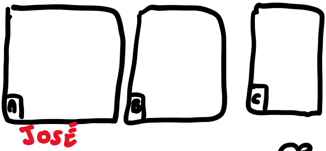
Tuesday, 20 September 2022 09:36

CONDITIONNING PARADOXS

$$P(A|B) \neq P(A|B')$$

* LET'S MAKE A DEAL

(MONTY HALL PARADOX)



- BEHIN 2 DOOR: GOAT
- ____ 1 DOOR: FERRARI

- MONTY HALL KNOW WHERE IS THE FERRARI

- MONTY HALL OPEN A LOOSING DOOR DIFFERENT THAT THE JOSÉ'S DOOR.

⇒ GO OUT

⇒ DO YOU WANT CHANGE YOUR CHOICE?

⇒ MARYLIN VON SAVANT

(QI 230)

⇒ SHE GIVE THE GOOD ANSWER

⇒ BIG DEBATE

* MONTY HALL

⇒ REAL LIFE "NUMERICAL" SIMULATION.

MOVIE: LAS VEGAS 21

VIDEO GAMES

NO-CHANGIST	CHANGIST
70%	30%

STARTING:

THE 3 DOOR WAS EQUI-PROBABLE

$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

↓

AFTER: REMAIN EQUI...

$(\frac{1}{2}, \frac{1}{2})$

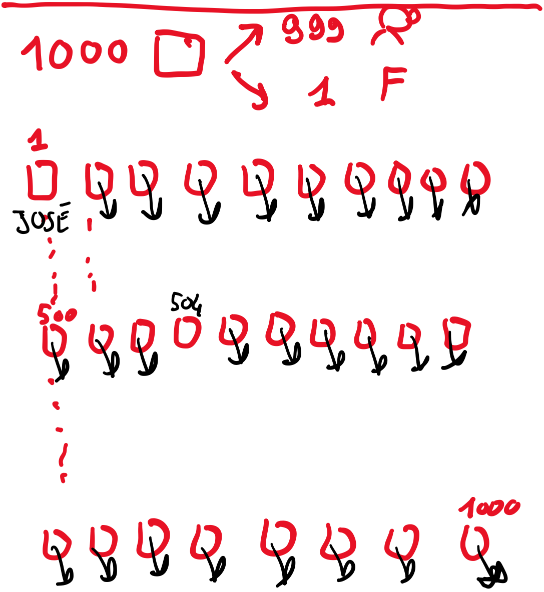
⇒ WE DON'T NEED TO CHANGE

STARTING:

* $\frac{2}{3}$ TO BE IN FRONT OF A LOOSING DOOR] ⇒ IF HE CHANGE, HE WIN

* $\frac{1}{3}$ ______ THE WINNER DOOR] ⇒ ______, HE LOSE

⇒ HE MUST CHANGE !!



F: R.V WHERE IS THE FERRARI $\in \{A, B, C\}$

P: R.V THE DOOR OPEN BY MONTY HALL $\in \{A, B, C\}$

⇒ JOSÉ IS IN FRONT OF THE DOOR A

$$\mathbb{P}[F=A \mid P=B]$$

$\frac{1}{2}$? $\frac{1}{3}$? ???

$$\mathbb{P}[F=A \mid P=B] = \frac{\mathbb{P}[F=A \cap P=B]}{\mathbb{P}[P=B]}$$

$$= \frac{\underbrace{\mathbb{P}[F=A]}_{\frac{1}{3}} \cdot \underbrace{\mathbb{P}[P=B \mid F=A]}_{\frac{1}{2}}}{\underbrace{\mathbb{P}(F=A)}_{\frac{1}{3}} \cdot \underbrace{\mathbb{P}(P=B \mid F=A)}_{\frac{1}{2}} + \underbrace{\mathbb{P}(F=B)}_{\frac{1}{3}} \cdot \underbrace{\mathbb{P}(P=B \mid F=B)}_{0} + \underbrace{\mathbb{P}(F=C)}_{\frac{1}{3}} \cdot \underbrace{\mathbb{P}(P=B \mid F=C)}_{1}}$$

$$\{P=B\} = \{P=B\} \cap \underset{\{F=A\} \cup \{F=B\} \cup \{F=C\}}{\underset{\shortparallel}{\Omega}}$$

$$= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 1}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + 0 + \frac{1}{3}}$$

$$= \frac{1}{1+2} = \frac{1}{3}$$

$$\boxed{\left\{\begin{array}{l} \mathbb{P}[F = A \mid P = B] = \frac{1}{3} \;!! \\ \mathbb{P}[F = B \mid P = B] = 0 \\ \mathbb{P}[F = C \mid P = B] = \frac{2}{3} \end{array}\right.}$$

$\Rightarrow$ WE MUST TO CHANGE

$$* \, \mathbb{P}(F = A \mid F \neq B)$$

$$\parallel$$

$$\frac{\mathbb{P}(F=A \cap F\neq B)}{\mathbb{P}(F\neq B)}$$

$$\parallel$$

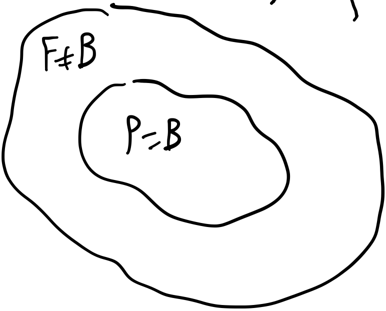
$$\frac{\mathbb{P}(F=A)}{\mathbb{P}(F=A \cup F=C)} = \frac{\frac{1}{3}}{\frac{2}{3}}$$

$$= \boxed{\frac{1}{2}}$$

$$\mathbb{P}(F=A \mid \boxed{P=B}) = \frac{1}{3}$$

$$\mathbb{P}(F=A \mid \boxed{F\neq B}) = \frac{1}{2}$$

$$\begin{tikzcd}
\boxed{P=B} \arrow[r, Rightarrow] & \boxed{F\neq B} \arrow[l, Rightarrow, bend left, "\times" description]
\end{tikzcd}$$



⇒ WE MUST TO CONDITIONATE WITH RESPECT TO THE MOST PRECISE INFORMATION

- $S(F) \overset{!}{=} \ln 3$
- $S(F|P) = \ln 3 - \frac{2}{3}\ln 2$
- $\boxed{S(F:P) \overset{?}{=} \underbrace{S(F)} - \underbrace{S(F|P)}}$

INFORMATION GIVEN BY THE PRESENTATER

DUAL PARADOX: PRISONERS PARADOX

GILBERT MIGUEL JAVI

⇒ JOSEPHINE

⇒ CUNEGONDE (BEST FRIEND)

- *

* CAN YOU TELL ME BETWEEN M AND J A MAN THAT J D'ONT CHOUSE.

* "MIGUEL IS NOT CHOSEN"

* $\boxed{\frac{1}{3} \Rightarrow \frac{1}{2}}$ $\boxed{\boxed{\text{THIS IS TRUE ??}}}$

$$\begin{cases} P(L=G \mid C=M) = \frac{1}{3} \\ P(L=J \mid C=M] = \frac{2}{3} \end{cases}$$