



On ENSO Theory

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ENSO Summer School

What is a scientific theory?

Scientific theory, systematic ideational structure of broad scope, conceived by the human imagination, that encompasses a family of empirical (experiential) laws regarding regularities existing in objects and events, both observed and posited. **A scientific theory is a structure suggested by these laws and is devised to explain them in a scientifically rational manner.**

<https://www.britannica.com/science/scientific-theory>. Accessed 7 July 2022.

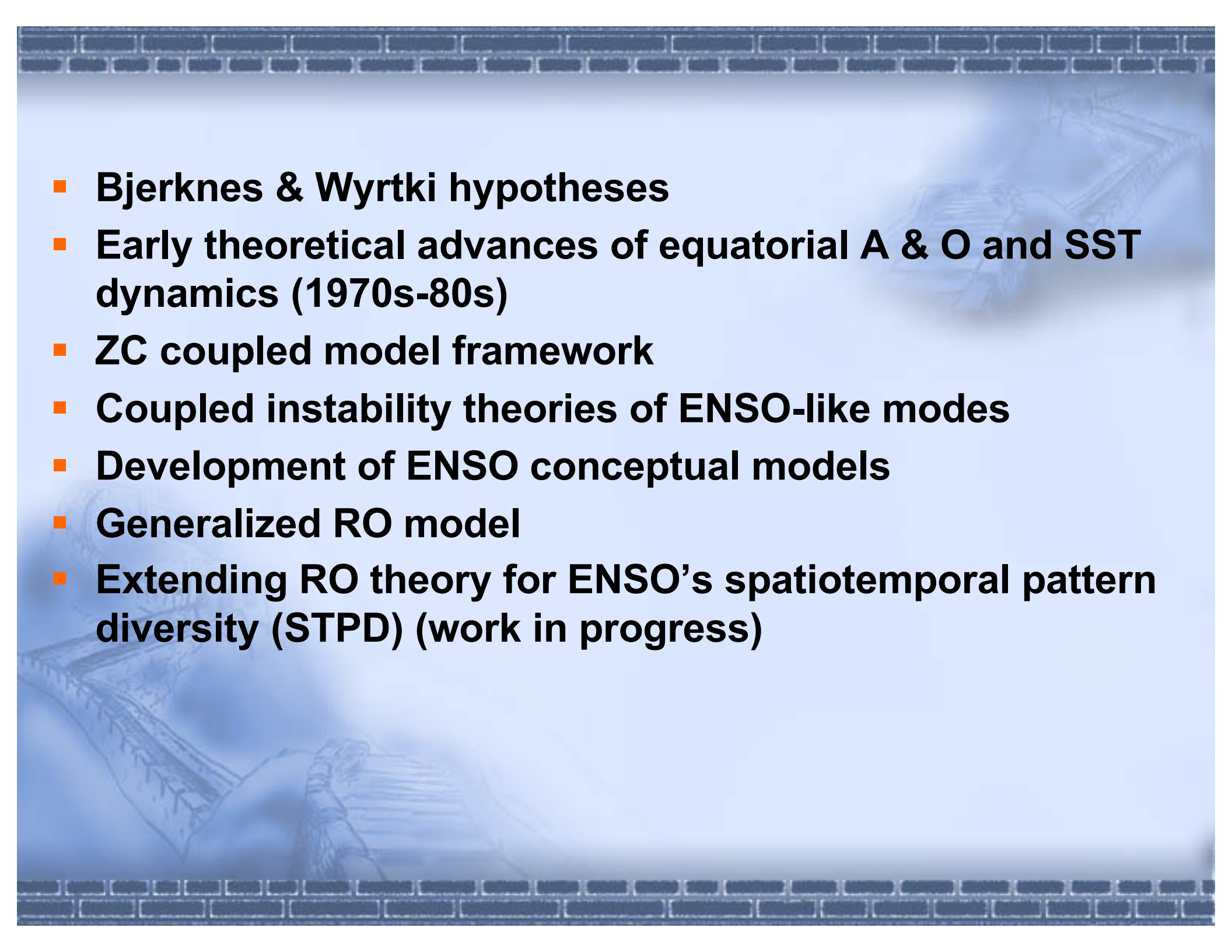
For an example: ENSO's Recharge Oscillator Theory is a model deduced from the A-O basic laws to explain observed features of ENSO

Key Questions for ENSO Theory

- What control ENSO's (1) growth rate and main periodicity, (2) phase locking, (3) amplitude (4) asymmetry, (5) spatiotemporal pattern diversity (STPD) ? and ?

A Goal of the 2-lectures

A brief and partial account of ENSO theories with a focus on RO theory's inception and generalization and possible extensions?

- 
- **Bjerknes & Wyrтки hypotheses**
 - **Early theoretical advances of equatorial A & O and SST dynamics (1970s-80s)**
 - **ZC coupled model framework**
 - **Coupled instability theories of ENSO-like modes**
 - **Development of ENSO conceptual models**
 - **Generalized RO model**
 - **Extending RO theory for ENSO's spatiotemporal pattern diversity (STPD) (work in progress)**

- In 1969, Bjerknes made the first hypothesis that El Niño/La Niña grow due to a positive coupled feedback. But it took another 10+ years to gain the understanding of the basic dynamics tropical atmospheric and ocean circulations.
- The 1982-83 event allowed Wyrski (1985) to hypothesize ENSO's phase transition due to ocean content charge/discharge. It kicked off a golden era of ENSO theoretic studies till today:
- Some landmark papers in 80s

J P. McCreary, MWR 1983

* Cane and Zebiak 1984,85, 87 **(CZ model)**

Philander et al 1984

Gill 1985 (who died shortly after)

* Suarez and Schopf 1987, Batisitti and Hirst 1989 **(DO model)**

In 1990s-2000s

Neelin 1991: SST mode,

* Jin et al 1993: unified theory for coupled modes,

Jin et al 1994; Tzipermann et al; Chang et al 1994; ENSO Chaos theory

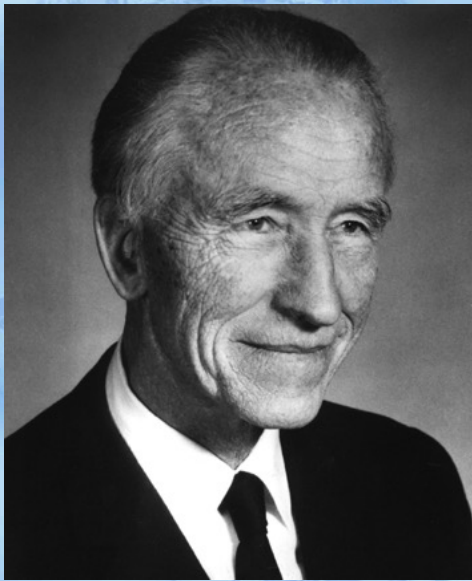
* Jin et al 1997a, b **Recharge oscillator theory, duality of RO and WO**

In 2000s-2020s →

- **ENSO mode's sensitivity to climate mean states**
(An and Jin 2000; Fedorov and Philander 2000)
- **ENSO asymmetry**
(Kang and Kug 2001; Jin and An 2003.....)
- **BJ instability index for ENSO**
(Jin et al 2007, * Jin et al 2020)
- **Noise induce instability of ENSO**
(Jin et al 2006)
- **ENSO phase-locking**
(Jin 1996; ... Chen and Jin 2021; Kim and An 2021)
- **ENSO combination mode**
(Stuecker et al 2013, 1+f, 1-f)
- * **Nonlinear/Noise induced Regime Transition for ENSO STPD**
(Geng and Jin 2022)

Bjerknes Hypothesis of Coupled Instability for E&L :

Weakening/strengthening of the Sothern Oscillation/WC/trade wind warms /cools SST that in turn weakens/strengthens the trade wind further!



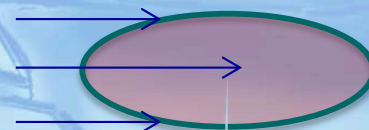
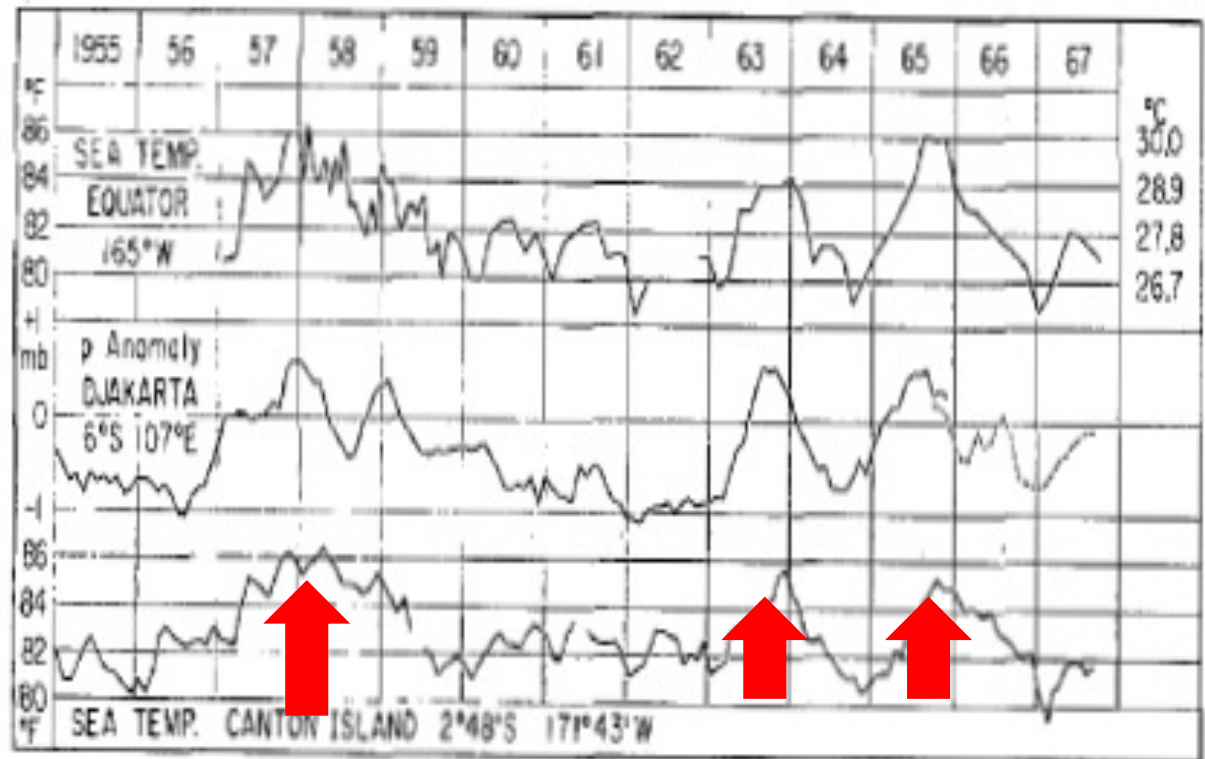
Bjerknes hypothesized that ENSO SST drives wind anomalies associated SO/WC to feed back positively on ENSO SST (via upwelling). He was unsure how E&L turn about.

