On ENSO Theory

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ENSO Summer School

From Inception to Generalization of ENSO Recharge Oscillator Theory

- Bjerknes & Wyrtki hypotheses
- Early theoretical advances of equatorial A & O and SST dynamics (1970s-80s)
- ZC coupled model framework
- Coupled Instability theories of ENSO-like modes (Jin 93)
- Development of ENSO conceptual models
- A Generalized RO model
- Extending RO theory for ENSO's spatiotemporal pattern diversity (STPD)

Conceptual models for ENSO

ENSO instability theories of 80-90 established the firm ground for mechanism ENSO' main pattern, growth and oscillatory nature! Different attempts have Been made to propose various conceptual models, we will focus on three of them:

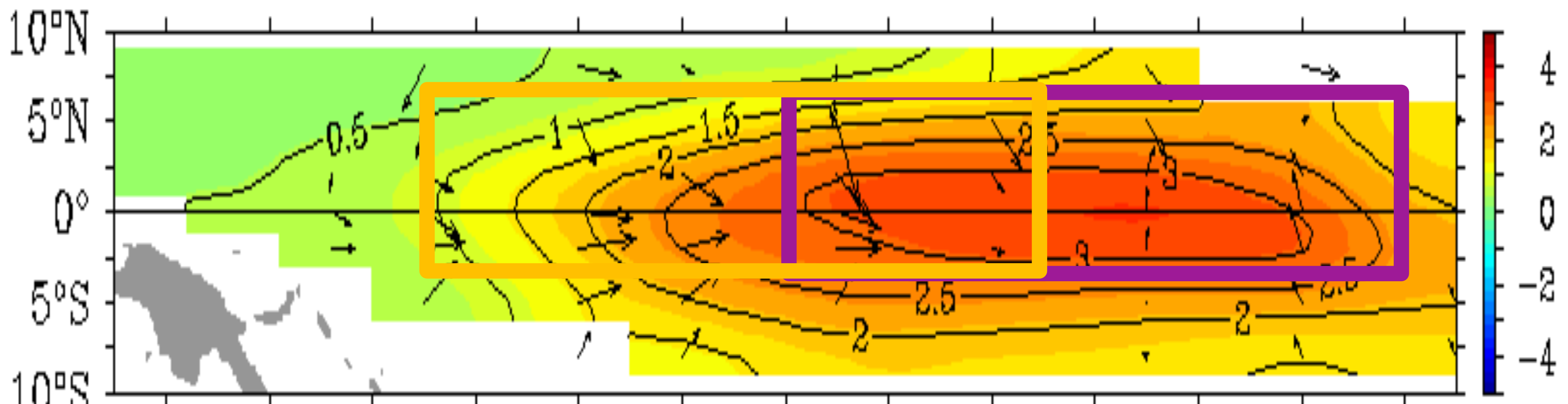
Key simplifications:

Using Nino3.4 (1 dof) to characterize ENSO SST

Describing ocean dynamic memory in three different ways

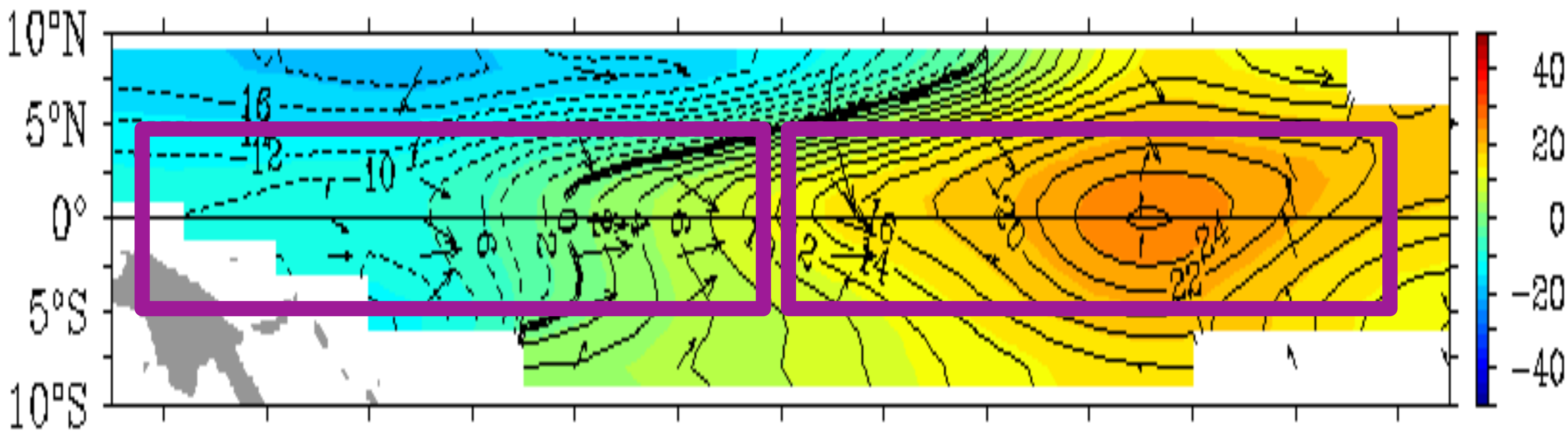
- **Delayed Oscillator (BH 1989)**
  forced RW wave as a negative feedback on SST with a delay (∞ dof)
- **Wave Oscillator (CMZ 1990)**
  gravest ocean basin mode consisting of all RWs + KW (∞ dof)
- **Recharge Oscillator (Jin 1997ab)**
  warm-pool heat content recharge/discharge as a slow adjustment (1 dof)

* advective reflective oscillator (Picaut et al 1997) can be considered as a version of RO.
* conceptual models with empirical equations not derived based basic laws are not included.



