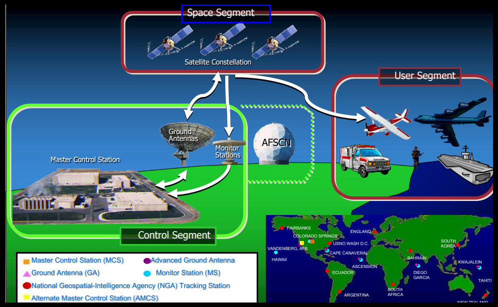
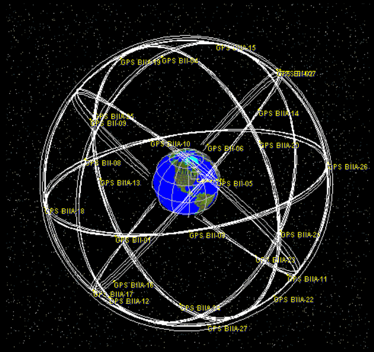




GNSS Fundamentals

African Capacity Building Workshop on Space Weather Effects on GNSS6 Oct 2022

Dr. John F. Raquet

Director, IS4S-Dayton

Integrated Solutions for Systems

SECTION 1: GPS HISTORY

GPS History

- **Navigation technology has always been a military (and commercial) asset**
  - Example: Seagoing clock[1]
- **Ability to determine position from satellites known since Sputnik**
- **Predecessors to GPS**
  - Navy TRANSIT system
    - Measured Doppler shift of satellites in polar orbits
    - Required stationary (or slowly moving) vehicles
  - NRL TIMATION satellites
    - First to orbit precise clocks
    - Provided precise time transfer between points on the Earth
    - Provided side-tone ranging capability
  - Air Force Project 621B
    - Demonstrated ranging based on pseudorandom noise (PRN)
    - Allowed all satellites to transmit at same frequency
- **GPS Joint Program Office (JPO) formed in 1973**
  - Combining TIMATION with Project 621B created the Navigation Technology Satellite (NTS-1 & NTS-2)
    (Note: NTS-2 was designated the first GPS Phase I SV)

TRANSIT Satellite[2]

TIMATION Satellite[3]

Navigation Technology Sat (NTS)
(result of TIMATION
and 621B)[3]

[1]Info can be found at http://www.oldnewspublishing.com/harrison.htm
[2]Image from https://www.patrick.af.mil/heritage/6555th/6555ch4/images/wcgtsz.jpg
[3]Images from http://code8200.nrl.navy.mil/nts.html

SECTION 2: TIME-OF-ARRIVAL POSITIONING (TRILATERATION)

- **How can a receiver figure out where it is?**

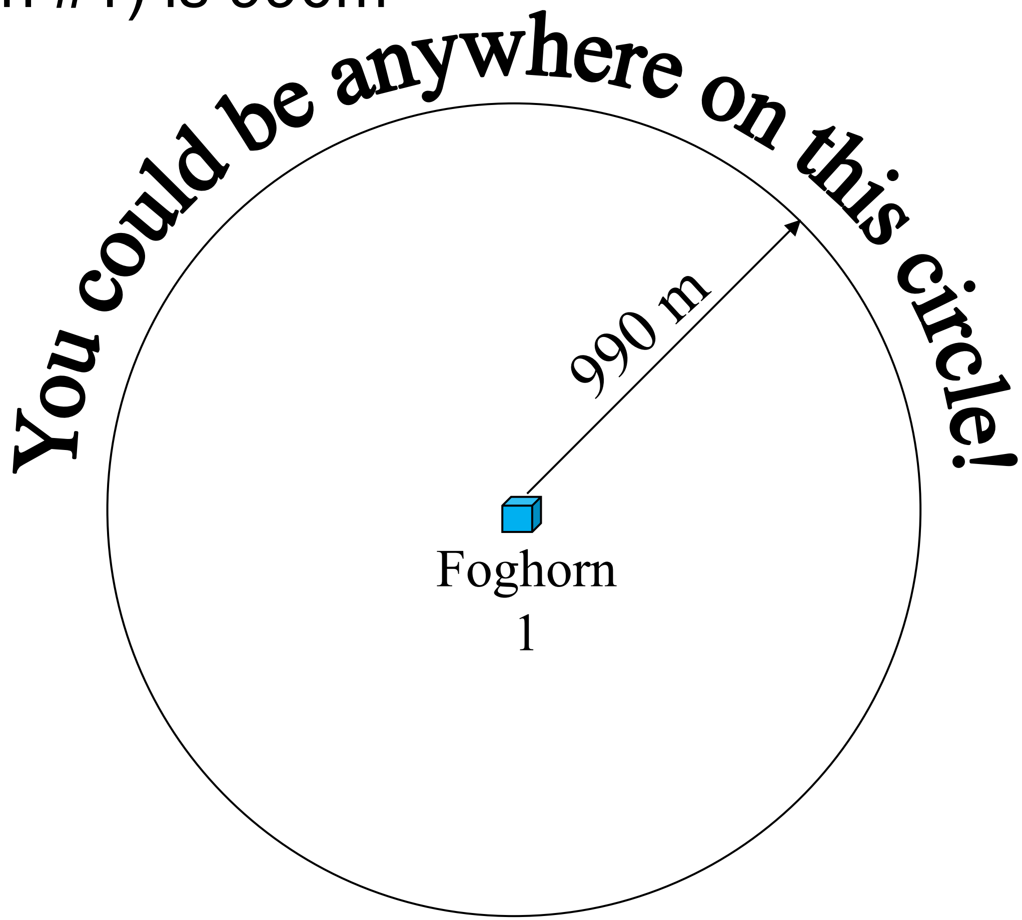
Ranging Using Time-Of-Arrival

- **Time-of-arrival (TOA) is one method that can be used to perform positioning**
- **Basic concept**
  - You must know
    - When a signal was transmitted
    - How fast the signal travels
    - Time that the signal was received
  - Then you can determine how far away you are from the signal emitter

- **Foghorn example**
  - Assume there is a foghorn that goes off at exactly 12:00:00 noon every day
  - You know that the velocity of sound around the foghorn is 330 m/sec
  - You have a device that measures the time when the foghorn blast is received, and it says it heard a foghorn blast at 12:00:03
  - What is the distance between the foghorn and the foghorn "receiver"?
  - Now that you know how far you are from the foghorn, the question is, "Where are you?"

Two-Dimensional Positioning Using Single Range Measurement

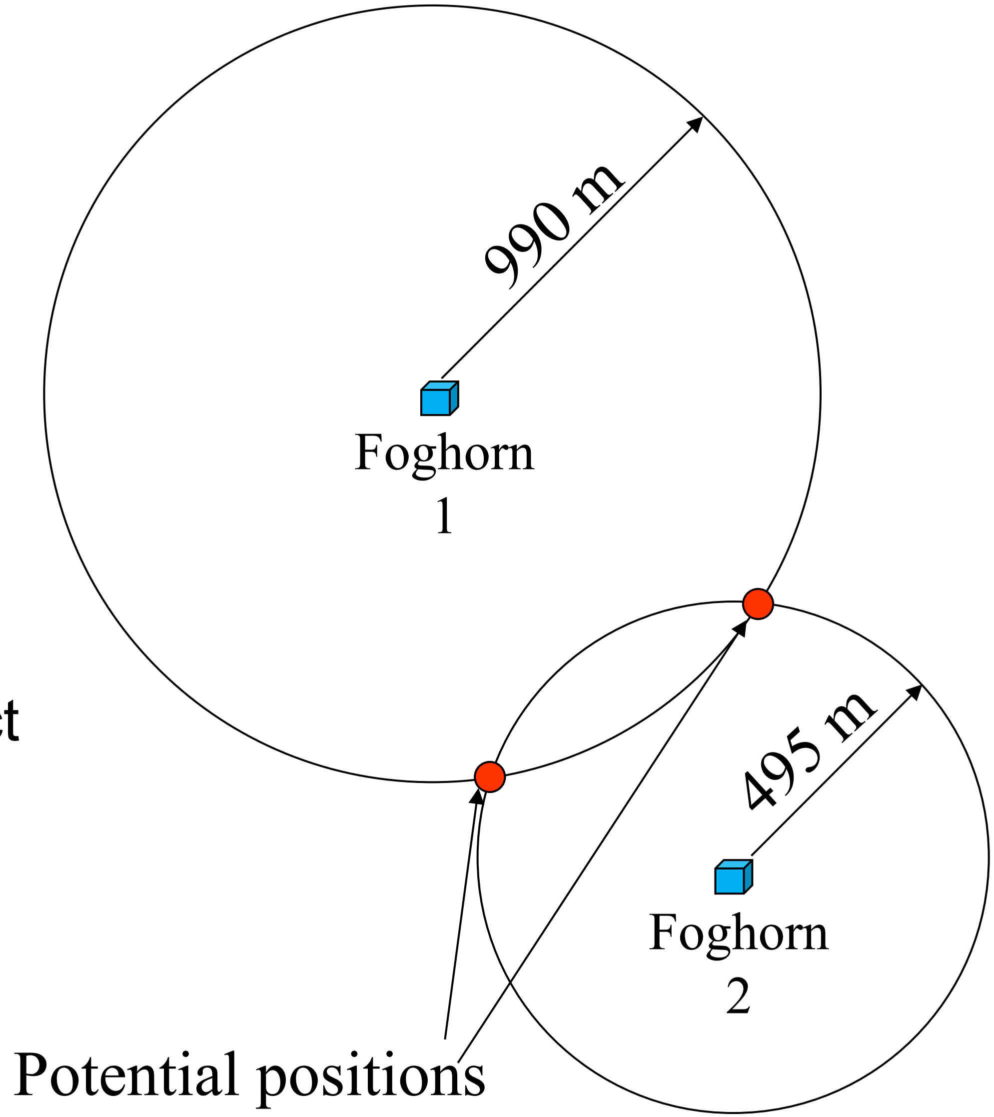
- Range between you and the foghorn (we'll call it foghorn #1) is 990m



- Unable to determine exact position in this case

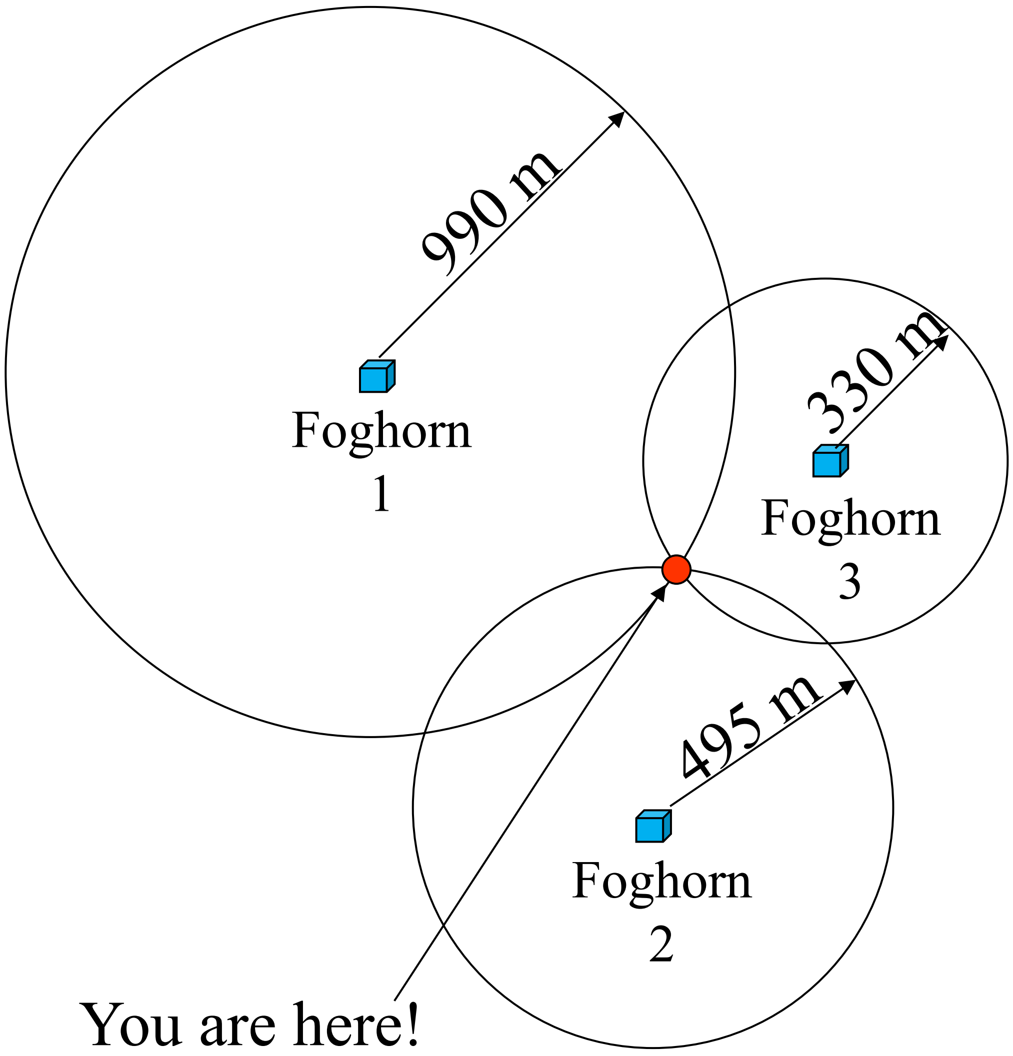
Two-Dimensional Ranging Using Two Measurements

- Now, you take a measurement from foghorn #2 at 12:00:01.5 (for a range of 495 m)
- Yields two potential solutions
  - How would you determine the correct solution?



Resolving Position Ambiguity Using Three Measurements

- You get a third measurement from foghorn #3 at 12:00:01 (Range = 330 m)
  - Now there's a unique solution



Receiver Clock Errors (one way time transfer)

- The foghorn example assumed that the foghorn “receiver” had a perfectly synchronized clock, so the measurements were perfect
- What happens if there is an unknown receiver clock error?
- Effect on range measurement
  - Without clock error

$$R = v_{sound} \Delta t$$

$R =$ range

$v_{sound} =$ velocity of sound

$\Delta t =$ transmit/receive time difference

  - With clock error δt

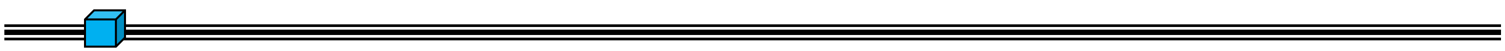
$$R' = v_{sound} \left(\Delta t + \delta t \right)$$

where

$R' =$ range with error (pseudo-range)

Receiver Clock Errors One-Dimensional Example (1/3)

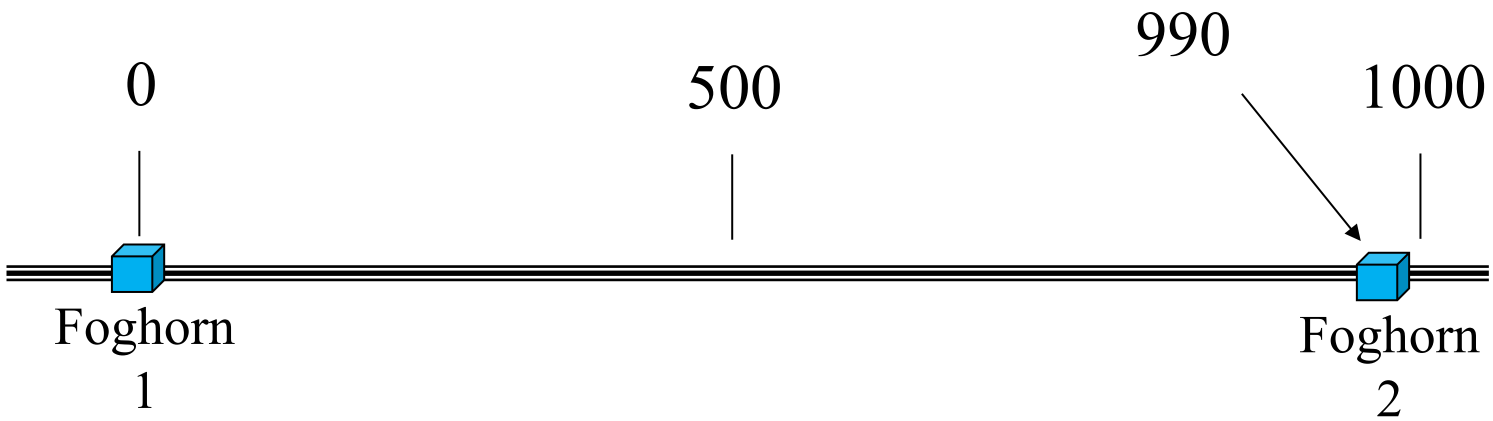
- Now, we'll look at the foghorn example, except in only one dimension
  - The foghorn(s) and receiver are constrained to be along a line
  - We want to determine the position of the receiver on that line



- If the receiver measured a signal at 12:00:10, where is it on the line?
- Now, assume an unknown clock bias δt in the clock used by the foghorn receiver
  - Your foghorn receiver measures a foghorn blast at 12:00:10
  - What can you say about where you are?

Receiver Clock Errors One-Dimensional Example (2/3)

- Clearly, more information is needed
- Assume that there is a second foghorn located 990 m away from the first



- You receive a signal from the second foghorn at 12:00:09
- What can you tell about where you are at this point?

Receiver Clock Errors One-Dimensional Example (3/3)

- Here are the measurements we have:

$$\text{Pseudorange 1} = 330 \times 10 = 3300 = R_1'$$

$$\text{Pseudorange 2} = 330 \times 9 = 2970 = R_2'$$

- From the pseudorange equation:

$$R_1' = v_{sound}\left(\Delta t_1 + \delta t\right) = \quad x \quad + v_{sound}\delta t = 3300$$

$$R_2' = v_{sound}\left(\Delta t_2 + \delta t\right) = 990 - x + v_{sound}\delta t = 2970$$

- Rearranging terms we get

$$x + v_{sound}\delta t = 3300$$

$$x - v_{sound}\delta t = -1980$$

- We can then solve for the two unknowns

$$\delta t = 8 \text{ seconds}$$

$$x = 660 \text{ m}$$

Does this work?

Receiver Clock Errors Extending to Three Dimensions

- **In the single-dimensional case**
  - We needed two measurements to solve for the two unknowns, x and δt.
  - The quantities x and (990 - x) were the "distances" between the position of the receiver and the two foghorns.
- **In three-dimensional case**
  - We need four measurements to solve for the four unknowns, x, y, z, and δt.
  - The distances between receiver and satellite are not linear equations (as was case in single-dimensional case).
  - The four equations to be solved simultaneously, for pseudorange measurements $R_1' \ldots R_4'$ and transmitter positions $(x_1,y_1,z_1)\ldots(x_4,y_4,z_4)$:

$$R_1' = \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} + c\delta t$$

$$R_2' = \sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2} + c\delta t$$

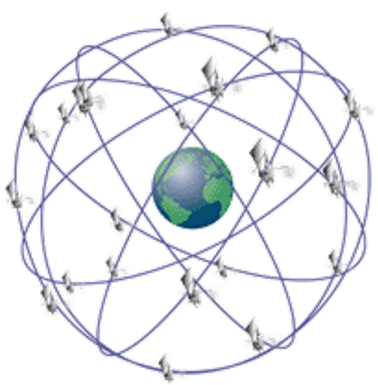
$$R_3' = \sqrt{(x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2} + c\delta t$$

$$R_4' = \sqrt{(x-x_4)^2 + (y-y_4)^2 + (z-z_4)^2} + c\delta t$$

Things You Need to Know/Assumptions

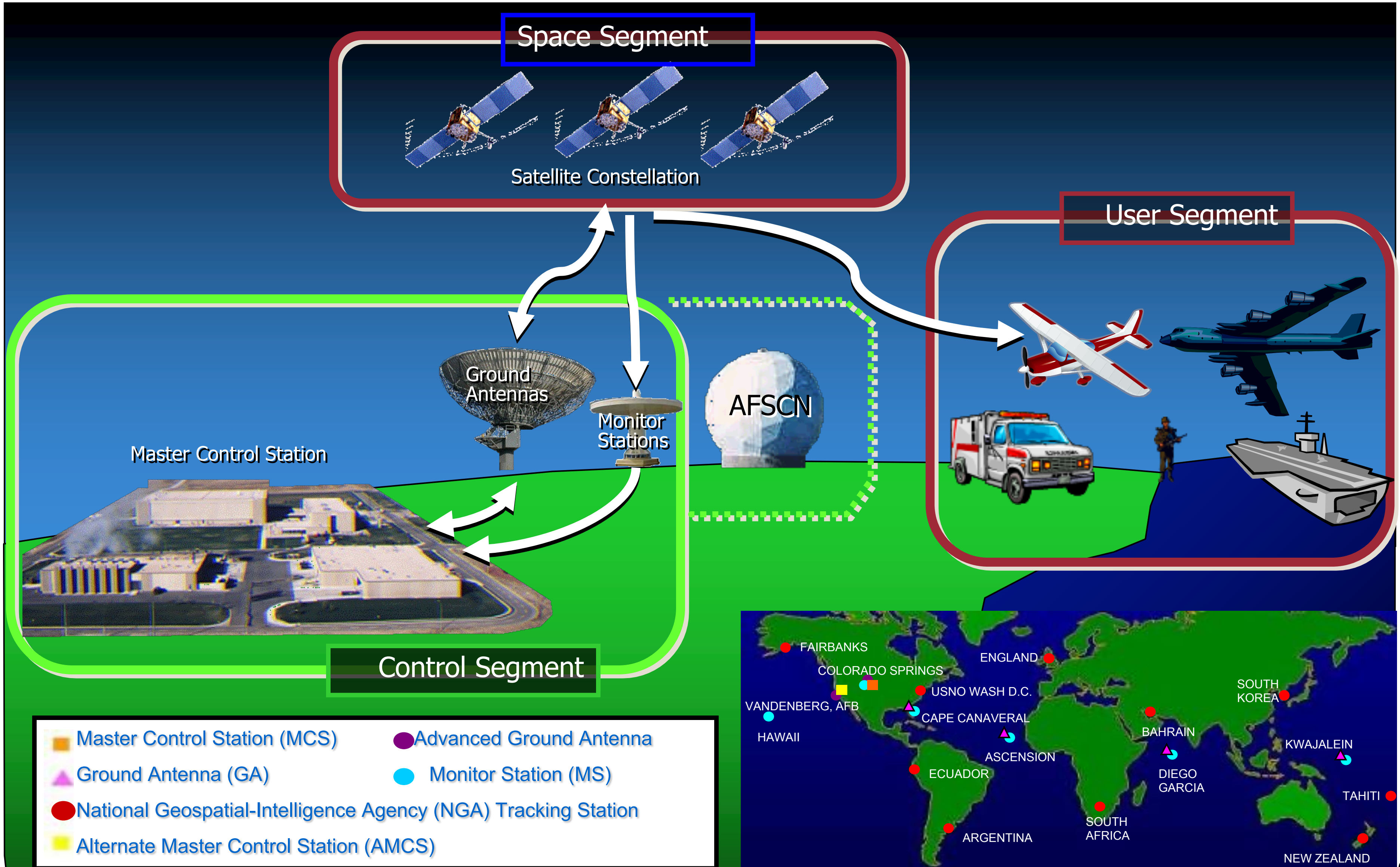
- **Positions of the transmitters**
- **Speed of the signal**
- **All signals received at the same time (so is a single receiver clock error)**
- **Transmit time of the signal (including accounting for transmitter clock errors)**

SECTION 3:
GPS SYSTEM OVERVIEW



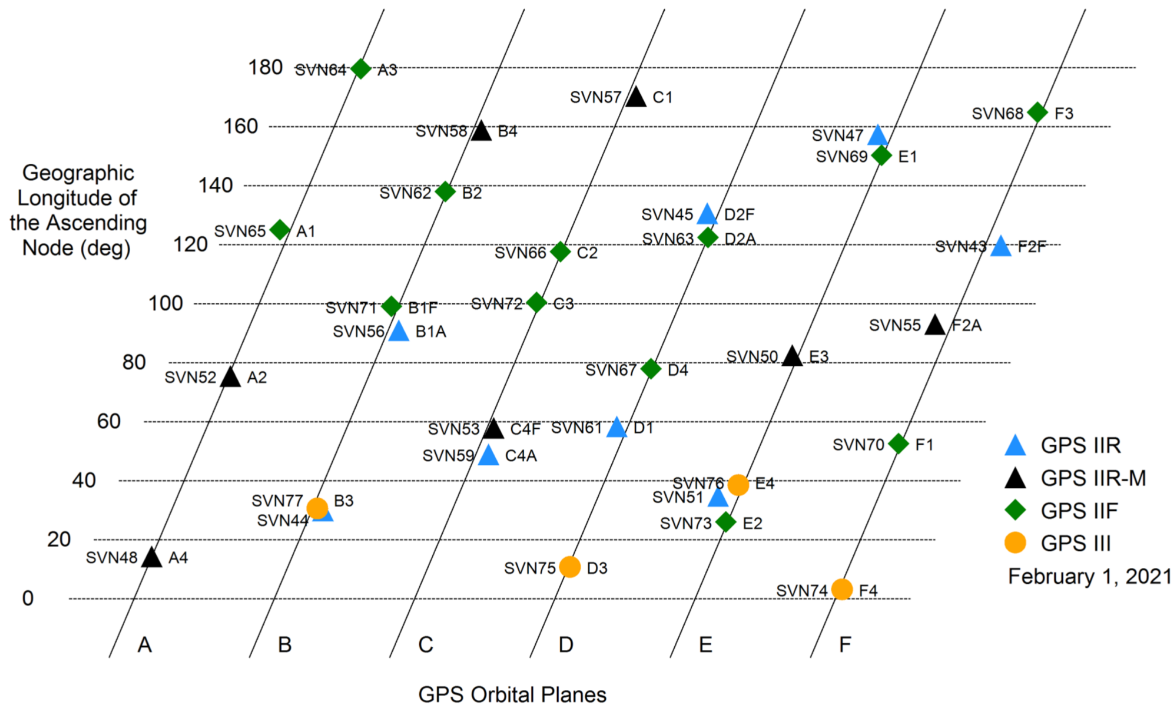
- Three segments of GPS system
- Differential GPS
- GPS performance

GPS Overview: Three Interactive Segments



GPS - Space Segment

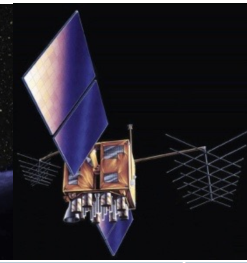
- **Nominally, there are 24 active satellites**
  - Originally “21 operational and 3 active spares” (but distinction not really made any more)
  - Current Constellation described as the “24+3”
  - Have been 30+ satellites recently



https://www.navcen.uscg.gov/pdf/gps/current.pdf

- **Orbit characteristics**
  - Six orbital planes
  - Four SVs per plane nominally
  - 55° inclination angle

Space Segment –
Satellite Characteristics

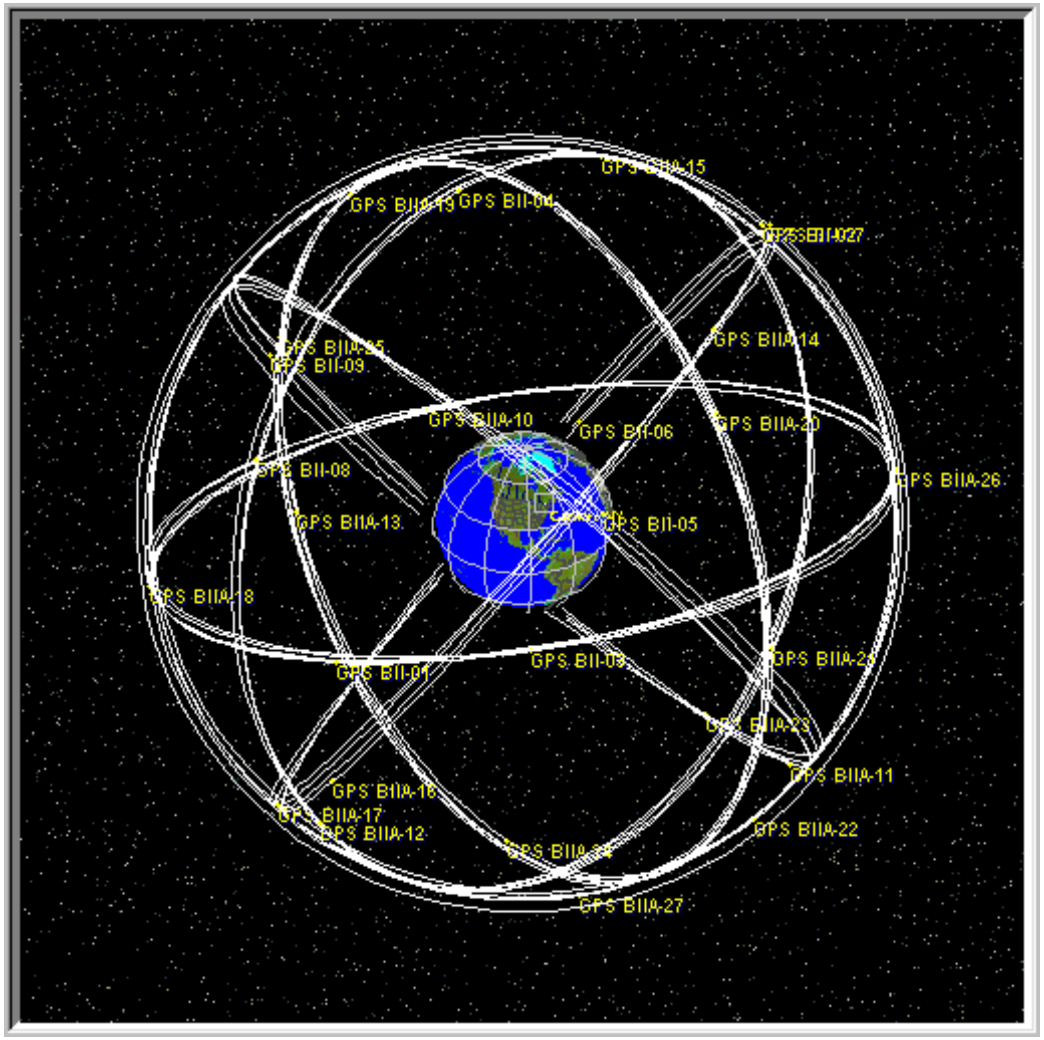




	II/IIA	IIR	IIR-M	IIF	III
Number SV's	28	13	8	12	32
First/Last Launch	1990-1997	1997-2004	2005-2009	2010-2016	2018-present
Satellite Weight (Kg)	900	1100	1100	844	2161
Power (W)	1100	1700	1700		
Design Life (Years)	7.5	7.5	7.5	12	15
In Use (as of Aug 2021)	0	8	7	12	4
L1 Signals	C/A, P(Y)	C/A, P(Y)	C/A, P(Y), M	C/A, P(Y), M	C/A, P(Y), M, L1C
L2 Signals	P(Y)	P(Y)	P(Y), L2C, M	P(Y), L2C, M	P(Y), L2C, M
L5 Signals	-	-	-	L5	L5