ATMOSPHERIC TELECONNECTIONS FROM THE EQUATORIAL PACIFIC¹

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ABSTRACT



$$u \approx a T$$

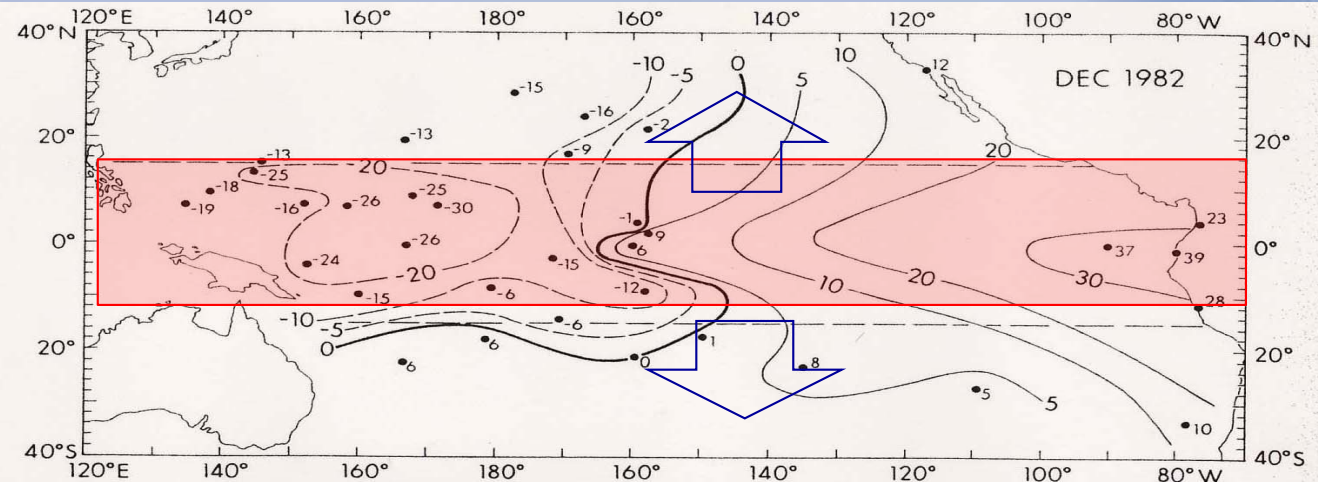


Fig. 2. Sea level anomaly in December 1982 (centimeters). Circles denote sea level stations; values are the deviation of sea level in December 1982 from the seven-year average December sea level 1975 to 1981. The dashed lines at 15°N and 15°S give the area used for the integration of upper layer volumes.

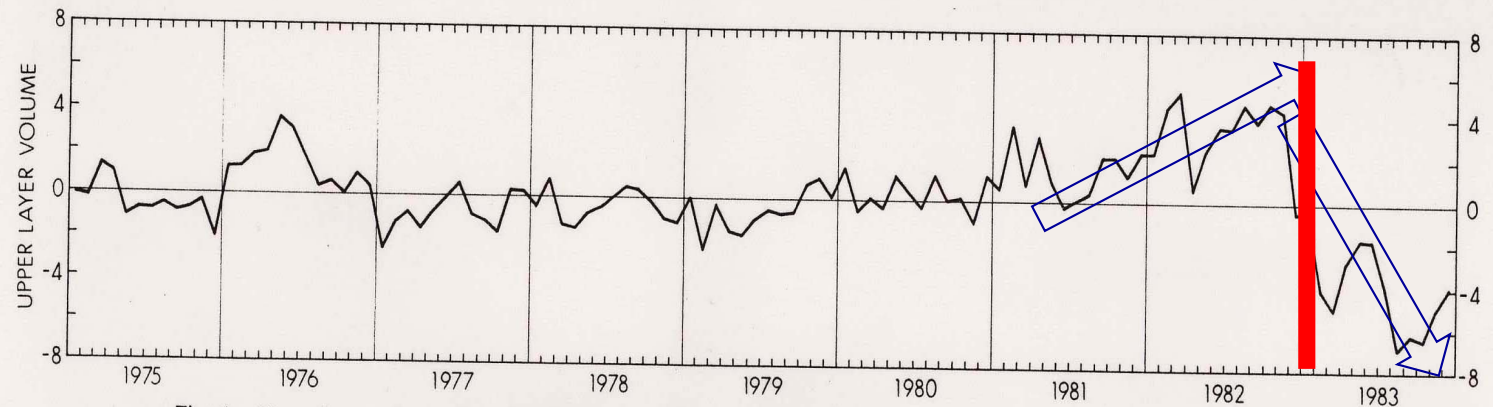


Fig. 4. Upper layer volume of the tropical Pacific (10^{14} m^3) from 1975 to 1983 relative to its mean value of about $70 \times 10^{14} \text{ m}^3$.

Wyrтки Hypothesis of Recharging and discharging
of warm pool heat content for E-L transition

Established Fundamental laws which ENSO and all motions of A-O systems obeys !

The primitive equations for atmospheric circulation:

Variables: (u, v, w, q, p, T, ρ)

$$p = RT_v \rho$$

$$C_p \frac{dT}{dt} - \frac{1}{\rho} \frac{dp}{dt} = J$$

$$\frac{dq}{dt} = Q_q$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$$

$$\frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} = -\frac{\nabla p}{\rho} - \vec{g} - \nu \Delta \vec{V}$$

- **Five / Six time derivatives \rightarrow**
- **6 types of eigen values = “waves”**
- **2 for sound-waves**
- **2 for gravity-waves (including K-wave inertial G-waves)**
- **1 for Rossby wave (including MRG)**
- **1 for moisture mode / wave (MJO)**
- **various instabilities for weather making phenomena such as fronts and cyclones etc.**

7 equations , 6 prognostic equations+ one algebraic equation

Many more equations for concentrations of chemical species aerosols, water droplets, ice crystals, ...can be added.

The primitive equations for oceanic circulation:

Variables: (u, v, w, s, p, T, ρ)

$$\rho = \rho_0 - \alpha_w (T - T_0) + \beta_w (s - s_0)$$

$$\frac{dT}{dt} = J$$

$$\frac{ds}{dt} = \nabla \cdot \kappa \nabla s$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} = -\frac{\nabla p}{\rho} - \vec{g} - \nu \Delta \vec{V}$$

- 6 time derivatives \rightarrow
- 6 types of eigen values = “waves”
- 2 for sound-waves
- 2 for gravity-waves (including K-wave inertial G-waves)
- 1 for Rossby wave (including MRG) K+R wave to form ocean modes in basins.
- TH modes, SST mode,
- various instabilities and motions of various scales.

Many more equations for concentrations of chemical species , biological species, ...can be added.

When Atmosphere-ocean system are coupled, new instabilities and phenomena are born, ENSO is one of the most noted of such.

Early theoretical advances of equatorial A & O and SST dynamics (1970s-80s)

- Tropical atmospheric wave dynamics and response to heating)

T Matsuno 1966, Gill A 1980

- Equatorial oceanic wave dynamics & response to winds

Cane M and E. Sarachick 1976-83

McPhaden, M.J., 1981 McCreary, J.P., 1981

- Thermodynamics of mixed-layer SST anomaly

Hasselmann K. (1976) $\frac{\partial T}{\partial t} = -rT + \xi$

$$\cancel{\frac{\partial u}{\partial t}} - fv + \frac{\partial \phi}{\partial x} = \underline{-ru}$$

$$\cancel{\frac{\partial v}{\partial t}} + fu + \frac{\partial \phi}{\partial y} = \underline{-rv}$$

$$\cancel{\frac{\partial}{\partial t}} \phi + gH_e \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = \underline{-Q_a - \varepsilon \phi}$$

Gill (1980) assumed the dominance of the first baroclinic mode response and a string linear damping

**Gill-Matsuno Model
for atmospheric response to heating
become key component of CZ ENSO
model by additional consideration of
relating forcing to ENSO SST induced
convective heating (Zebiak 1982)**

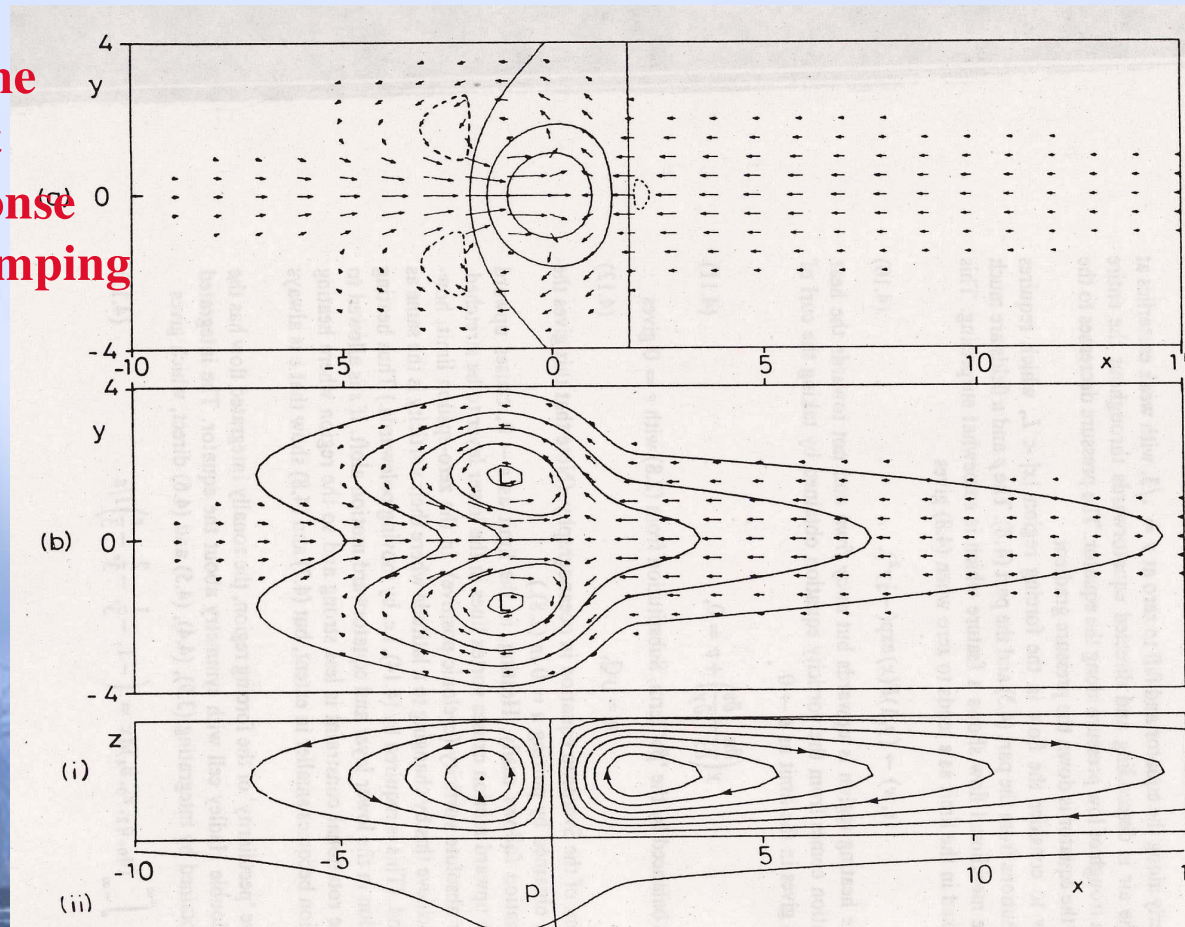


Figure 1. Solution for heating symmetric about the equator in the region $|x| < 2$ for decay factor $\varepsilon = 0.1$.