November 2015 Anomalies

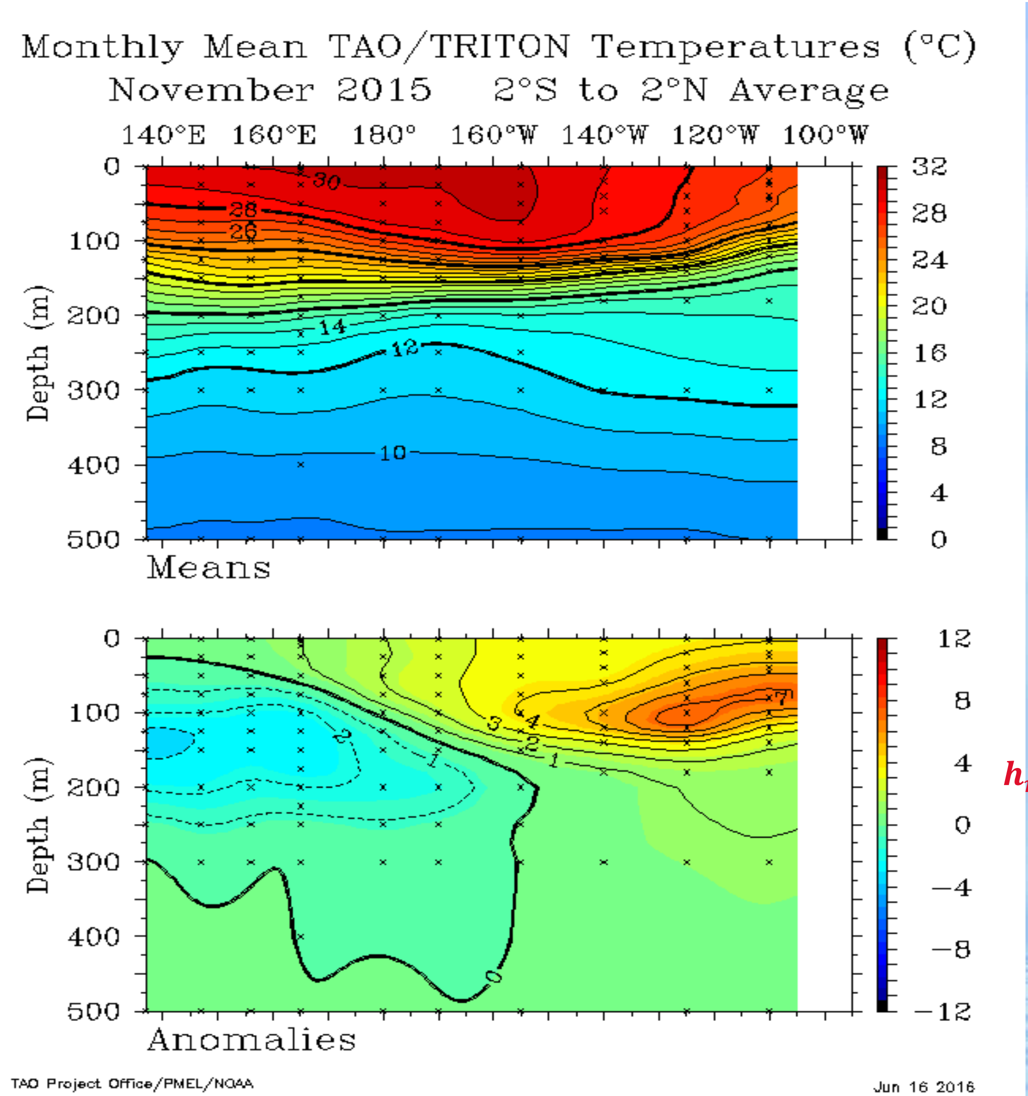
$$\tau_x \approx aT'$$



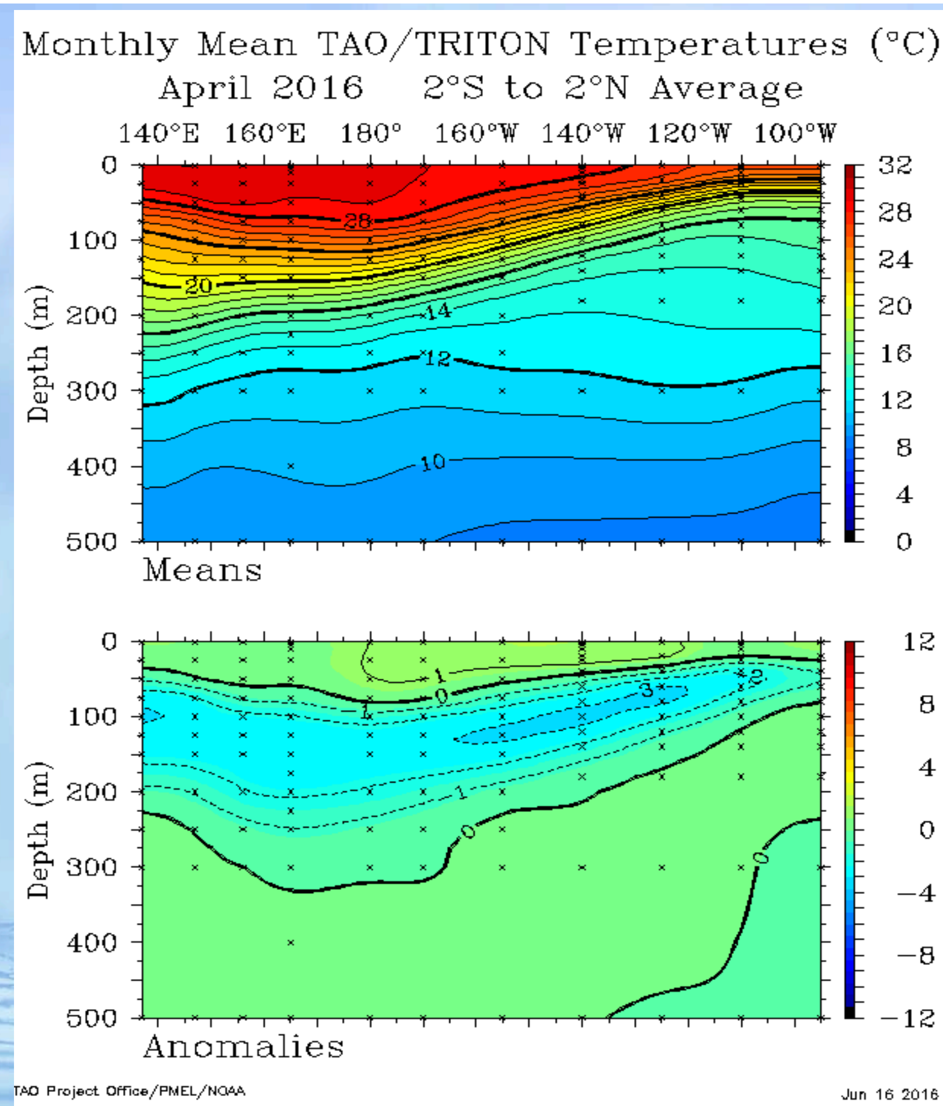
November 2015 Anomalies

$$\partial h / \partial x \approx b\tau_x$$

$$h_E = h_w + abLT$$



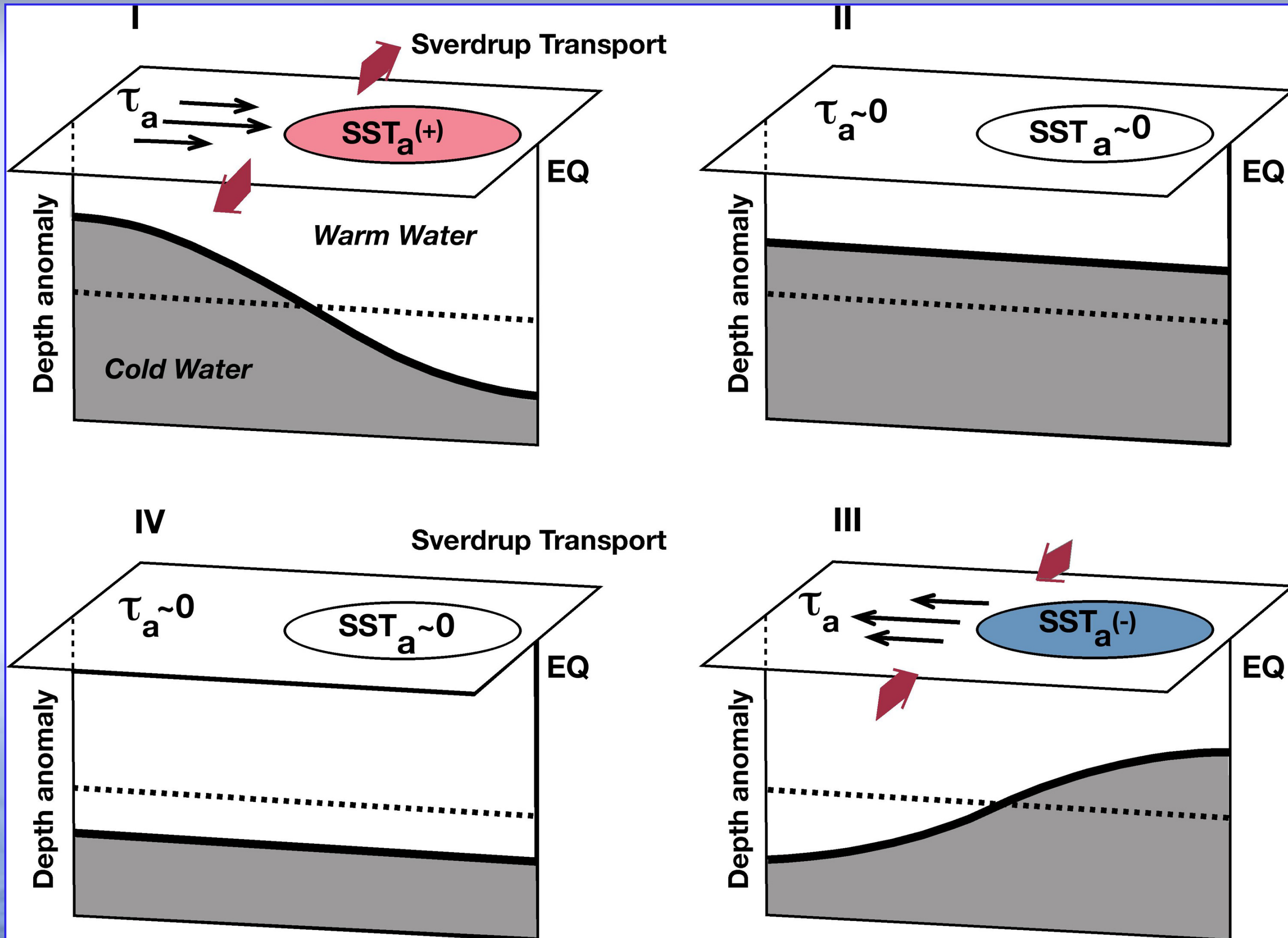
h_m



let: $h_m = (h_E + h_w)/2$, $h_E = h_w + \text{abLT} = h_m + \text{abLT}/2$,

$$\frac{\partial T}{\partial t} \approx \cdots - \bar{W}\frac{\partial T}{\partial z} \approx -\frac{\bar{W}}{H}(T - \gamma h_E) \approx \frac{\bar{W}}{H}(\gamma \text{abL} - 1)\text{T} + \frac{\bar{W}}{H}\gamma h_w$$

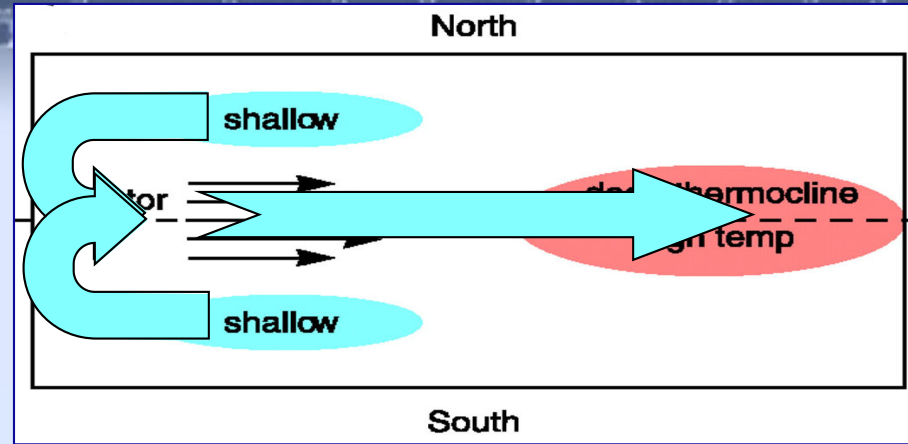
$$= \frac{\bar{W}}{H}(\gamma \text{abL}/2 - 1)\text{T} + \frac{\bar{W}}{H}\gamma h_m$$



What controls mean of the equatorial heat content of ENSO?

1) by slow recharge/discharge mode → RO
2) by forced R-Wave → DO
3) by graved ocean basin mode → WO

1) Delayed Oscillator (DO)



derived based on CZ model (BH 1989)

$$h_w = -\beta_h\, aT(\ t-\eta)$$

$$h_E = -\beta_h\ aT(\ t-\eta) + \text{abLT}$$

$$\frac{\partial T}{\partial t} \approx \cdots - \bar{W}\frac{\partial T}{\partial z} \approx -\frac{\bar{W}}{H}(T-\gamma h_E) \approx \frac{\bar{W}}{H}(\gamma\text{abL}-1)T - \frac{\bar{W}}{H}\gamma\beta_h\, aT(\ t-\eta)$$

When uncoupled (a=0) , SST eq. reduced to the KH damped SST mode. When coupled, forced R-wave reflected at western boundary provides a delayed negative feedback. In this case, equatorial mean thermocline is

$$h_m = (h_E+h_w)/2 = -\beta_h\, aT(\ t-\eta) + \text{abLT(t)}/2$$

For example: at the time of T from positive to negative transition, h_m is at discharged condition! Thus DO is a kind of RO too!

Weakness :

1) **Ocean dynamics is heavily distorted which is especial at weaking coupling limit (small a)**
2) **introducing delay into the mode brings infinite degrees of freedom into otherwise simple model.**
3) **highly sensitive dependence of periodicity to parameter delay.**

LINEAR SOLUTIONS

$$\frac{\partial T}{\partial t} = -b\,T(t-\tau) + c\,T$$

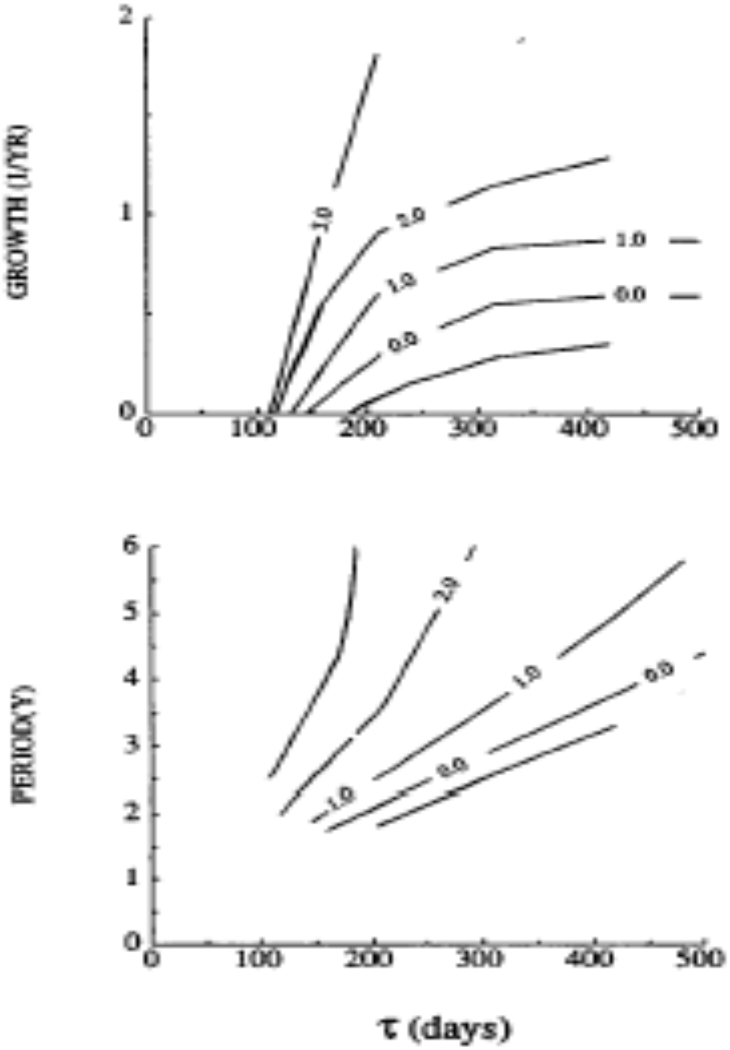


FIG. 3. The growth rate (yr^{-1}) (a) and period (yr) (b) of the coupled system vs time lag τ, for $b = 3.9\ yr^{-1}$ at $\tau = 180$ days. Each curve is for a difference reference value for c (in yr^{-1}) (for the Pacific $c = 2.2\ yr^{-1}$).