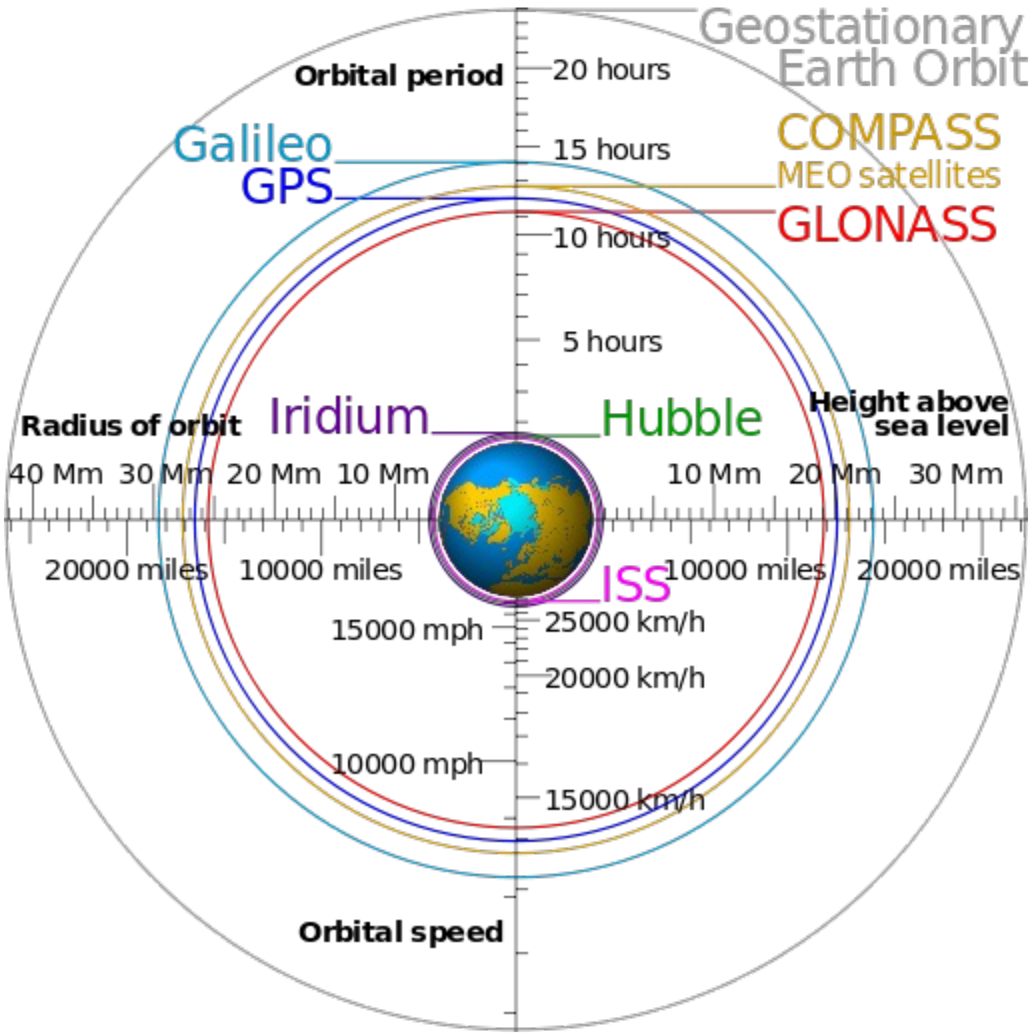
*Estimates

Sources: ftp://tycho.usno.navy.mil/pub/gps/gpsb2.txt
Misra and Enge, *Global Positioning System: Signals, Measurements, and Performance, 2001*
http://www.deagel.com/C3ISTAR-Satellites/GPS-Block-IIR_a000238003.aspx
http://www.deagel.com/C3ISTAR-Satellites/GPS-Block-IIF_a000238004.aspx
https://www.gps.gov/systems/gps/space

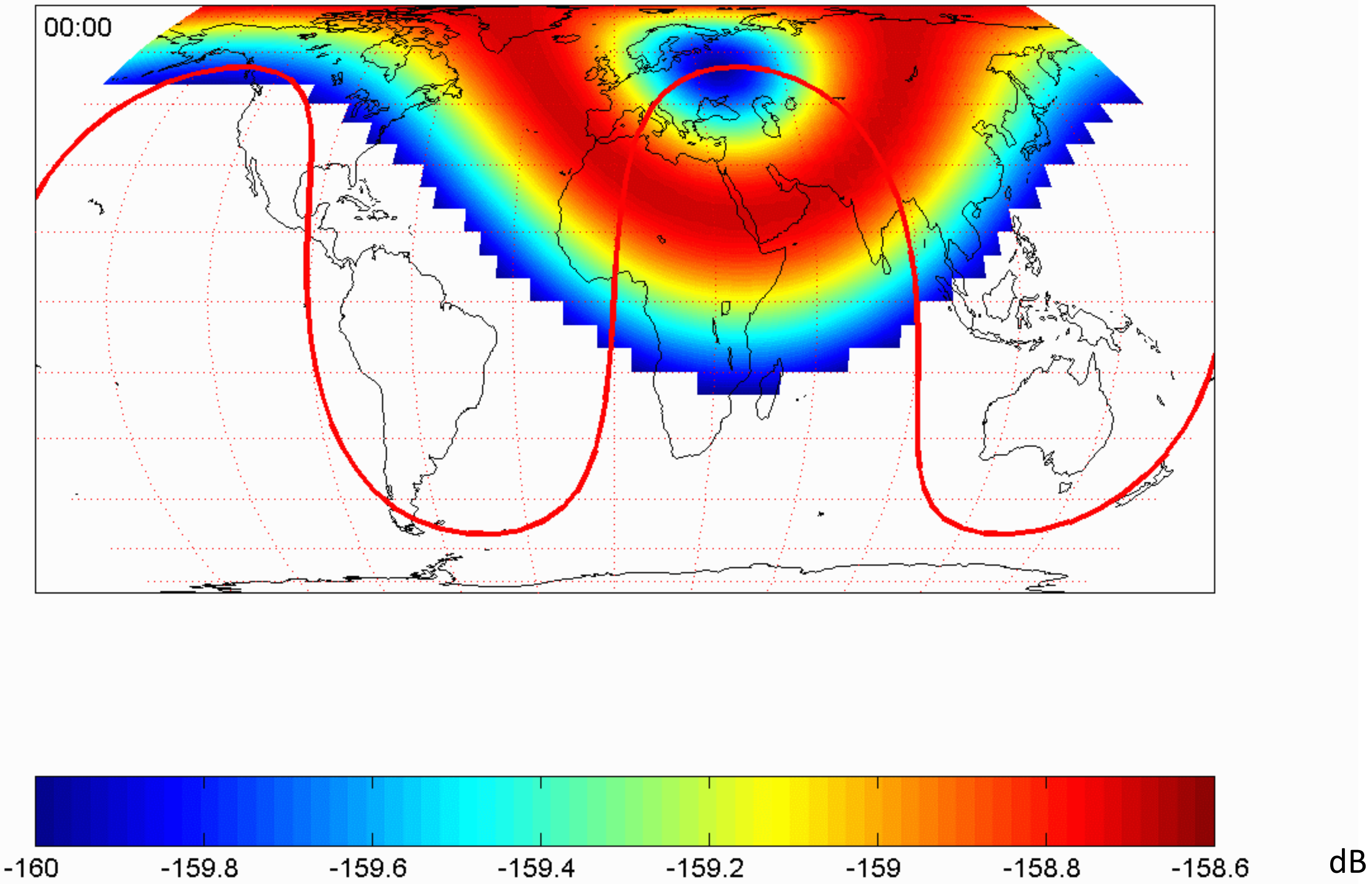
Space Segment –
GPS Constellation as Viewed from Space



Comparison of GPS to Other Satellite Orbits

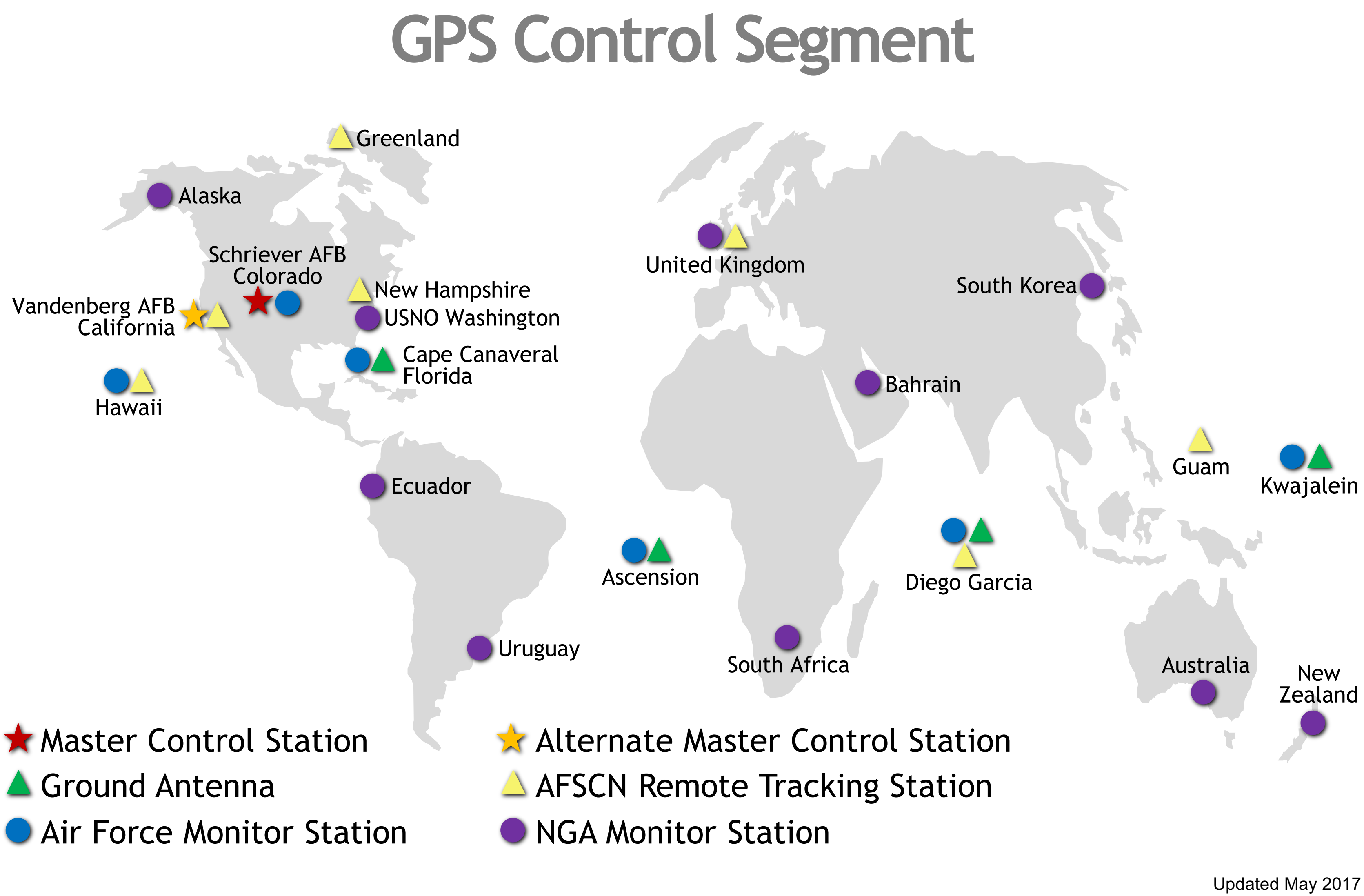


Space Segment - Representative GPS Ground Track



D. Goldstein, GPS Joint Program Office

Control Segment



Obtained from http://www.gps.gov/systems/gps/control/

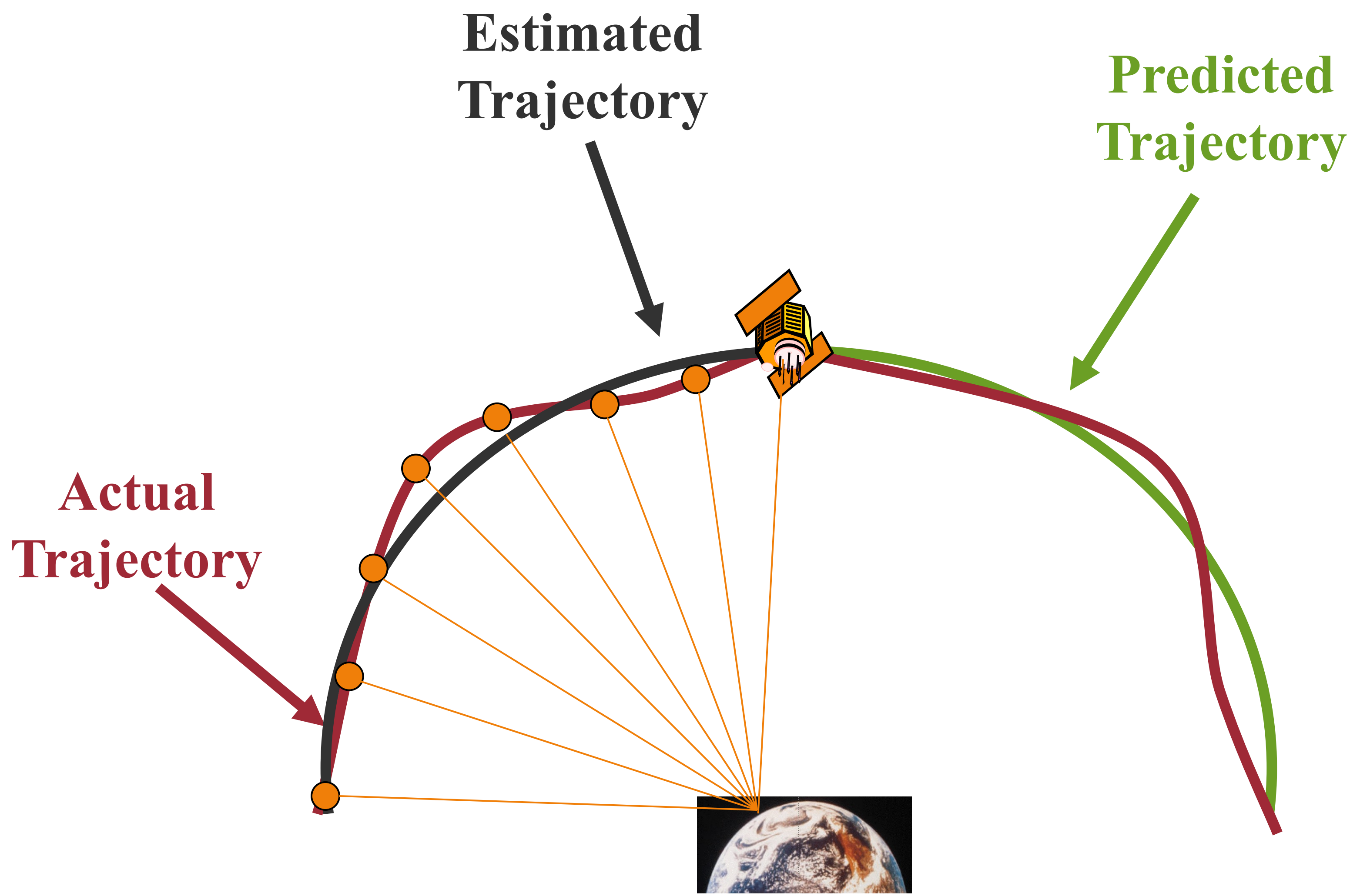
GPS - Control Segment

- **GPS Master Control Station (MCS) located at Schriever AFB, CO (2nd Space Operations Squadron, or 2SOPS)**
  - Manages constellation (flies satellites)
  - Monitors GPS system performance
  - Calculates data sent over the 50 bps navigation message
    - Orbit ephemeris data
    - Satellite clock error correction coefficients
    - Ionospheric model parameters
    - System status
    - GPS time information
- **Communications with satellite using S-band data link**
  - Types of communication
    - Satellite control
    - Navigation message upload
  - S-band communications are intermittent

Galileo Control Segment

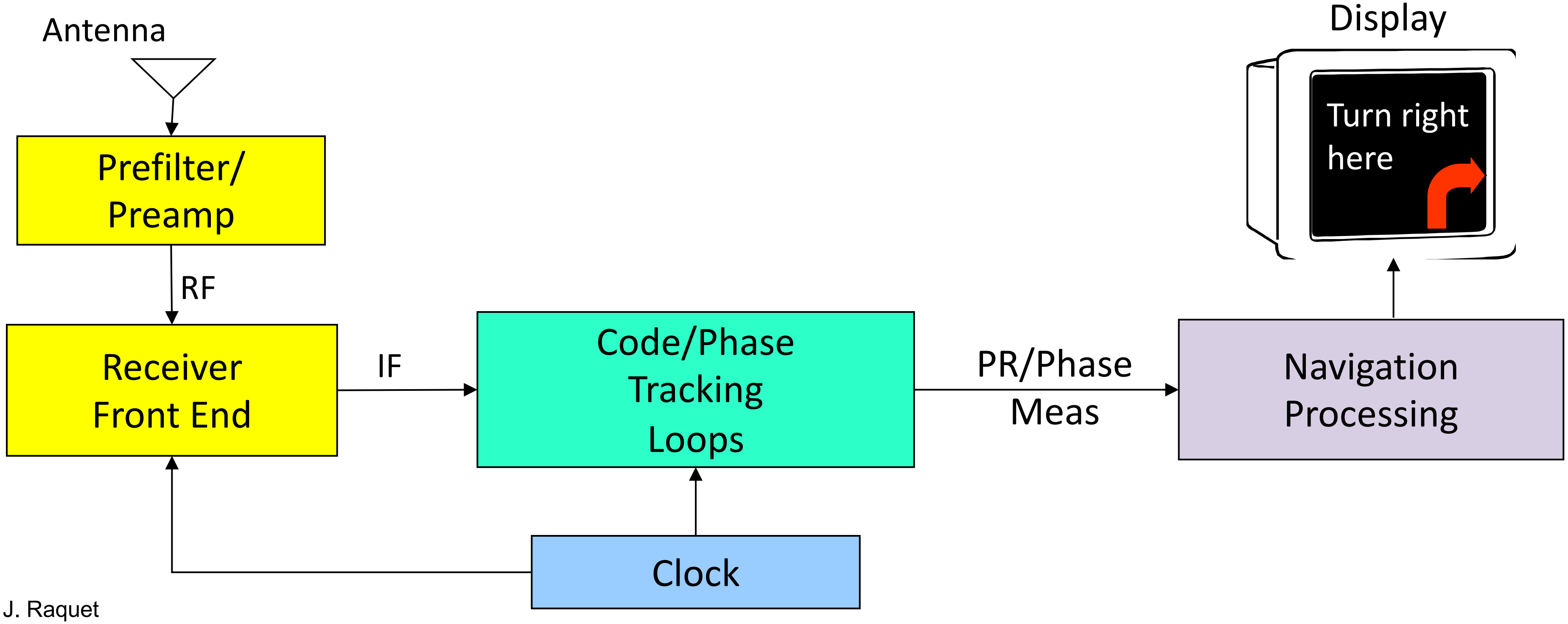
https://www.esa.int/ESA_Multimedia/Images/2018/03/Galileo_s_global_ground_segment

Control Segment – Trajectory Estimation/Prediction



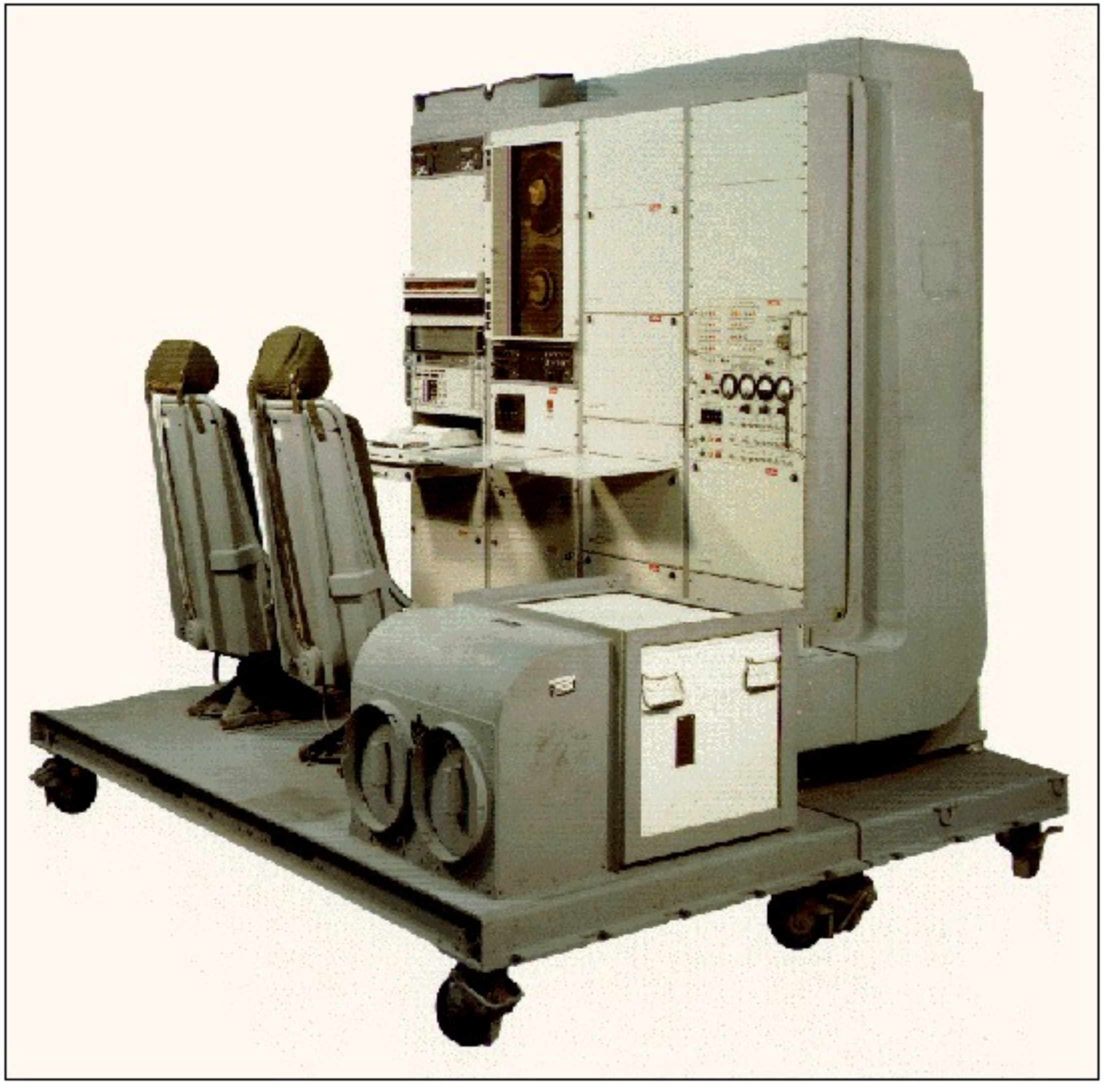
GPS - User Segment

- **User segment consists of all GPS receivers**
  - Space
  - Air
  - Ground
  - Marine
- **Typical GPS receiver components**



First Military User Equipment

First GPS MUE receiver developed under government contract by Rockwell Collins, circa 1977.



http://gpsworld.com/trends-in-gpspnt-user-equipment/

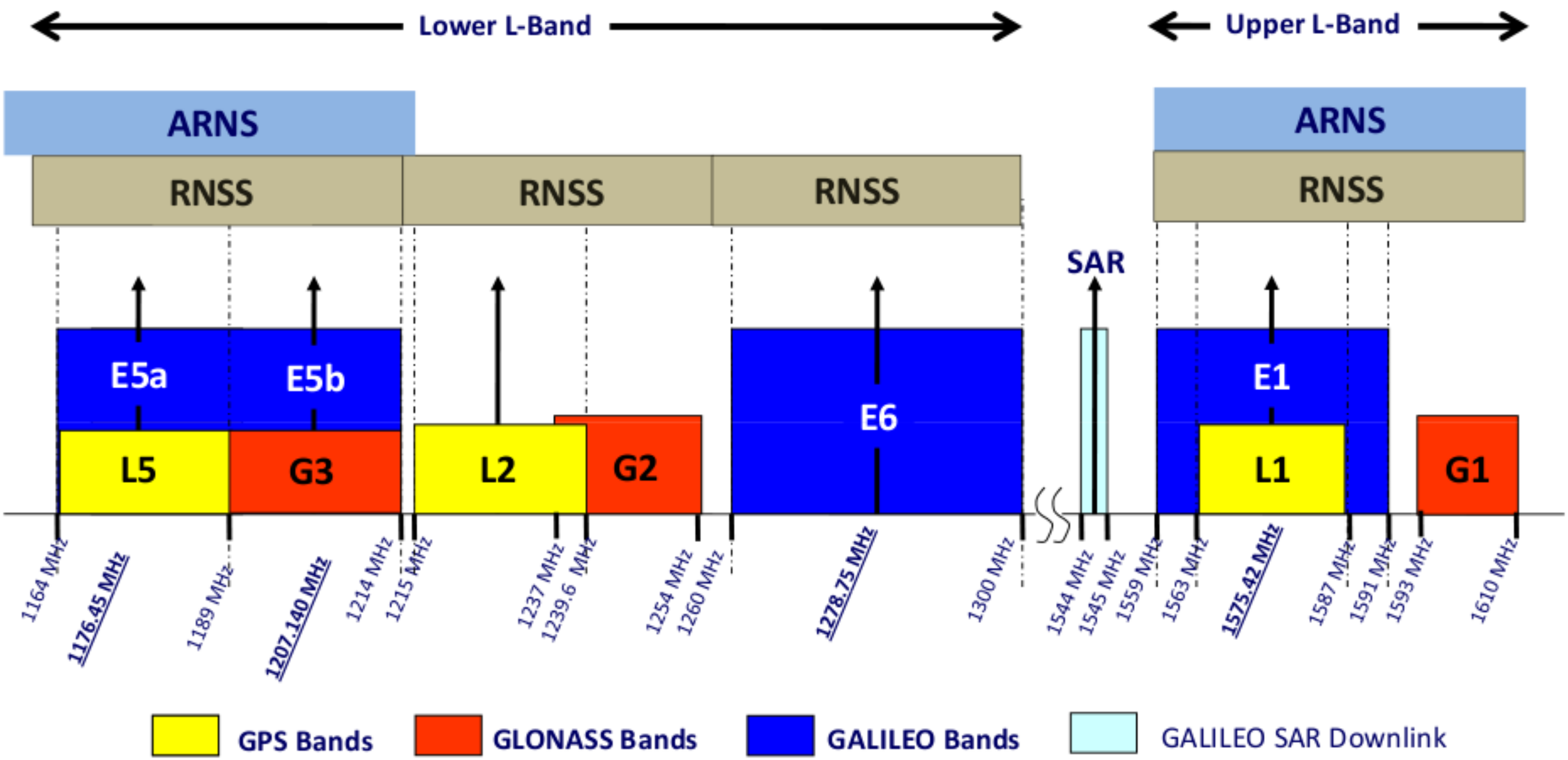
First Significant Transportable Civilian GPS Receiver

TI 4100 NAVSTAR Navigator Multiplex Receiver
designed by Phil Ward for Texas Instruments (1981)