(a) Contours of vertical velocity w (solid contours are 0.3, 0.6, broken contour is -0.1) superimposed on the velocity field for the lower layer. The field is dominated by the upward motion in the heating region where it has approximately the same shape as the heating function. Elsewhere there is subsidence with the same pattern as the pressure field.

(b) Contours of perturbation pressure p (contour interval 0.3) which is everywhere negative. There is a trough at the equator in the easterly regime to the east of the forcing region. On the other hand, the pressure in the westerlies to the west of the forcing region, though depressed, is high relative to its value off the equator. Two cyclones are found on the north-west and south-west flanks of the forcing region.

(c) The meridionally integrated flow showing (i) stream function contours, and (ii) perturbation pressure. Note the rising motion in the heating region (where there is a trough) and subsidence elsewhere. The circulation in the right-hand (Walker) cell is five times that in each of the Hadley cells shown in (c).

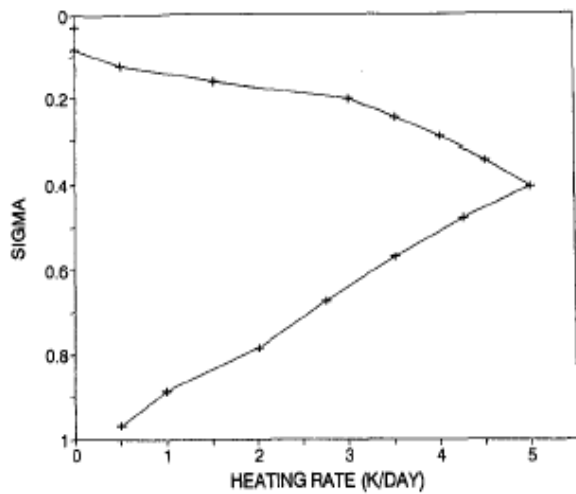
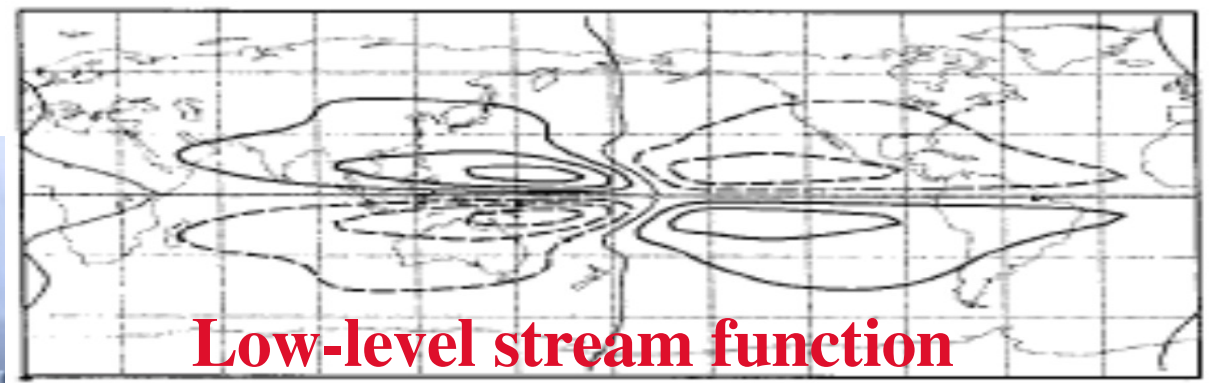
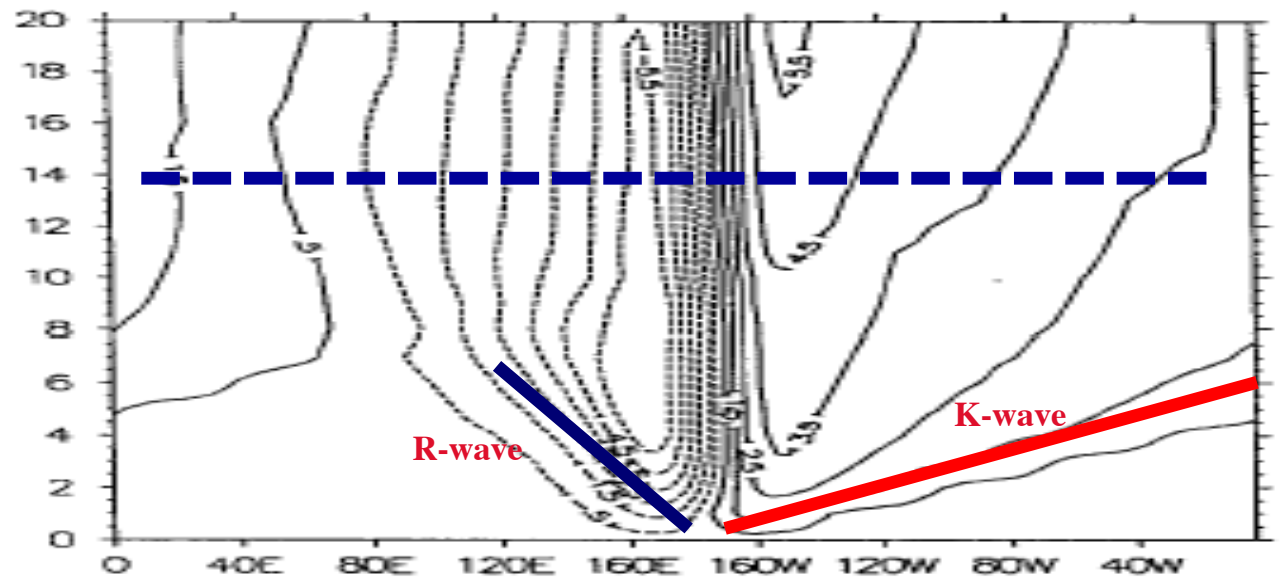
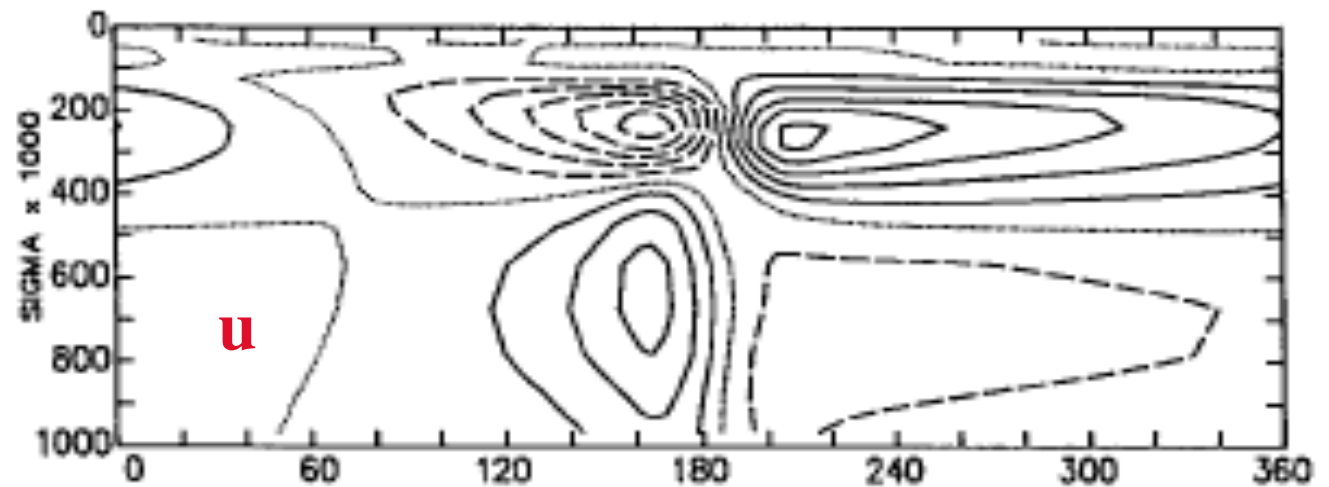


FIG. 1. The vertical profile of the diabatic heating at the center of the source. The plus signs indicate the values at model levels.

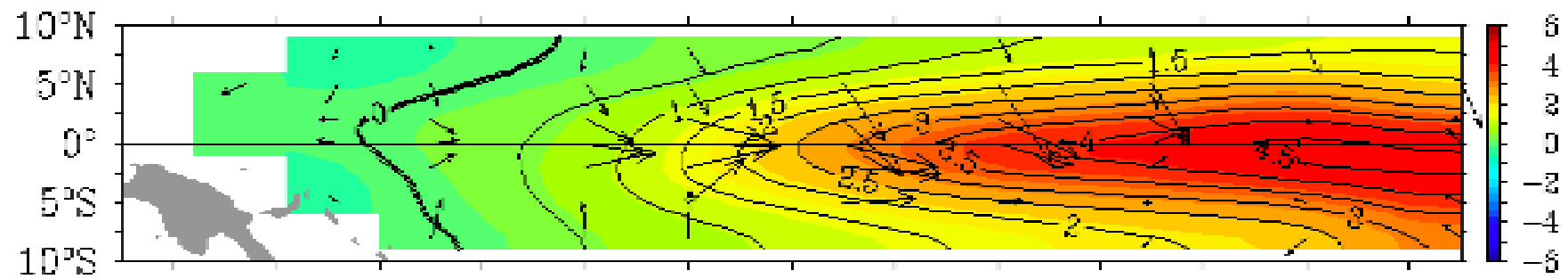
**Primitive GCM simulations
by Jin and Hoskins 1995
verified Gill solutions.**

**Reaching quasi-equilibrium
in about two weeks.**

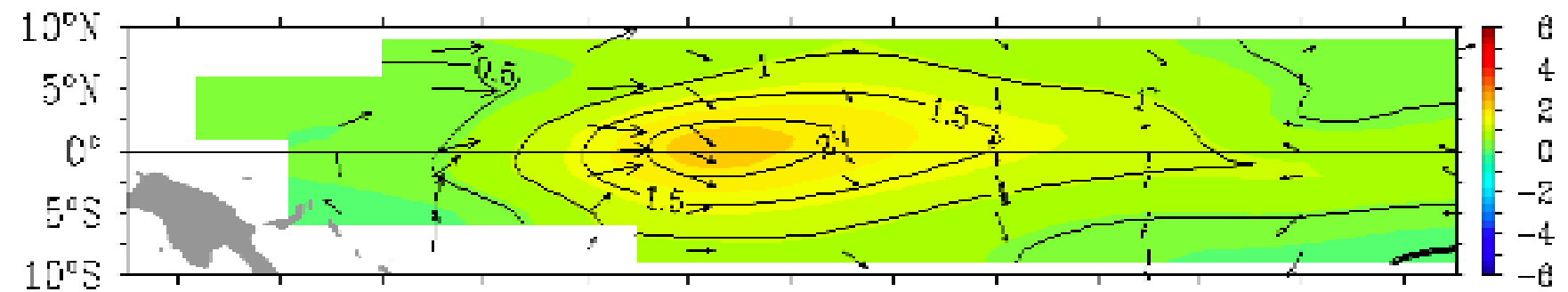
**But model heating
profile may affect surface winds
(Wu's 1999)**