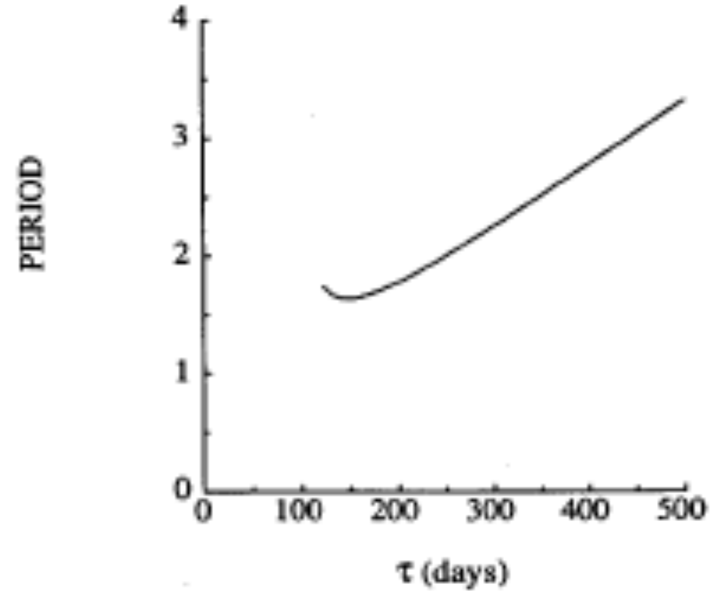


FIG. 17. The final state amplitude, T_0, (in °C) (a) and period (yr) (b) of the coupled system vs time lag τ, for $b = 3.9\ yr^{-1}$, $e = 0.07\ C^{-2}\ yr^{-1}$. Each curve is for a different reference value for c (in yr^{-1}).

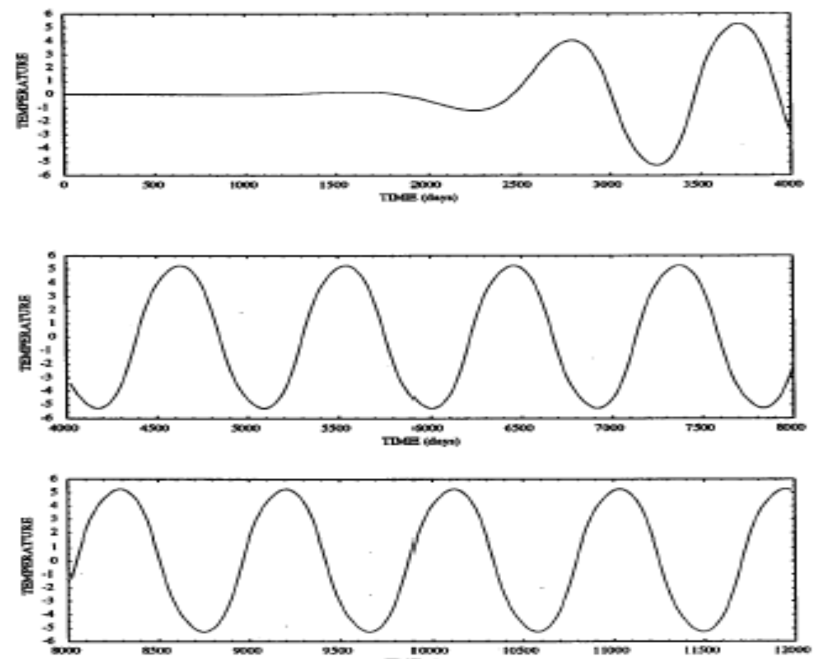


FIG. 19. Integration of (4.6) with the basic state values $b = 3.9\ yr^{-1}$, $c = 2.2\ yr^{-1}$, $e = 0.07\ C^{-2}\ yr^{-1}$, and $\tau = 180$ days. Temperature is plotted (in °C) vs time (days) for $r = 0.66$.

2) Wave Oscillator (WO) ,

With a simplified version of ZC model at "fast SST limit", CMZ 1990 solved analytical coupled mode $\frac{\partial T}{\partial t} \approx 0$, $T = \gamma h_E$ del solution and found the gravest ocean basin mode is destabilized and dramatically slowdown to an interannual ENSO mode.

Weakness:

a) fast SST limit ignores SST dynamics,

b) WO's period is ultrasensitive to A-O coupling.

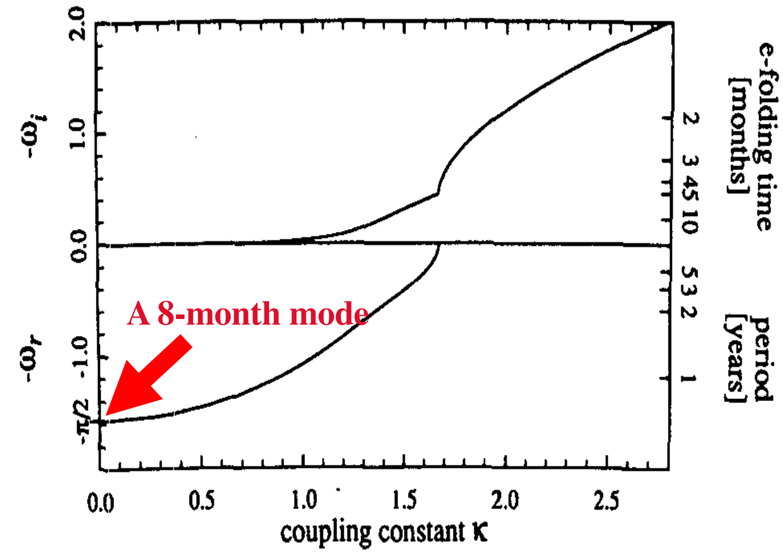


FIG. 1. Dependence of growth rate $-\omega_i$ and frequency ω_r on the coupling constant κ. Forcing parameters: $\mu = 0.1$, $x_1, = 0.25$, $x_2 = 0.75$.

3) Recharge Oscillator

Jin 1996 (Science) and Jin 1997a,b (JAS) were original submitted to JAS as one paper: "Mixed SST-Ocean Dynamics Mode Theory for ENSO" . Prof Mark Cane who reviewed the MS suggested to revise it into three papers: one for broad audience, another to focus on conceptual RO model with " little" math, third with mathematical deduction of RO from Jin 1993 under strip-down approximation.

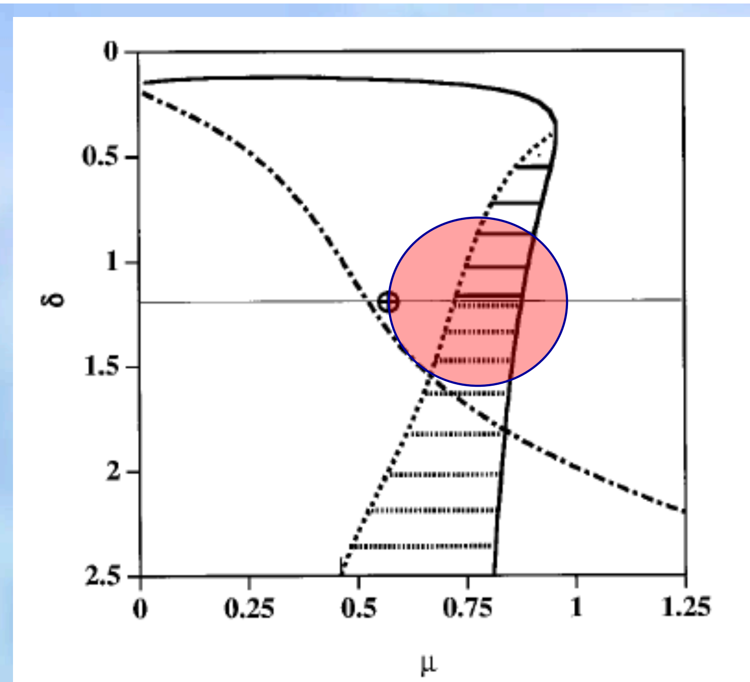
Jin 1996: A Unified Conceptual Model of Pacific cold and its instability to give RO for ENSO in a simplest possible 2-degree model. **(recommend to read)**

Jin 1997a Conceptual RO oscillator for ENSO

Jin 1997b Regimes of RO and WO in strip-down CZ model (simplest model to capture equatorial ocean gravest basin mode and recharge/discharge mode and their coupling with SST via A-O interaction to give rise to both RO and WO)

Jin 1997b solved the solutions of the ENSO mode regime in parameter space, analytically.

1) RO and WO can coexist in stable regime RO-DO degenerate to become one in the subcritical.
2) Only one of them become unstable and its frequency and growth rate are sensitivity to parameters in the middle regime.
3) The simplest RO model can be derived from the strip-down model.



Jin 1997a: The equatorial zonal mean or equivalently the warm-pool heat content, which is not full constrained by zonal wind stress , may be approximated by a slow adjustment process as the result of integrated effect of equatorial wave dynamics. Together with equatorial Sverdrup balance, it gives the simplest yet rather accurate description of ENSO related equatorial content response to ENSO.

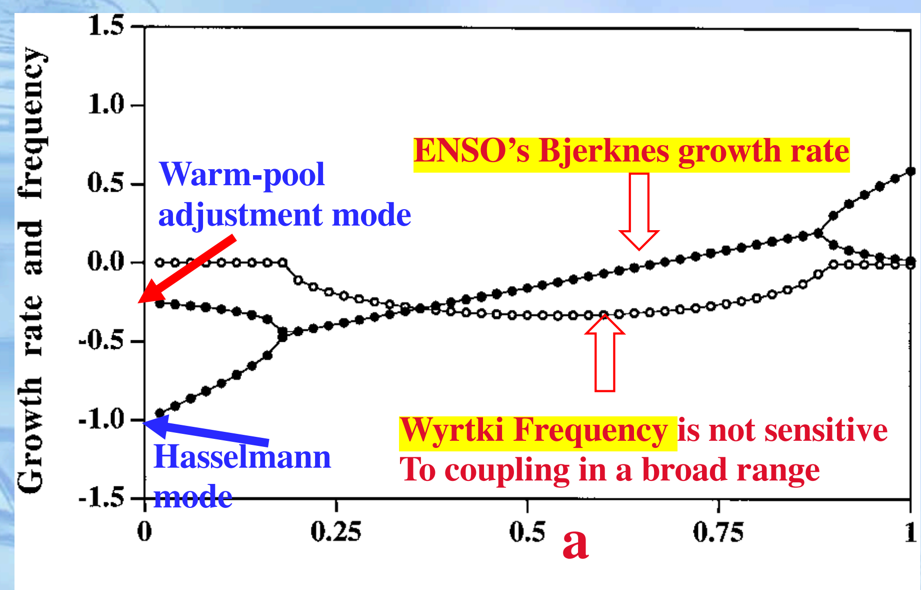
$$\frac{\partial h_w}{\partial t} = -rh_w - \theta_h aT$$

The SST equation can be written as

$$\frac{\partial T}{\partial t} \approx \frac{\bar{W}}{H}(\gamma abL-1)T + \frac{\bar{W}}{H}\gamma h_w + \ldots$$

which can be extended to included all other processes.