SECTION 4: SIGNAL STRUCTURE

- **So what do those satellites transmit anyway?**

Satellite Navigation Bands



ARNS : Aviation Radio Navigation Service

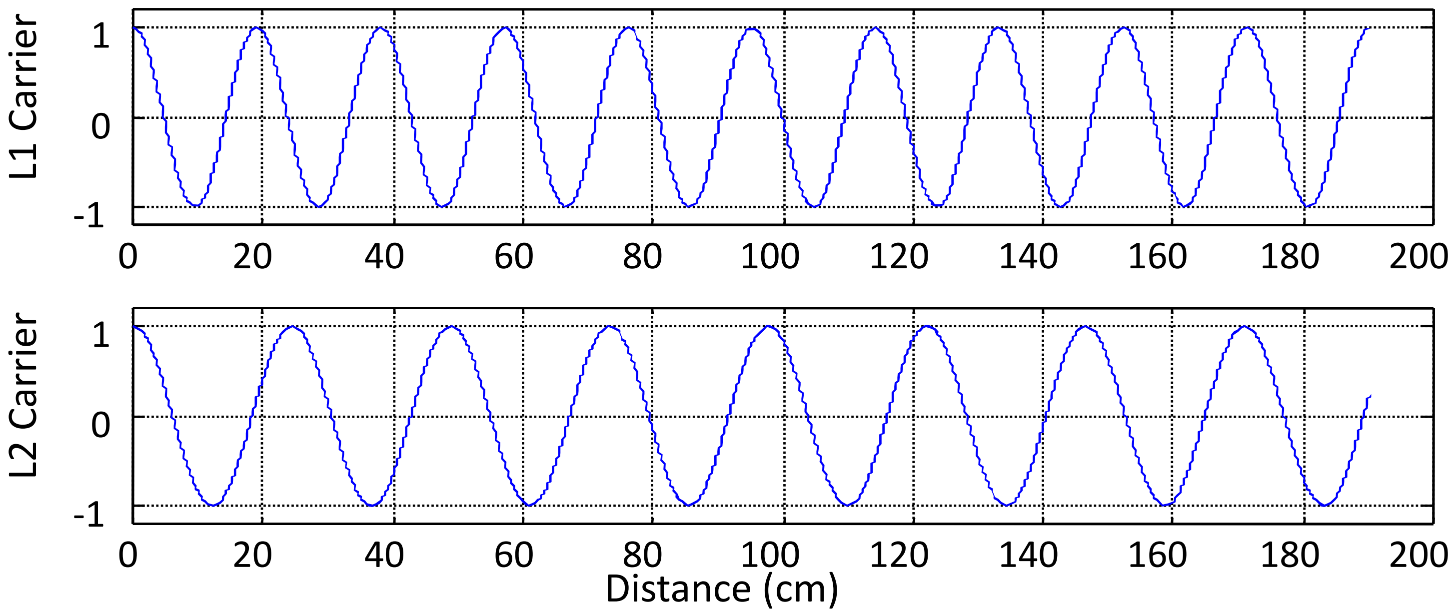
RNSS : Radio Navigation Satellite Service

http://www.navipedia.net/images/f/f6/GNSS_navigational_frequency_bands.png

GPS Carrier Frequencies

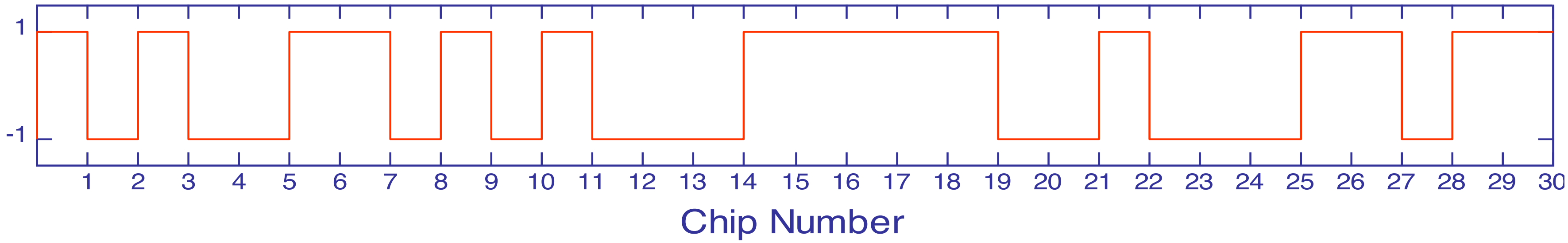
- **Fundamental frequency f_0 = 10.23 MHz**
- **GPS carrier (or center) frequencies**
  - f_{L1} = 1575.42 MHz = 154 f_0
  - f_{L2} = 1227.6 MHz = 120 f_0
  - f_{L5} = 1176.45 MHz = 115 f_0
- **Wavelengths of carriers**

$\lambda_{L1} = c/f_{L1} \approx$ 19.03 cm $\lambda_{L2} = c/f_{L2} \approx$ 24.42 cm $\lambda_{L2} = c/f_{L2} \approx$ 25.48 cm



GPS Pseudo-Random Noise (PRN) Codes

- **A PRN code is a binary sequence that appears to be random. Example:**



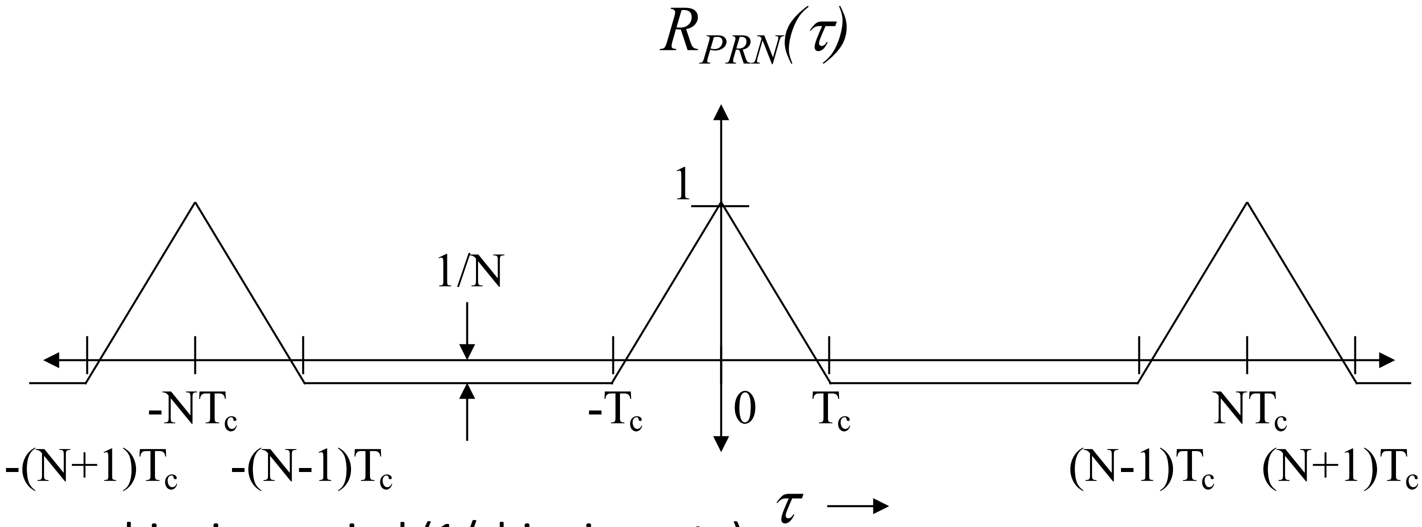
  - Not called a data bit, because it is not data being transmitted
  - The number of chips per second is called the “chipping rate”
- **PRN code sequence generated in hardware using a tapped feedback shift register**
  - Sequence of bits where the new bit is generated by an exclusive-or of two previous bits in the sequence
  - Easy to implement in hardware

GPS Signal Autocorrelation

- **Definition of autocorrelation for function *g(t)*:**

$$R(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)dt$$

- **Autocorrelation function for maximum length PRN sequence (code amplitude of +/- 1)**



T_c = chipping period (1/chipping rate)

NT_c = code repeat period (i.e., repeats after N chips)

(N = 1023 for C/A-code)

Legacy Signals: C/A and P-Codes

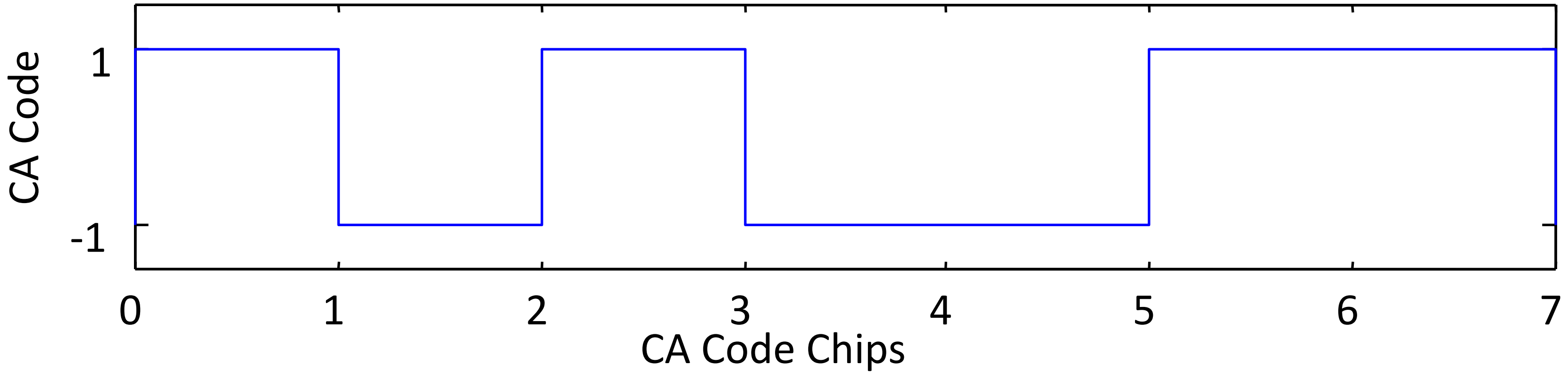
- **GPS uses two classes of codes**
  - Coarse-Acquisition (C/A) code
    - Intended for initial acquisition of the GPS signal
  - Precise (P) code
    - Higher chipping rate, so provides better performance
  - Comparison between C/A and P codes:

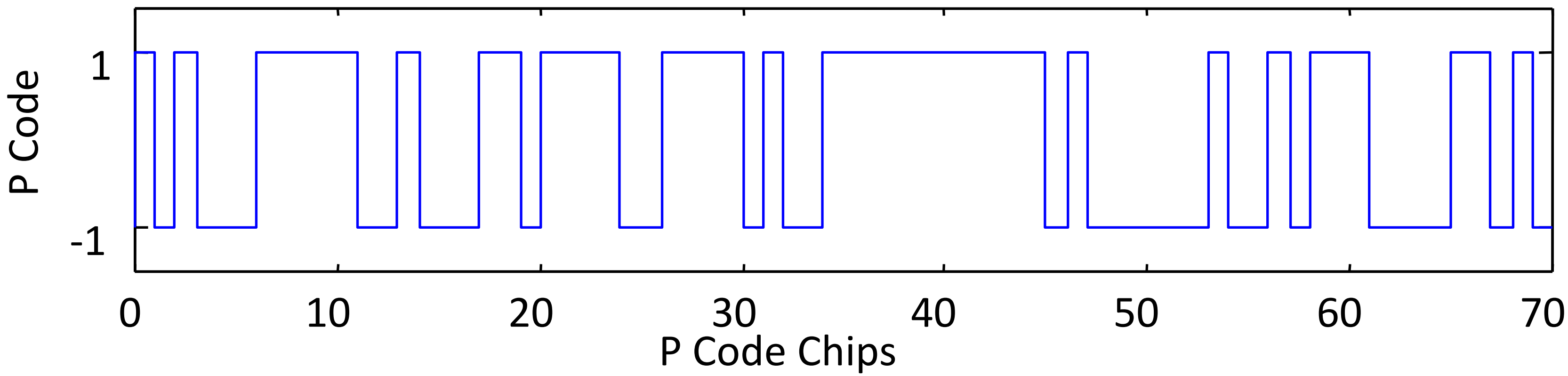
Parameter	C/A-Code	P-Code
Chipping Rate (chips/sec)	1.023×10^6	10.23×10^6
Chipping Period (nsec)	977.5 nsec	97.75 nsec
Range of One Chip	293.0 m	29.30 m
Code Repeat Interval	1 msec	1 week

- **It's more difficult to lock onto the P-code (due to length of code)**
  - Requires accurate knowledge of time
  - Normally, C/A-code locked onto first
    - Easier, since there's only 1ms to search over
    - Once locked onto C/A-code, receiver has accurate time information for locking onto P-code
  - Using accurate timing information to lock onto P-code without initial C/A-lock called "direct P(Y)-code acquisition"

Example C/A and P-Codes

- **Simulated C/A and P-Codes are given below.**
  - Note that the P-code chipping rate is 10 times higher than the C/A-code



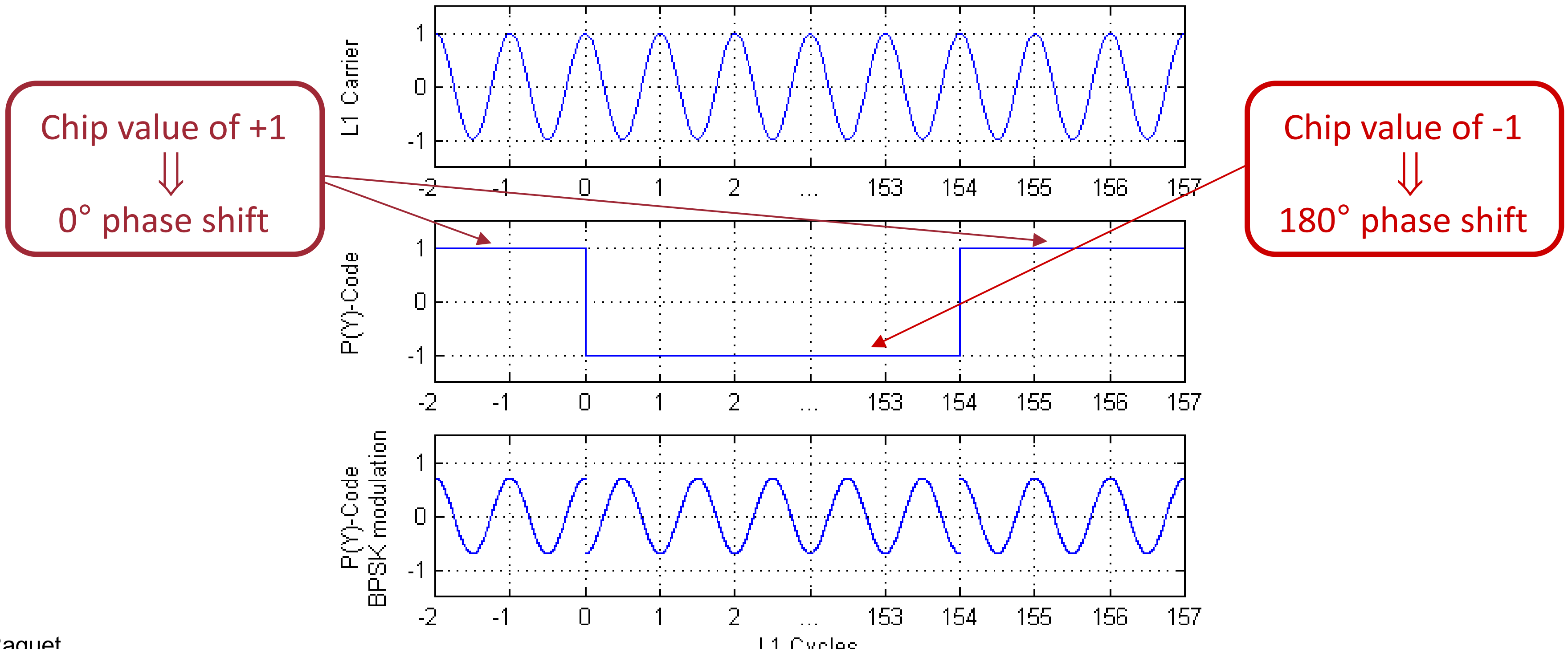


P-Code Encryption for Anti-Spoofing

- **P-code is unclassified and defined in ICD-GPS-200.**
- **Satellites don’t normally transmit P-code, however.**
  - P-code is encrypted by an encryption code
  - The encrypted P-code is called Y-code
    - Often referred to as P(Y)-code
    - Y-code is classified, so unauthorized users cannot
      - Directly lock onto the Y-code
      - Spoof the Y-code (i.e., make a fake signal that appears to be coming from a GPS SV)
    - Correlation techniques exist that allow advanced civilian receivers to lock onto P(Y)-code.
      - Degraded capability vs. direct Y-code tracking
      - Requires C/A-code lock

Code Modulation of Carrier

- **So far, we've covered**
  - GPS L1 and L2 carrier frequencies
  - C/A-code and P-code
- **These need to be combined through modulation**
  - GPS uses biphase shift key (BPSK) modulation



Legacy L1 and L2 Signal Breakdown (Legacy Signals)

- **Note: 50 bps navigation message modulated on all of the codes**
- **L1 signal**
  - P-code
  - C/A-code modulated on carrier that is 90° out of phase from P-code carrier

$$s_{L1}(t) = \overbrace{A_{P_{L1}} Y(t) N(t) \cos(\omega_1 t)}^{\text{P-Code}} + \overbrace{A_{C/A} CA(t) N(t) \sin(\omega_1 t)}^{\text{C/A-Code}}$$

$$N(t) = \text{50 bps navigation message}$$

$$A_{P_{L1}} = \text{Amplitude of L1 P-code signal} \approx \text{-163 dBW}$$

$$A_{C/A} = \text{Amplitude of C/A-code signal} \approx \text{-160 dBW}$$

$$\omega_1 = 2\pi f_{L1}$$

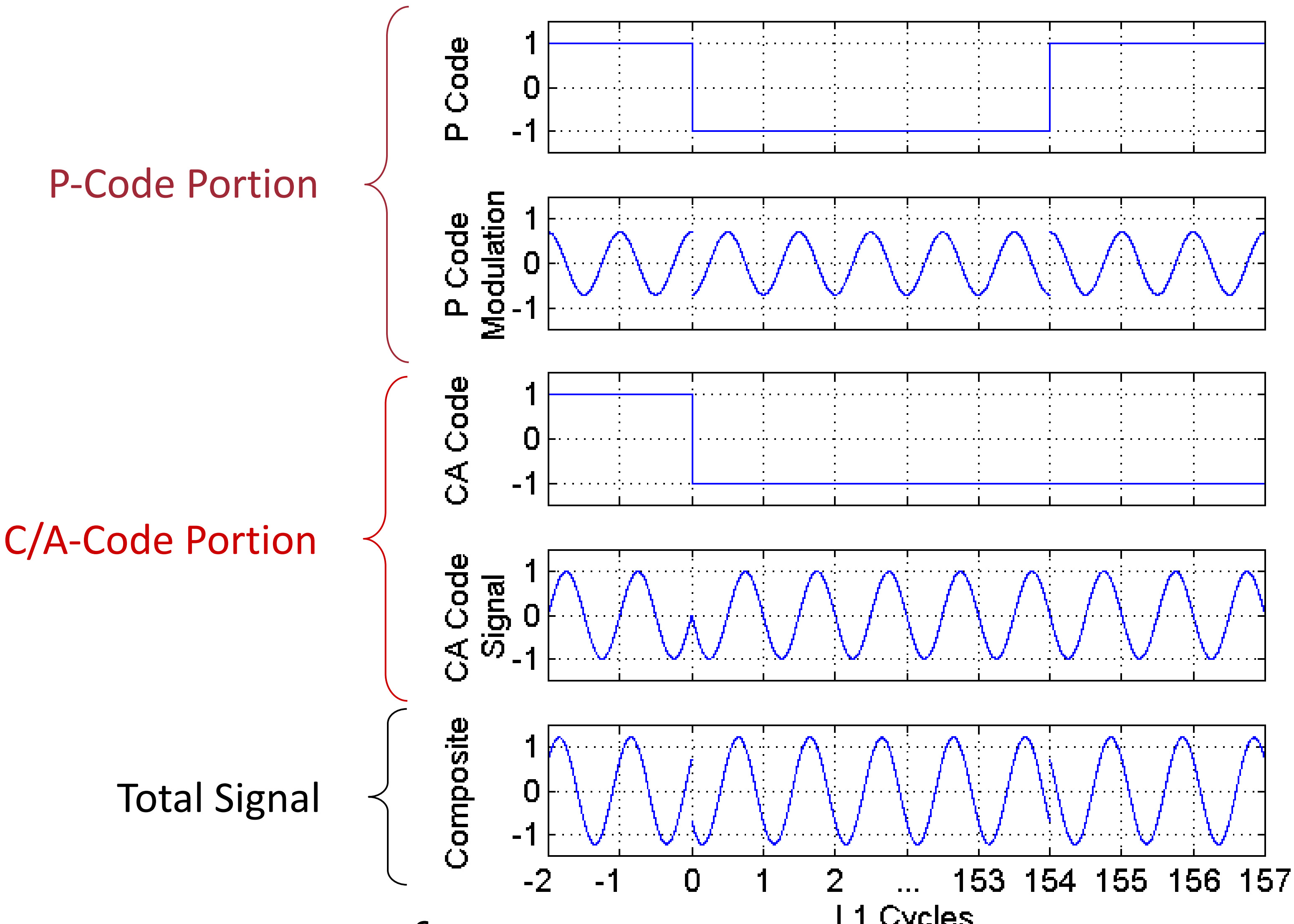
- **L2 signal**
  - P-code only

$$s_{L1}(t) = \overbrace{A_{P_{L2}} Y(t) N(t) \cos(\omega_2 t)}^{\text{P-Code}}$$

$$A_{P_{L2}} = \text{Amplitude of L2 P-code signal} \approx \text{-166 dBW}$$

$$\omega_2 = 2\pi f_{L2}$$

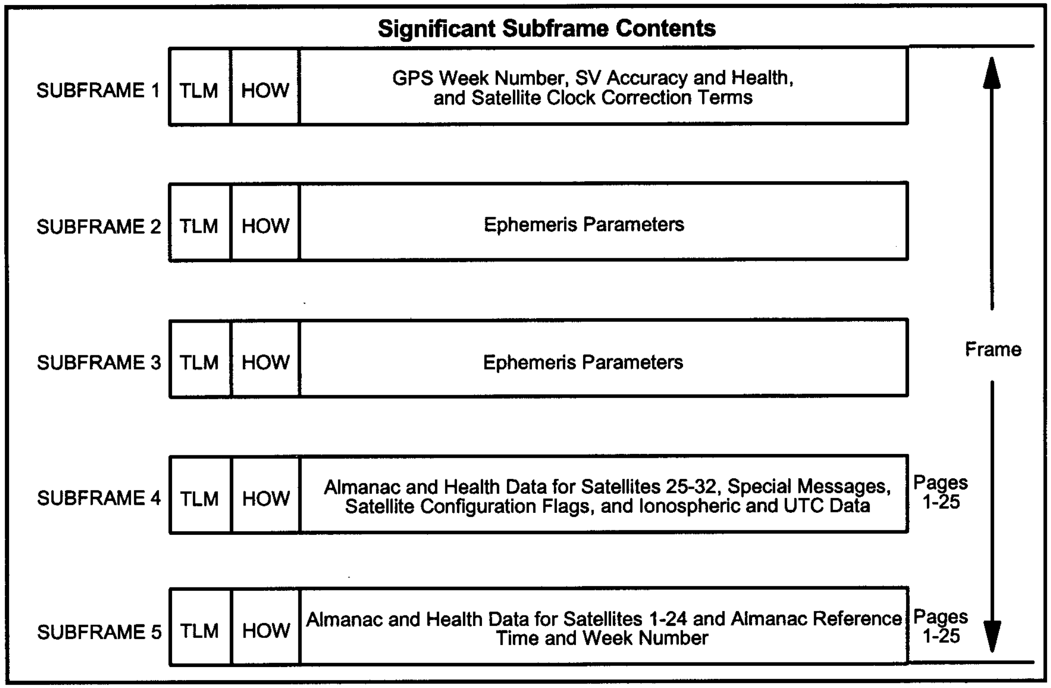
Sample of How L1 Signal is Generated



LNAV - Legacy GPS Navigation Message

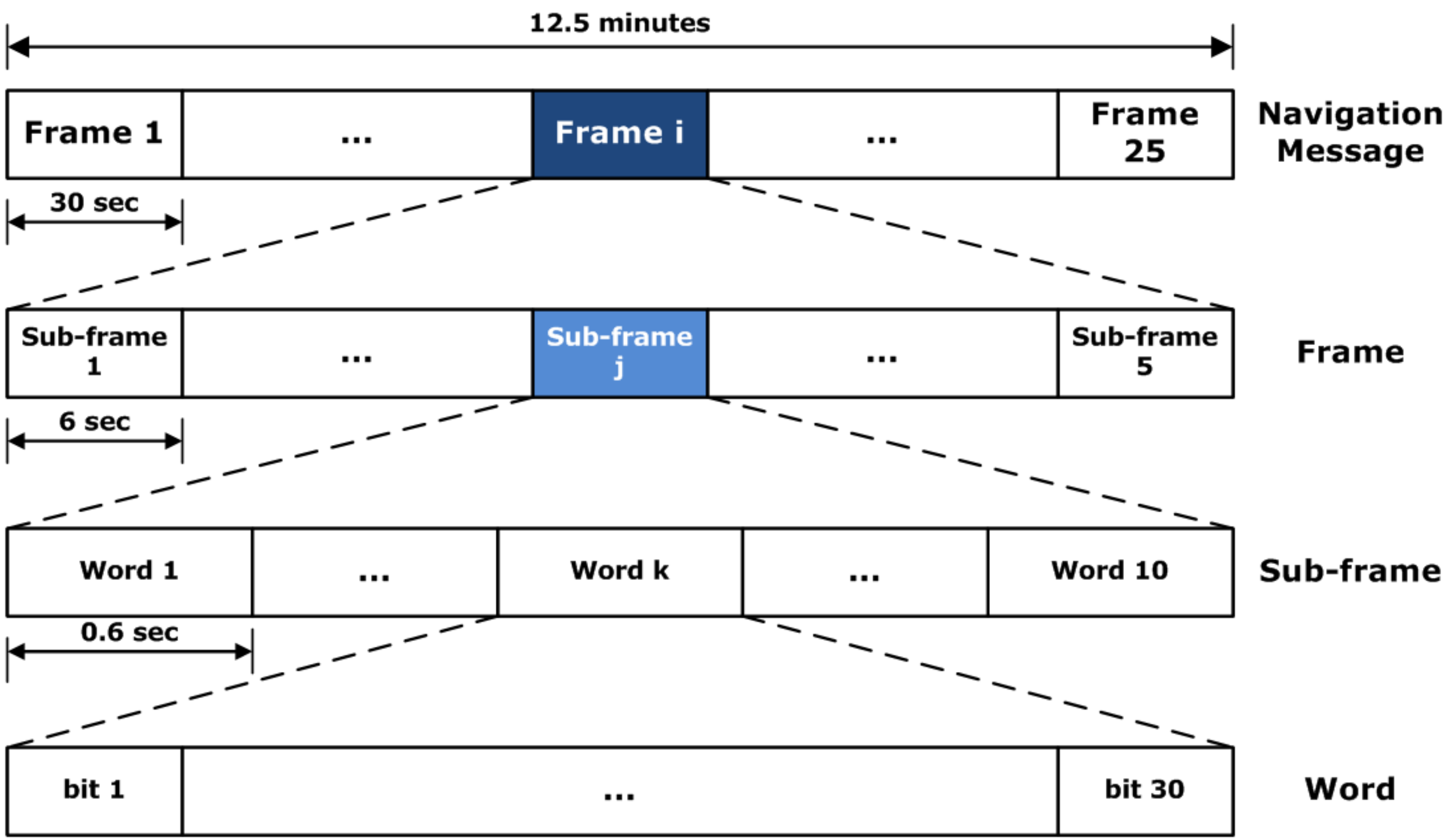
- **In addition to the C/A or P(Y)-codes, the signal is also modulated with the 50 bit/sec navigation message**
  - One "frame" is 1500 bits (30 seconds), and is broken into 5 300-bit "sub-frames" (6 seconds each):

Subframes 4 and 5 change each frame
(Total of 12.5 minutes to cycle through all messages)



- **Navigation message is combined with code**
  - For 1/0 representation: exclusive-or
  - for 1/-1 representation: multiplication