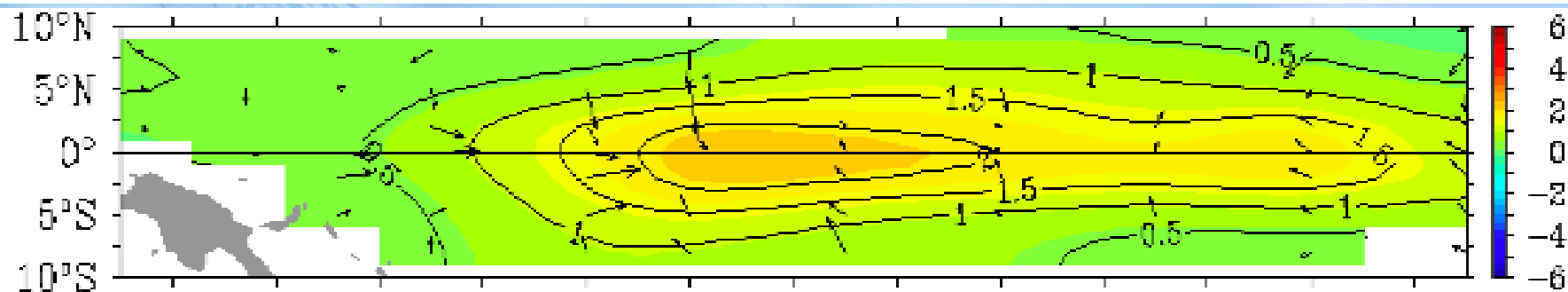
Low-level stream function



November 1997 Anomalies



November 1994 Anomalies



November 2002 Anomalies

The linear reduced-gravity/first baroclinic mode model forced by wind stress

$$\frac{\partial u}{\partial t} - fv + g \frac{\partial h}{\partial x} = -\cancel{ru} + \tau_x / H$$

Equatorial Sverdrup Balance

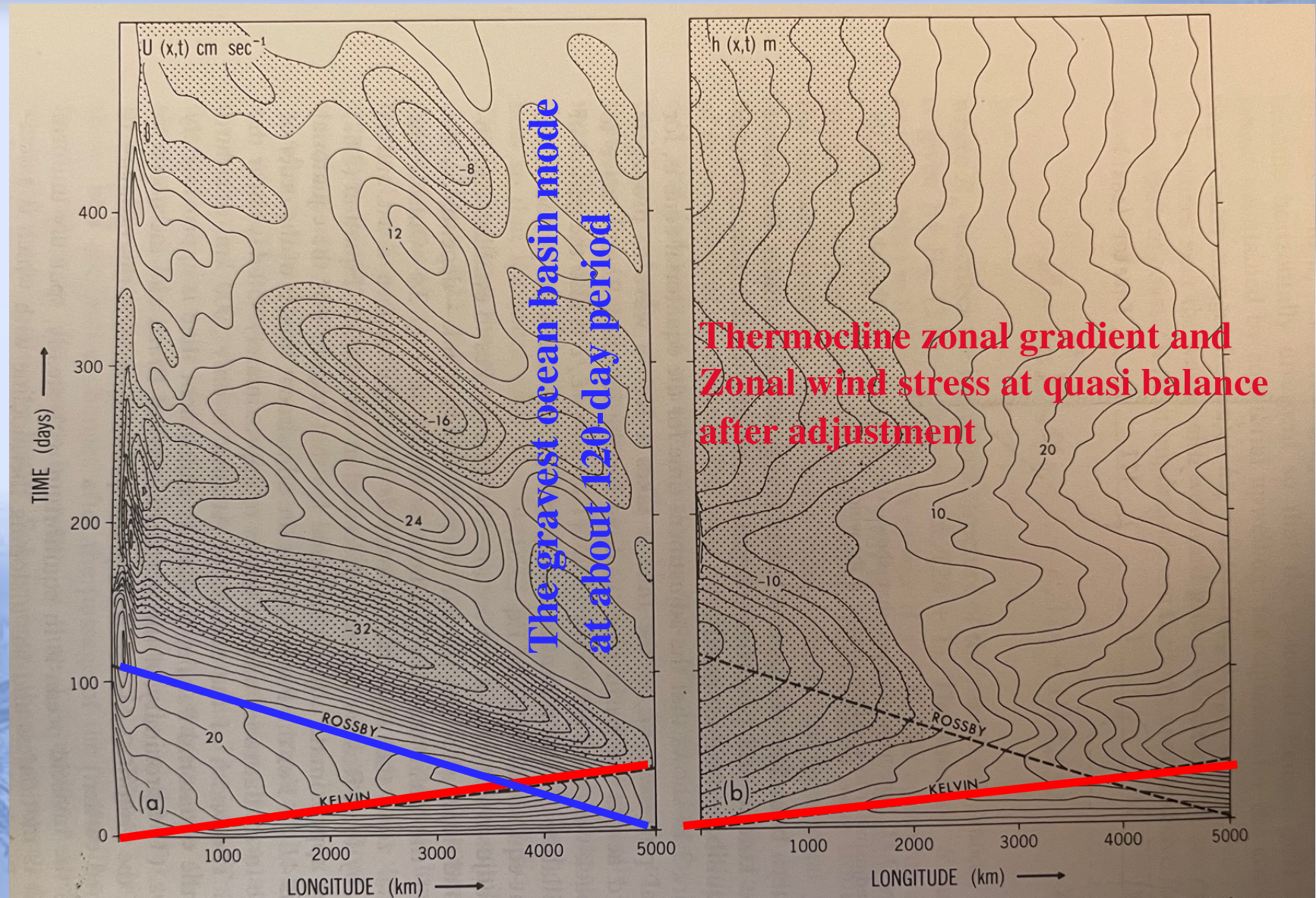
$$\cancel{\frac{\partial v}{\partial t}} + fu + g \frac{\partial h}{\partial y} = -\cancel{rv} + \cancel{\tau_y} / \cancel{H}$$

Quasi-geostrophic balance in y-direction

$$\frac{\partial}{\partial t} h + H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = -\cancel{\epsilon} h$$

(then only true prognostic equation is h)

Philander 1990 p140



K and R wave propagations, reflections, basin-mode oscillations, quasi-steady balance, slow adjustment (years)

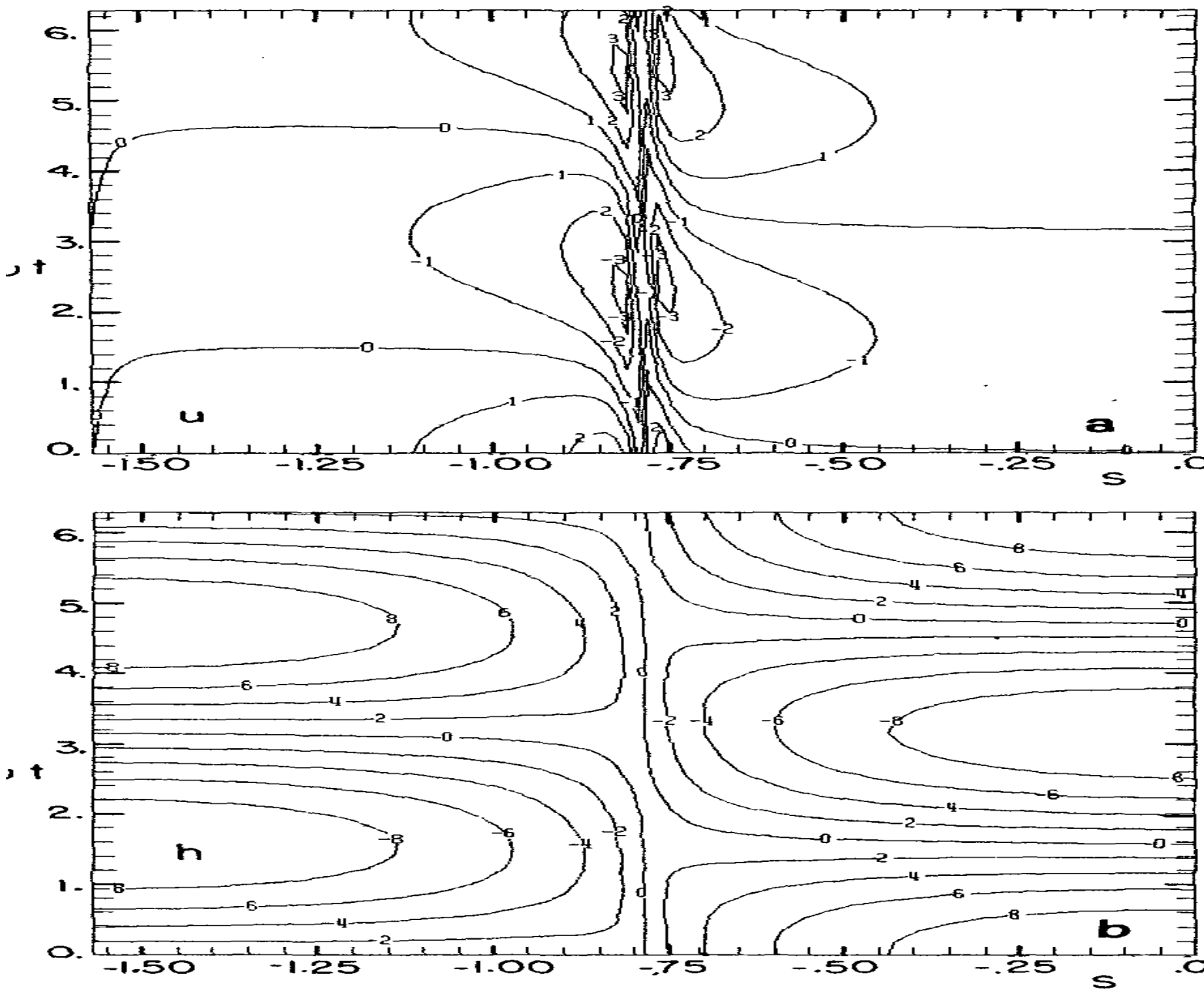
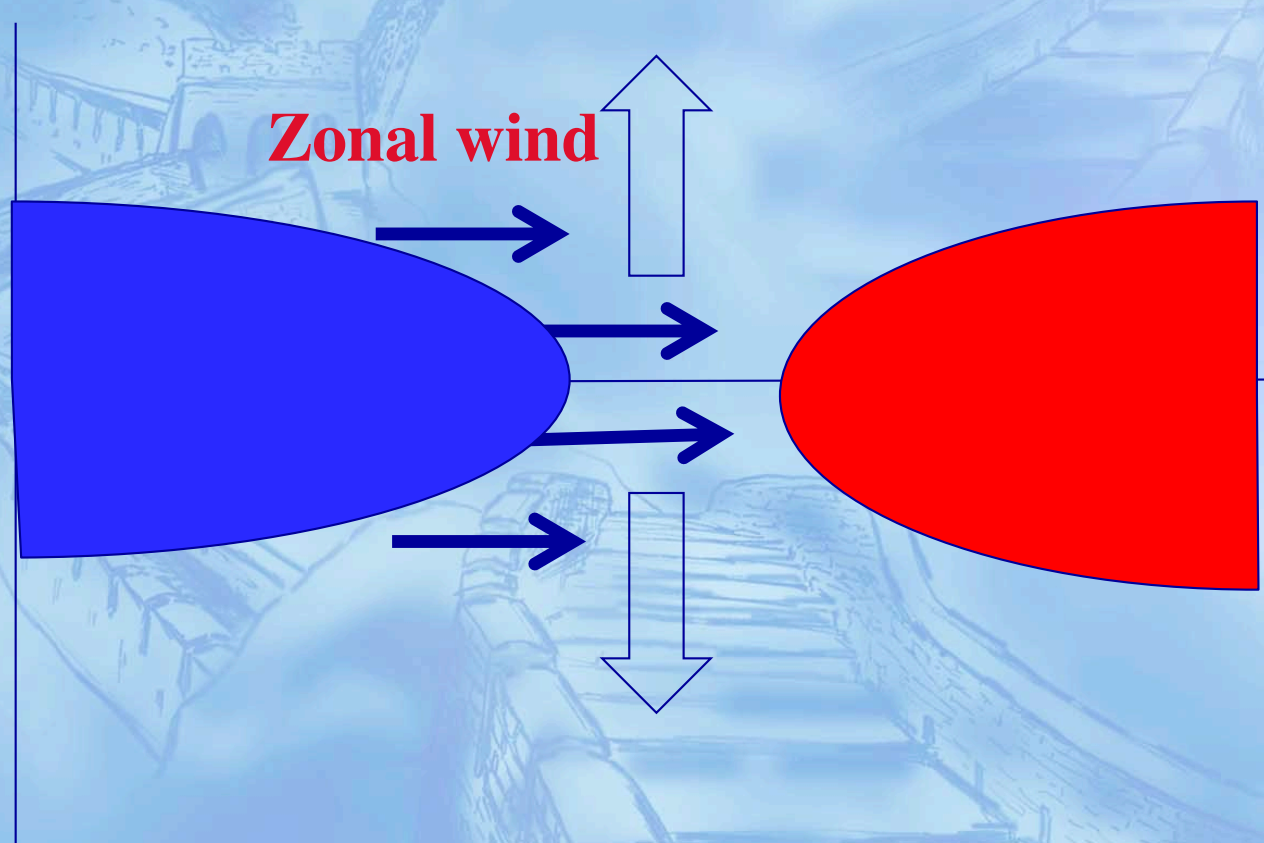


