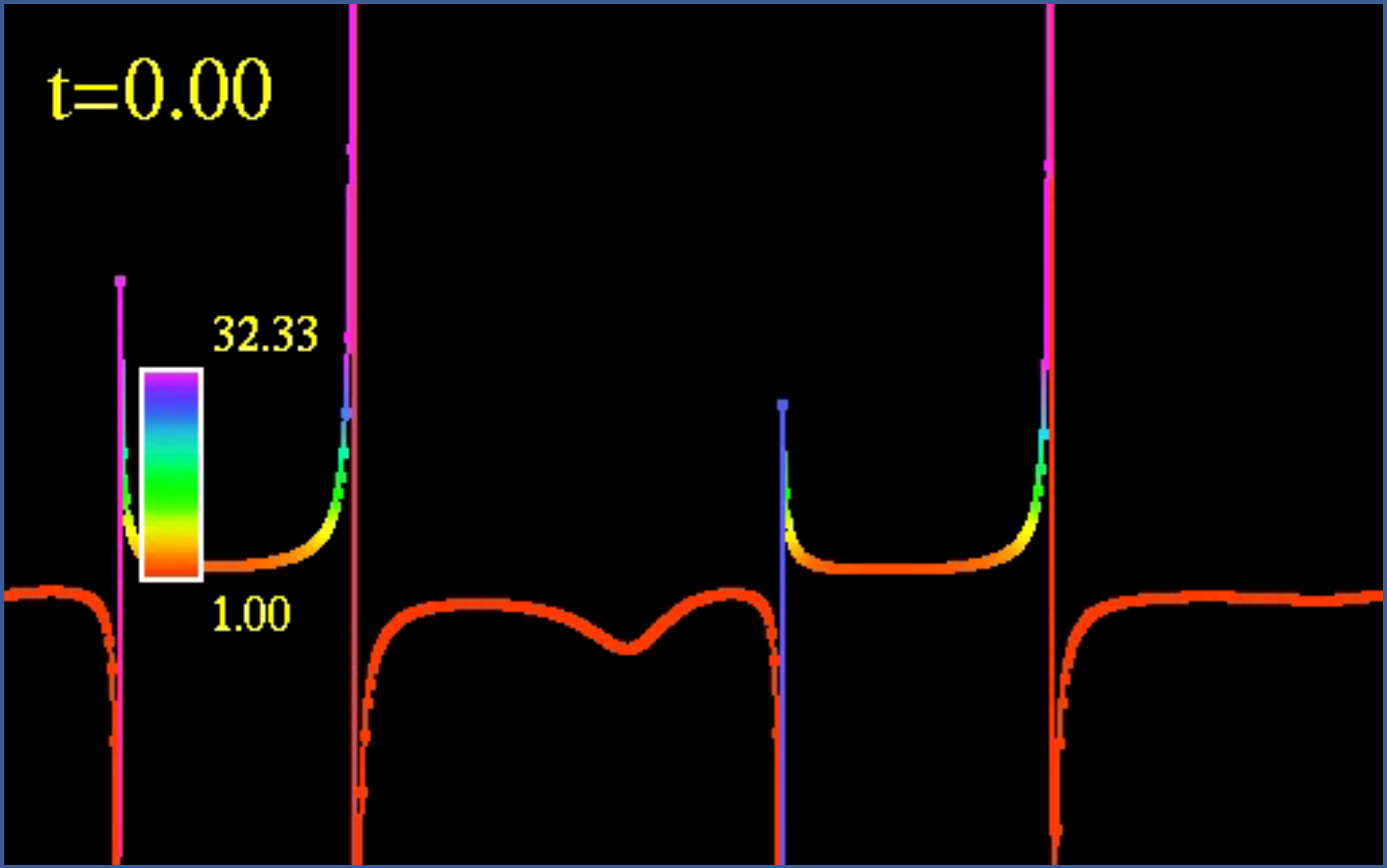
Note that spikes are not being smoothed out – that they seem to "disappear" after some time is an artifact of having converted the data to a lo-res uniform mesh for visualization purposes