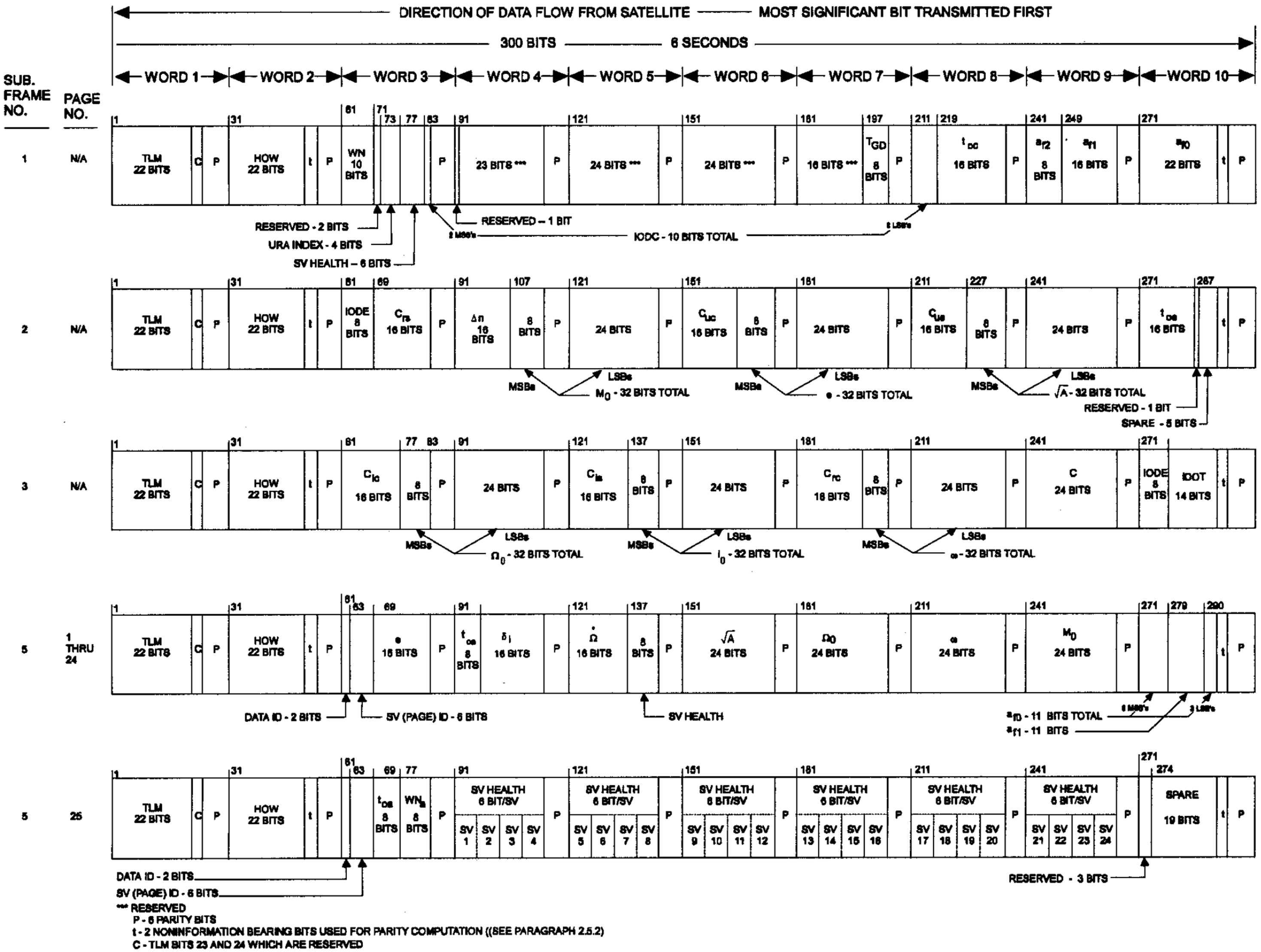
LNAV Structure



LNAV is rigid, fixed-length structure—not much ability to adapt

https://gssc.esa.int/navipedia/index.php/GPS_Navigation_Message

Data Format of Subframes 1, 2, 3, and 5



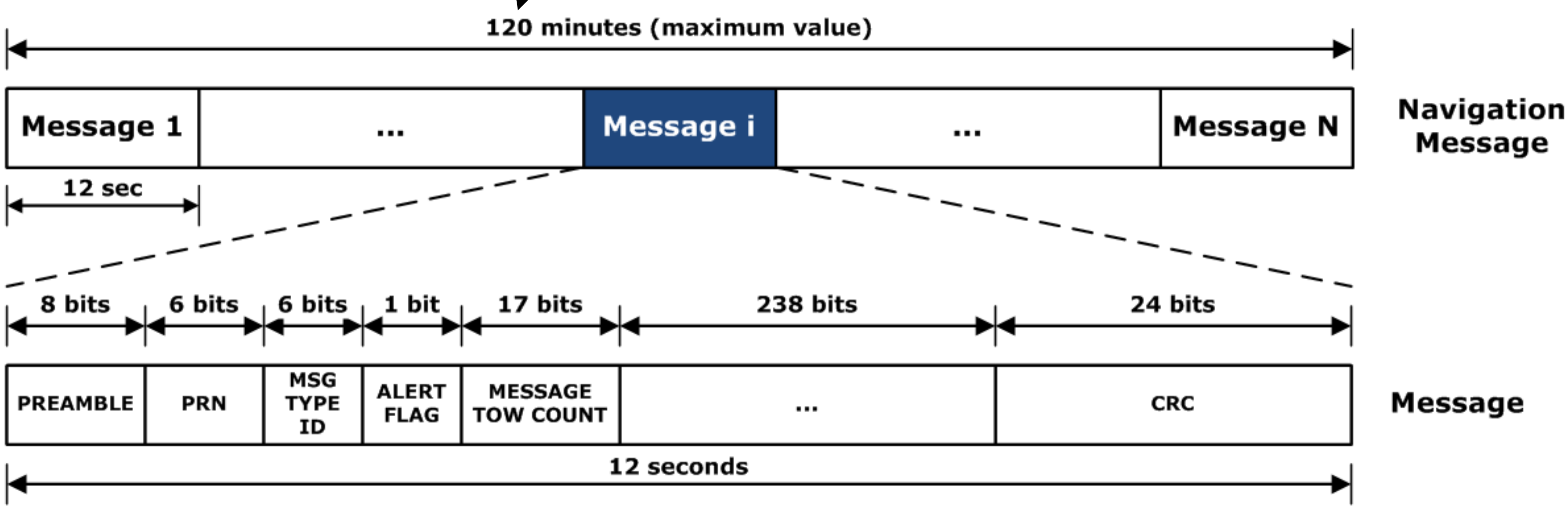
Obtained from SPS Signal Spec (http://www.spacecom.af.mil/usspace/gps_support/documents/SPSMAIN.pdf)

CNAV – Civil Navigation Message

- For L2C and L5 signals
- Much more flexible than current LNAV structure
- Enables more accurate orbit representation
- More modern coding approaches (forward error correction, convolutional code, CRC)
- GPS week now only repeats every 157 years (compared to 19.6 years for part of LNAV message)
- Additional messages (such as GPS-GNSS time offsets)

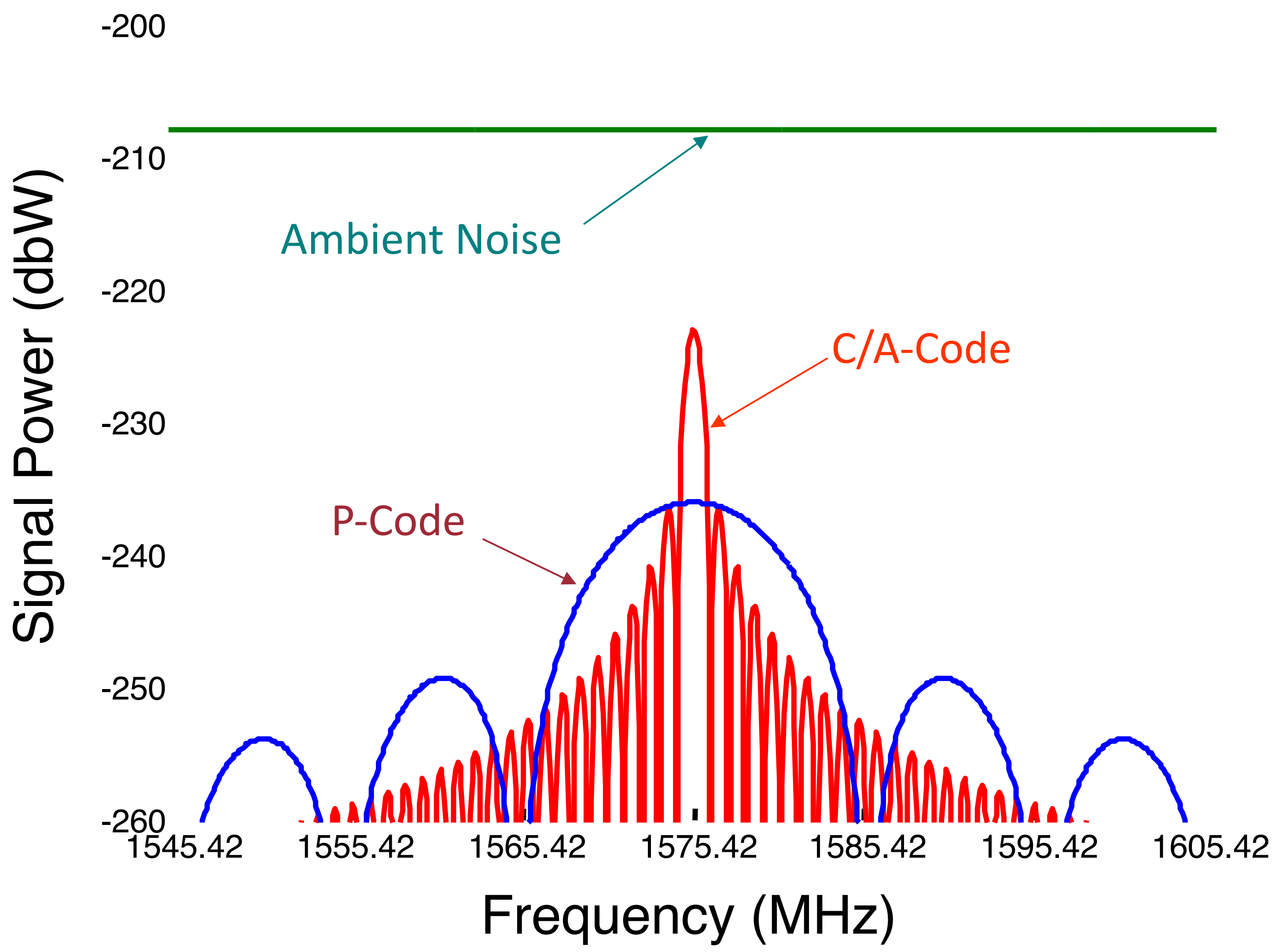
CNAV Structure

For L2C. This value is 60 minutes for L5

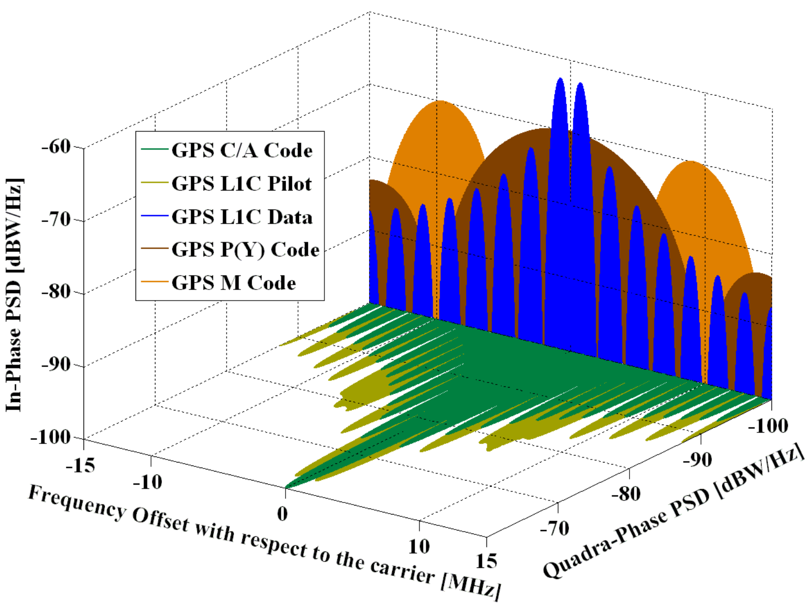


https://gssc.esa.int/navipedia/index.php/GPS_Navigation_Message

Comparison of GPS C/A-Code and P-Code Power Spectral Densities with Noise



GPS L1 Signals



GNSS System	GPS	GPS		GPS	GPS
Service Name	C/A	L1C		P(Y) Code	M-Code
Centre Frequency	1575.42 MHz	1575.42 MHz		1575.42 MHz	1575.42 MHz
Frequency Band	L1	L1		L1	L1
Access Technique	CDMA	CDMA		CDMA	CDMA
Signal Component	Data	Data	Pilot	Data	N.A.
Modulation	BPSK(1)	TMBOC(6,1,1/11)		BPSK(10)	$BOC_{sin}(10,5)$
Sub-carrier frequency [MHz]	-	1.023	1.023 & 6.138	-	10.23
Code frequency	1.023 MHz	1.023 MHz		10.23 MHz	5.115 MHz
Primary PRN Code length	1023	10230		$6.19 \cdot 10^{12}$	N.A.
Code Family	Gold Codes	Weil Codes		Combination and short-cycling of M-sequences	N.A.
Secondary PRN Code length	-	-	1800	-	N.A.
Data rate	50 bps / 50 sps	50 bps / 100 sps	-	50 bps / 50 sps	N.A.
Minimum Received Power [dBW]	-158.5	-157		-161.5	N.A.
Elevation	5°	5°		5°	5°

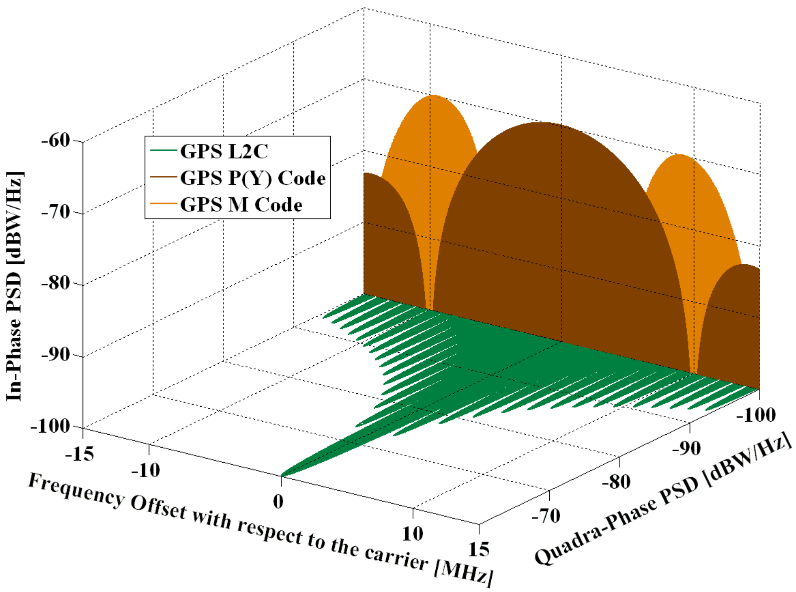
http://www.navipedia.net/index.php/GPS_Signal_Plan

Modernized GPS Signals

- **L2C – Block IIR-M SV's and later**
  - Contains CM and CL Codes (Civilian Moderate and Long)
    - CM has CNAV Data Modulation
    - CL has NO Data Modulation
  - CNAV is half rate of 'standard NAV' and has several important improvements including Forward Error Correction and information to link GPS to other GNSS systems
- **M – Block IIR-M SV's and later**
  - Centered on L1 and L2 frequencies
  - Binary Offset Carrier (BOC) 5.2 w/ bandwidth of 24 MHz
  - Carrier MNAV data (similar to CNAV)
- **L5 – Block IIF (and tested on late IIR-M's)**
  - Two ranging codes transmited- I5 and Q5 (in-phase and quad)
  - I5 and Q5 10,230 bit sequences transmitted at 10.23 MHz

– **L1C – GPS IIIA**

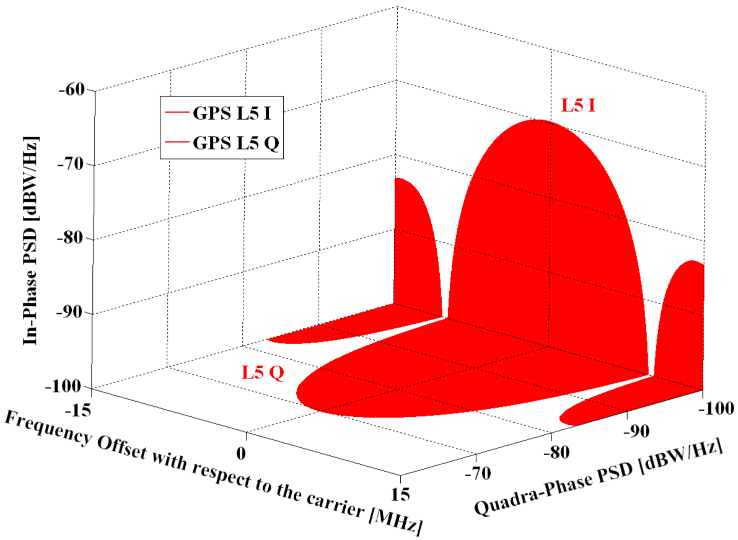
GPS L2 Signals



GNSS System	GPS	GPS	GPS	GPS
Service Name	L2 CM	L2 CL	P(Y) Code	M-Code
Centre Frequency	1227.60 MHz	1227.60 MHz	1227.60 MHz	1227.60 MHz
Frequency Band	L2	L2	L2	L2
Access Technique	CDMA	CDMA	CDMA	CDMA
Spreading modulation	BPSK(1) result of multiplexing 2 streams at 511.5 kHz		BPSK(10)	BOCsin(10,5)
Sub-carrier frequency	-	-	-	10.23 MHz
Code frequency	511.5 kHz	511.5 kHz	10.23 MHz	5.115 MHz
Signal Component	Data	Pilot	Data	N.A.
Primary PRN Code length	10,230 (20 ms)	767,250 (1.5 seconds)	6.19 x 1012	N.A.
Code Family	M-sequence from a maximal polynomial of degree 27		Combination and short-cycling of M-sequences	N.A.
Secondary PRN Code length	-	-	-	N.A.
Data rate	IIF 50 bps / 50 sps IIR-M Also 25 bps 50 sps with FEC	-	50 bps / 50 sps	N.A.
Minimum Received Power [dBW]	II/IIA/IIR -164.5 dBW IIR-M -161.5 dBW IIF -161.5 dBW		II/IIA/IIR -164.5 dBW IIR-M -161.4 dBW IIF -160.0 dBW	N.A.
Elevation	5°		5°	5°

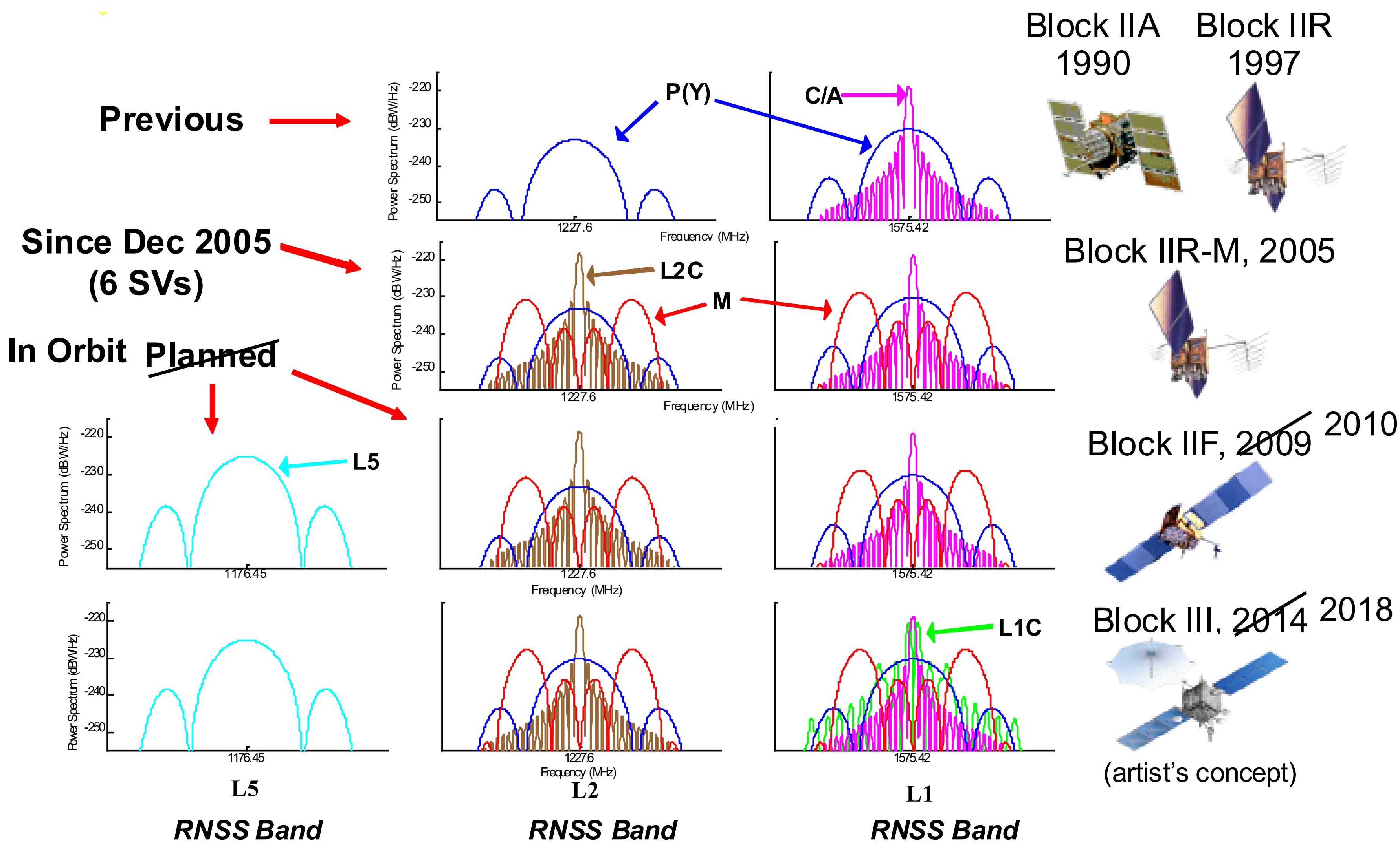
http://www.navipedia.net/index.php/GPS_Signal_Plan

GPS L5 Signals



GNSS System	GPS	GPS
Service Name	L5 I	L5 Q
Centre Frequency	1176.45 MHz	1176.45 MHz
Frequency Band	L5	L5
Access Technique	CDMA	CDMA
Spreading modulation	BPSK(10)	BPSK(10)
Sub-carrier frequency	-	-
Code frequency	10.23 MHz	10.23 MHz
Signal Component	Data	Pilot
Primary PRN Code length	10230	10230
Code Family	Combination and short-cycling of M-sequences	
Secondary PRN Code length	10	20
Data rate	50 bps / 100 sps	-
Minimum Received Power [dBW]	-157.9 dBW	-157.9 dBW
Elevation	5°	5°

GPS Signal Modernization



SECTION 5: GPS RECEIVER MEASUREMENTS

- **What does the receiver measure?**

GNSS Measurements (Overview)

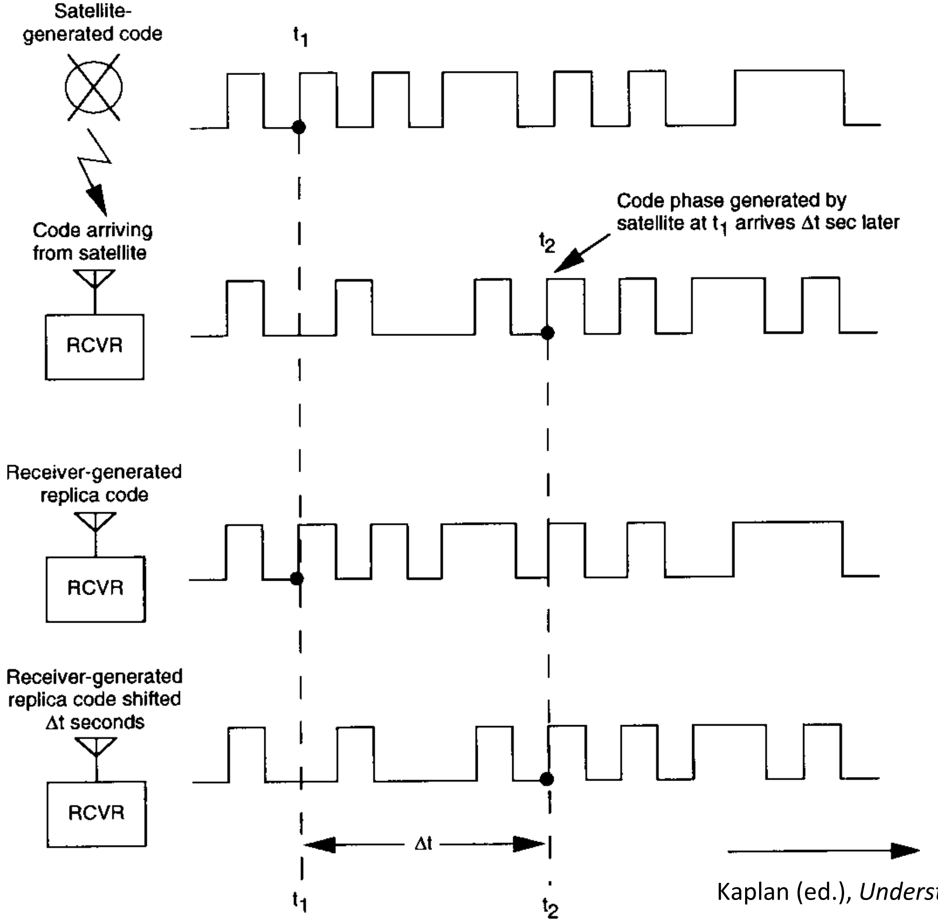
- **Each separate tracking loop typically can give 4 different measurement outputs**
  - Pseudorange measurement
  - Carrier-phase measurement (sometimes called integrated Doppler)
  - Doppler measurement
  - Carrier-to-noise density C/N_0
- **Actual output varies depending upon receiver**
  - NovAtel, Trimble, Leica, etc. give them all
  - RCVR-3A gives just C/N_0
- **Note: We're talking here about *raw measurements***
  - Almost all receivers generate navigation processor outputs (position, velocity, heading, etc.)

Measurement Rates and Timing

- **Most receivers take measurements on all channels/tracking loops simultaneously**
  - Measurements time-tagged with the receiver clock (receiver time)
  - The time at which a set of measurements is called a data epoch.
- **The data rate varies depending upon receiver/application. Typical data rates:**
  - Static surveying: One measurement every 30 seconds (120 measurements per hour)
  - Typical air, land, and marine navigation: 0.5-2 measurement per second (most common)
  - Specialized high-dynamic applications: Up to 50 measurements per second (recent development)

Pseudorange Measurement

- **Pseudorange is a measure of the difference in time between signal transmission and reception**

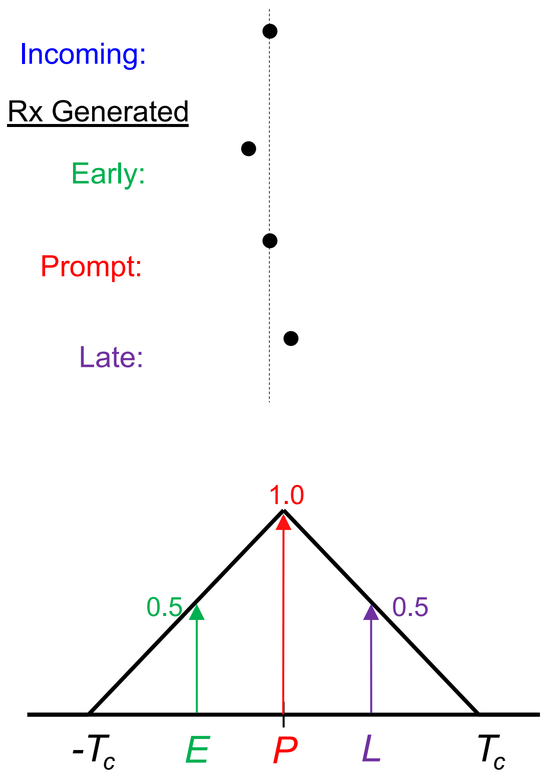


Kaplan (ed.), *Understanding GPS: Principles and Applications*, Artech House, 1996

How the PRN Code is Tracked

Aligned

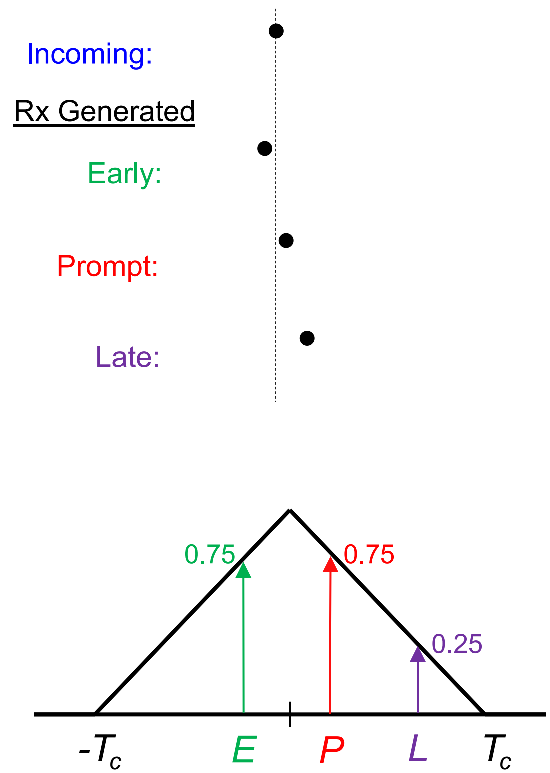
$\tau = 0$



E – L = 0

0.25 Chip Early

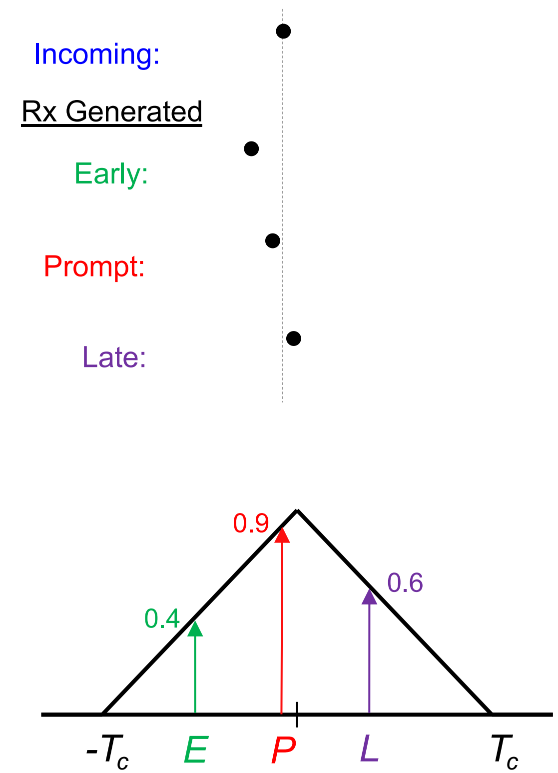
$\tau = 0.25$



E – L = 0.5

0.1 Chip Late

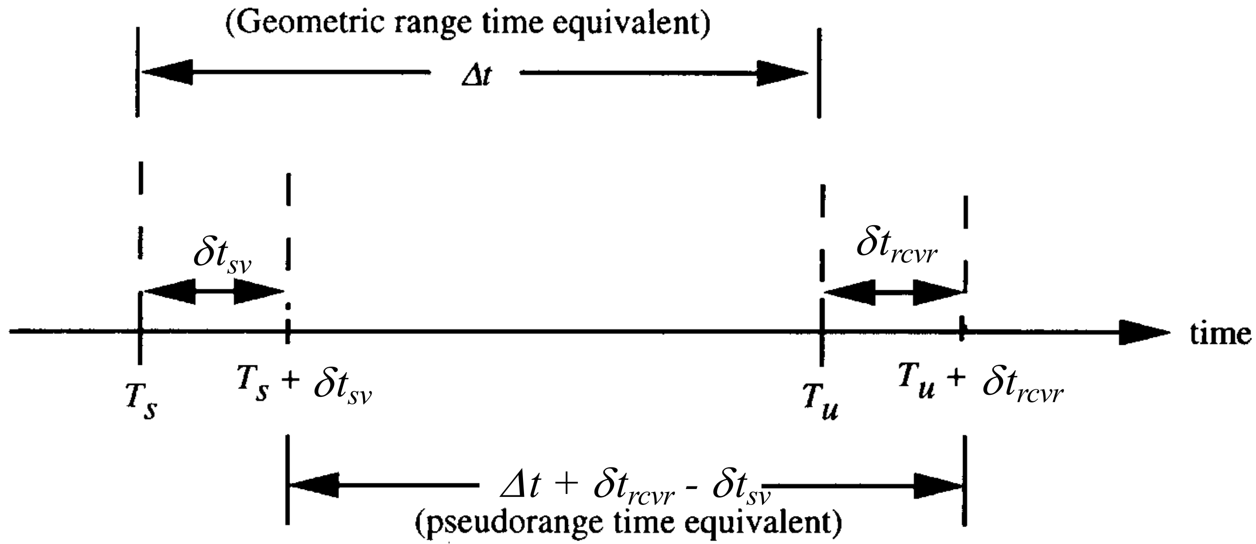
$\tau = -0.1$



E – L = -0.2

Effect of Clock Errors on Pseudorange

- **Since pseudorange is based on time difference, any clock errors will fold directly into pseudorange**