FIG. 3. Values of (a) u and (b) h along the equator ($y = 0$) for one full period.

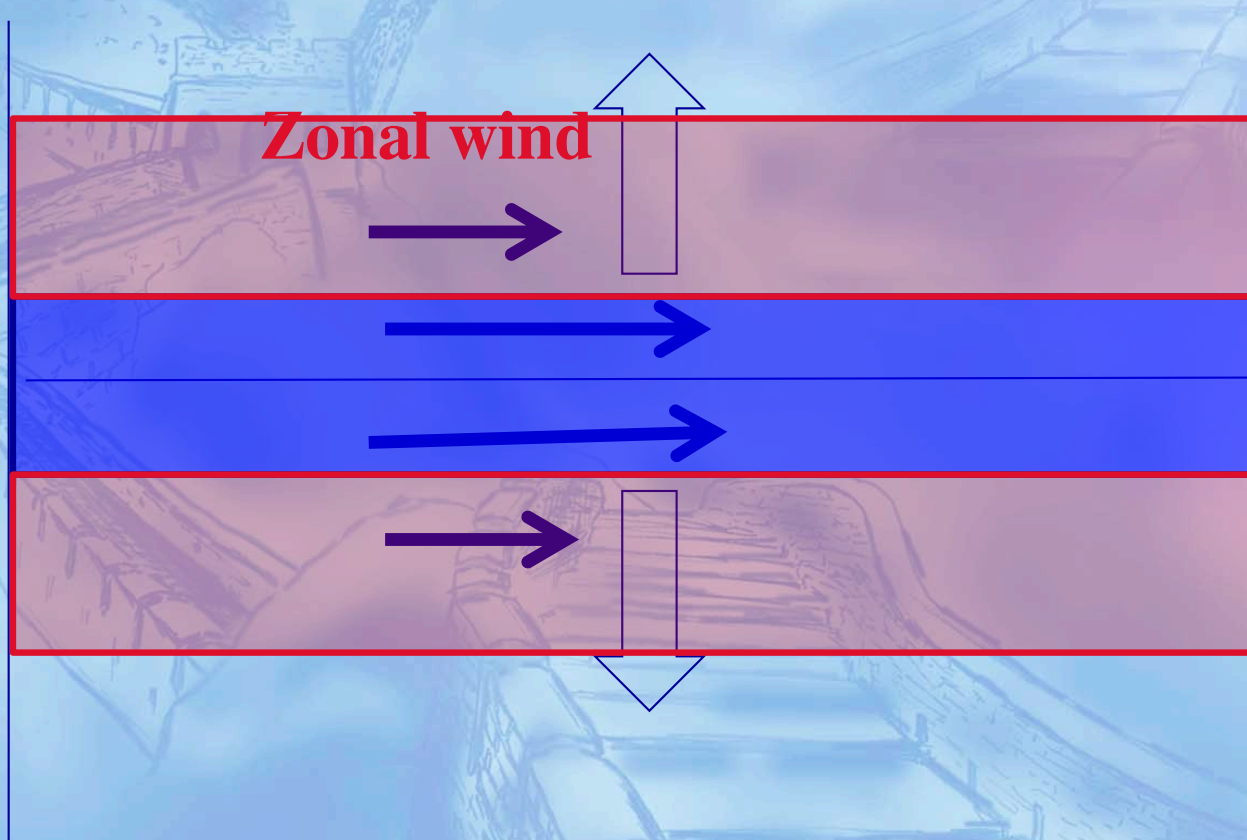
Moore and
Cane 1981
First found it.

Cane et al
1990 &
Jin et al 1993
showed this
mode can be
transformed
into ENSO—
like mode
under some
conditions!

- The equatorial Sverdrup balance produce thermocline tilt
- The off-equatorial wind stress curl produce discharging of mass/heat content transport



The discharging will producing meridional pattern of thermocline with 90 phase delay at the equatorial region



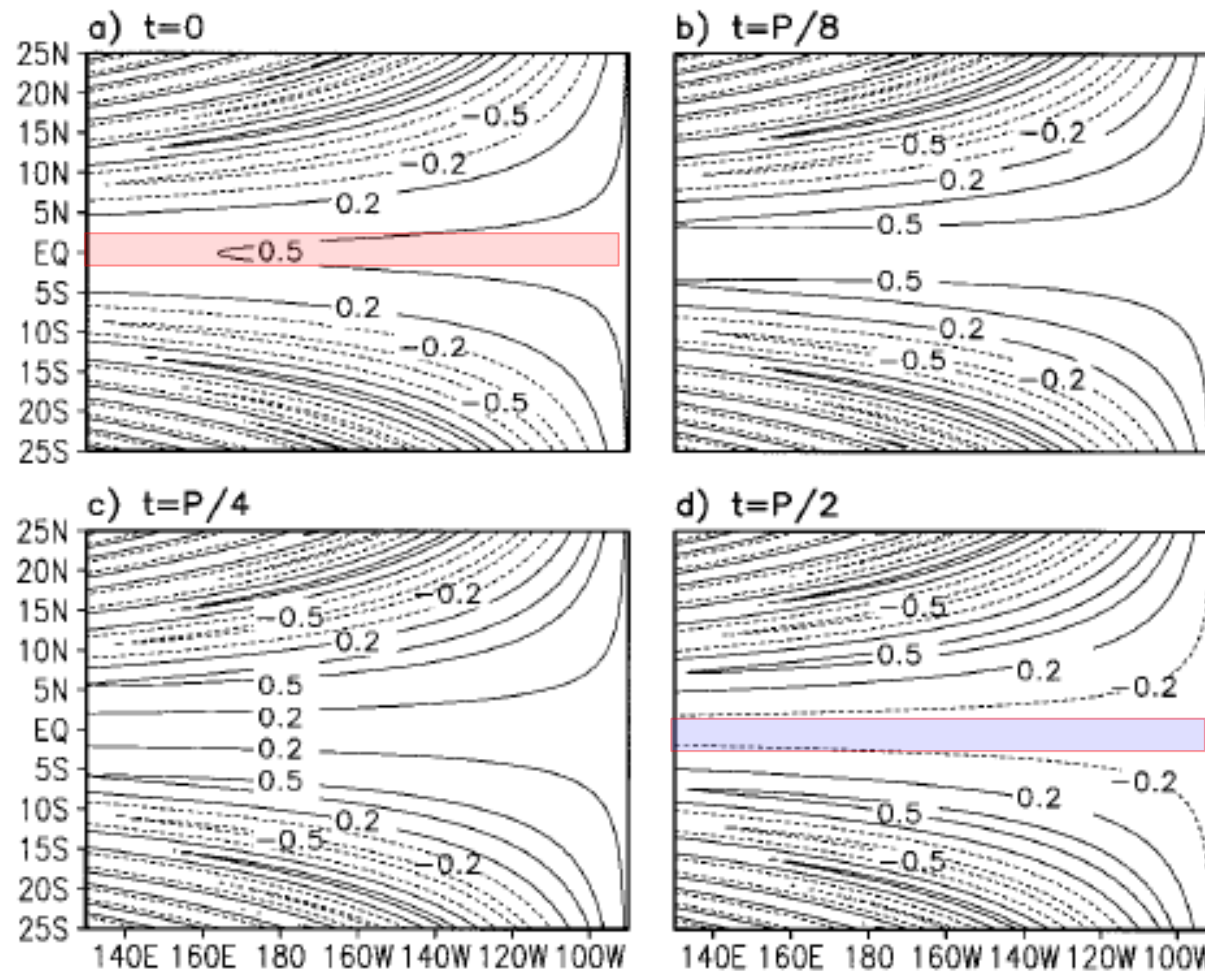
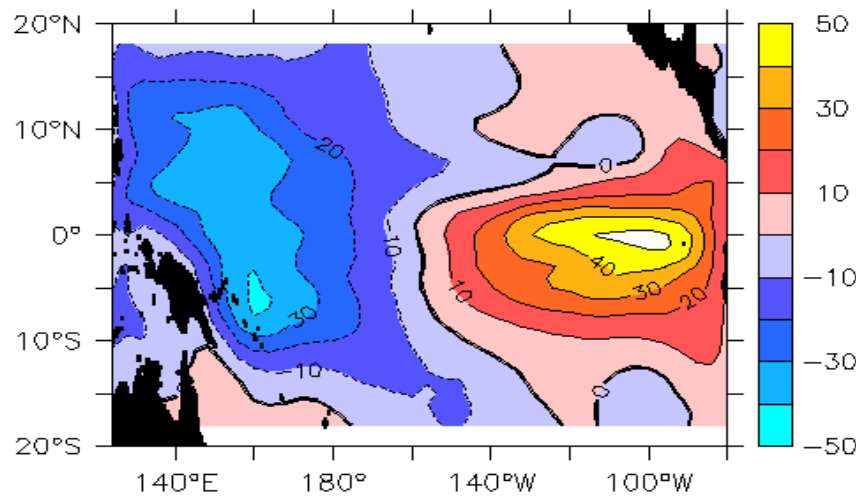


FIG. 3. The thermocline of the solution in Fig. 2b ($k_y = 0.14$) at the different phases of the oscillation throughout a half period.