Hubble-normalized lapse ℵ

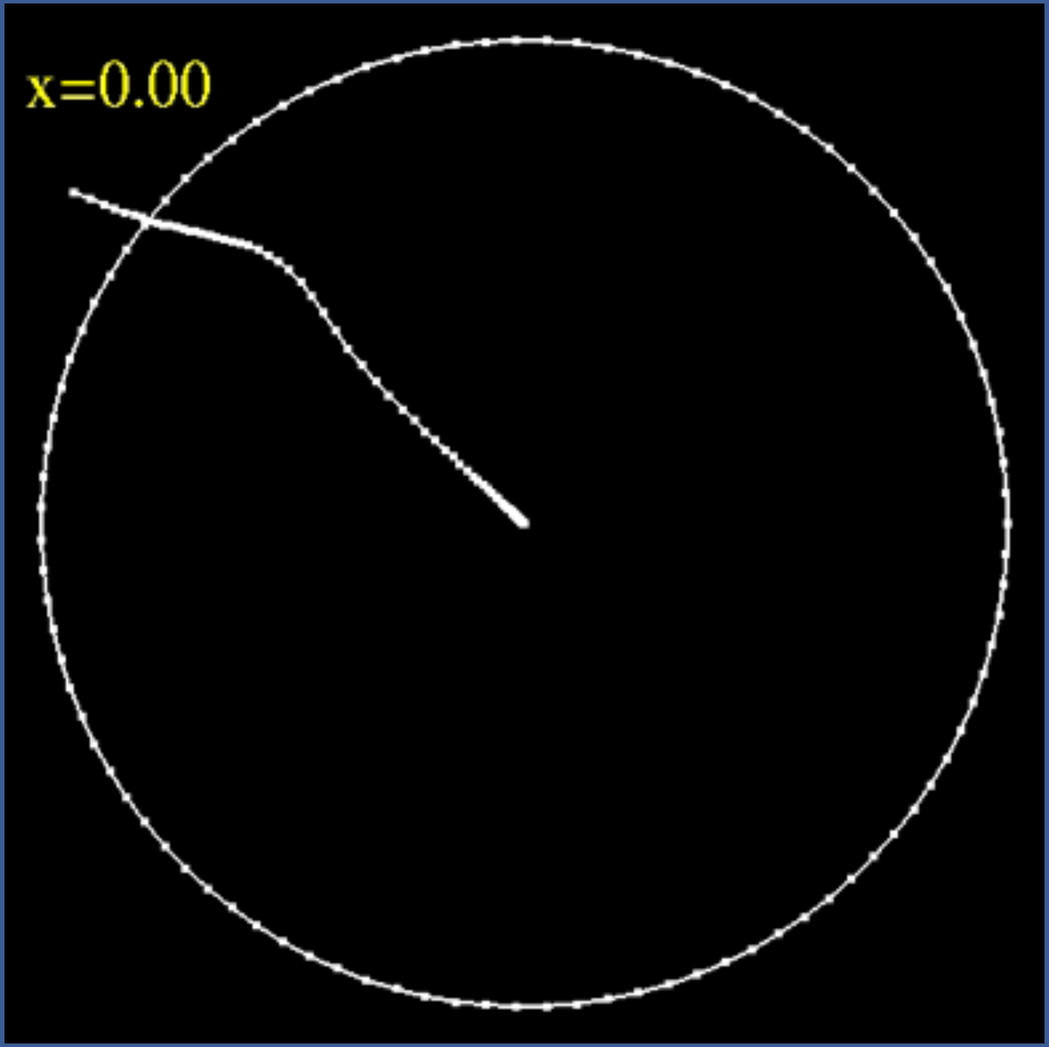
- In CMC slicing, as used in evolution, the proper volume element S of $t=constant$ slices at late times scales as $S \propto e^{-3\aleph t}$



Effective equation of state parameter w



State space orbits



- Each frame of the animation shows

  $\Sigma_- = (\Sigma_{11} - \Sigma_{22})/2/\sqrt{3}$

  as a function of

  $\Sigma_+ = 1/2\ (\Sigma_{11} + \Sigma_{22})$

  along an *x=constant* wordline, scanning from *x=0 to x=2*π.

- A point on the circle is Kasner (unstable), the center is flat FRLW (stable), points within an inner circle of radius $1/\sqrt{3}$ (not shown) are the mixed Kasner-like scalar field spacetimes (unstable).

- A trajectory flowing to the center thus represents evolution to a locally smooth, isotropic geometry

Formalism

- Solve the Einstein field equations

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$$

where the stress-energy tensor is sourced by a scalar field with ekpyrotic potential

$$V(\phi) = -V_0 e^{-k\phi}$$

- We expand the equations using the *orthonormal-frame formalism with Hubble-normalized variables* (Uggla et al, 2003)
  - the metric is defined in terms of a set of four linearly independent 1-forms ω^a, which are dual to an orthonormal "tetrad" $\mathbf{e}_a$, with $\mathbf{e}_0$ being timelike and the 3 $\mathbf{e}_\alpha$ spacelike:

$$ds^2 = \eta_{ab}\boldsymbol{\omega}^a\boldsymbol{\omega}^b, \qquad \eta_{ab} = \mathrm{diag}[-1,1,1,1]$$

Formalism - geometry

- Choosing coordinates where there is no vorticity in the time-like vector field $\mathbf{e}_0$, and the spatial frame $\mathbf{e}_a$ is non-rotating with no shift

$$\mathbf{e}_0 = N^{-1}\partial_t, \qquad \mathbf{e}_\alpha = e_\alpha{}^i\partial_i$$

we can decompose the commutators of the tetrad as

$$[\mathbf{e}_0,\mathbf{e}_\alpha] = u_\alpha \mathbf{e}_0 - \left(H\delta_\alpha{}^\beta + \sigma_\alpha{}^\beta\right)\mathbf{e}_\beta$$

$$[\mathbf{e}_\alpha,\mathbf{e}_\beta] = \left(2a_{[\alpha}\delta_{\beta]}{}^\gamma + \varepsilon_{\alpha\beta\nu}n^{\nu\gamma}\right)\mathbf{e}_\gamma$$

where N is the lapse; du^α/dt is the acceleration, H the (Hubble) expansion rate, and $\sigma^{\alpha\beta}$ the shear of the time-like congruence; $n^{\alpha\beta}$ and a^α contain information about the spatial metric.

- Hubble normalized (scale invariant) gravitational variables are defined by

$$\left\{E_\alpha{}^i, \Sigma_{\alpha\beta}, A^\alpha, N_{\alpha\beta}, \aleph^{-1}\right\} \equiv \left\{e_\alpha{}^i, \sigma_{\alpha\beta}, a^\alpha, n_{\alpha\beta}, N^{-1}\right\}/H$$

- Choosing consta

Formalism - matter

- Scale invariant matter quantities are defined via

$$W = \aleph^{-1}\partial_t\phi$$
$$S_\alpha = E_\alpha{}^i\partial_i\phi$$
$$\overline{V} = V / H^2$$

- In terms of these variables the effective "equation of state" function w of the scalar field is

$$w \equiv \frac{P}{\rho} = \frac{\frac{1}{2}W^2 + \frac{1}{2}S_\alpha S^\alpha - \overline{V}}{\frac{1}{2}W^2 + \frac{1}{2}S_\alpha S^\alpha + \overline{V}}$$