- **Small clock errors can result in large pseudorange errors (since clock errors are multiplied by speed of light)**
- **Satellite clock errors (δt_{sv}) are very small**
  - Satellites have atomic time standards
  - Satellite clock corrections transmitted in navigation message
- **Receiver clock (δt_{rcvr}) is dominant error**

Kaplan (ed.), *Understanding GPS: Principles and Applications*, Artech House, 1996

Doppler Shift
(The "Original" Satellite Navigation)

- **For electromagnetic waves (which travel at the speed of light), the received frequency f_R is approximated using the standard Doppler equation**

$$f_R = f_T\left(1 - \frac{(\boldsymbol{v}_r \cdot \boldsymbol{a})}{c}\right)$$

f_R = received frequency (Hz)

f_T = transmitted frequency (Hz)

$\boldsymbol{v}_r$ = satellite-to-user relative velocity vector (m/s)

$\boldsymbol{a}$ = unit vector pointing along line-of-sight from user to SV

c = speed of light (m/s)

  – Note that $\boldsymbol{v}_r$ is the (vector) velocity difference

$$\boldsymbol{v}_r = \boldsymbol{v} - \dot{\boldsymbol{u}}$$

$\boldsymbol{v}$ = velocity vector for satellite (m/s)

$\dot{\boldsymbol{u}}$ = velocity vector for user (m/s)

- **The Doppler shift Δf is then**

$$\Delta f = f_R - f_T \quad \text{(Hz)}$$

Doppler Measurement

- **The GPS receiver locks onto the carrier of the GPS signal and measures the received signal frequency**
  - Relationship between true and measured received signal frequency:

$$f_{R_{meas}}$$

$$f_R = f_{R_{meas}}(1 + \delta \dot{t}_{rcvr})$$

$$f_R = \text{true received signal frequency (Hz)}$$

$$f_{R_{meas}} = \text{measured received signal frequency (Hz)}$$

$$\delta \dot{t}_{rcvr} = \text{receiver clock drift rate (sec/sec)}$$

  - Doppler measurement formed by differencing the measured received frequency and the transmit frequency:

$$\Delta f_{meas} = f_{R_{meas}} - f_T$$

  - Note: transmit frequency is calculated using information about SV clock drift rate given in navigation message

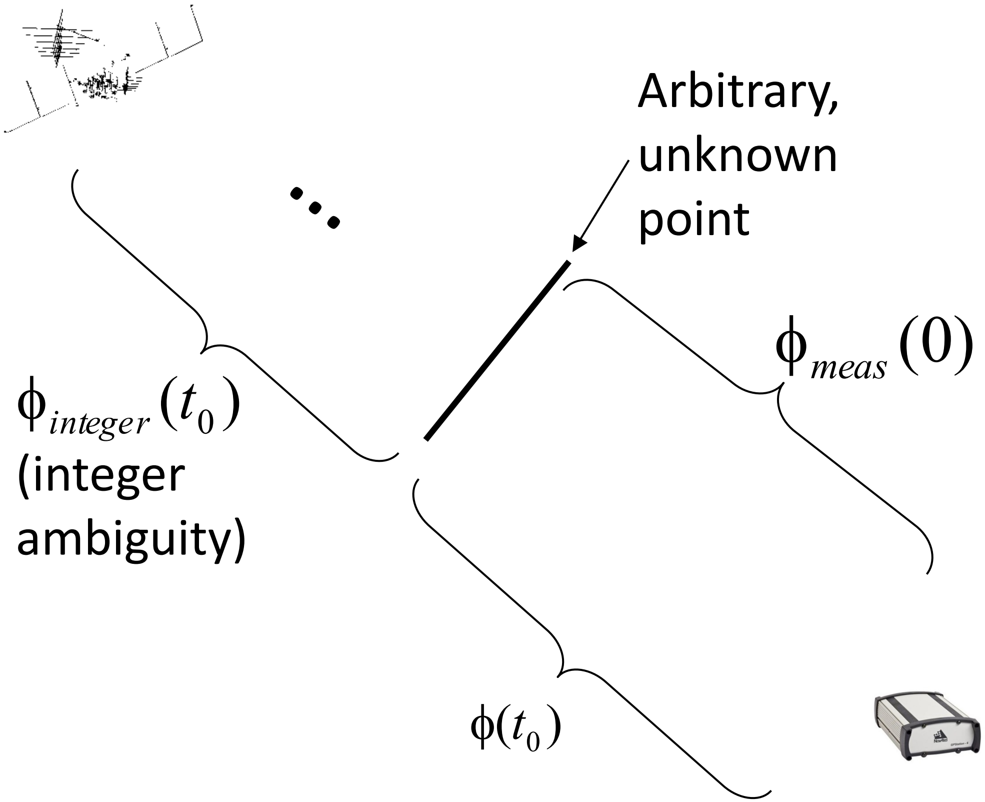
Carrier-Phase (Integrated Doppler) Measurement

- The carrier-phase measurement $\phi_{meas}(t)$ is calculated by integrating the Doppler measurements

$$\text{range}(t) = \underbrace{\int_{t_o}^{t} \Delta f_{meas}(t)dt + \phi(t_0)}_{\substack{\phi_{meas}(t) \\ \text{(can be measured by receiver)}}} + \phi_{integer}(t_0) + \text{clock error} + \text{other errors}$$

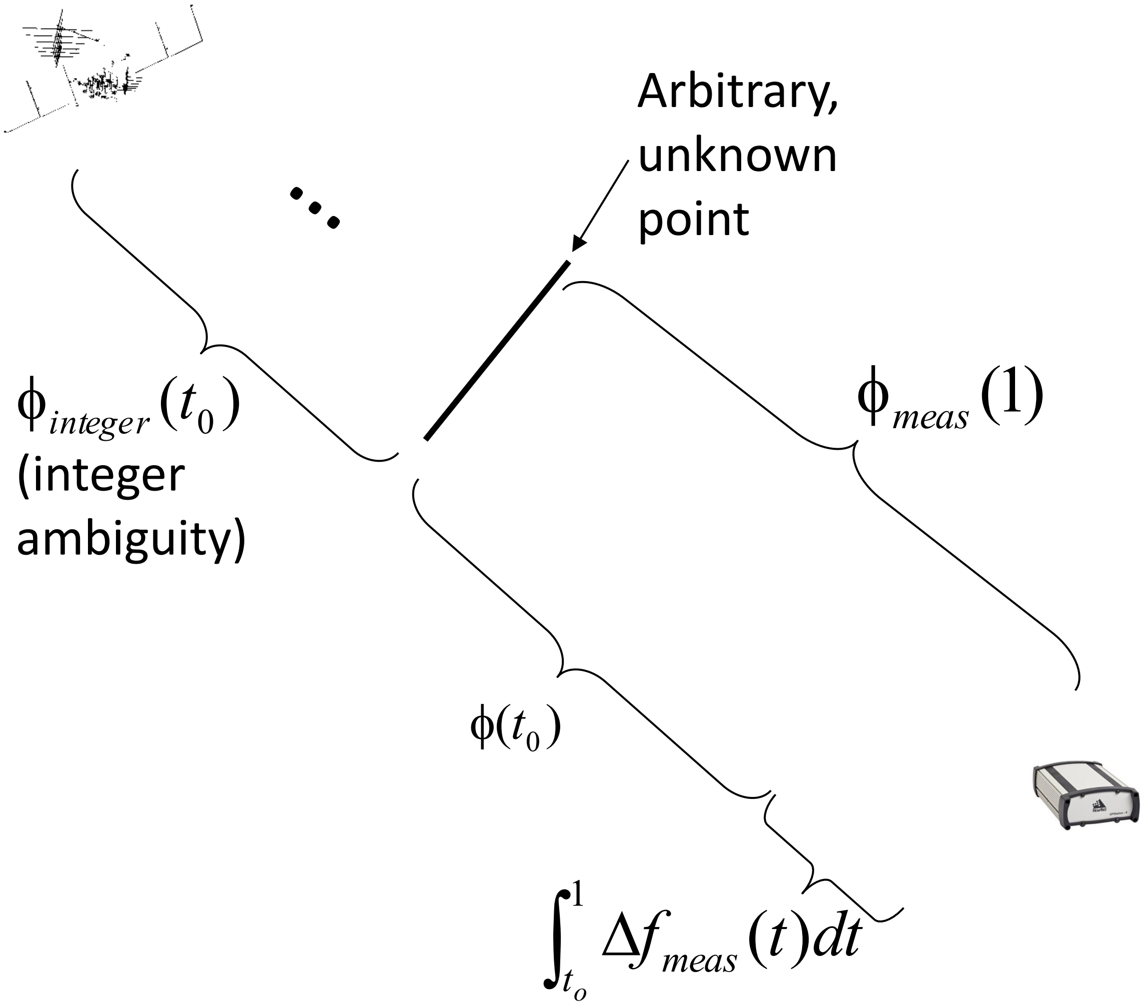
- The integer portion of the initial carrier-phase at the start of the integration ($\phi_{integer}(t_0)$) is known as the “carrier-phase integer ambiguity”
  - Because of this ambiguity, the carrier-phase measurement is not an absolute measurement of position
  - Advanced processing techniques can be used to resolve these carrier-phase ambiguites (carrier-phase ambiguity resolution)
- Alternative way of thinking: carrier-phase measurement is the “beat frequency” between the incoming carrier signal and receiver generated carrier.

Phase Tracking Example
At Start of Phase Lock (Time = 0 seconds)



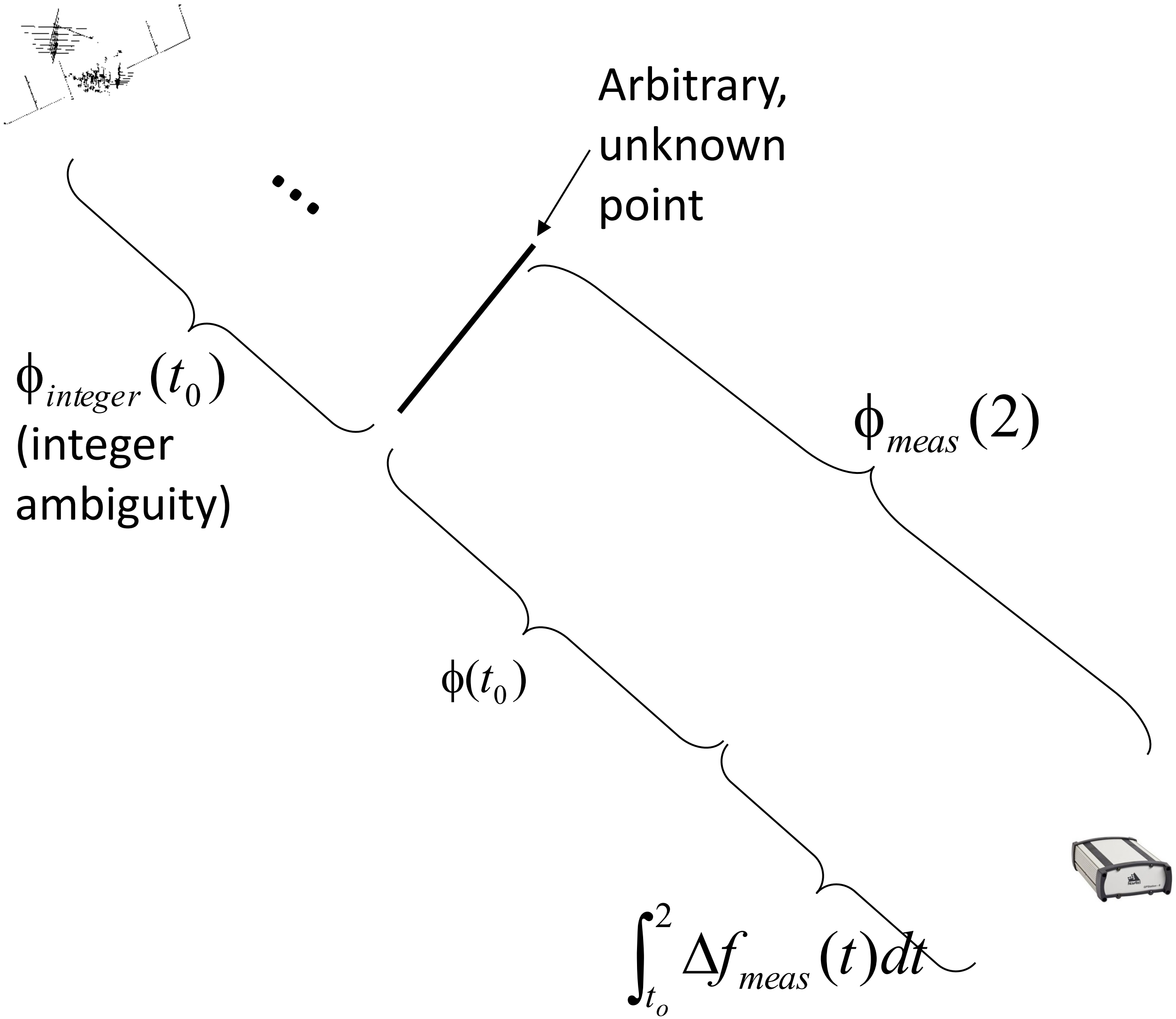
Ignoring clock and other errors

Phase Tracking Example
After Movement (for 1 Second)



Ignoring clock and other errors

Phase Tracking Example
After Movement (for 2 Seconds)



Ignoring clock and other errors

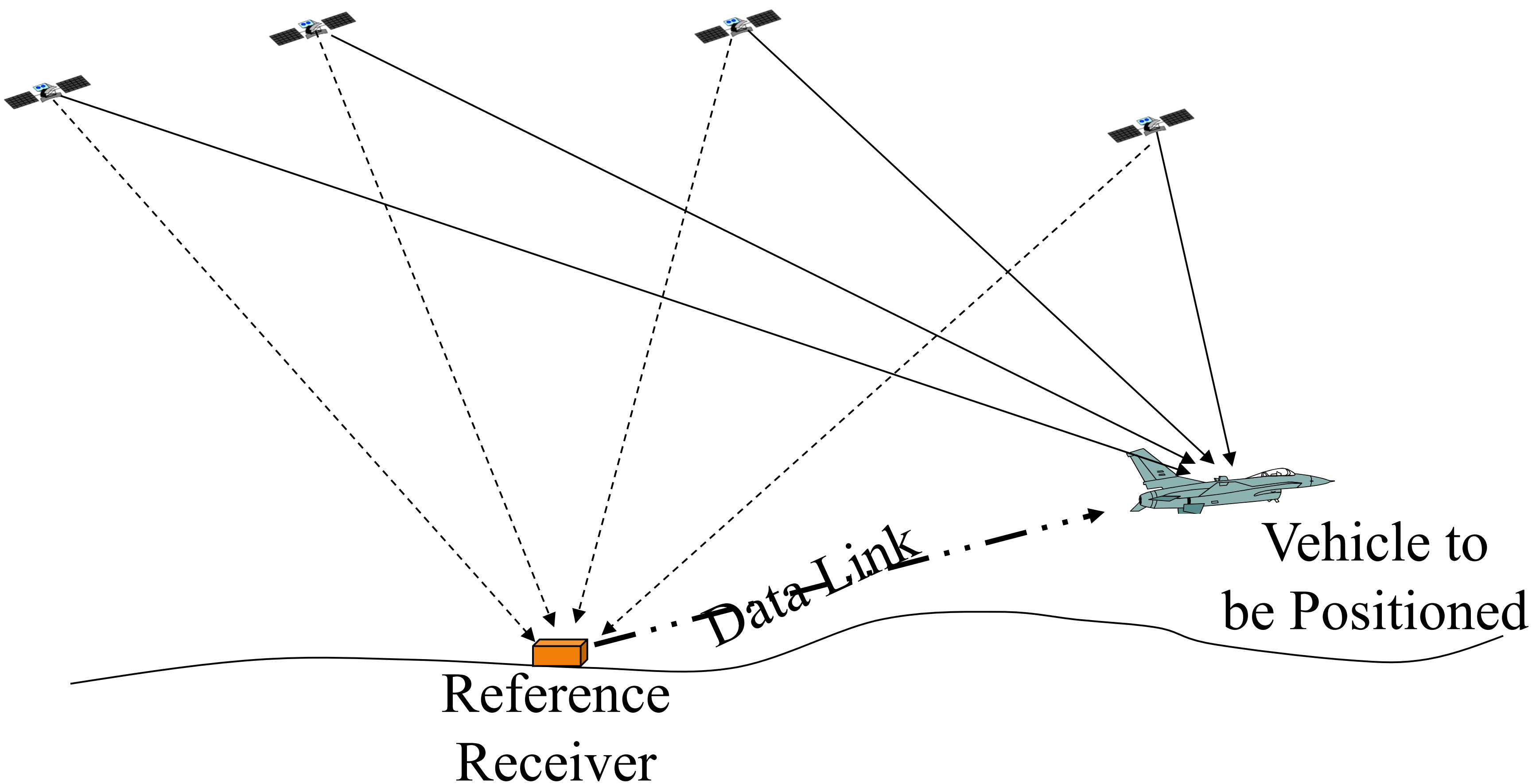
Comparison Between Pseudorange and Carrier-Phase Measurements

	Pseudorange	Carrier-Phase
Type of measurement	Range (absolute)	Range (ambiguous)
Measurement precision	~1 m	~0.01 m
Robustness	More robust	Less robust (cycle slips possible)

Carrier-to-Noise Density (C/N_0)

- **The carrier-to-noise density is a measure of signal strength**
  - The higher the C/N_0, the stronger the signal (and the better the measurements)
  - Units are dB-Hz
  - General rules-of-thumb:
    - $C/N_0 > 40$: Very strong signal
    - $32 < C/N_0 < 40$: Marginal signal
    - $C/N_0 < 32$: Probably losing lock (unless using high sensitivity receiver)
- **C/N_0 tends to be receiver-dependent**
  - Can be calculated many different ways
  - Absolute comparisons between receivers not very meaningful
  - Relative comparisons between measurements in a single receiver are very meaningful

Real-time Differential Concept



(For post-processing, data would be stored from reference receiver and aircraft and processed after the flight is over.)

Typical GPS Accuracy

- **How accurate is GPS?**
- **Definitive answer: It Depends!**

	Mode	Approximate Horizontal Accuracy (drms)
Stand-Alone	Civilian receiver, SA on (historical)	100 m
	Civilian receiver, SA off (current) / with WAAS	2-10 m / 0.5-1 m
	Military receiver (dual frequency)	2 m
Differential	Code differential	1 – 2 m
	Carrier-smoothed code differential	0.1 – 1 m
	Precise carrier-phase (kinematic)	1 – 2 cm
	Precise carrier-phase (static)	1 – 2 mm

SECTION 6: GLONASS

GLONASS Webstite (9 May 2022)

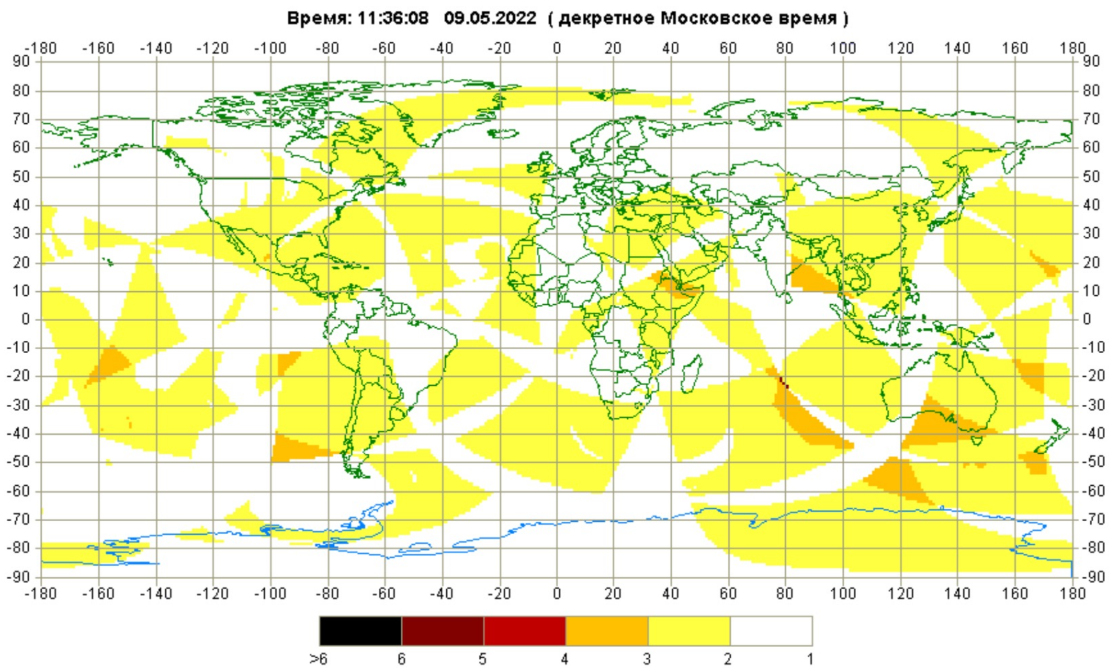
https://www.glonass-iac.ru

GLONASS constellation status at 09.05.2022

Total satellites in constellation	25
In operation	23
In commissioning phase	0
In maintenance	2
Under check by the Satellite Prime Contractor	0
Spares	0
In flight tests phase	0

Current values of position geometry factor PDOP on the Earth surface (angle ≥ 5°).

Date: 2022-05-09



SC GLONASS current position 11:36 (UTC+3) 09.05.22

orbital plane #1 orbital plane #2 orbital plane #3

GLONASS Signal

- **GLONASS uses Frequency Division Multiple Access (FDMA)**
  - All satellites transmit same PRN code, but at different frequencies
  - More costly receiver design
  - Better interference rejection
    - Interference at a given frequency will affect only the satellite transmitting at that frequency
    - Cross-correlation between PRN codes is not an issue
- **Like GPS, each GLONASS satellite transmits on two L-band carrier frequencies (L1 and L2)**
  - L1 includes 0.511 MHz CA-code and 5.11 MHz P-code
  - L2 includes 5.11 MHz P-code
  - 50 bps navigation message modulated onto L1 and L2
  - CA-code has 1ms repeat rate
  - P-code has 1s repeat rate
    - Actual maximal-length P-code repeats at 6.57s intervals, but it’s truncated at 1s

GLONASS Frequencies

- **Carrier frequencies**

$$f_{L1} = 1602 + 0.5625K \text{ MHz}$$

$$f_{L2} = 1246 + 0.4375K \text{ MHz}$$

  – Frequency shift underway to move GLONASS out of radio astronomy band
    - Until 1998: K = 0 to 12
    - 1998-2005: K = -7 to 12
    - After 2005: K = -7 to 4
  – Frequency sharing by anti-podal satellites
  – Ratio of L1 to L2 frequencies is 9/7
  – Note that adjacent CA-codes operate near the “null” of each other
  – Adjacent satellites have cross-correlation levels not exceeding 48 dB
    - Better than GPS

GLONASS Navigation Messages

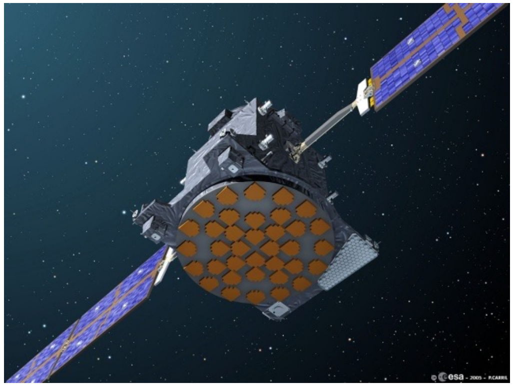
- **Like GPS, GLONASS transmits a 50 bps navigation message that's modulated on CA-code and P-code**
- **Unlike GPS, the GLONASS navigation message is different for CA-code and P-code**
  - CA-code navigation message
    - Precise ephemeris (position, velocity, and acceleration rather than Keplerian parameters)
      - Time to acquire: 30 seconds
    - Almanac data (Keplerian parameters)
      - - Time to acquire: 2.5 minutes
    - Epoch timing
    - Synchronization bits
    - Error correction bits
    - Satellite health
    - Age of data
  - P-code navigation message
    - Not published, but empirically studied
    - Precise ephemeris
      - Time to acquire: 10 seconds
    - Almanac data
      - Time to acquire: 12 minutes

Comparison Between GPS and GLONASS

	GLONASS	GPS
Number of Satellites	24	24
Number of orbital planes	3	6
Spacing within orbital plane	45 deg	varied
Orbital inclination	64.8 deg	55 deg
Orbital radius	25,510 km	26,560 km
Orbital period	11 hours, 15 min	11 hours, 58 min
Ground track repeat	8 siderial days 1 siderial day for next slot	1 siderial day
Datum	PZ-90	WGS-84
Time reference	UTC(SU)	UTC(USNO)
Access method	FDMA	CDMA
Carrier frequencies	L1: 1602+0.5625K L2: 1246+0.4375K K=-7 to 12 (4)	L1: 1575.42 L2: 1227.60
Code	CA-code on L1 P-code on L1 and L2	CA-code on L1 P(Y)-code on L1 and L2
Code frequency	CA-code: 0.511 MHz P-code: 5.11 MHz	CA-code: 1.023 MHz P-code: 10.23 MHz
Crosscorrelation interference	-48 dB	-21.6 dB
Number of code elements	CA-code: 511 P-code: 5110000	CA-code: 1023 P-code: 2.35E14
Selective availability	No	Yes
Anti-spoofing	No	Yes
Navigation message rate	50 bps	50 bps
Navigation message length	2.5 min	12.5 min

SECTION 7: GALILEO

Galileo Overview



Graphic: ESA

- **Constellation**
  - 30 Satellites (MEO)
  - 56 deg inclination
- **Signals**
  - Generally reusing GPS frequency spectrum
  - Dual frequency planned for standard users from the beginning
- **Levels of service**
  - **Open Access Navigation**: This will be 'free to air' and for use by the mass market; Simple timing and positioning down to 1m.
  - **Commercial Navigation** (Encrypted): High accuracy to the cm; Guaranteed service for which service providers will charge fees.
  - **Safety Of Life Navigation**: Open service; For applications where guaranteed accuracy is essential; Integrity messages will warn of errors.
  - **Public Regulated Navigation** (Encrypted): Continuous availability even in time of crisis; Government agencies will be main users.
  - **Search And Rescue**: System will pick up distress beacon locations; Feasible to send feedback, confirming help is on its way.