A 4-year uncoupled RO mode of tropical Pacific basin, Jin 2001
It signifies the slow dynamics of equatorial heat recharge and discharge.

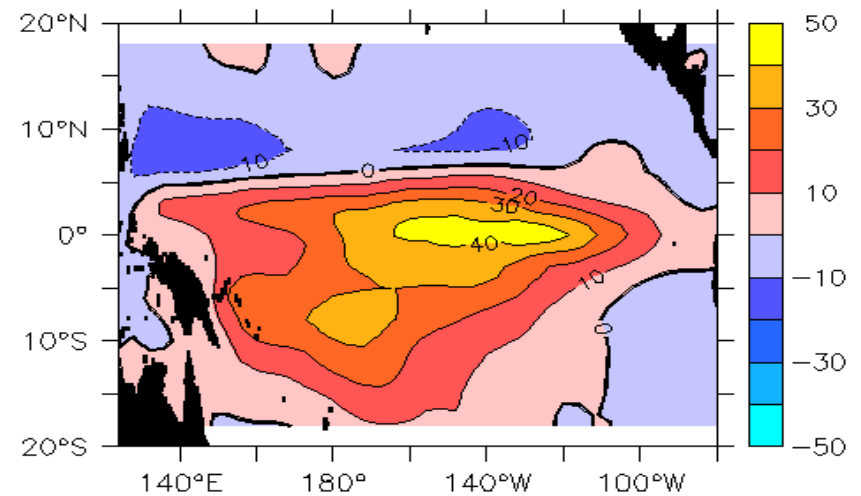
Thermocline Depth (20 ° C)

Z₂₀ EOF Mode 1 spatial structure (m) (36.1%)



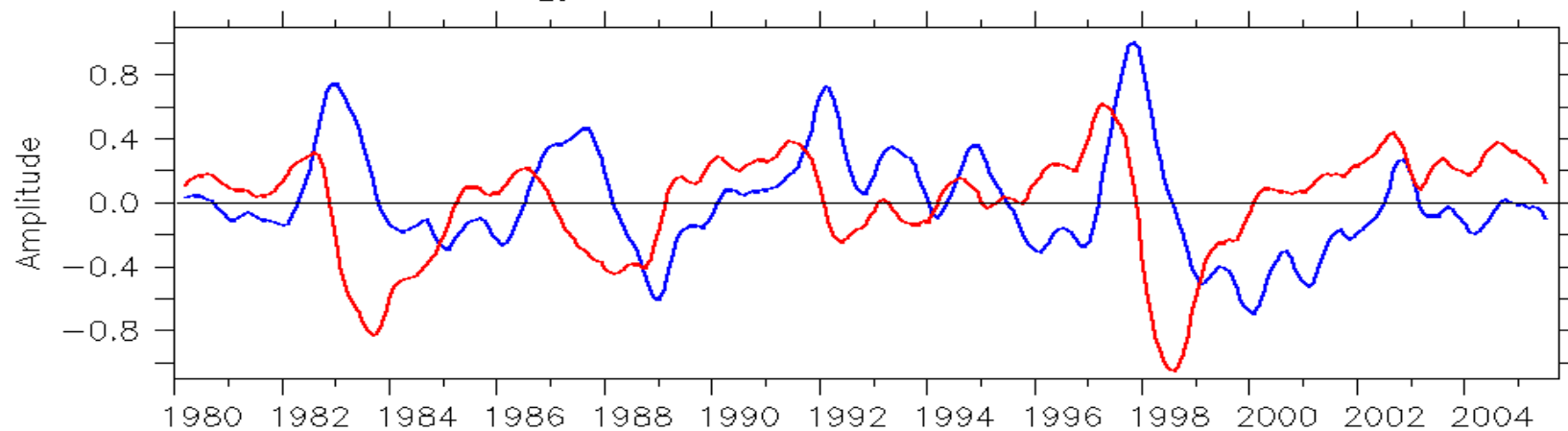
“Tilting mode”

Z₂₀ EOF Mode 2 spatial structure (m) (23.8%)



“Recharge Mode”

Z₂₀ EOF Mode 1 and Mode 2 amplitude



$$h_{eE} = r_w h_{nW} + \Delta x A_M$$

Meinen & McPhaden, 2000



ZC coupled model framework

Approximations

The “right” approximations are necessary for modeling, more so for understanding.

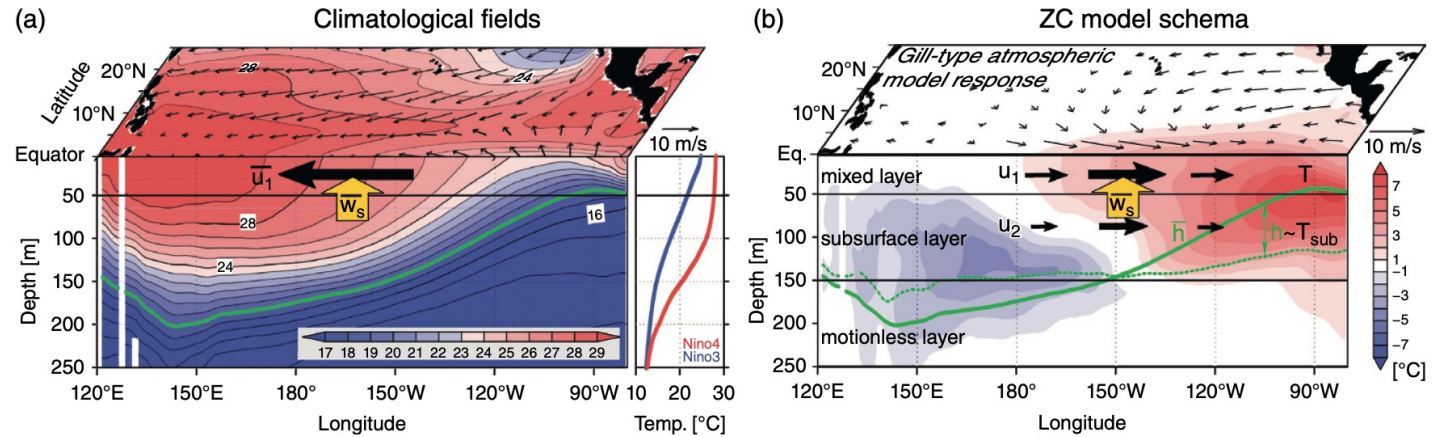
E.g. Richardson’s “failure” in 1924 vs Chaney’s success in 1946

Key approximations relevant to ENSO studies

- 1) Quasi-steadiness and quasi-linear response of equatorial atmosphere to ENSO’ SST anomalies, Gill-type approximation for vertical structures, parameterizations of convection and boundary processes (friction) etc.
- 2) “Slow” dynamic adjustment of equatorial ocean circulation and mass/heat content to ENSO’s winds, reduced gravity approximation.
- 3) “Slow” SST dynamics and thermodynamics, heat flux and subsurface parameterizations

Cane-Zebiak coupled model was the first to utilize these “good” and simplifying approximations to form a great framework of intermediate complexity for modeling ENSO and understanding its essential dynamics!

An Ingenus Construction of CZ model Framework (1984, 85)



$$\frac{\partial u}{\partial t} - \beta_0 y v = -g' \frac{\partial h}{\partial x} + \frac{\tau^x}{\rho H} - ru \quad (\text{A2a})$$

$$\beta_0 y u = -g' \frac{\partial h}{\partial y} + \frac{\tau^y}{\rho H} - rv \quad (\text{A2b})$$

$$\frac{\partial h}{\partial t} + H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = -rh, \quad (\text{A2c})$$

where $\mathbf{u} = H^{-1}(H_1 \mathbf{u}_1 + H_2 \mathbf{u}_2)$. The subscripts 1 and 2 indicate that within the upper layer ocean, there is an embedded mixed layer with fixed depth H_1 (50 m) and an underlying subsurface layer with fixed depth H_2 (100 m). The equations governing the velocity shear (\mathbf{u}_s) between layer 1 and 2 are

$$r_s u_s - \beta_0 y v_s = \frac{\tau^x}{\rho H_1}, \quad (\text{A3a})$$

$$r_s v_s + \beta_0 y u_s = \frac{\tau^y}{\rho H_1}, \quad (\text{A3b})$$

where $\mathbf{u}_s = \mathbf{u}_1 - \mathbf{u}_2$.

The mixed-layer SSTA is governed by the heat budget

$$\frac{\partial T}{\partial t} = -\mathbf{u}_1 \cdot \nabla (\bar{T} + T) - \bar{\mathbf{u}}_1 \cdot \nabla T - \{M(\bar{w}_s + w_s) - M(\bar{w}_s)\} \times \bar{T}_z$$

**Two and half layer upper ocean model
with embedded mixed-layer SST dynamics**

$$-M(\bar{w}_s + w_s) \frac{T - T_e}{H_1} - \alpha_s T, \quad (\text{A4a})$$

$$T_e = \gamma T_{sub} + (1 - \gamma) T, \quad (\text{A4b})$$

$$T_{sub}(h) = \begin{cases} T_1 \left\{ \tanh \left[b_1 (\bar{h} + h) \right] - \tanh(b_1 \bar{h}) \right\}, & h > 0 \\ T_2 \left\{ \tanh \left[b_2 (\bar{h} - h) \right] - \tanh(b_2 \bar{h}) \right\}, & h < 0 \end{cases}, \quad (\text{A4c})$$

where \bar{T} , $\bar{\mathbf{u}}_1$, and \bar{w}_s are the mean SST, horizontal mixed-layer ocean currents, and upwelling. Parameters and their values used in the model are listed in Table A.

coupled with a Gill-type Atmospheric Model

$$\varepsilon_a u_a^n - \beta_0 y v_a^n = -(p^n / \rho_0)_x, \quad (\text{A1a})$$

$$\varepsilon_a v_a^n + \beta_0 y u_a^n = -(p^n / \rho_0)_y, \quad (\text{A1b})$$

$$\varepsilon_a (p^n / \rho_0) + c_a^2 \left[(u_a^n)_x + (v_a^n)_y \right] = -\dot{Q}_s - \dot{Q}_1^{n-1}, \quad (\text{A1c})$$

$$\dot{Q}_s = (\alpha T) \exp \left[(\bar{T} - 30) / 16.7 \right], \quad (\text{A1d})$$

$$\dot{Q}_1^n = \beta \left[M(\bar{c} + c^n) - M(\bar{c}) \right]. \quad (\text{A1e})$$

- It gives rise a linear quasi-equilibrium relationship between ENSO's zonal wind stress and SST anomalies with a linear operator!

$$\tau_x = \rho_a H_a r u_a = \mu L_a(T)$$

ZC model atmospheric model component is largely linear. It fails to include important convective threshold nonlinearities, for example.

- It includes embedded two-layers for considering the vertical shear in current, which is approximated by Ekman current, and mixed layer for SST thermodynamics and dynamics
- Under half geostrophic balance, upper ocean current u can be determined from h , v can be determined from h and wind stress. All other variables are diagnostically related to h and T which determines ENSO's wind stress. Thus, only two key prognostic variables of CZ framework are $(\frac{\partial h}{\partial t}, \frac{\partial T}{\partial t})$, both of which are associated with memories (h , T) and with coupling via atmosphere, a new oscillatory ENSO mode is generated in the tropical pacific coupled CZ model.