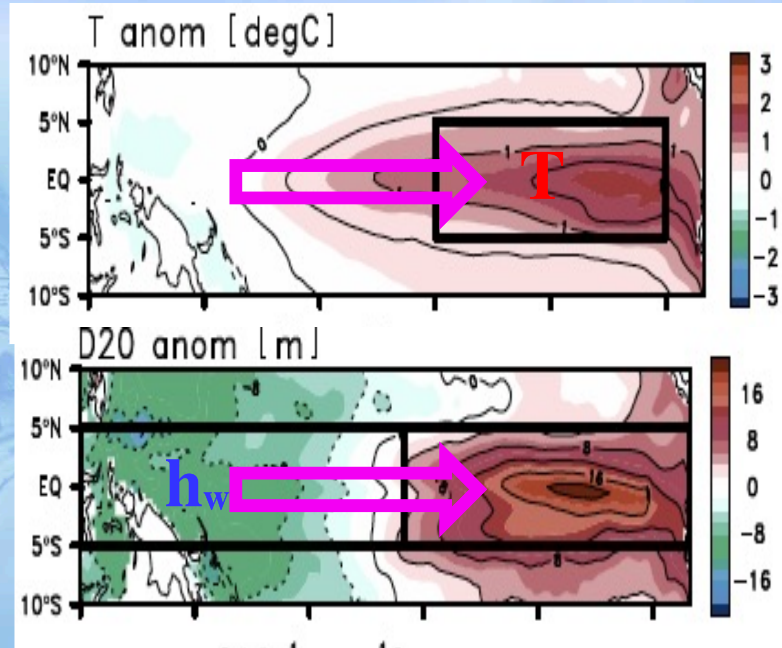
- Simplest possible description of ENSO mode
- When a=0, the model captures physics of u H- SST mode, Ocean Recharge/adjustment mode
- RO is born out of the coupling of the two physical modes

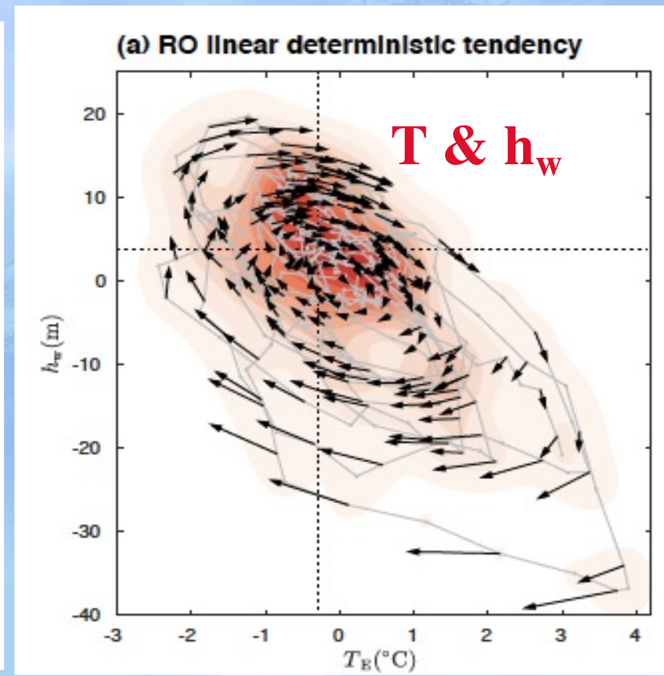


Recharge Oscillator Theory

$$\frac{dT}{dt} = RT + F_1 h_w + \ldots\ldots$$

$$\frac{dh_w}{dt} = -\varepsilon h_w - F_2 T$$





ENSO Phase Digram (Jin-diagram (1997)

Growth rate: $\frac{R-\varepsilon}{2}$ **Bjerknes index**

Frequency: $\varpi = \sqrt{F_1 F - (R+\varepsilon)^2 / 4}$ **Wyrtki index**

RO theory of Jin 1996, 1997

BJ instability index Jin et al 2006, Wyrtki frequency index Lu et al 2017

Generalized RO model

Bjerknes-Wyrtki-Jin (BWJ) Index

$$\frac{dT_E}{dt} = RT_E + F_1 h_w,$$
$$\frac{dh_w}{dt} = -\varepsilon h_w - F_2 T_E.$$

growth rate frequency

$$BWJ = \frac{(R-\varepsilon)}{2} + i\sqrt{F_1F_2 - \frac{(R+\varepsilon)^2}{4}}$$

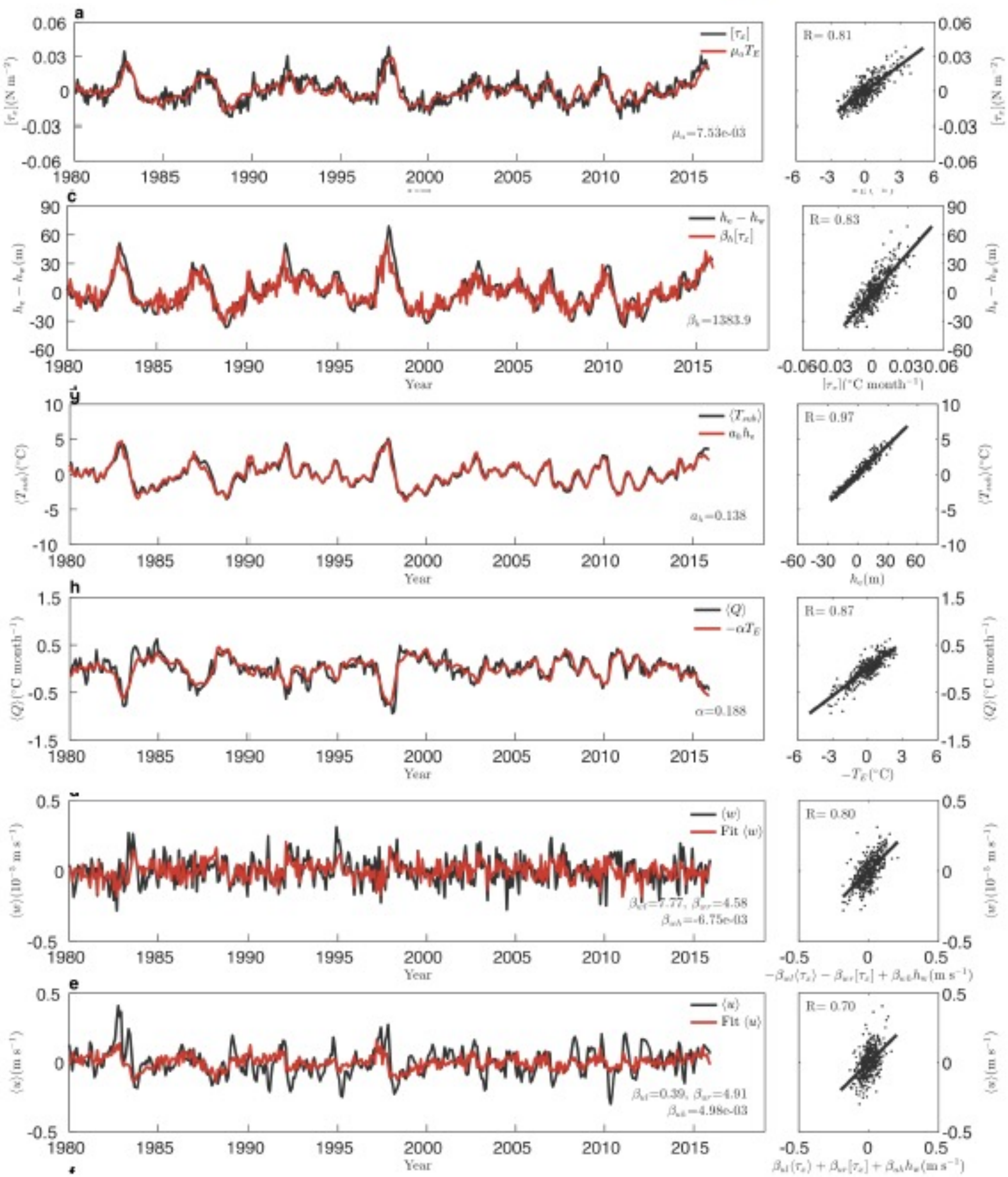
Linearized mixed layer SST tendency equation

$$\frac{\partial T_E}{\partial t} \approx \underbrace{-\left(\frac{\langle \bar{u}\rangle}{L_x} + \frac{\langle -2y\bar{v}\rangle}{L_y^2} + \gamma\frac{\langle M(\bar{w})\bar{w}\rangle}{H_m}\right)}_{\text{Dynamic damping (DD)}} T_E + \underbrace{\langle Q\rangle/(\rho_0 C_p H_m)}_{\text{Thermal damping(TD)}}$$
$$\underbrace{+\gamma\frac{\langle M(\bar{w})\bar{w}\rangle}{H_m}\langle T_{sub}\rangle}_{\text{Thermcline feedback(TH)}} \quad \underbrace{-\langle u\rangle\langle\partial_x\bar{T}\rangle}_{\text{Zonal advective feedback(ZA)}} \quad \underbrace{-\langle v\rangle_A\langle\partial_y\bar{T}\rangle_A - \langle w\rangle\langle M(\bar{w})\partial_z\bar{T}\rangle}_{\text{Ekman feedback}(EK)}.$$

$$R = \underbrace{-\left(\frac{\langle \bar{u}\rangle}{L_x} + \frac{\langle -2y\bar{v}\rangle}{L_y^2} + \gamma\frac{\langle M(\bar{w})\bar{w}\rangle}{H_m}\right)}_{\text{DD}} \underbrace{-\alpha}_{\text{TD}}$$
$$\underbrace{+\mu_a\beta_h a_h\gamma\frac{\langle M(\bar{w})\bar{w}\rangle}{H_m}}_{\text{TH}} + \underbrace{\mu_a\beta_u\langle-\partial_x\bar{T}\rangle}_{\text{ZA}} + \underbrace{\mu_a\beta_v\langle-\partial_y\bar{T}\rangle_A + \mu_a\beta_w\langle M(\bar{w})\partial_z\bar{T}\rangle}_{\text{EK}}$$

$$F_1 = \underbrace{a_h\gamma\frac{\langle M(\bar{w})\bar{w}\rangle}{H_m}}_{\text{TH}} + \underbrace{\beta_{uh}\langle-\partial_x\bar{T}\rangle}_{\text{ZA}} + \underbrace{\beta_{vh}\langle-\partial_y\bar{T}\rangle_A - \beta_{wh}\langle M(\bar{w})\partial_z\bar{T}\rangle}_{\text{EK}}$$

Balanced relations for deriving BWJ index



$$[\tau_x] = \mu_a T_{\mathrm{E}}$$

$$h_{\mathrm{e}} - h_{\mathrm{w}} = \beta_h [\tau_x]$$

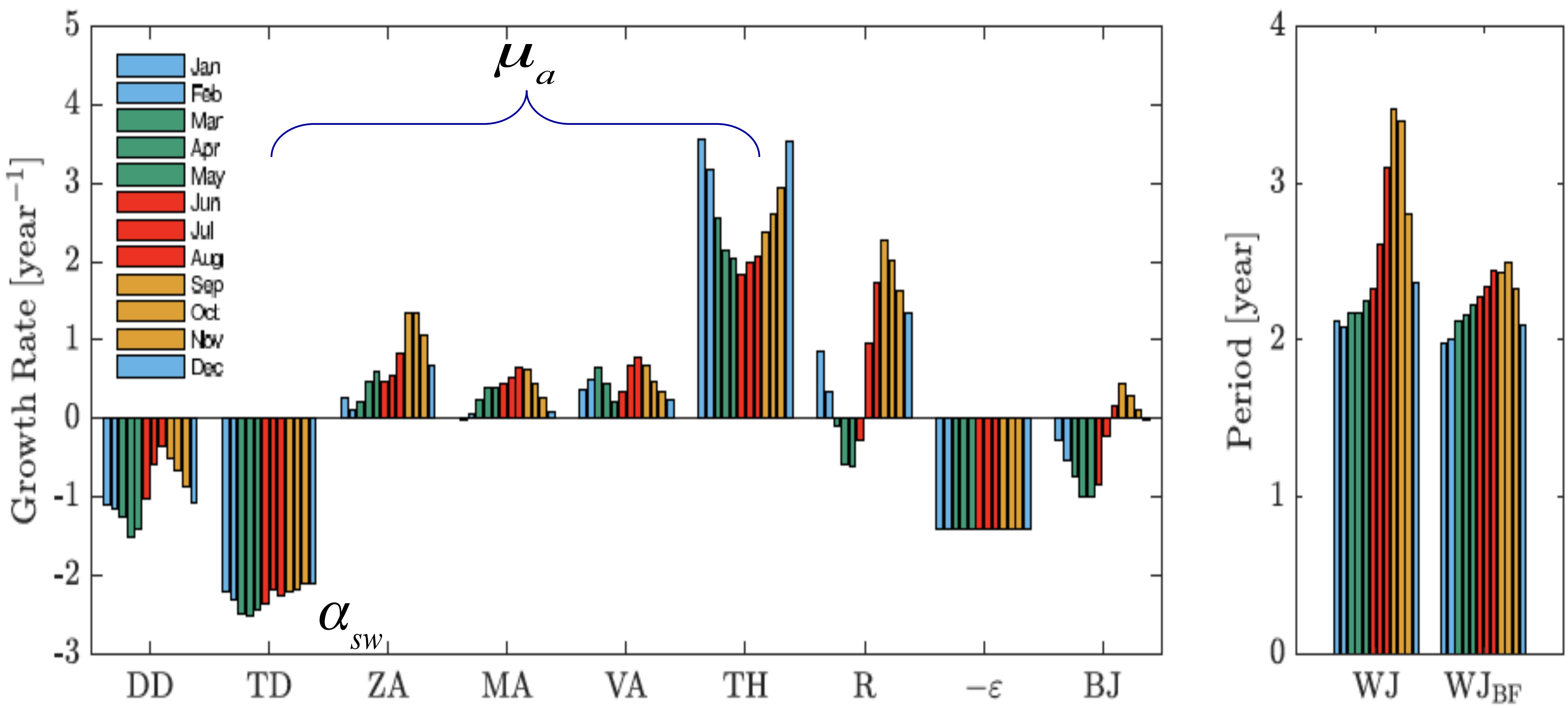
$$\langle T_{sub} \rangle = a_h h_e$$

$$\langle Q \rangle / (\rho_0 C_p H) = -\alpha T_{\mathrm{E}}$$

$$\langle w \rangle = -\beta_{wr} [\tau_x] + \beta_{\mathrm{wh}} h_{\mathrm{w}}$$