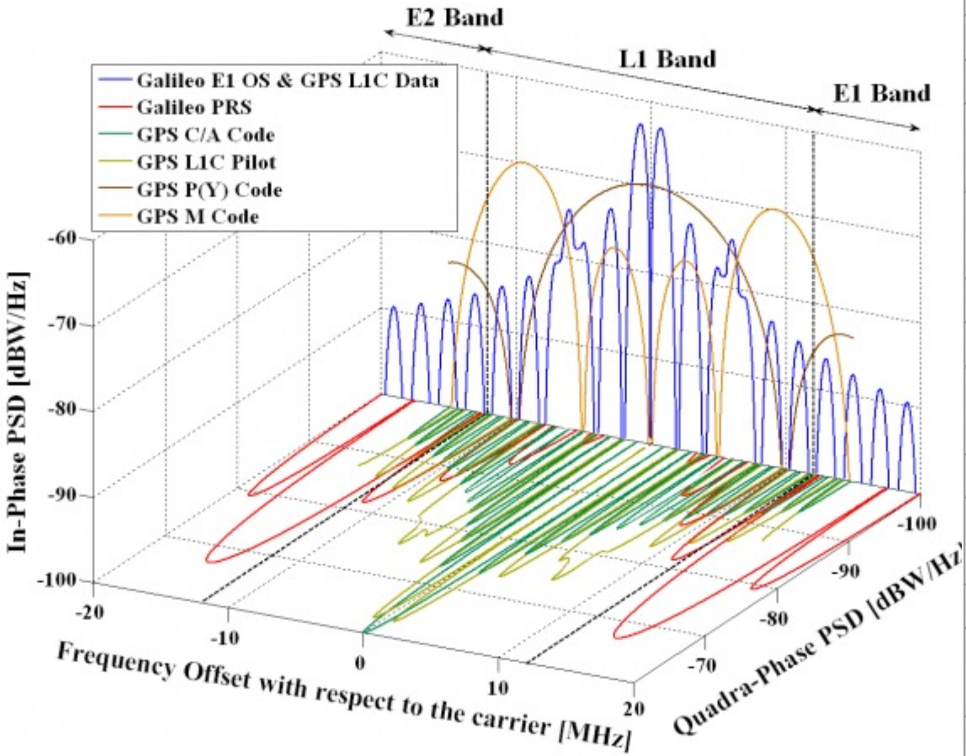
Galileo Satellite History (current as of 9 May 2022)

Summary of satellites, as of 7 December 2021

Block	Launch period	Satellite launches			In operation and healthy
		Full success	Failure	Planned	
GIOVE	2005–2008	2	0	0	0
IOV	2011–2012	4	0	0	3
FOC	From 2014	22	2[α]	10	21
G2G	From 2024	0	0	12	0
Total		28	2	22	24

α. ^ Partial failure

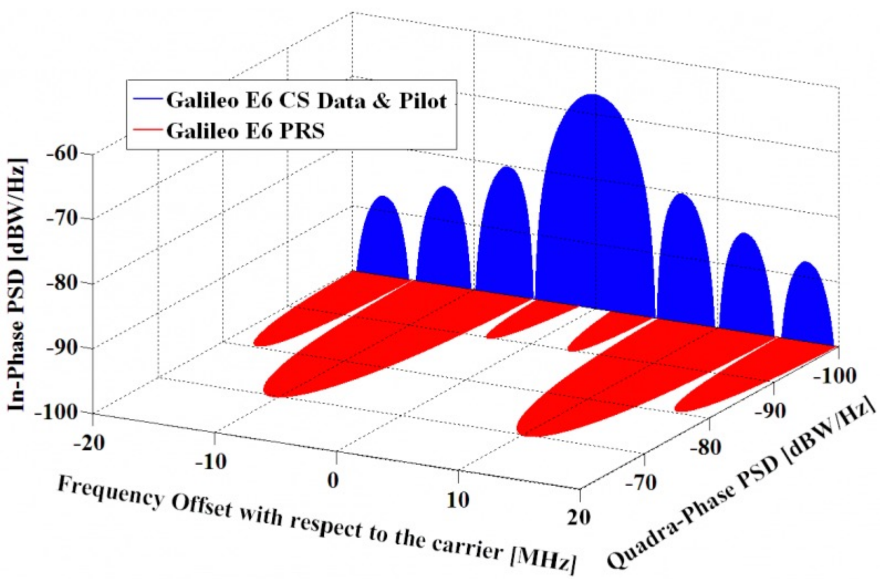
Galileo (and GPS) L1 Signals



GNSS System	Galileo	Galileo	Galileo
Service Name	E1 OS		PRS
Centre Frequency	1575.42 MHz		
Frequency Band	E1		
Access Technique	CDMA		
Spreading modulation	CBOC(6,1,1/11)		$BOC_{cos}(15,2.5)$
Sub-carrier frequency	1.023 MHz and 6.138 (Two sub-carriers)		15.345 MHz
Code frequency	1.023 MHz		2.5575 MHz
Signal Component	Data	Pilot	Data
Primary PRN Code length	4092		N/A
Code Family	Random Codes		N/A
Secondary PRN Code length	-	25	N/A
Data rate	250 sps	-	N/A
Minimum Received Power [dBW]	-157		N/A
Elevation	10°		N/A

http://www.navipedia.net/index.php/GALILEO_Signal_Plan

Galileo E6 Band

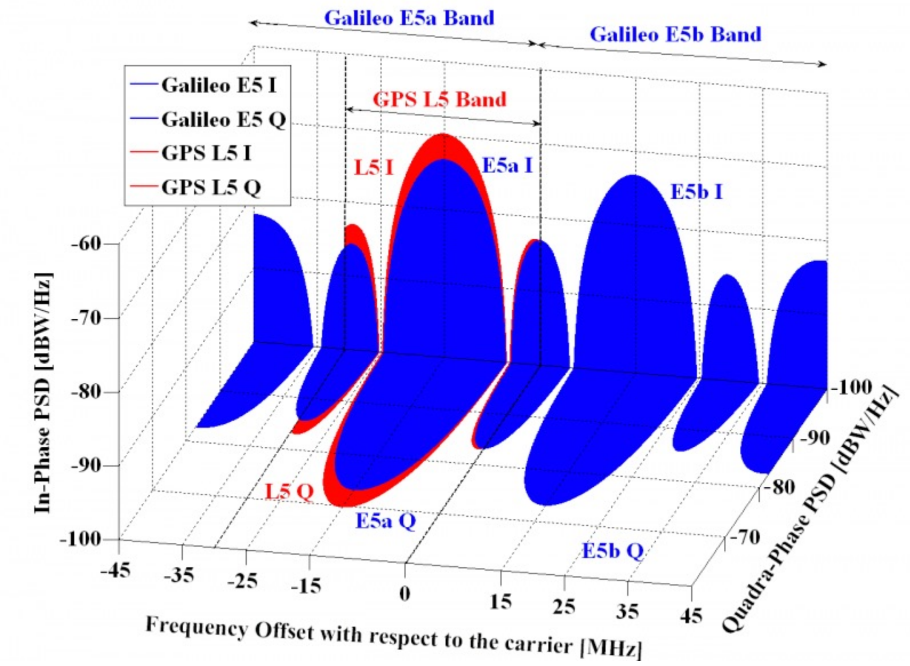


GNSS System	Galileo	Galileo	Galileo
Service Name	E6 CS data	E6 CS pilot	E6 PRS
Centre Frequency	1278.75 MHz		
Frequency Band	E6		
Access Technique	CDMA		
Spreading modulation	BPSK(5)	BPSK(5)	$BOC_{cos}(10,5)$
Sub-carrier frequency	-	-	10.23 MHz
Code frequency	5.115 MHz		
Signal Component	Data	Pilot	Data
Primary PRN Code length	5115	5115	N/A
Code Family	Memory codes		N/A
Secondary PRN Code length	-	100	N/A
Data rate	1000 sps	-	N/A
Minimum Received Power [dBW]	-155		N/A
Elevation	10°		N/A

CS – Commercial Service
PRS – Public Regulated Service

http://www.navipedia.net/index.php/GALILEO_Signal_Plan

Galileo E5 Band



GNSS System	Galileo	Galileo	Galileo	Galileo
Service Name	E5a data	E5a pilot	E5b data	E5b pilot
Centre Frequency	1191.795 MHz			
Frequency Band	E5			
Access Technique	CDMA			
Spreading modulation	AltBOC(15,10)			
Sub-carrier frequency	15.345 MHz			
Code frequency	10.23 MHz			
Signal Component	Data	Pilot	Data	Pilot
Primary PRN Code length	10230			
Code Family	Combination and short-cycling of M-sequences			
Secondary PRN Code length	20	100	4	100
Data rate	50 sps	-	250 sps	-
Minimum Received Power [dBW]	-155 dBW		-155 dBW	
Elevation	10°		10°	

http://www.navipedia.net/index.php/GALILEO_Signal_Plan

SECTION 7: BEIDOU

BeiDou (China)

- **Regional System (Beidou 1)**
  - A signal is transmitted skyward by a remote terminal.
  - Each of the geostationary satellites receive the signal.
  - Each satellite sends the accurate time of when each received the signal to a ground station.
  - The ground station calculates the longitude and latitude of the remote terminal, and determines the altitude from a relief map.
  - The ground station sends the remote terminal's 3D position to the satellites.
  - The satellites broadcast the calculated position to the remote terminal.
- **Global System (Beidou 2, formally called "Compass")**
  - 35 SV constellation planned (5 GEO, 27 MEO, 3 IGSO SVs)
  - Public service – 10 m accuracy
  - Licensed military service

BeiDou Satellite History

Summary of satellites, as of 23 June 2020

Block	Launch period	Satellite launches			Currently in orbit and healthy
		Success	Failure	Planned	
1	2000–2006	4	0	0	0
2	2007–2019	20	0	0	12
3	2015–present	35	0	0	30
Total		59	0	0	42

https://en.wikipedia.org/wiki/BeiDou_Navigation_Satellite_System

Final BeiDou Constellation

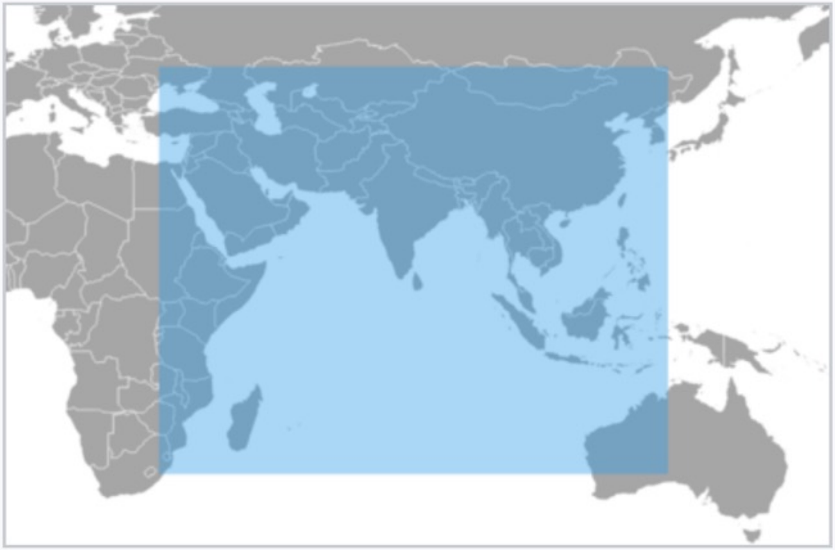
	Geostationary Orbit	Inclined Geosynchronous Orbit)	Medium Earth Orbit
Orbit parmts.	**GEO**	**IGSO**	**MEO**
Semi-Major Axis (Km)	42164	42164	27878
Eccentricity	0	0	0
Inclination (deg)	0	55	55
RAAN (deg)	158.75E, 180E, 210.5E, 240E,260E	218E,98E,338E	
Argument Perigee	0	0	--
Mean anomaly (deg)	0	218E:0,98E:120,338E:240	
# Sats	5	3	27
# Planes	1	3	3

Final BeiDou Constellation

http://www.navipedia.net/index.php/BeiDou_Space_Segment

Indian Regional Satellite Navigation System (IRNSS)

- **Sometimes called NavIC** (acronym for **Nav**igation with **I**ndian **C**onstellation)
- **Constellation of geosynchronous satellites**
  - 3 in geostationary orbit
  - 5 in inclined geosynchronous orbit
- **Stand-alone system**
  - Does not require any other GNSS
- **Continuous coverage over India and surrounding areas**



Coverage of NavIC

https://en.wikipedia.org/wiki/Indian_Regional_Navigation_Satellite_System

SECTION 8: AUGMENTATION SYSTEMS

Augmentation Systems

- **WAAS (Wide Area Augmentation System) – US**
  - Declared operationsl in Jul 2003
  - 3 geostationary satellites
  - 38 reference stations
- **MSAS (MTSAT Satellite based Augmentation System) – Japan**
  - Declared operational in September, 2007
  - 2 geostationary satellites
- **EGNOS (European Geostationary Navigation Overlay Service) – Europe**
  - Declared operational in Oct 2009
  - 3 geostationary satellites
  - 40 reference stations
- **GAGAN (GPS Aided GEO Augmented Navigation) – India**
  - Declared operational in July, 2013
  - 3 geostationary satellites
  - 15 reference stations
- **QZSS (Quasi-Zenith Satellite System) – Japan**
  - Declared operational in Nov 2018
  - Four satellites
  - Highly elliptical “tundra” orbits (so satellites linger over Japan)
- **Are others**

SBAS Coverage Map

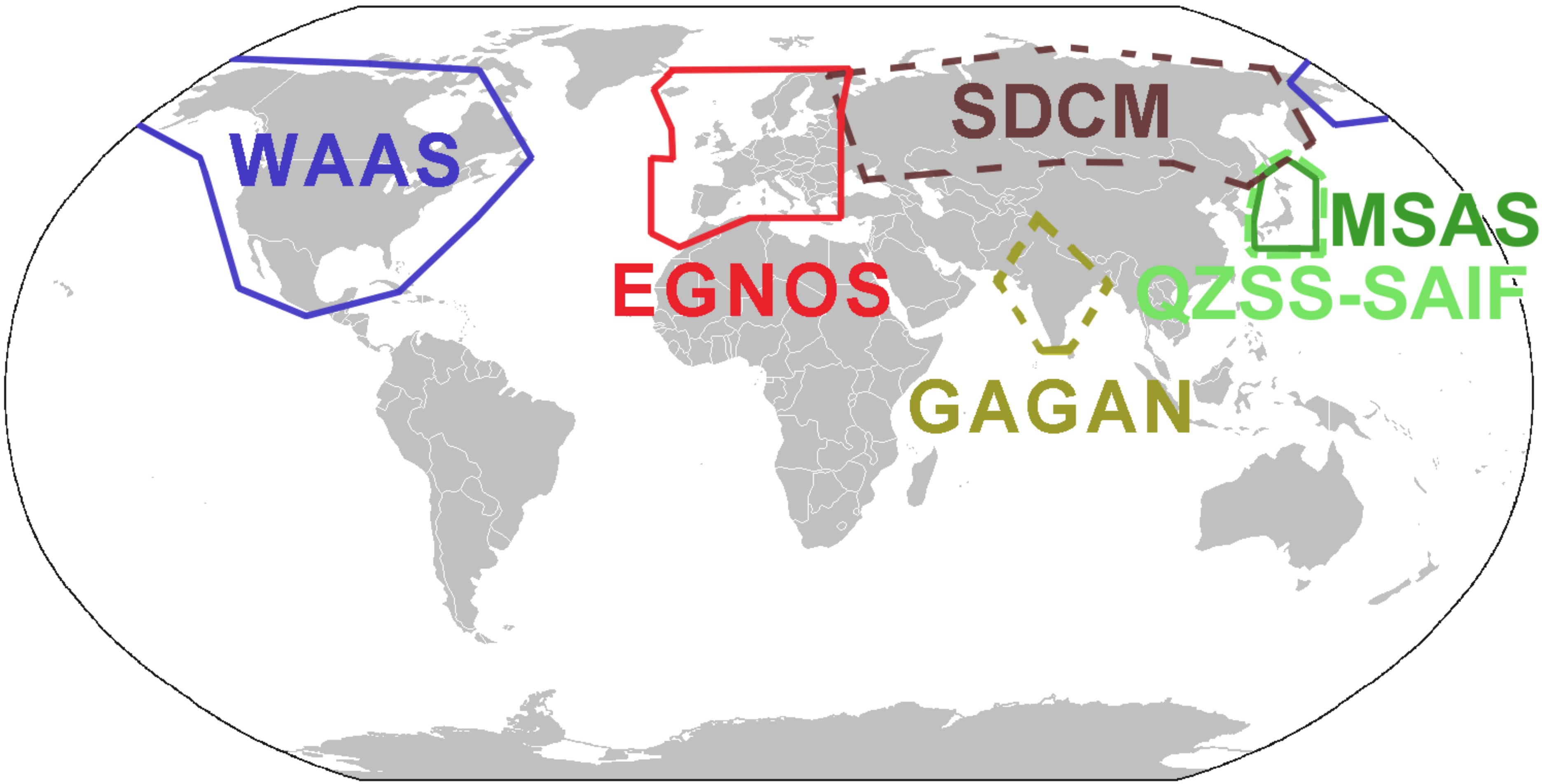


Image from Wikimedia Commons: https://commons.wikimedia.org/wiki/File:SBAS_Service_Areas.png

SBAS Satellites

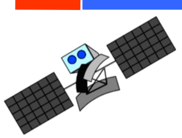
SBAS	SATELLITE	ORBIT LONGITUDE	PRN	SIGNALS
EGNOS	Inmarsat-3-F2/AOR-E	15.5° W	120	L1
	Astra 5B	31.5° E	123	L1/L5
	Artemis	21.5° E	124	L1
	Inmarsat-4-F2	25° E	126	L1
	SES-5	5° E	136	L1/L5
GAGAN	GSAT-8	55° E	127	L1/L5
	GSAT-10	83° E	128	L1/L5
MSAS	MTSAT-1R	140° E	129	L1
	MTSAT-2	145° E	137	L1
QZSS	QZS-1	135° E	183	L1
SDCM	Luch-5A	167° E	140	L1
	Luch-5B	16° W	125	L1
	Luch-5V	95° E	141	L1
WAAS	Intelsat Galaxy 15 (CRW)	133° W	135	L1/L5
	TeleSat Anik F1R (CRE)	107.3° W	138	L1/L5
	Inmarsat-4-F3 (AMR)	98° W	133	L1/L5

 http://gpsworld.com/wp-content/uploads/2012/08/GPSWorld_Almanac_Aug2015_v2.pdf

SECTION 9: GNSS MEASUREMENT ERRORS

Pseudorange Errors

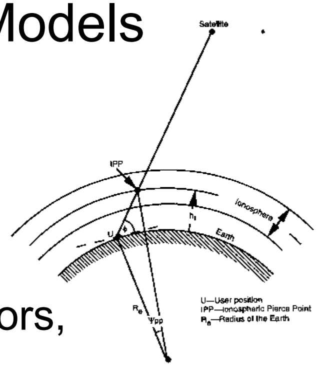
- The pseudorange measurement includes
  - True (geometric) range between receiver and satellite
  - Satellite clock error (or residual error after SV clock correction Δt_{sv} is applied)
  - Receiver clock error
  - Other errors due to atmosphere, selective availability, receiver hardware, etc (δt_D)
    - Lumped together in δt_D term



- For single point positioning
  - Receiver clock error is estimated
  - Other errors are ignored (and cause errors in position and time solution)
  - It’s important to understand these errors

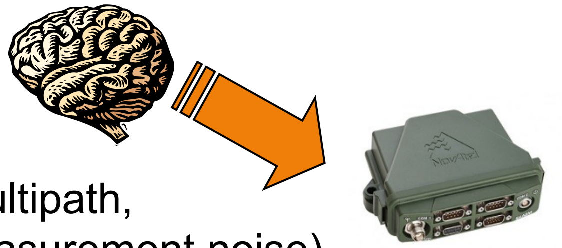
Ways of Correcting Additional Errors

Apply Error Models



(atmospheric errors, multipath(?))

Improved Receiver Processing Algorithms



(multipath, measurement noise)

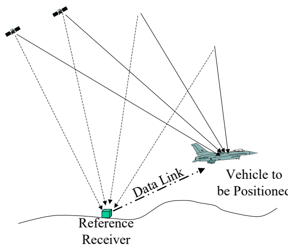
Use Externally Generated Data

SP3

Ionospheric Grid

(SV position, ionosphere)

Differential GPS



(SV position, atmospheric errors, clock errors)

Pseudorange Error Equation

- **Pseudorange ρ is**

$$\rho = r + c(\delta t_u - \delta t_{sv} + \delta t_D)$$

$r = c(T_u - T_s) = c\Delta t =$ geometric range

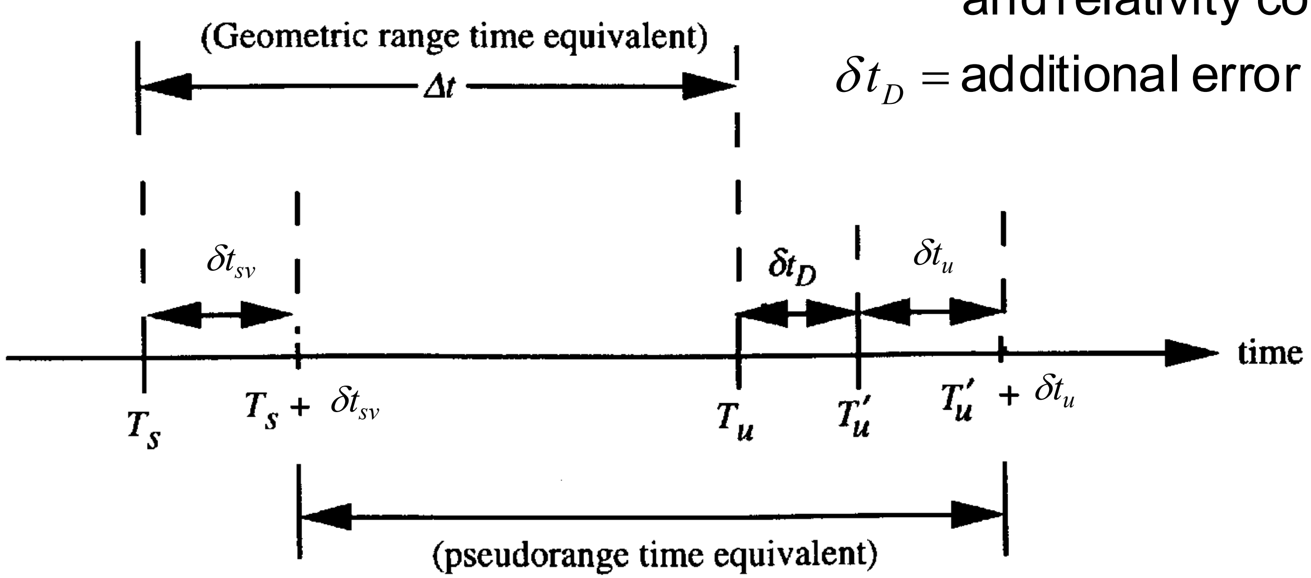
$T_u =$ time signal would have been received if there were no errors (theoretical)

$T_s =$ true signal transmit time

$\delta t_u =$ receiver clock error

$\delta t_{sv} =$ satellite clock error (after polynomial and relativity corrections applied)

$\delta t_D =$ additional error effects



Kaplan, *Understanding GPS: Principles and Applications*, Artech House, 1996

Pseudorange Errors

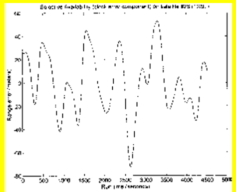
- **All non-clock errors are combined in the δt_D error term**

$$\delta t_D = \delta t_{SA} + \delta t_{iono} + \delta t_{trop} + \delta t_{noise\&res} + \delta t_{mp} + \delta t_{hw}$$

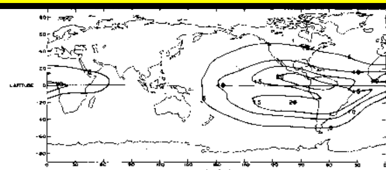
$$\delta t_{SA} = \text{Selective Availability degradation}$$
$$\delta t_{iono} = \text{delay due to ionosphere}$$
$$\delta t_{trop} = \text{delay due to troposphere}$$
$$\delta t_{noise\&res} = \text{receiver noise and resolution error}$$
$$\delta t_{mp} = \text{multipath error}$$
$$\delta t_{hw} = \text{hardware errors (inter-channel or inter-frequency biases)}$$

- **Additionally, satellite position error will contribute to an error in the solution**
  - Not actually a measurement error, but “seems” like a measurement error
- **Each error will be covered separately**
  - Emphasis on pseudorange errors
  - Carrier-phase errors often related

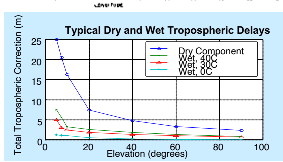
GPS Measurement Errors



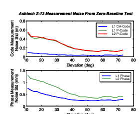
Selective Availability



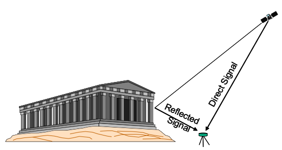
Ionosphere



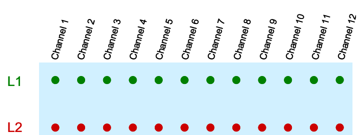
Troposphere



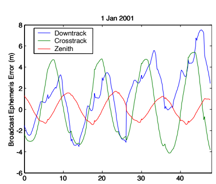
Receiver noise



Multipath



Hardware errors



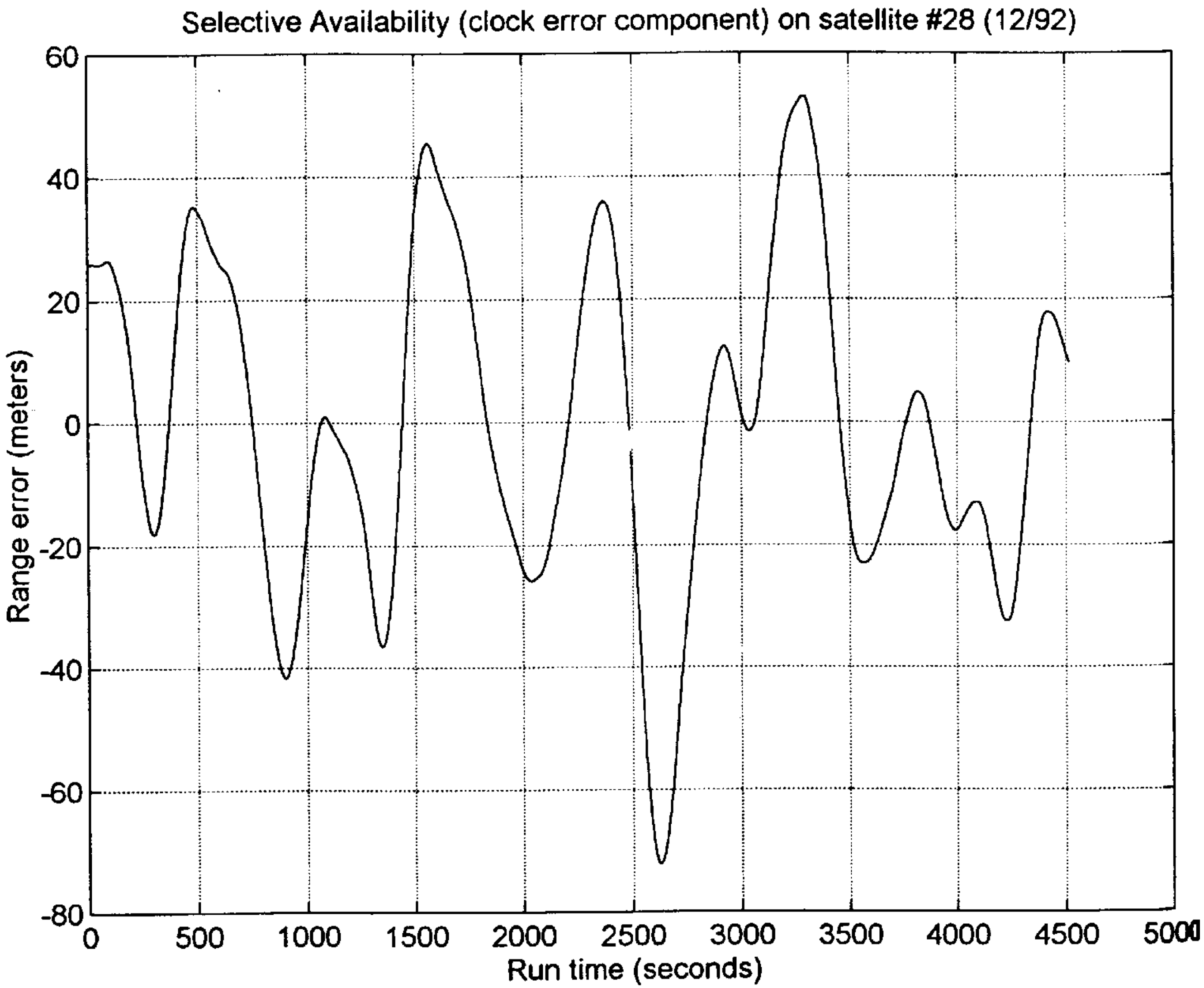
Broadcast ephemeris error

Selective Availability

- **Two components**
  - Epsilon (ε): Errors in the broadcast ephemeris values
    - There are times when this is not apparently implemented (based upon comparisons with precise orbits)[1]
  - Dither (δ): Intentionally changing the frequency of the satellite clock
- **Dominant error source for non-authorized users**
- **Characteristics**
  - From observation
    - Random oscillations
    - Periods of 4-12 minutes
    - Error up to 70 m
  - From DOD/DOT Signal Specification Issues Technical Group (1994)
    - Range rate bound: Not to exceed 2 m/s
    - Range acceleration bound: Not to exceed 19 mm/s^2
    - Range acceleration: 8 mm/s^2 (2σ)
    - Can be changed by National Command Authority

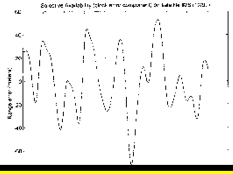
[1] J. Raquet Braasch, et al., "Improved Modeling of GPS Selective Availability", *Proc. Of 1993 ION National Technical Meeting*, San Francisco, CA, 1993

Representative Selective Availability Plot

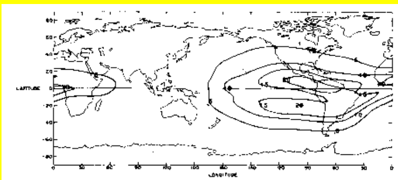


Kaplan, *Understanding GPS: Principles and Applications*, Artech House, 1996.