- CZ model was the first realistically simulated ENSO's basic ENSO features and successfully predicted 1986-87 El Niño. The recognition about the crucial importance of equatorial ocean wave dynamics for ENSO was insightful. All CGCM became able to capture ENSO by resolving the equatorial wave guide. It demarked the new era for dynamic climate predication, envision first by **Shukla (1981)** (implied by Hasselmann 1976)

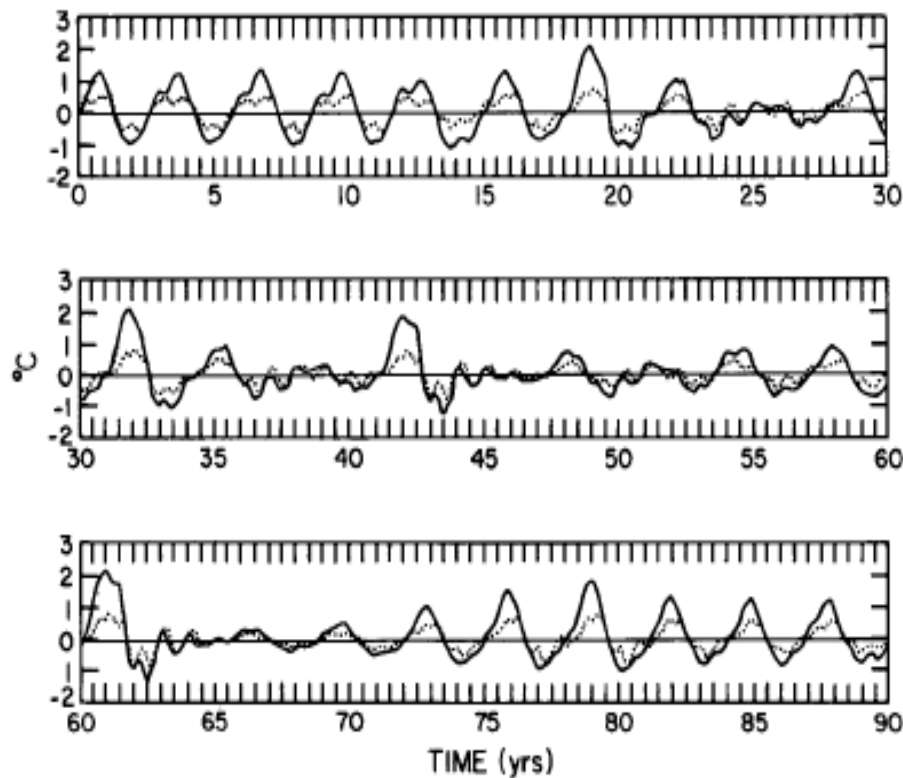


FIG. 1. Area-averaged SST anomalies for the 90-year model simulation. The solid line is NINO3 (5°N-5°S, 90°-150°W), and the dotted line is NINO4 (5°N-5°S, 150°W-160°E).

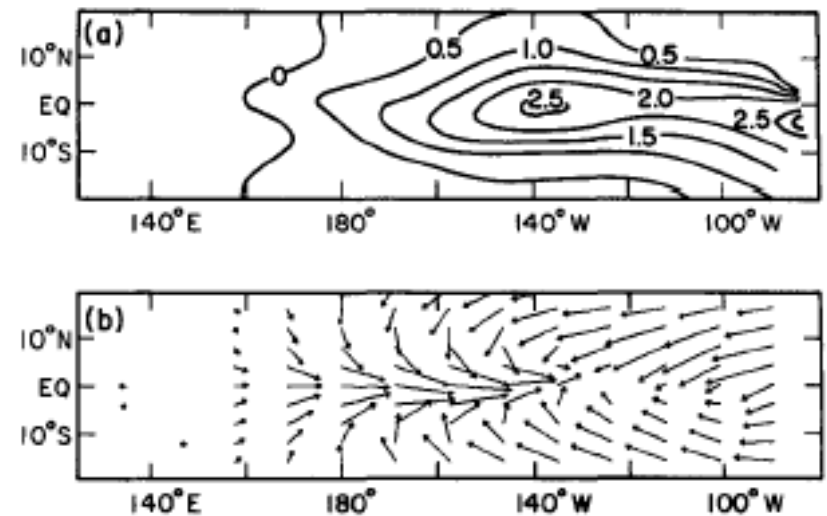


FIG. 8. As in Fig. 4, except for December of year 31.

When zonal mean h is removed in the integration, a permanent El Nino state is simulated as expected by Bjerknes. Only With with zonal mean h included, E/L oscillates which provided first support of the Wyrтки hypothesis!

***CZ model confirmed Bjerknes-Wyrтки hypothesis for ENSO and served as a firm foundation for ENSO theory/conceptual models to be established!**

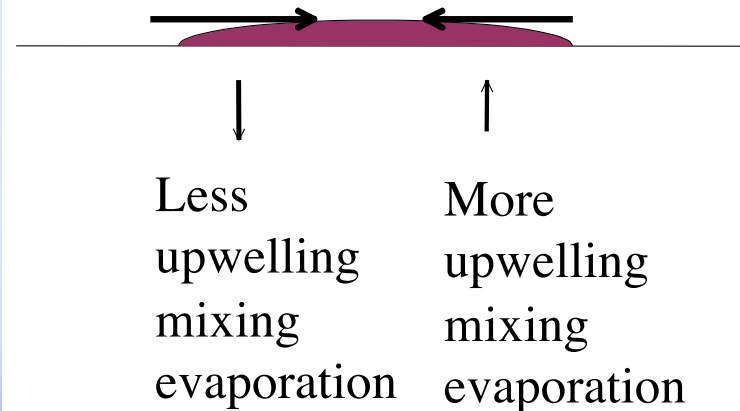
ENSO instability theory

- Except the earlier work (e.g. Philander et al, 1984, Gill 1985, Hirst 1986, main works of **ENSO instability theory are based on** CZ model framework. It has been a basic framework from which the the delayed oscillator conceptual model were derived by Battisti and Hirst (1989), wave oscillator theory were proposed by Cane et al 1990, and a unified coupled linear instability theory for ENSO by Jin et al 1993a,c) and recharge oscillator theory were put forward by Jin 97a,b), among other theoretical ENSO studies.

SST mode—an extension of locally damped SST damping mode of Hasse 1976 via considerations of nonlocal heat flux feedback or mixing or Ekman flow feedback. Neelin 1991, Jin et al 1993 considered Ekman, Xie 1994 consider wind-evaporation, or ocean mixing. All three processes can be formulated in a mathematical identical form with additive coefficients.

A west propagating coupled SSTA mode

without oceanic R-K waves!
Or fast wave limit!



Weakness: this mode plays a role in the cold tongue annual cycle, it is not essential for ENSO

Coupled wave mode, Hirst, 1986

Consider a simple case:

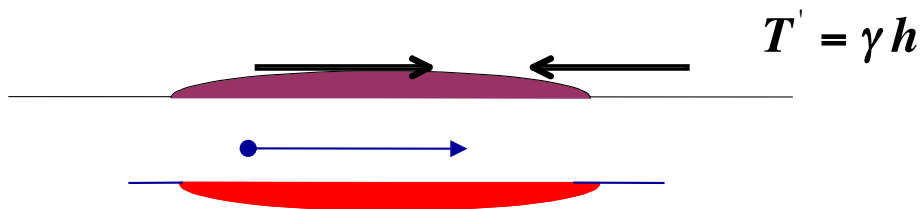
$\bar{w} / H_1 \approx \infty$, (strong upwelling in climate mean state)

then the SST equation

becomes: $T' \approx \gamma h$

Fast SST limit !

Wind stress and SSTa



Thermocline and ocean currents

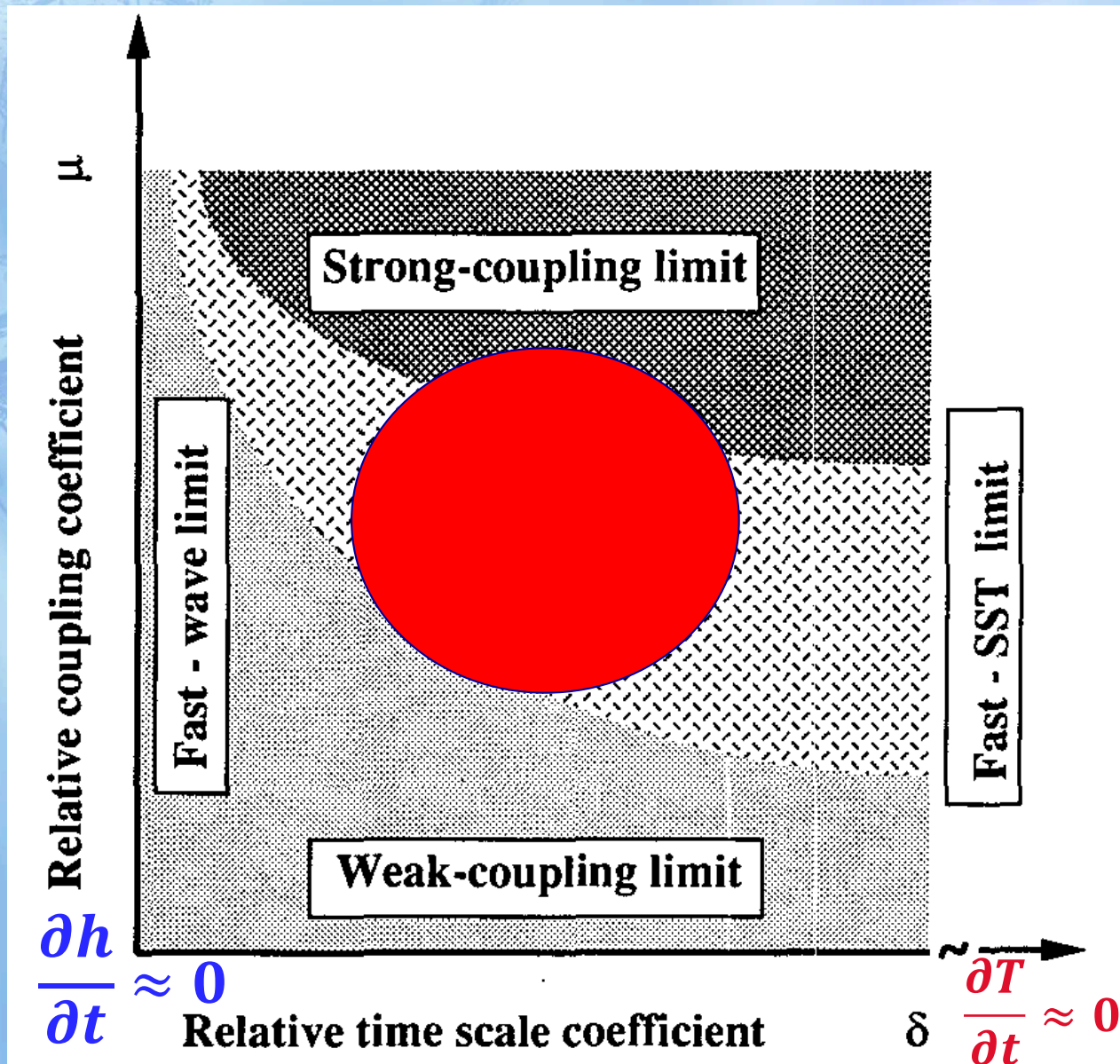
Coupling slows down Kelvin wave

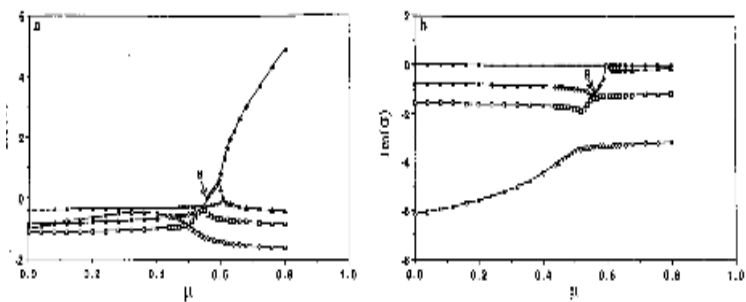
In case of wind stress SSTA phase shift is less than 90 degree, the K-wave slows down , also grow!