$$\langle u \rangle = \beta_{ur} [\tau_x] + \beta_{\mathrm{uh}} h_{\mathrm{w}}$$

Components & Seasonality of BW Index



Jin et al 2020

Generalized Recharge Oscillator Theory

(Jin et al 2020)

$$\frac{dT}{dt} = (R_0 + \underline{R_a(t)})T + F_1 h_W \underline{- cT^3 + bT^2} + \underline{\sigma(1 + BT)\xi}$$

$$\frac{dh_W}{dt} = -\varepsilon h_W - F_2 T$$

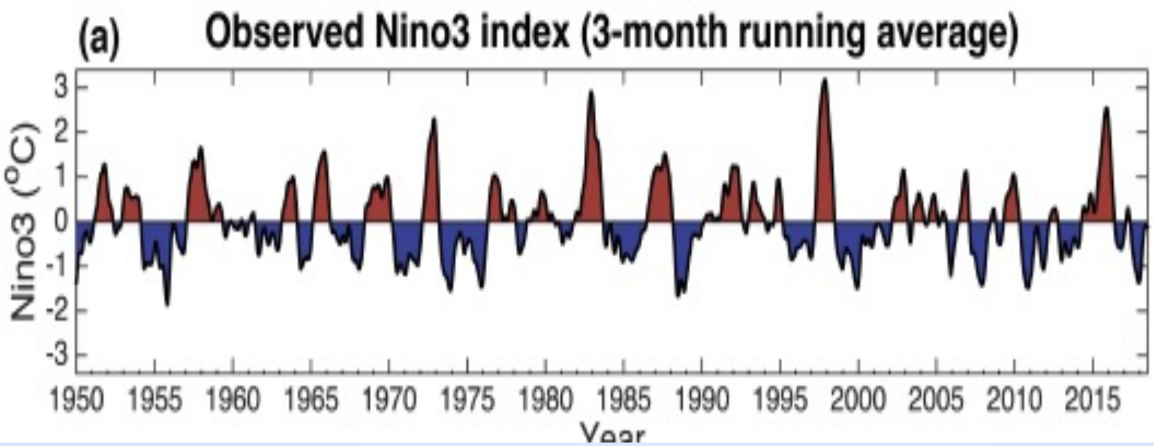
1) **Annual cycle modulation**

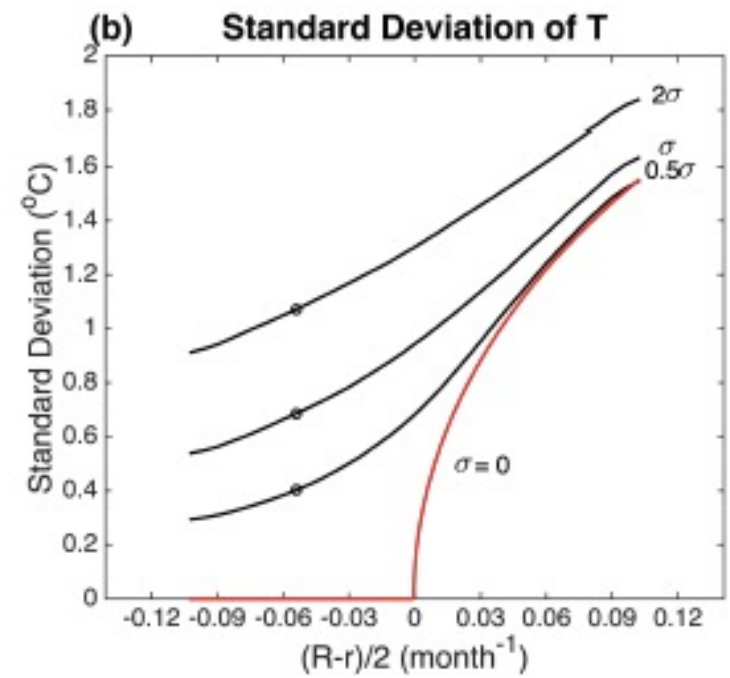
2) **Nonlinear Dynamic heating**

3) **State-dependent noise forcing**

What controls ENSO amplitude?

(Standard deviation of Nino3.4)



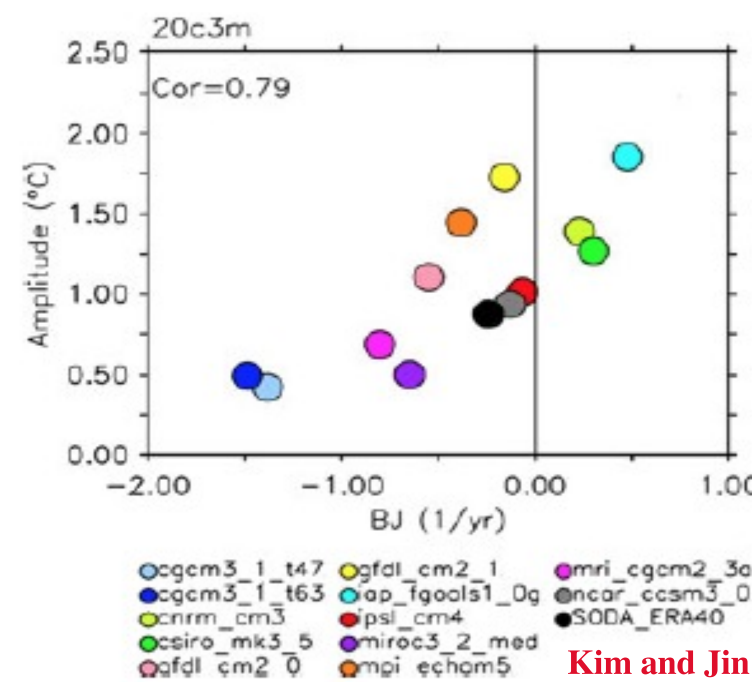


$$\frac{dT}{dt} = RT + \varpi h_W - cT^3 + \sigma\xi$$

$$\frac{dh}{dt} = -\varepsilon h_W - \varpi T$$

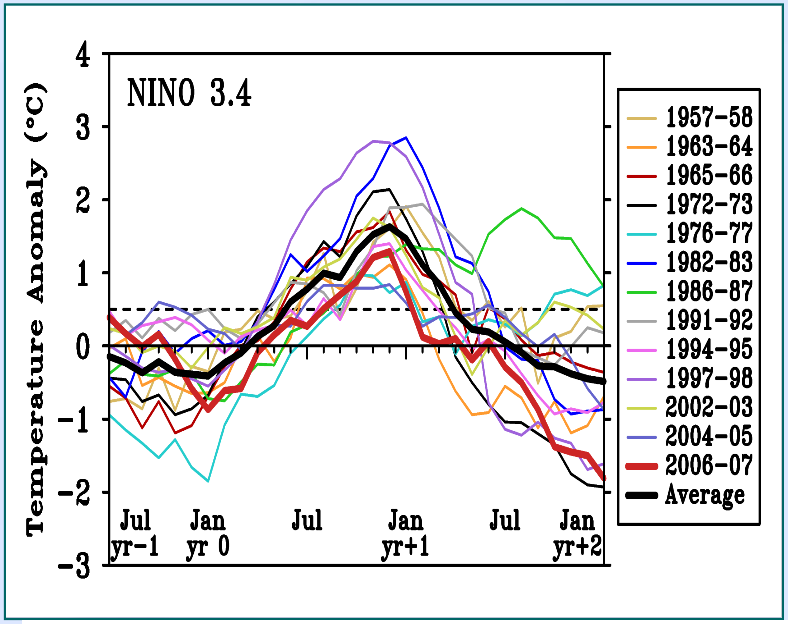
Jin et al 2020

$$(R - \varepsilon + \delta)\overline{T^2} - cK\overline{T^2}^2 + \sigma^2 = 0$$



Kim and Jin 2010

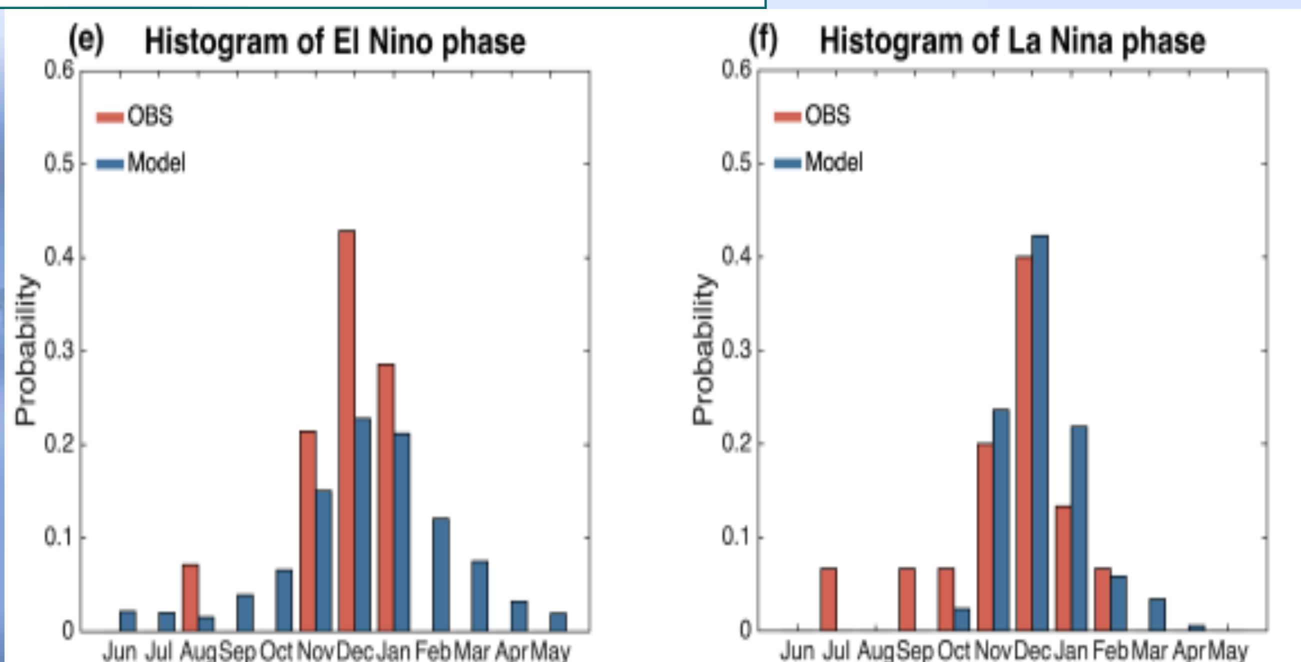
What Controls ENSO Phase Locking?