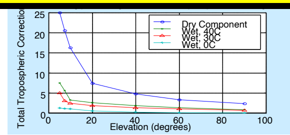
GPS Measurement Errors



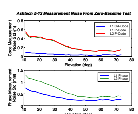
Selective Availability



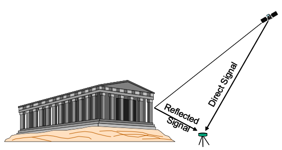
Ionosphere



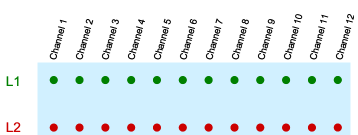
Troposphere



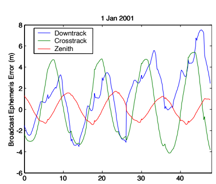
Receiver noise



Multipath



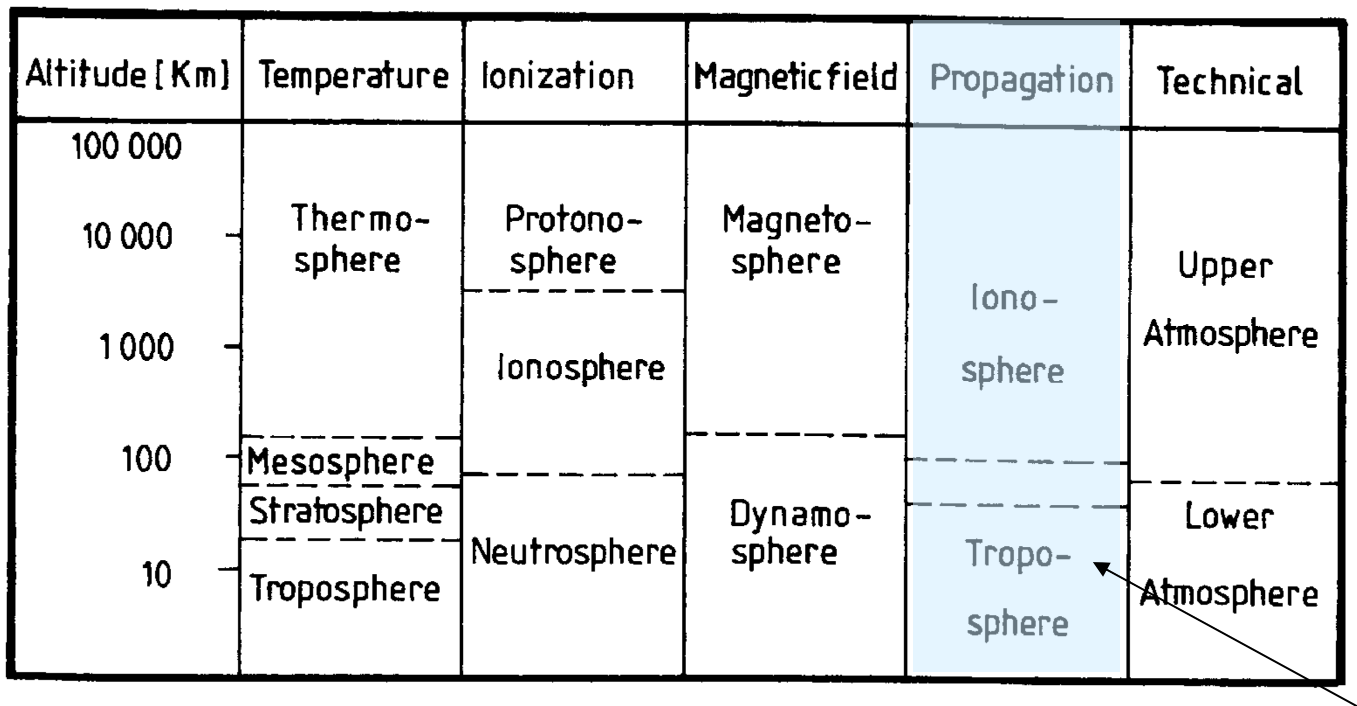
Hardware errors



Broadcast ephemeris error

Atmospheric Errors with GPS

- **There are many different ways to describe the atmosphere**

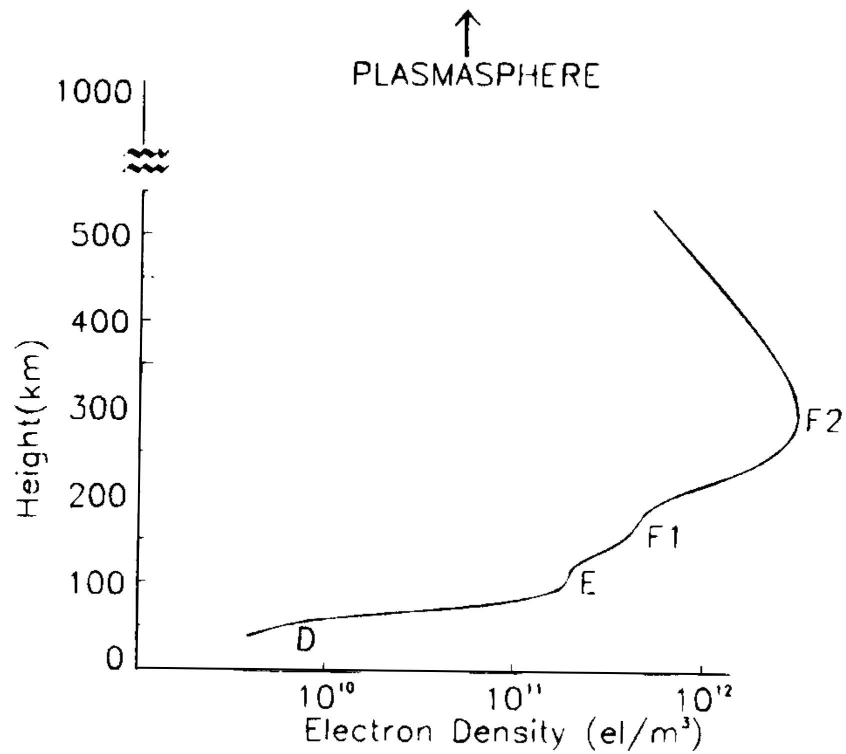


Seeber, *Satellite Geodesy*, Walter de Gruyter & Co., 1993.

- **For GPS, atmosphere usually divided up into troposphere and ionosphere**
  - Troposphere: neutral (uncharged) atmosphere
  - Ionosphere: free ions

Ionosphere

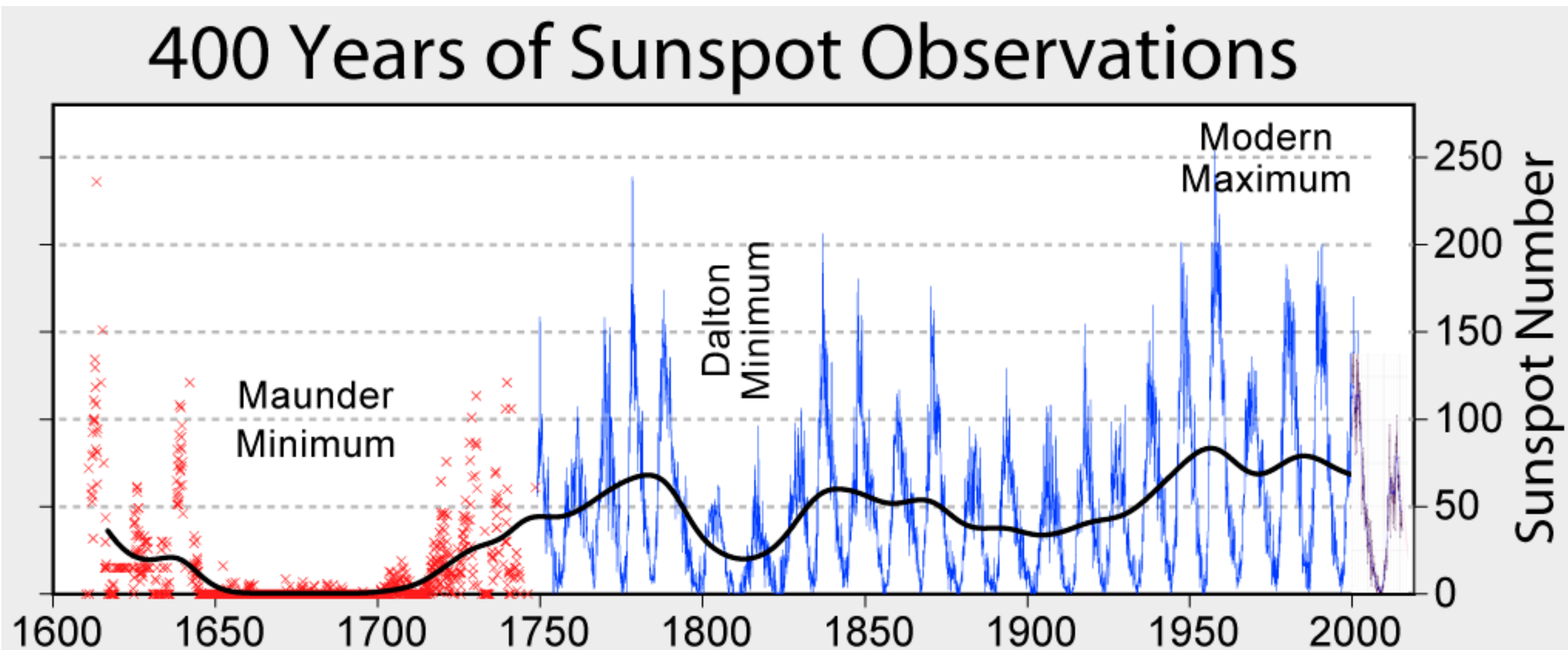
- Weakly ionized plasma that can affect radio waves
  - Caused by ultraviolet radiation from the sun
  - Maximum density between 300-400 km
  - Effects
    - Code delay (phase advance)
    - Doppler shift
    - Refraction (bending) of radio wave
    - Amplitude or phase scintillations



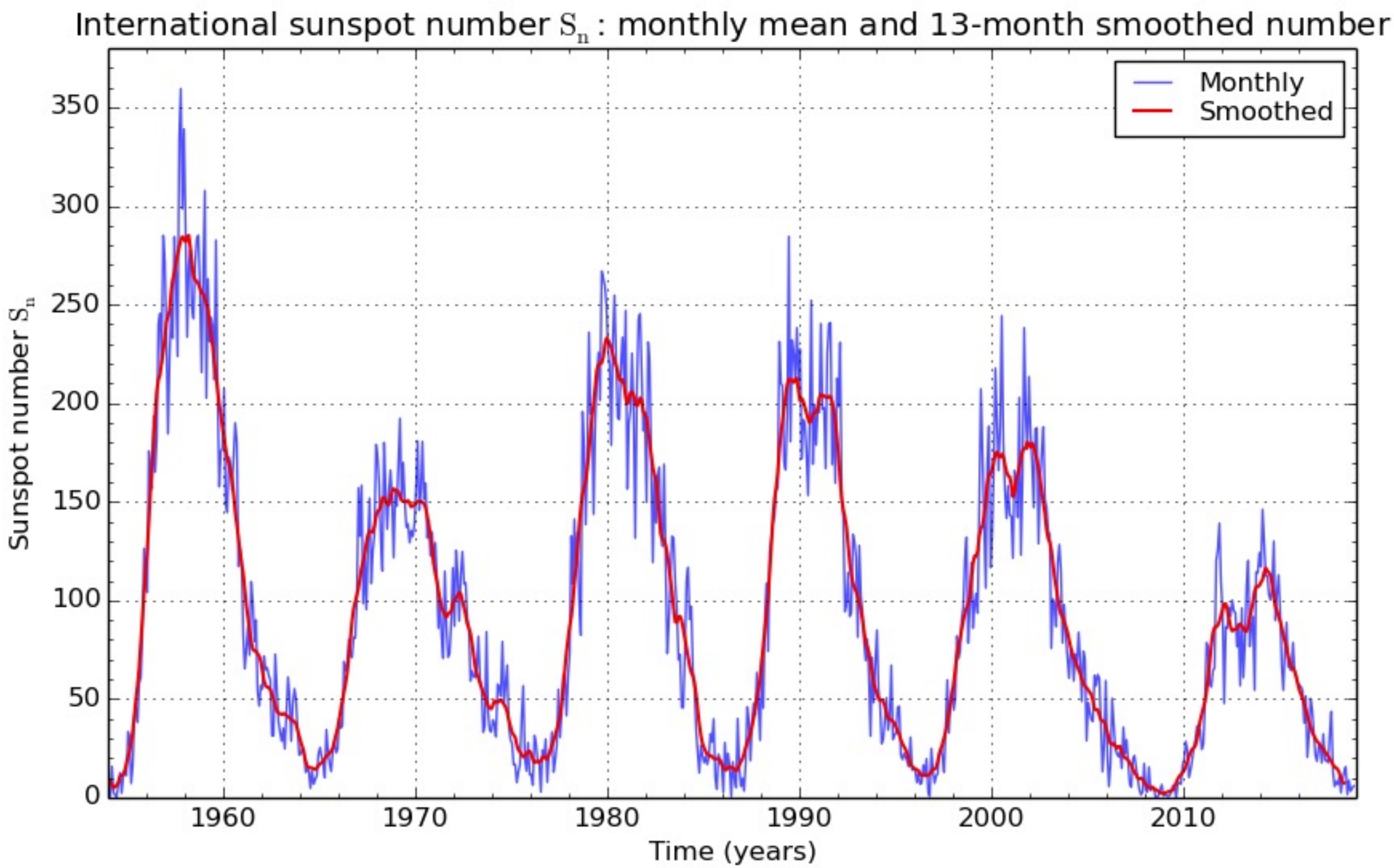
J. Klobuchar, *Ionospheric Effects on GPS*, Chapter 12 of Parkinson et al (ed.), *Global Positioning System: Theory and Aplications*, AIAA, 1996.

Solar Cycle

- **Ionospheric activity is directly related to the solar cycle**
  - Measured by sunspot number
  - Varies on ~11 year cycle
  - In peak around 2014

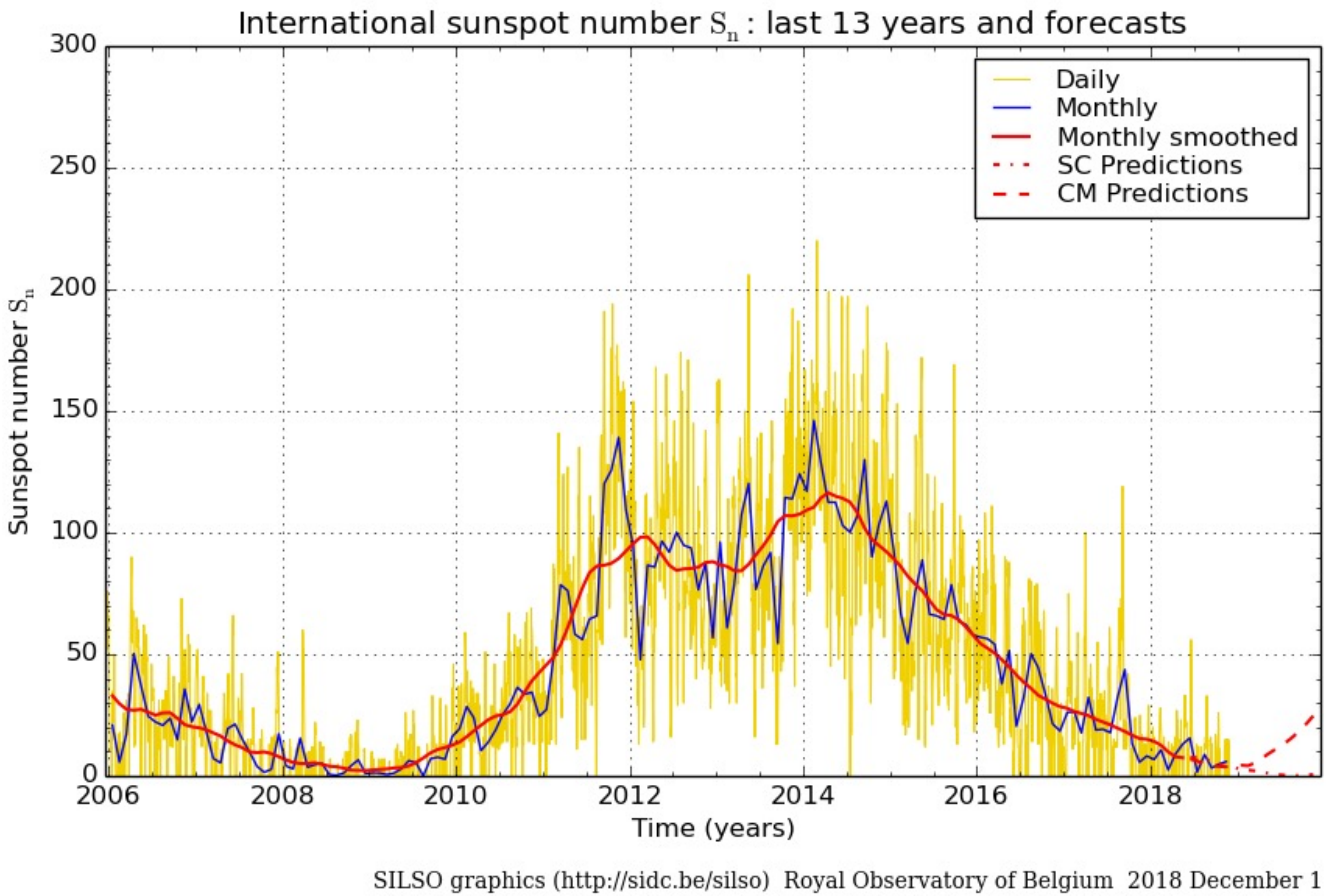


Recent Solar Cycles



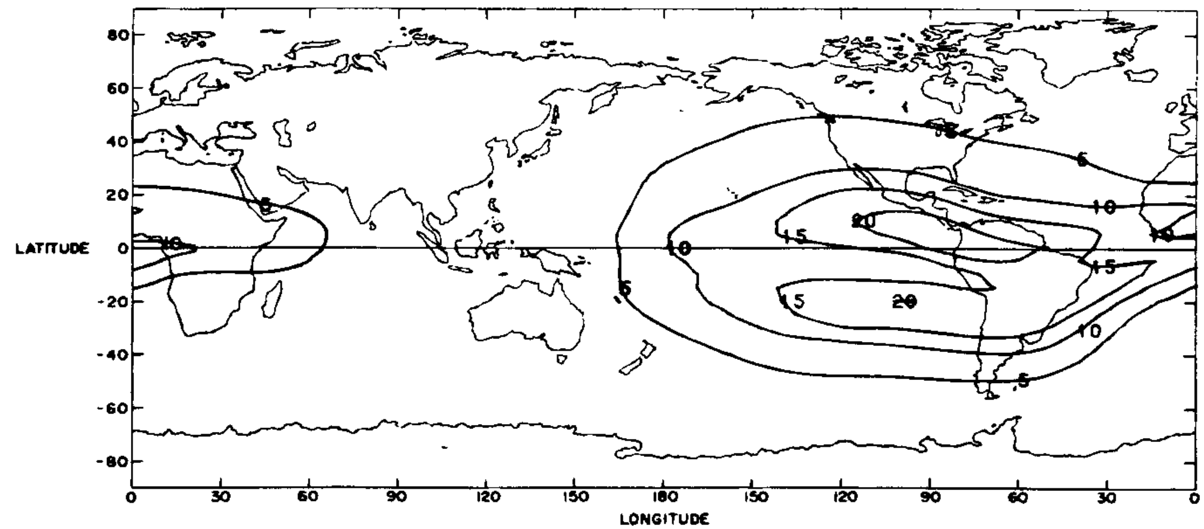
SILSO graphics (http://sidc.be/silso) Royal Observatory of Belgium 2018 December 1

Sunspot Numbers – More Detailed View, Including One Year Predictions

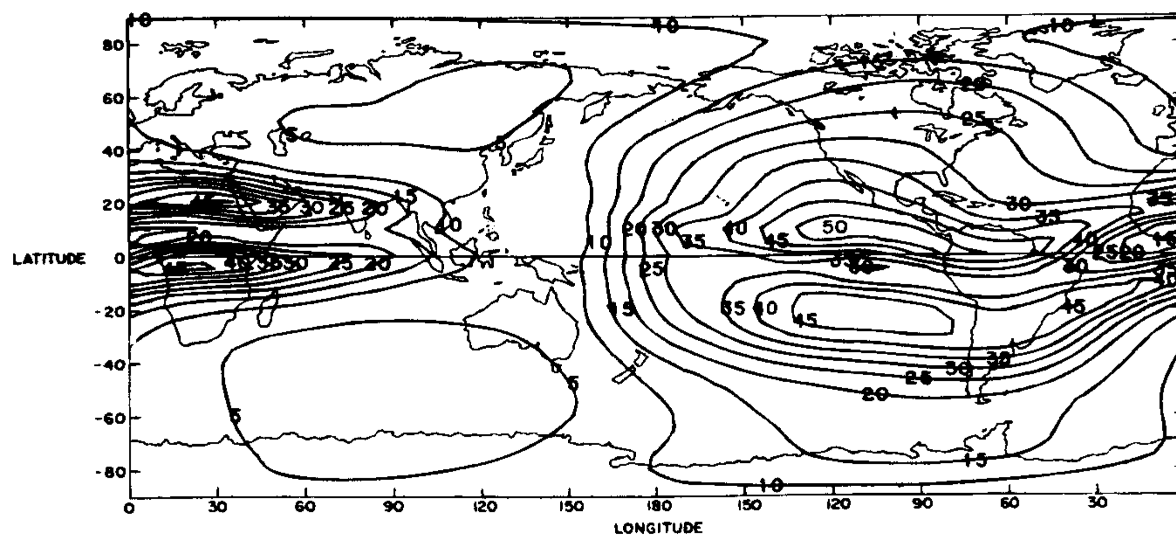


Spatial and Temporal Variation of L1 Ionospheric Delay

Zenith L1 Ionospheric Time Delay (ns) - Solar Minimum (sunspot number = 10)

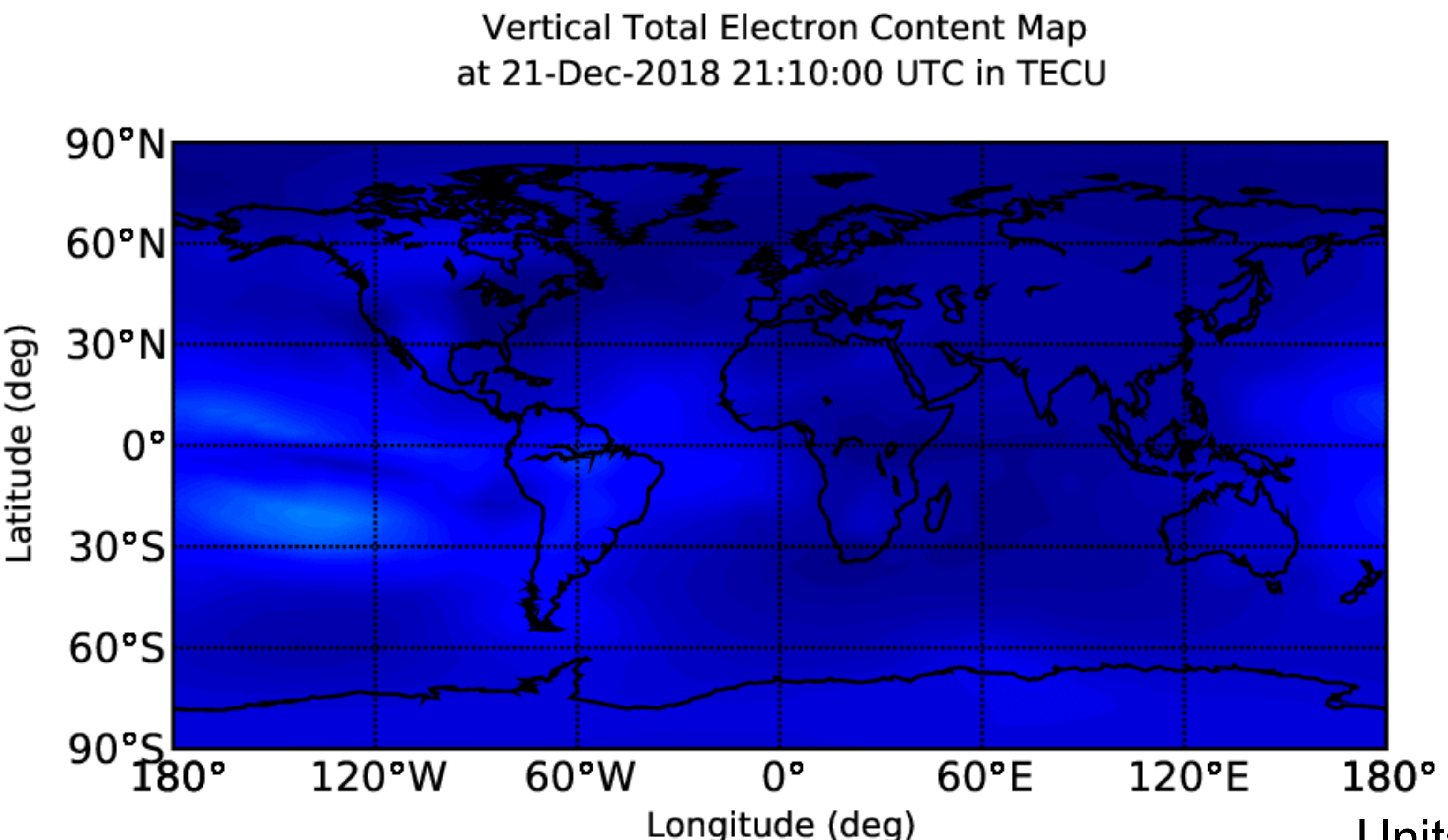


Zenith L1 Ionospheric Time Delay (ns) - Solar Maximum (sunspot number = 153)



J. Klobuchar, *Ionospheric Effects on GPS*, Chapter 12 of Parkinson et al (ed.), *Global Positioning System: Theory and Aplications*, AIAA, 1996.