Weakness: R-K waves are no longer separated wave solutions with basin boundary.

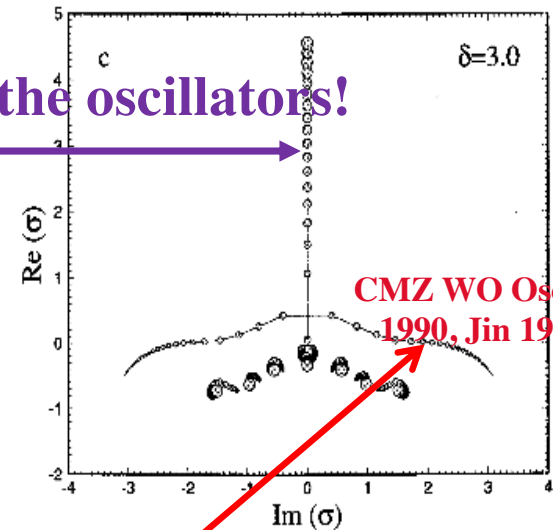
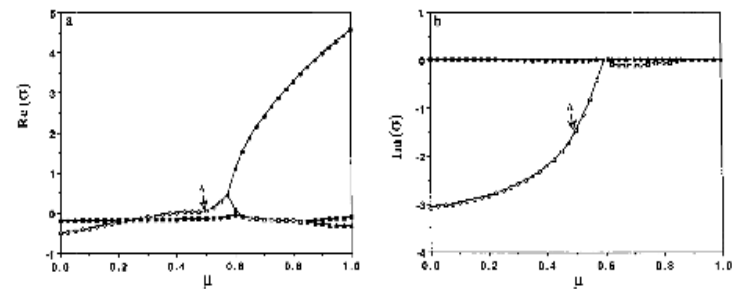
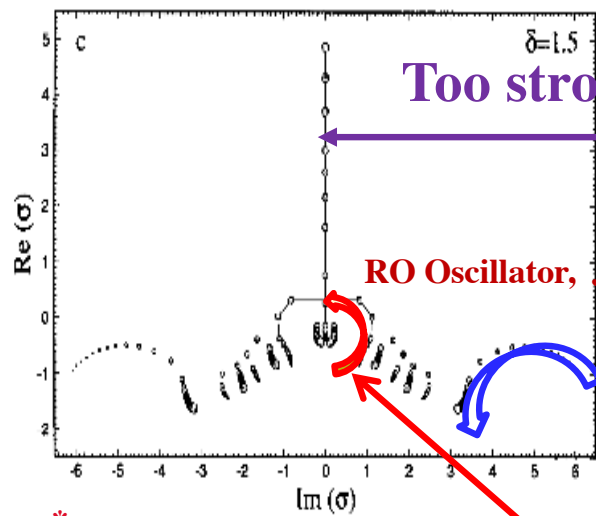
ENSO mode regimes : Jin et al 1993

- Full Eigen solutions of the CZ model dynamics except N-S hemispheric asymmetry is omitted.
- Analytical solutions at 4 limits of broad 2-d parameter space





Too strong coupling destroy the oscillators!



*
FIG. 10. As in Fig. 10 except $\delta = 1.5$ (slice 4 of Fig. 9) and (a) growth rates of the four leading modes; (b) frequencies (negative or zero $\text{Im}(\sigma)$) of the four leading modes; symbols denoting the same branch between (a) and (b) change at the 2-degeneracies for clarity (to distinguish from branches that cross without degeneracy) but mode properties are continuous; (c) Collective plot of the four leading modes. Label A marks the point for which the eigenstructure is shown in Fig. 17.

FIG. 13. As in Fig. 10 except for $\delta = 3.0$ (slice 5 of Fig. 9). (a) Growth rates of the two leading modes; (b) frequencies (negative or zero $\text{Im}(\sigma)$) of the two leading modes; (c) collective plot of the five leading modes. The label A marks the point for which the eigenstructure is shown in Fig. 14.

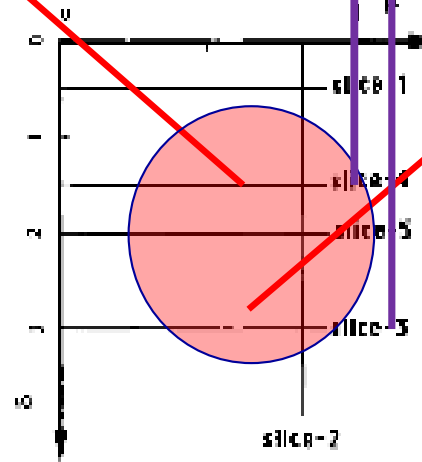


FIG. 9. Schematic of the μ - δ plane (coupling coefficient and relative time-scale coefficient) showing the slices for which eigenvalues will be presented in Figs. 10, 11, 13, 15, and 16. Slices are labeled in order of presentation.

- The RO Oscillator solution has quasi-stationary T, h.
- Zonal mean h is ahead of SST in terms of phase as envisioned by Wyrтки 85 (* but I was not aware of Wyrтки 85 paper then)

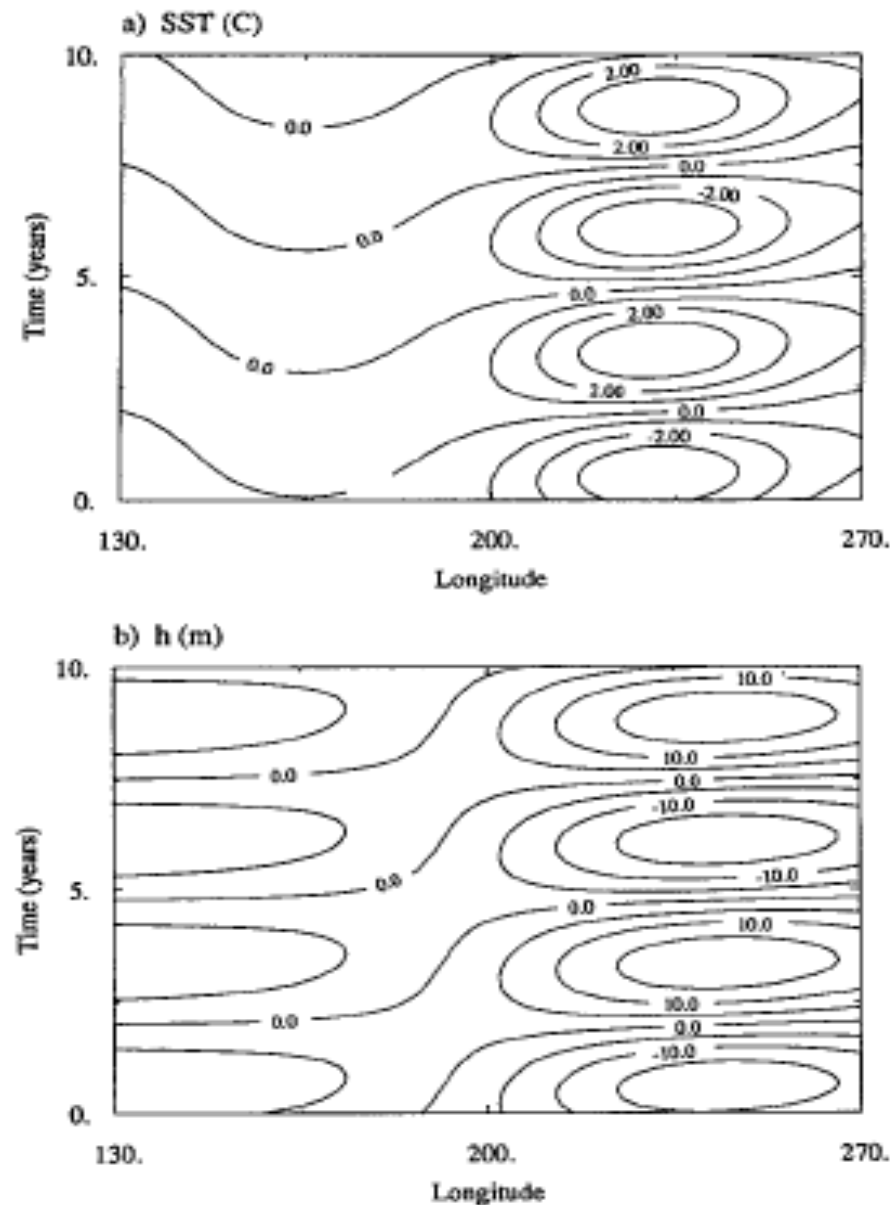


FIG. 17. As in Fig. 14 except for the point labeled B in Fig. 15 ($\mu = 0.55$, $\delta = 1.5$).

Some key points that we have learned from instability theories from 80-90s based on CZ framework, as summarized by the unified theory of Jin et al 1993:

- 1) There are two class of modes of relevance to AC/ENSO: (i) SST mode and coupled ocean “wave” or better coupled ocean basin/recharge mode!**
- 2) ENSO-like mode develops in regions of equatorial upwelling and strong zonal SST gradient and vertical ocean temperature gradient . It’s time scale depends on both SST dynamics($\frac{\partial T}{\partial t}$) and ocean dynamics ($\frac{\partial h}{\partial t}$) , which are coupled together via atmospheric coupling.**
- 3) In the middle regime between fast-wave and fast SST limits and weak and strong coupling limits, a leading ENSO mode’s frequency and growth rate are sensitive to the relative contributions of SST dynamics and ocean wave dynamics with RO and WO modes tend to dominate under different conditions.**

A) These points were further demonstrated analytically in Jin 97a, b;

B) Explored in the basic states space by An and Jin 2000; Fedorov and Philander (2000),

C) Explore by Geng (2021), Geng and Jin (2022), Jin 2022 for understanding ENSO STPD !

From Inception to Generalization of ENSO Recharge Oscillator Theory

- Bjerknes & Wyrski hypotheses
- Early theoretical advances of equatorial A & O and SST dynamics (1970s-80s)
- ZC coupled model framework
- Coupled Instability theories of ENSO-like modes (Jin 93)
- **Development of ENSO conceptual models**
- **A Generalized RO model**
- **Extending RO theory for ENSO's spatiotemporal pattern diversity (STPD)**

To be continued after a 5 minute break!