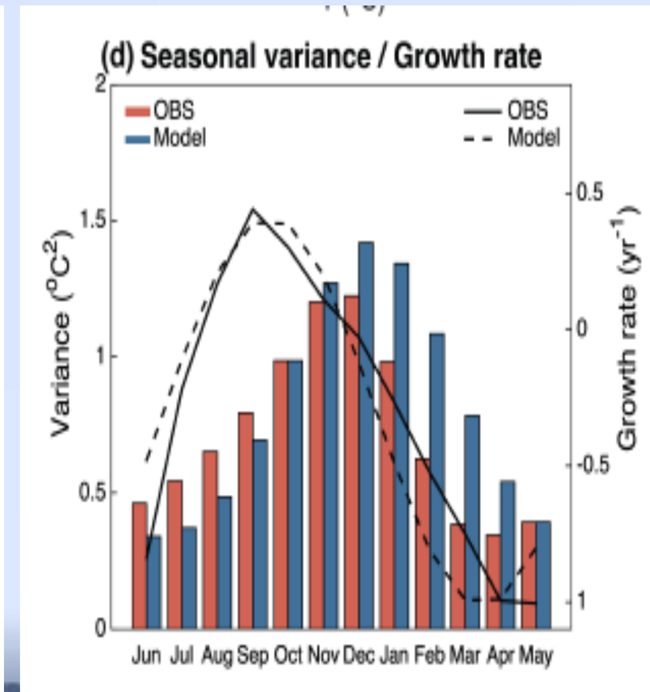


$$\frac{dT}{dt} = (R_0 + \underline{R_a(t)})T + \varpi h_W - cT^3 + \sigma\xi$$

$$\frac{dh}{dt} = -\varepsilon h_W - \varpi T$$

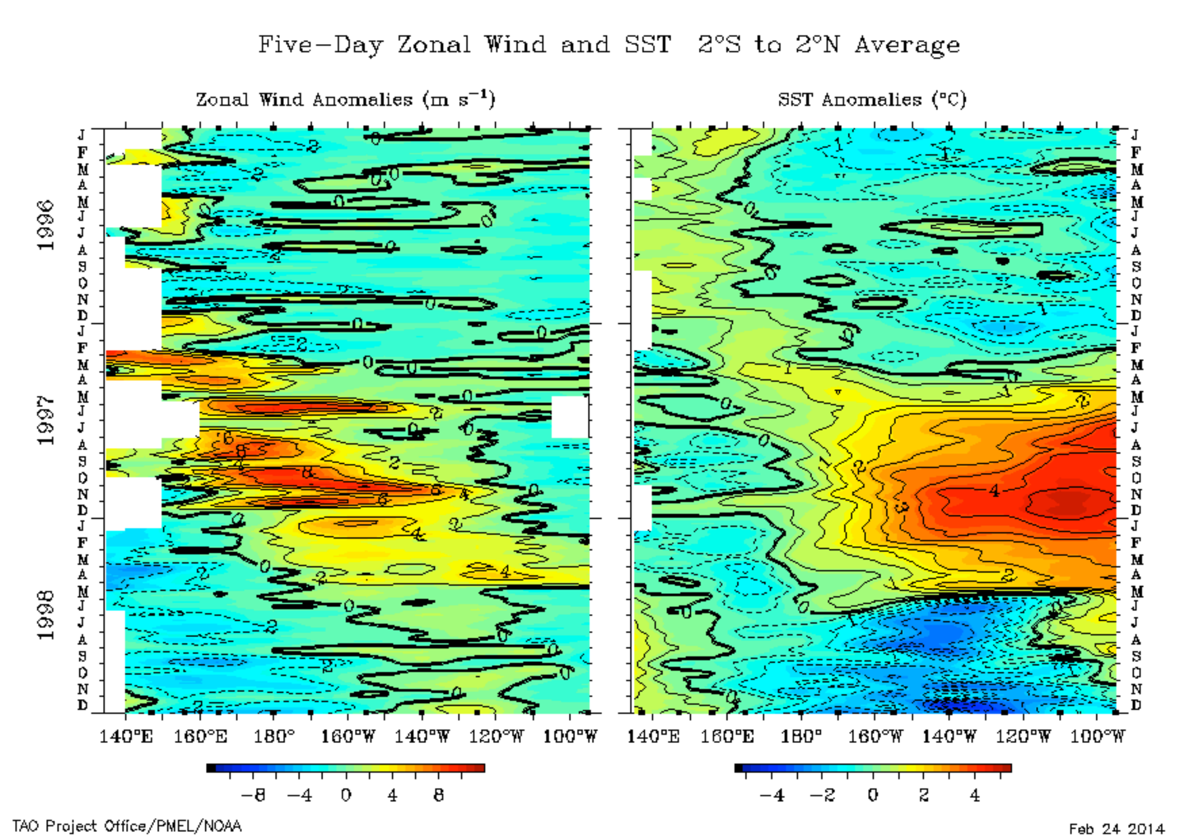
Seasonal modulation in ENSO growth rate





Jin et al 2020

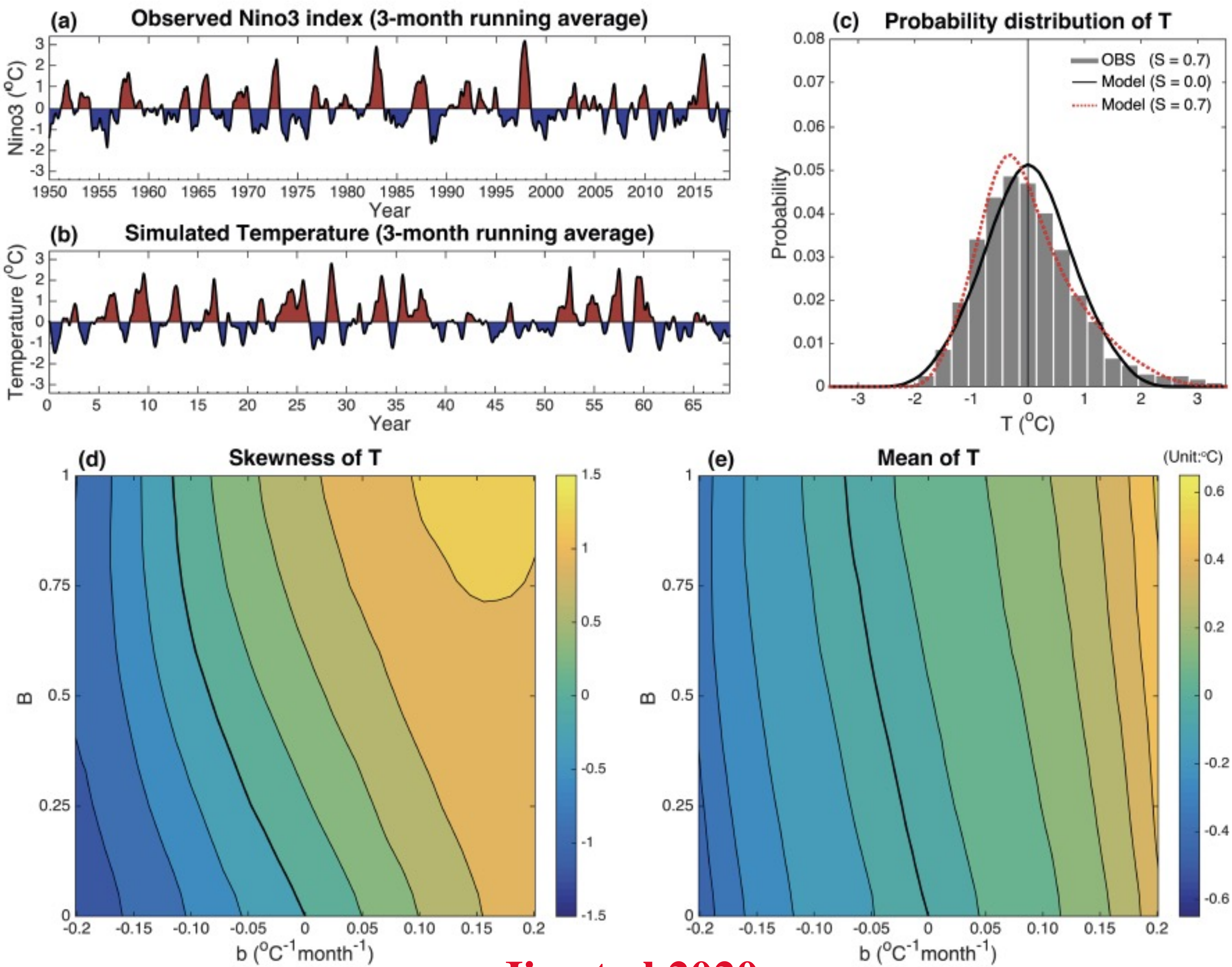
What controls ENSO asymmetry ?



1) **ENSO-state-dependent noise forcing (WWEs and TIWs)**
2) **Symmetry breaking nonlinearity !**

$$\frac{dT}{dt} = (R_0 + R_a(t))T + \varpi h_W - cT^3 \underline{+bT^2} + \sigma(1 + \underline{BT})\xi$$

$$\frac{dh}{dt} = -\varepsilon h_W - \varpi T$$



Jin et al 2020

ENSO Complexity

- What is the fundamental dynamics for ENSO pattern spatiotemporal pattern diversity (STPD)?

ENSO spatial pattern diversity is beyond current RO conceptual model due to inadequate dof for SST. But what are the other key elements necessary to be taken into considerations to capture ENSO STPD?

What controls ENSO pattern diversity and multi-timescale complexity?

2009

2006

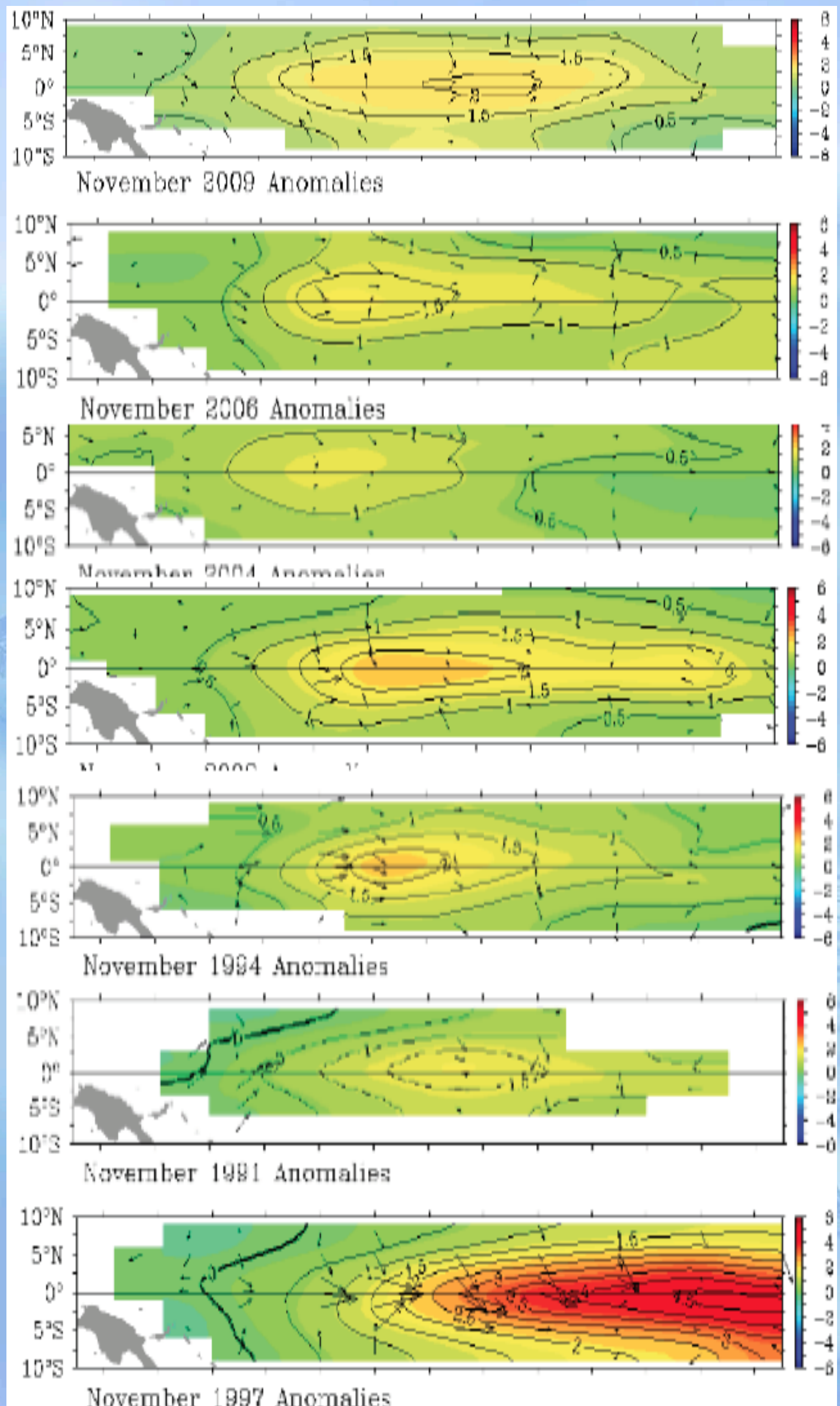
2004

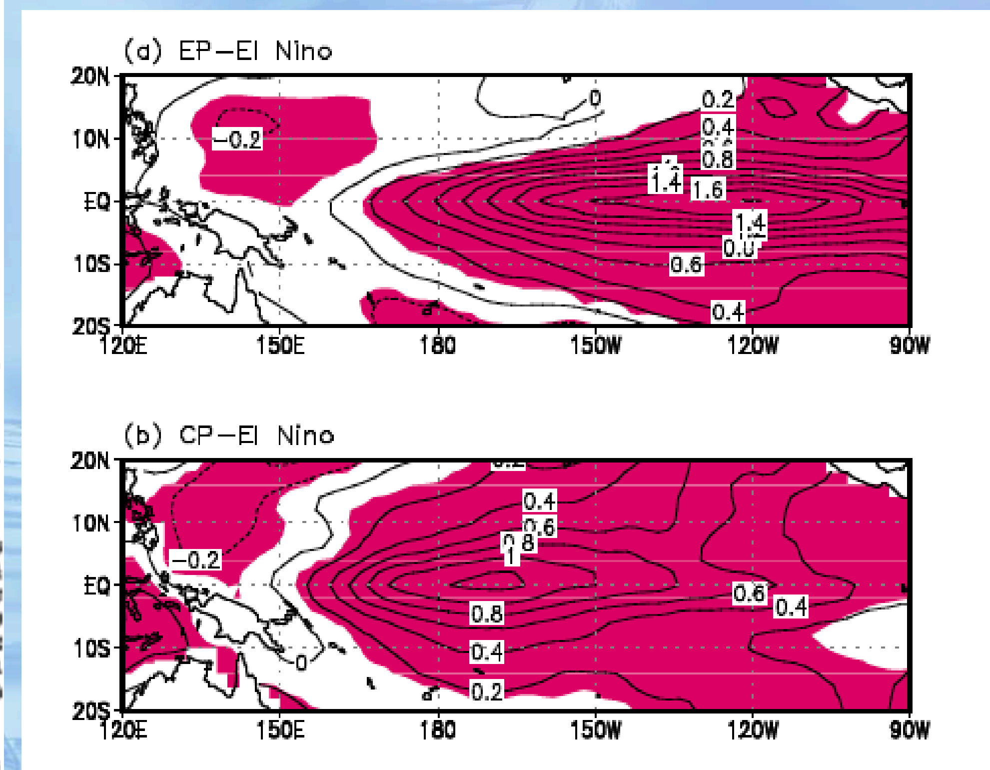
2002

1994

1991

1997





Yeh et al (2009)

Existing theoretical studies:

1. Two ENSO modes (BJ 2008; XJ 2018 had an error occurred !). There is only one unstable modes under relative realistic conditions of basic state and parameter settings
2. a) One mode + extreme events via convective threshold nonlinearity (Choi et al 2014; Takahashi et al 2018)

   b) One mode + nonlinear zonal advection (Chen et al 2016)

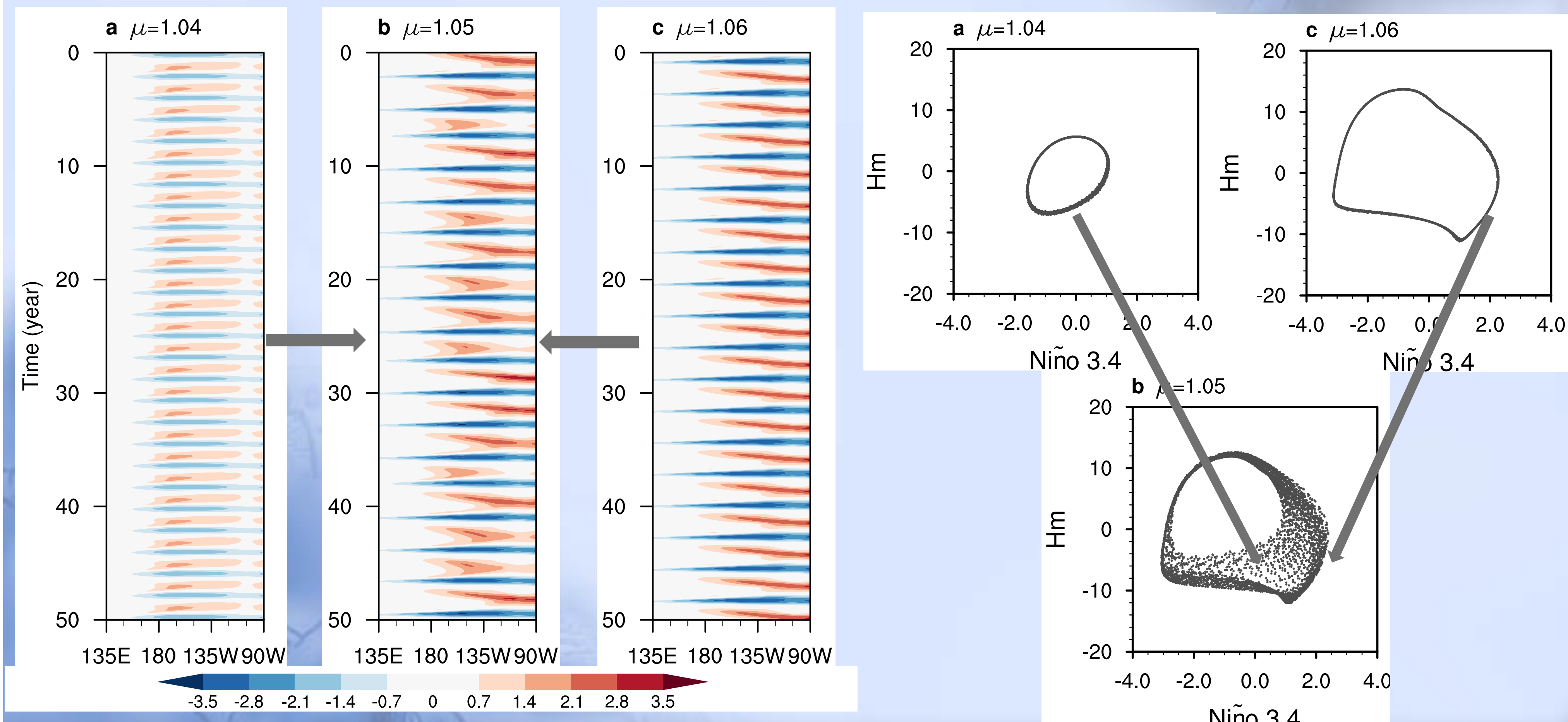
   c) One mode + decadal modulation + stochastic forcing (Feng et al 2018, 2021)

   d) Basic-state modulations of the ENSO mode

   e) One ENSO mode + nonlinear instability (Geng and Jin 2022; Jin 2022)

New Hypothesis : One linear ENSO mode can lead to ENSO diversity via a nonlinear regime transition pathway (L-C Geng's Thesis) in a new version of CZ model

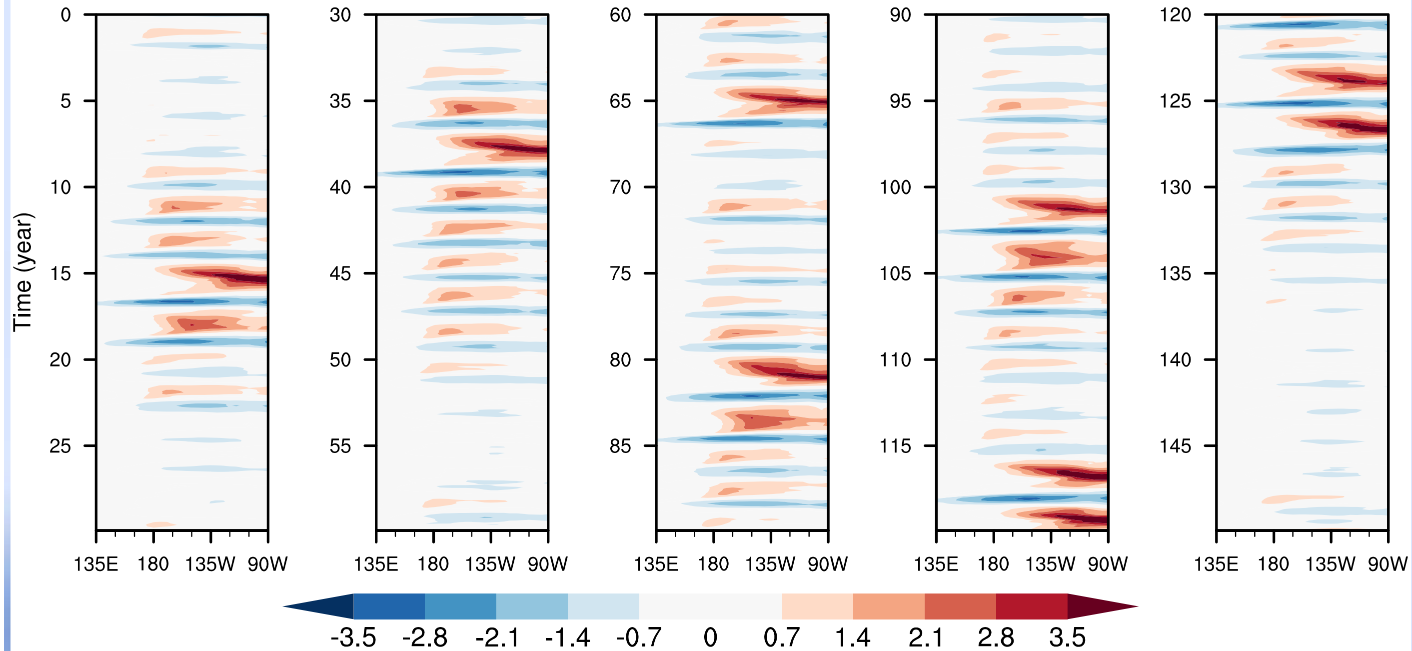
Namely: the nonlinear instabilities of the ENSO RO oscillator gives rises to the co-existence of CP&EP El Niño via ENSO's nonlinear regulations to the climate state and annual cycle Geng & Jin (2022).



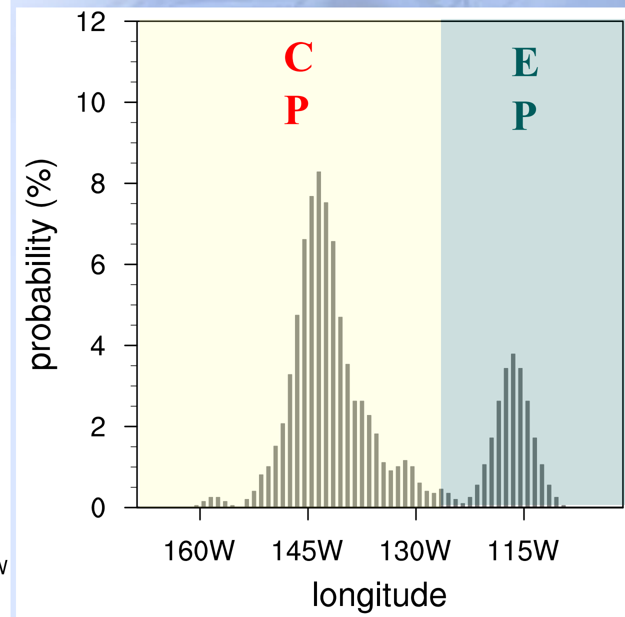
linear (CP) regime **diversity regime** **nonlinear (EP) regime**

ENSO diversity in Geng's R-CZ with stochastic forcing

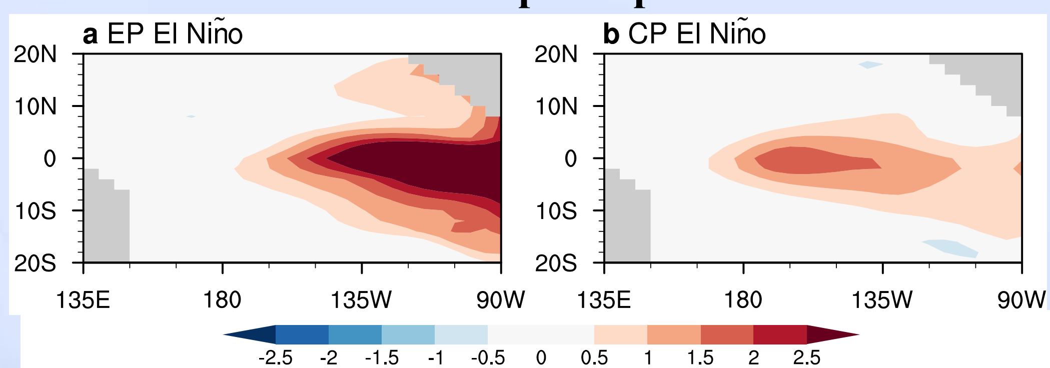
Hovmöller diagram of equatorial SSTA



PDF for centroid longitude λ_c

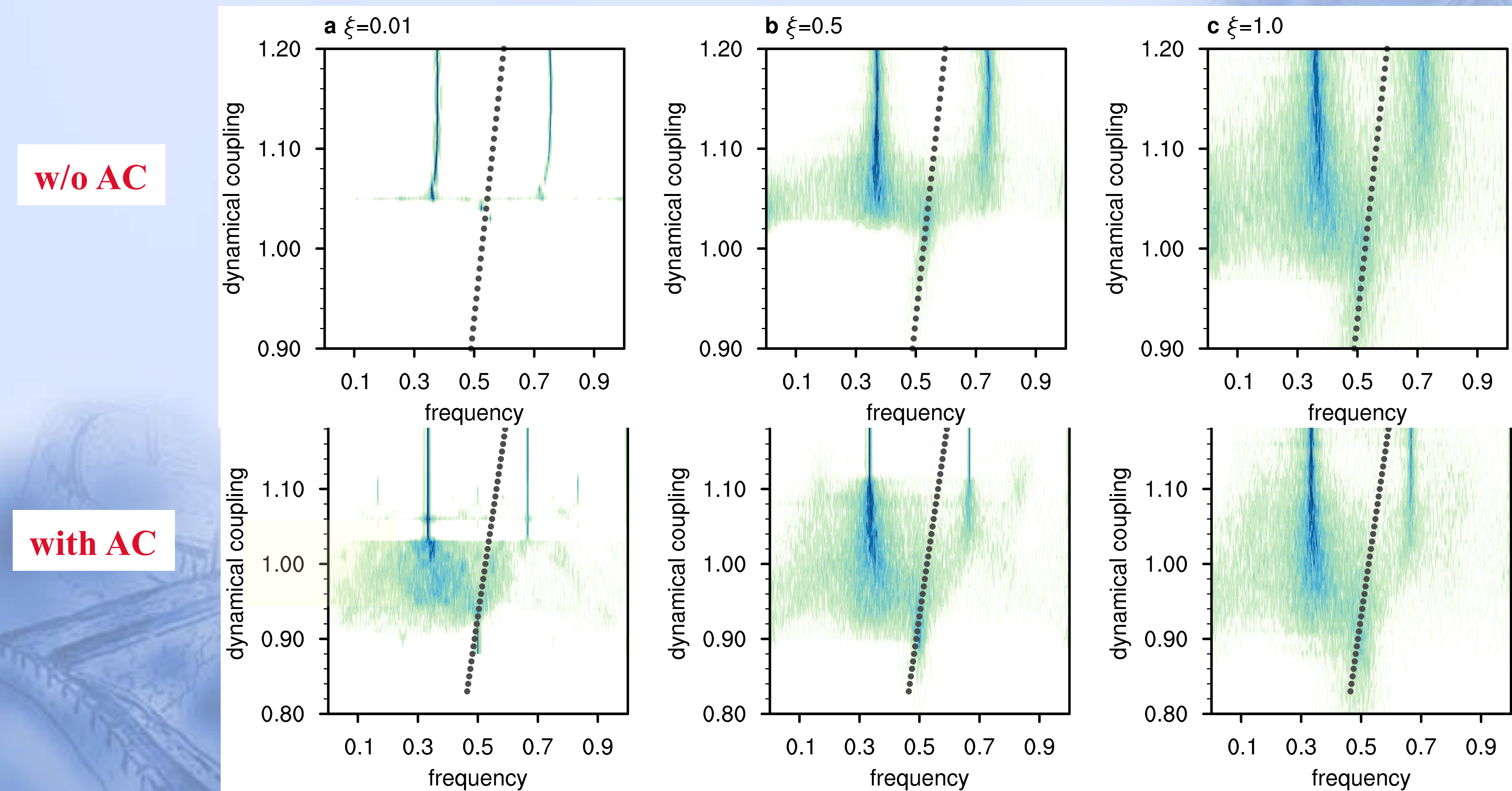


spatial pattern



- **EP and CP El Niño as two preferred states**
- **decadal variability of ENSO behavior**

ENSO Diversity regime broadened by the interaction with seasonal cycle (via Devil staircases) &/or stochastic forcing

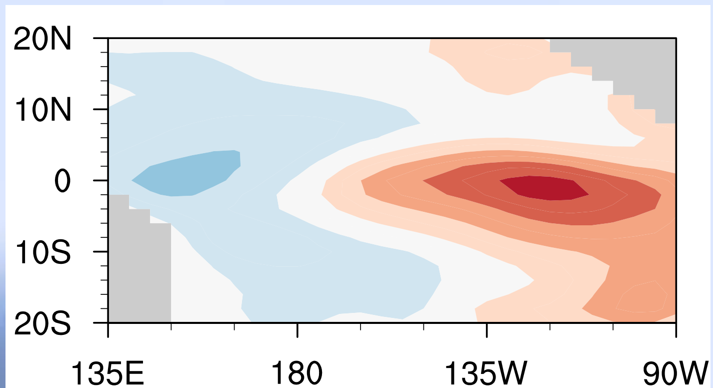


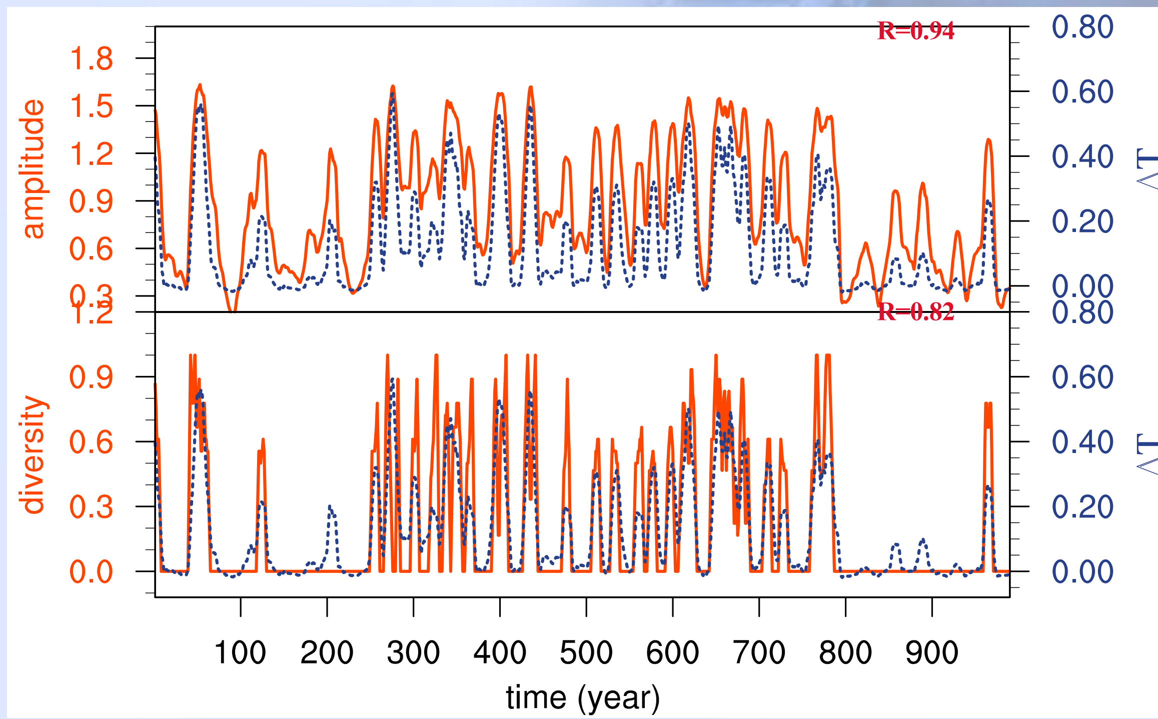
power spectra of Nino 3.4

With AC: more complex ENSO behavior and more ubiquitous ENSO diversity even without stochastic forcing

Close relationship between ENSO behavior and mean state

EOF1 (90%) of 10-year running average SST





- **Two the key constrains for ENSO STPD to occur:**
  1. **leading ENSO mode is close to or slightly unstable**
  2. **leading ENSO mode has mixed features of RO and WO and thus its pattern and frequency are sensitive depending on modest changes in the basic state.**
- **The strong double constrains make it hard for GCMs to capture ENSO STPD which may affect ENSO's predictability.**
- **A minimum model for capturing ENSO STPD need to at least to have 2 dofs for SST + 3 dofs for ocean dynamics or a RO5, which is a conjecture proposed by Jin 2022.**
- **It is unclear whether strong deterministic low-order chaos or chaos + noise are essential for ENSO STPD are essential for ENSO STPD remains to be explored.**
- **How ENSO STPD affect the limit of ENSO predictbility?**
- **……**

Summary

- The simple RO theory, derived with a hierarchy approaches via first-principle based CZ framework with both realism and simplicity, serves as a solid conceptual paradigm for understanding basic features of ENSO: including its growth rate, frequency, amplitude, phase-locking, and even asymmetry.
- It may be further expanded to a low-order conceptual theory to capture ENSO's spatiotemporal pattern diversity and other higher-order dynamics, which may shield some new light on the limit of ENSO's predictability as well.