Sample Ionosphere Plot
(Sunspot Number = 0)

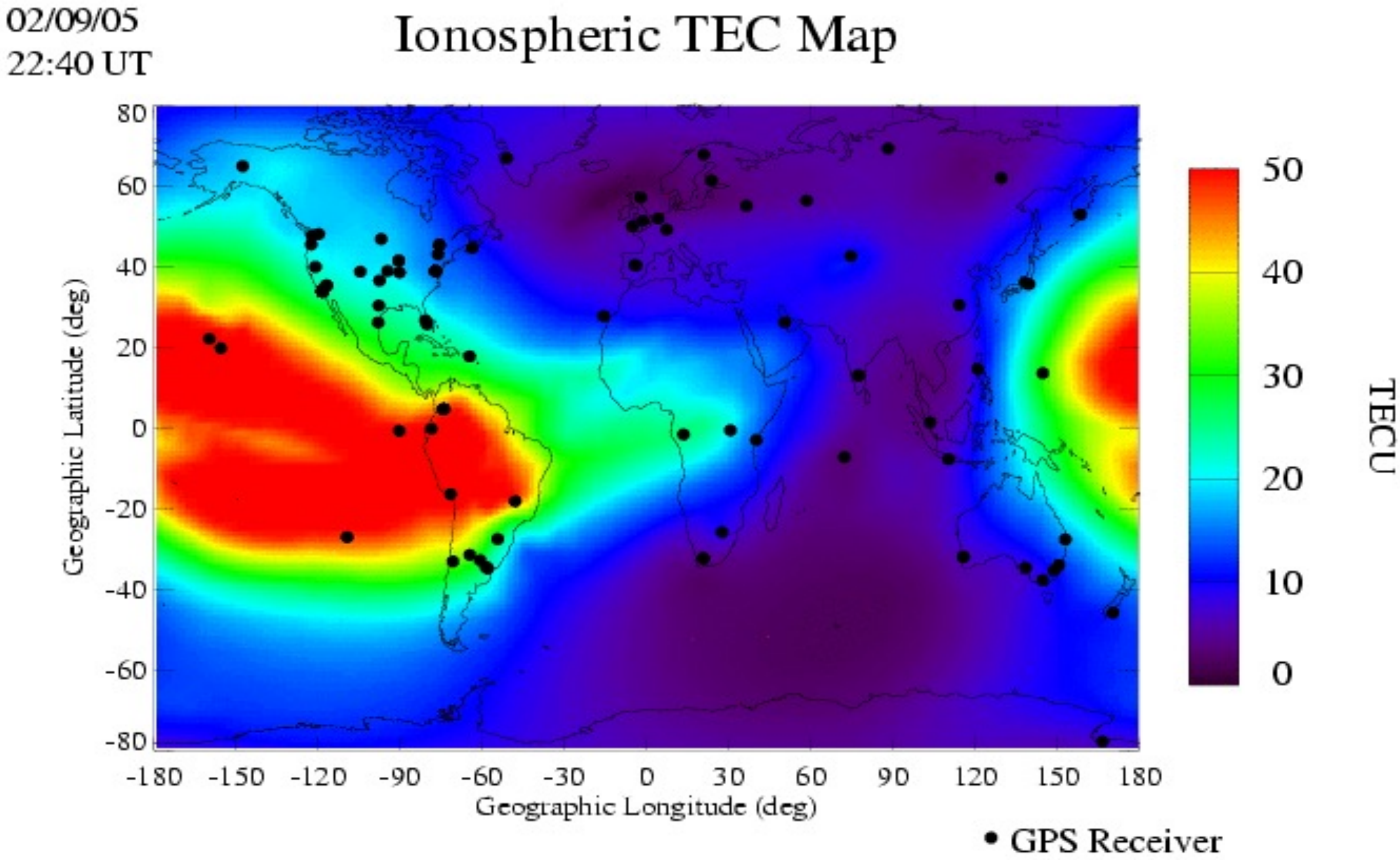


Units of TECU
(=1x10^{16} TEC)

(1 TECU = 0.16 m
pseudorange error on L1)

Obtained from https://iono.jpl.nasa.gov/latest_rti_global.html
Sunspot Number obtained from http://www.sidc.be/silso/datafiles

More Typical Ionosphere Plot
(Sunspot Number = 58)



http://www.arizonaenergy.org/AirEnergy/map20050209_224000.gif
Sunspot Number obtained from http://www.sidc.be/silso/datafiles

Ionospheric Effects on GPS Pseudorange Measurement

- **Total Electron Content (TEC)**
  - Number of electrons in a 1m x 1m rectangular column from the receiver up through the ionosphere (units of electrons/m^2)

$$TEC = \int_{SV}^{User} n_e dl$$

$$n_e = \text{electron density}$$

- **Group (pseudorange) delay**

$$\Delta S_{iono,g} = \frac{40.3\ \text{TEC}}{f^2}$$

$$\Delta S_{iono,g} = \text{group (pseudorange) delay (m)}$$

$$f = \text{carrier frequency (L1 or L2 for GPS)}$$

- **Carrier-phase delay**

$$\Delta S_{iono,p} = -\frac{40.3\ \text{TEC}}{f^2}$$

  - Carrier-phase is actually *advanced* by ionosphere by same magnitude as

Use of Dual Frequency Measurements to Calculate Ionospheric Delay

- **L1 ionospheric delay calculated by**

$$\Delta S_{iono,corr_{L1}} = \left(\frac{f_2^2}{f_2^2 - f_1^2} \right) (\rho_{L1} - \rho_{L2})$$

$\Delta S_{iono,corr_{L1}} =$ L1 Ionospheric Delay (m)

$f_1, f_2 =$ L1 and L2 carrier frequencies

$\rho_{L1}, \rho_{L2} =$ L1 and L2 pseudorange measurements

- **L2 ionospheric delay can be calculated by**

$$\Delta S_{iono,corr_{L2}} = \left(\frac{f_1}{f_2} \right)^2 \Delta S_{iono,corr_{L1}}$$

- **Ionospheric-free pseudorange:**

$$\rho_{IF} = \frac{\rho_{L2} - \gamma \rho_{L1}}{1 - \gamma}, \qquad \gamma = \left(\frac{f_{L1}}{f_{L2}} \right)^2 = \left(\frac{77}{60} \right)^2$$

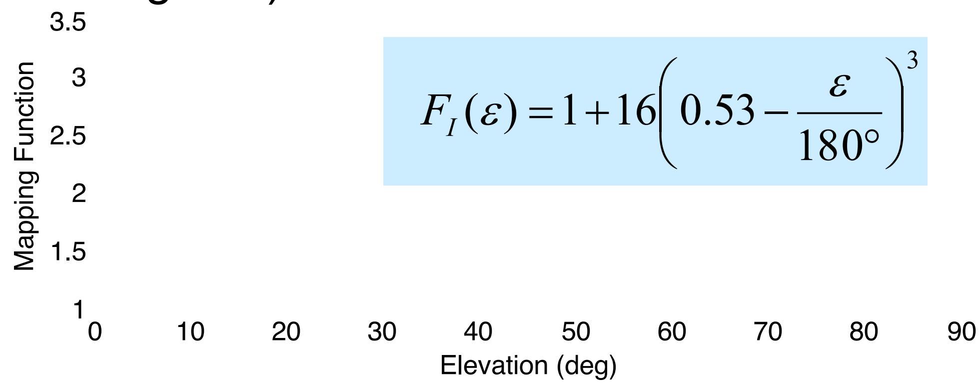
- **Multipath and measurement noise will corrupt this measurement of ionosphere**

Ionospheric Mapping Functions

- **Mapping function (or obliquity factor) is used to relate ionospheric error at elevation ε with ionospheric error at zenith:**

$$\delta t_{iono} = F_I(\varepsilon)\,\delta t_{iono,zenith}$$

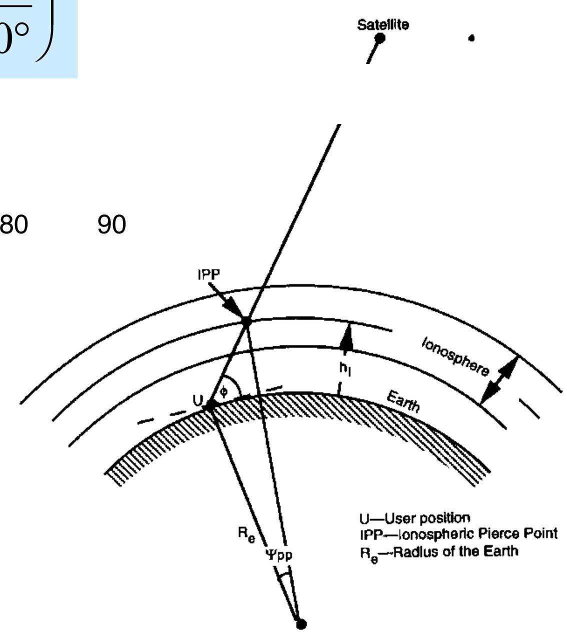
  - Mapping function used in Klobuchar model (from ICD-GPS-200) (elevation ε in degrees)



$$F_I(\varepsilon) = 1 + 16\left(0.53 - \frac{\varepsilon}{180^\circ}\right)^3$$

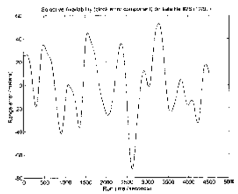
  - Mapping function used by WAAS

$$F_I(\varepsilon) = \left[1 - \left(\frac{R_e \cos\phi}{R_e + h_I}\right)^2\right]^{-\frac{1}{2}}$$

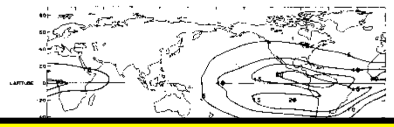


Kaplan, *Understanding GPS: Principles and Applications*, Artech House, 1996.

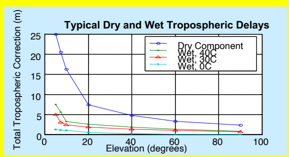
GPS Measurement Errors



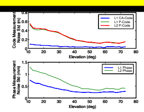
Selective Availability



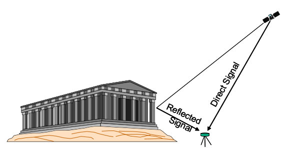
Ionosphere



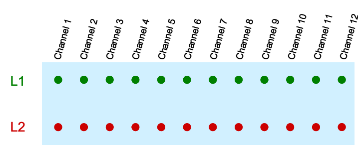
Troposphere



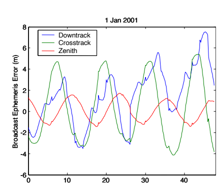
Receiver noise



Multipath



Hardware errors



Broadcast ephemeris error

Tropospheric Delay (δt_{trop})

- **Troposphere is defined as the neutral atmosphere**
  - Actual troposphere (~0-10 km)
    - Contributes about 75% of "tropospheric" error
  - Tropopause (~10-16 km)
  - Stratosphere (~16-50 km)

  } Contribute about 25% of error
- **Tropospheric delay expressed as**

$$\delta t_{trop} = \int_{path} (n-1)ds + \Delta_g$$

$n =$ refractive index $= \dfrac{c_v}{c_m}$

$c_v =$ velocity in vacuum

$c_m =$ velocity in medium (air in this case)

$\Delta_g =$ error due to path curvature

- **Sometimes refractivity (N) is used**

$$N = (n-1)\times 10^6$$

  - Average N on Earth's surface: 320

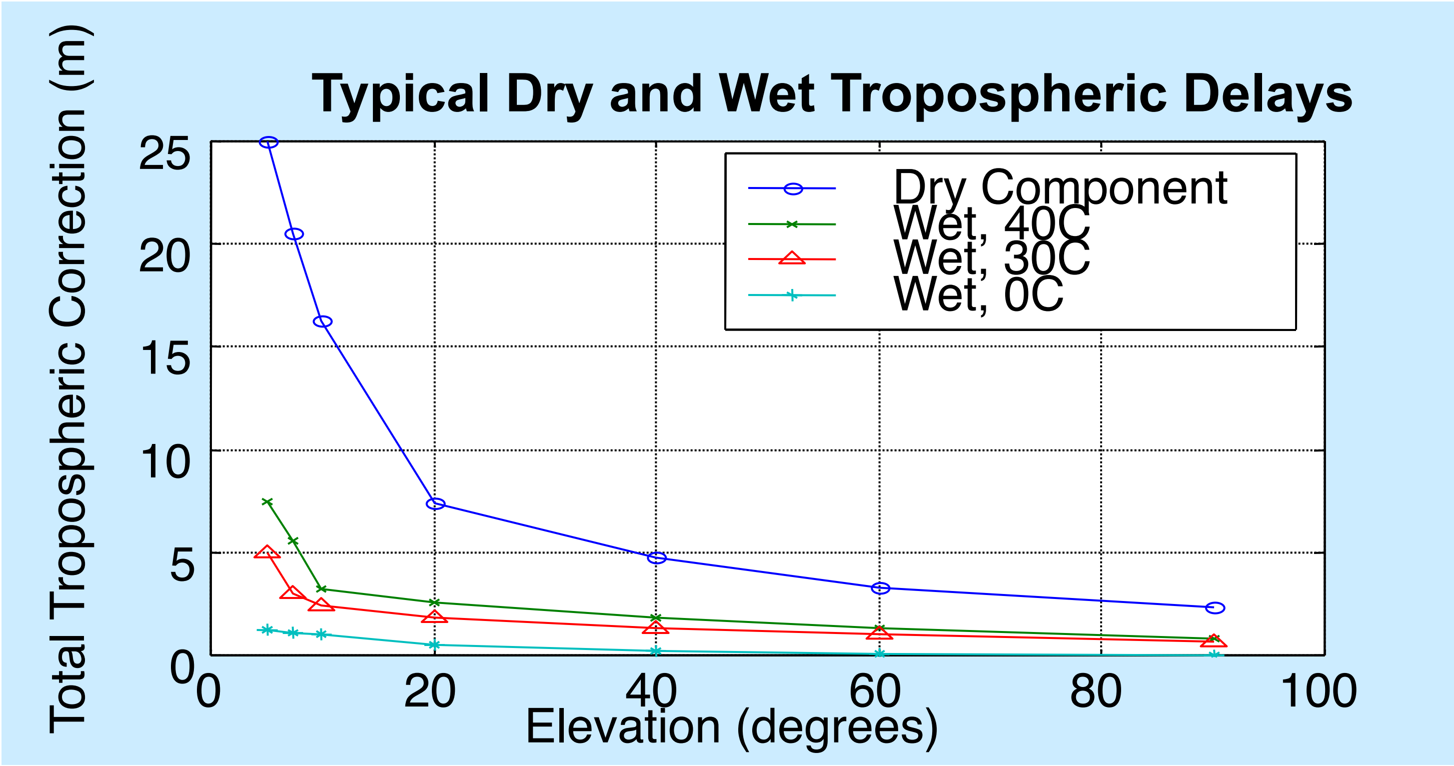
Characteristics of Tropospheric Error

- **Two different terms can be calculated**

Dry — 80-90% of total error; Predictable to 1% @ zenith

Wet — 10-20% of total error; Predictable to 10-20% @ zenith

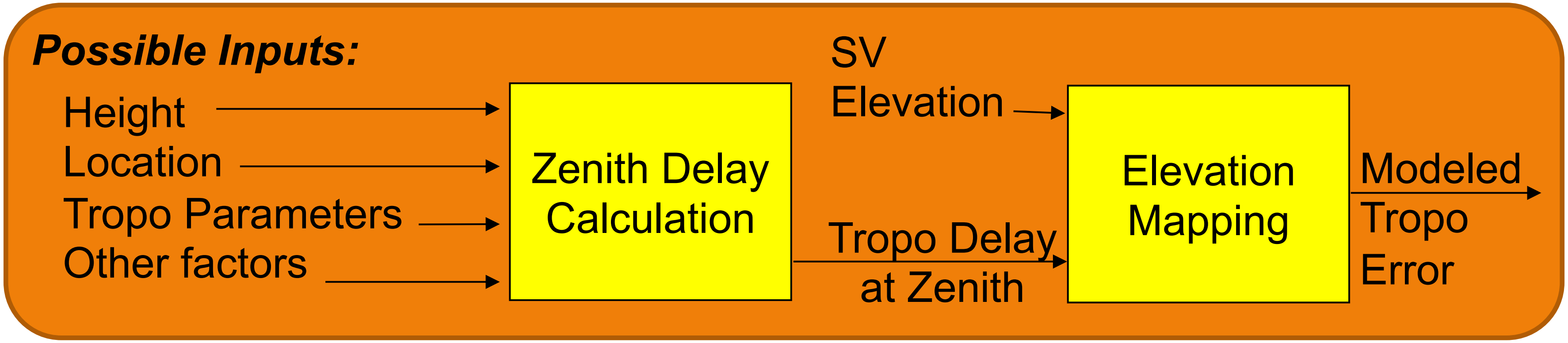
Total = Wet + Dry



Data from Black and Eisner, “Correcting Satellite Doppler Data for Tropospheric Effects”, *Proc. Of International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics*, National Technical Univ. of Athens, 1984.

Tropospheric Models

- Many models exist for modeling troposphere



- Most are accurate at elevation angles > 15°
- Can be significant differences for low elevations
- More common models:
  - Saastamoinen (Saastamoinen, "Contributions to the Theory of Atmospheric Refraction." In three parts. *Bulletin Geodesique*, No. 105, pp. 279-298; No. 106, pp. 383-397; No. 107, pp. 13-34.)
  - Black & Eisner (Black and Eisner, "Correcting Satellite Doppler Data for Tropospheric Effects." *Journal of Geophysical Research*, Vol. 89, No. D2, pp. 2616-2626.)
  - Marini & Murray (Marini and Murray, "Correction of Laser Range Tracking Data for Atmospheric Refraction at Elevations Above 10 Degrees." Goddard Space Flight Center Report X-591-73-351, NASA GSFC, Greenbelt, MD, 1973.)

Tropospheric Mapping Functions

- **Mapping function is used to relate tropospheric error at elevation ε with tropospheric error at zenith:**

$$\delta t_{trop} = F_T(\varepsilon)\delta t_{trop,zenith}$$

– Chao mapping function[1]

- Wet component:

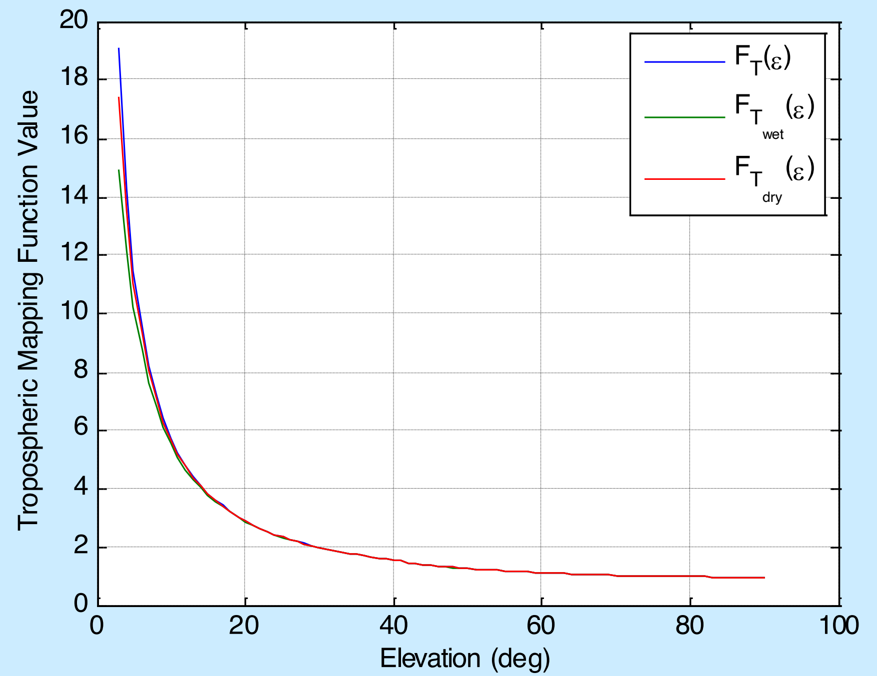
$$F_{T_{wet}}(\varepsilon) = \frac{1}{\sin\varepsilon + \dfrac{0.00143}{\tan\varepsilon + 0.0445}}$$

- Dry component

$$F_{T_{dry}}(\varepsilon) = \frac{1}{\sin\varepsilon + \dfrac{0.00035}{\tan\varepsilon + 0.017}}$$

– Simplified troposphere mapping function (useful for many applications)

$$F_T(\varepsilon) = \frac{1}{\sin\varepsilon}$$

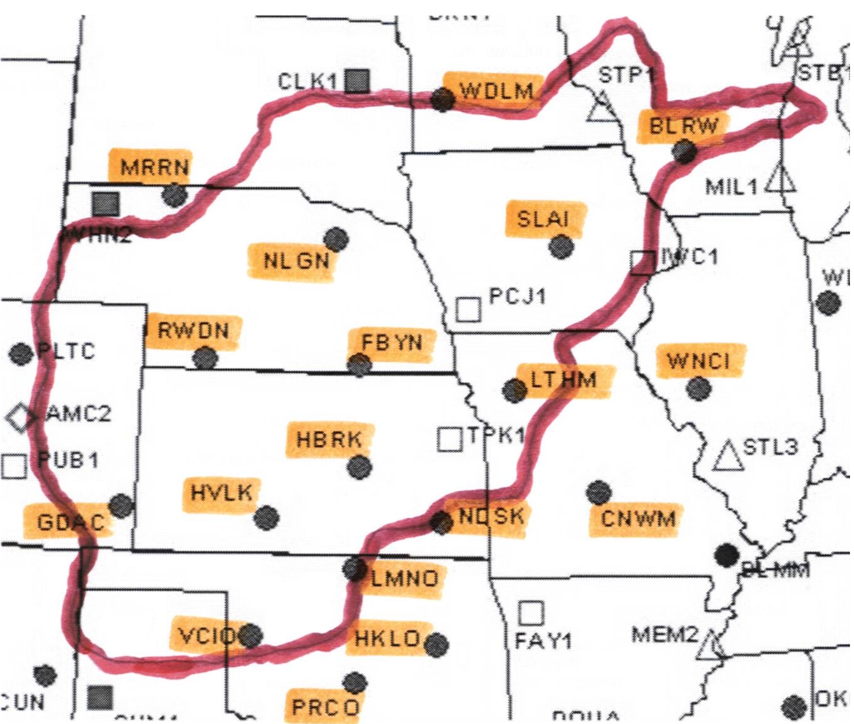


J Raque
[1] Chao, “The Tropospheric Calibration Model for Mariner Mars, 1971,” JPL TR 32-1587, Jet Propulsion Laboratory, 1974.

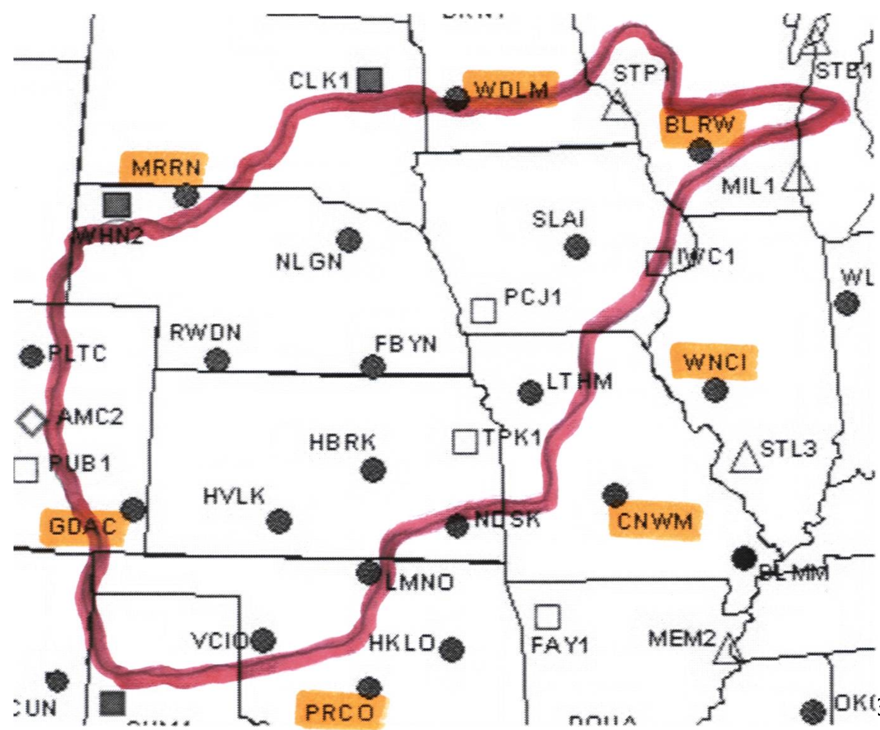
GPS Meteorology

- **GPS can be used to actually measure water vapor content of atmosphere**
- **Trial network (using CORS stations):**

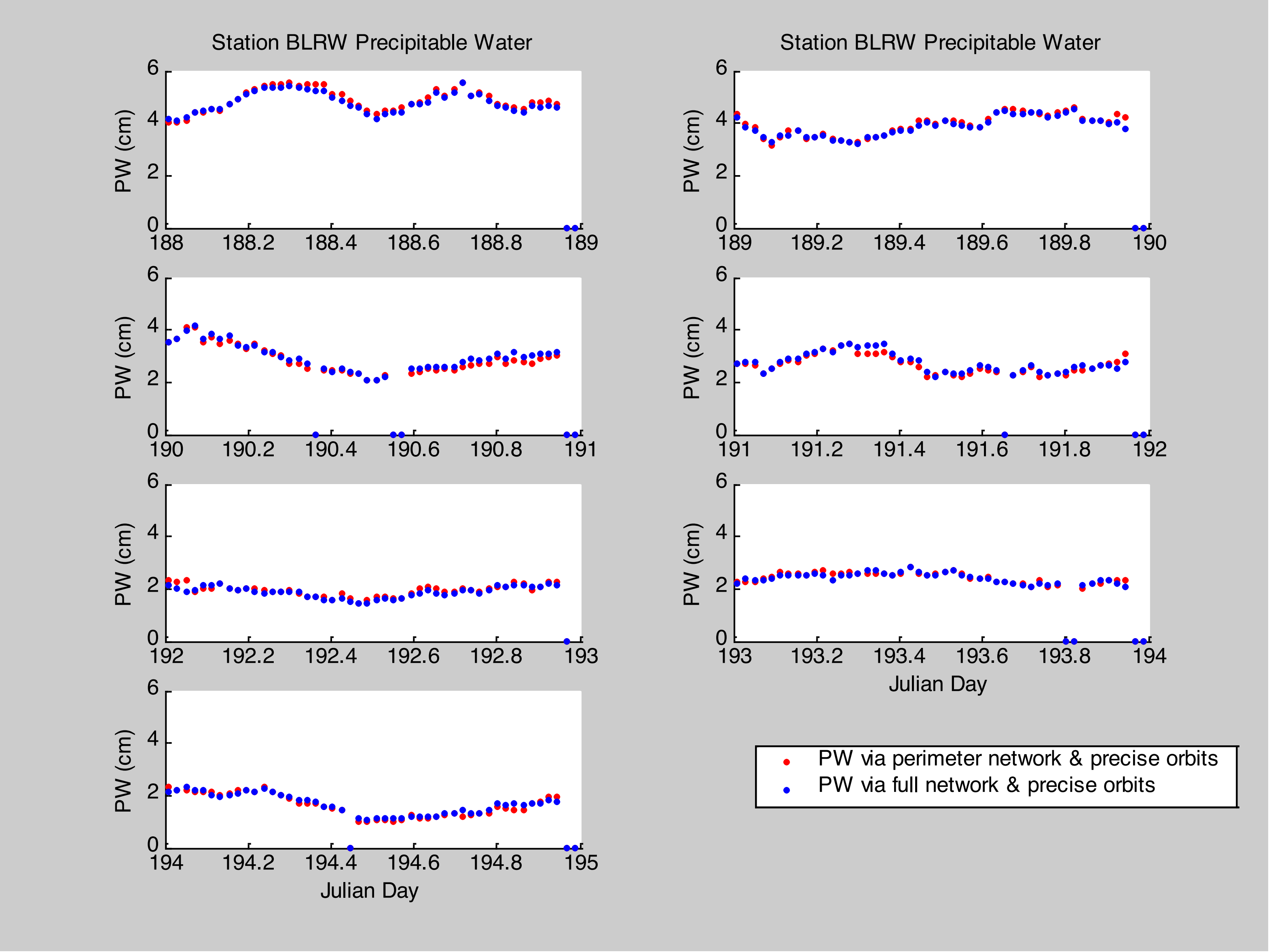
Full Network



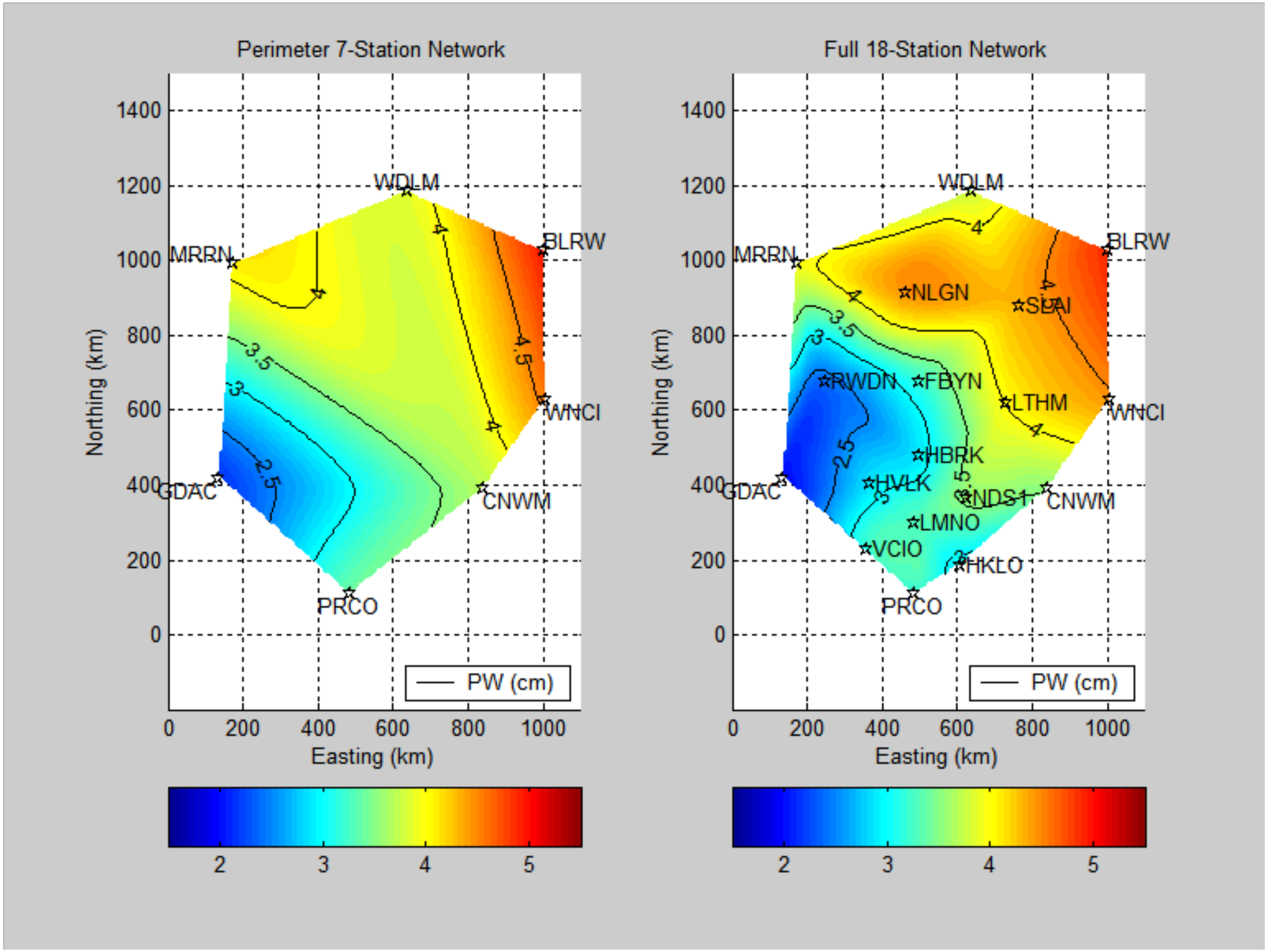
Degraded Network



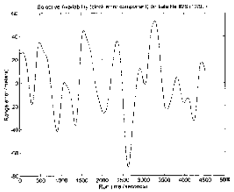
GPS Meteorology
Comparison Between Full and Degraded Networks



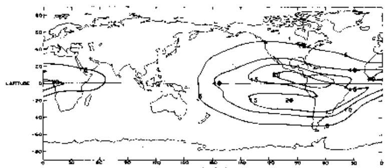
GPS Meteorology
Water Vapor Over the Network



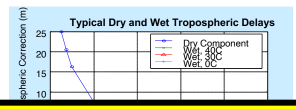
GPS Measurement Errors



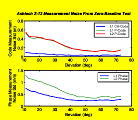
Selective Availability



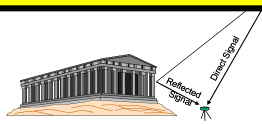
Ionosphere



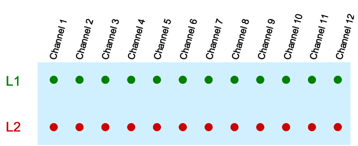
Troposphere



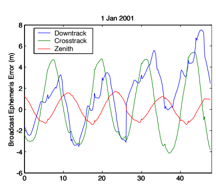
Receiver noise



Multipath



Hardware errors



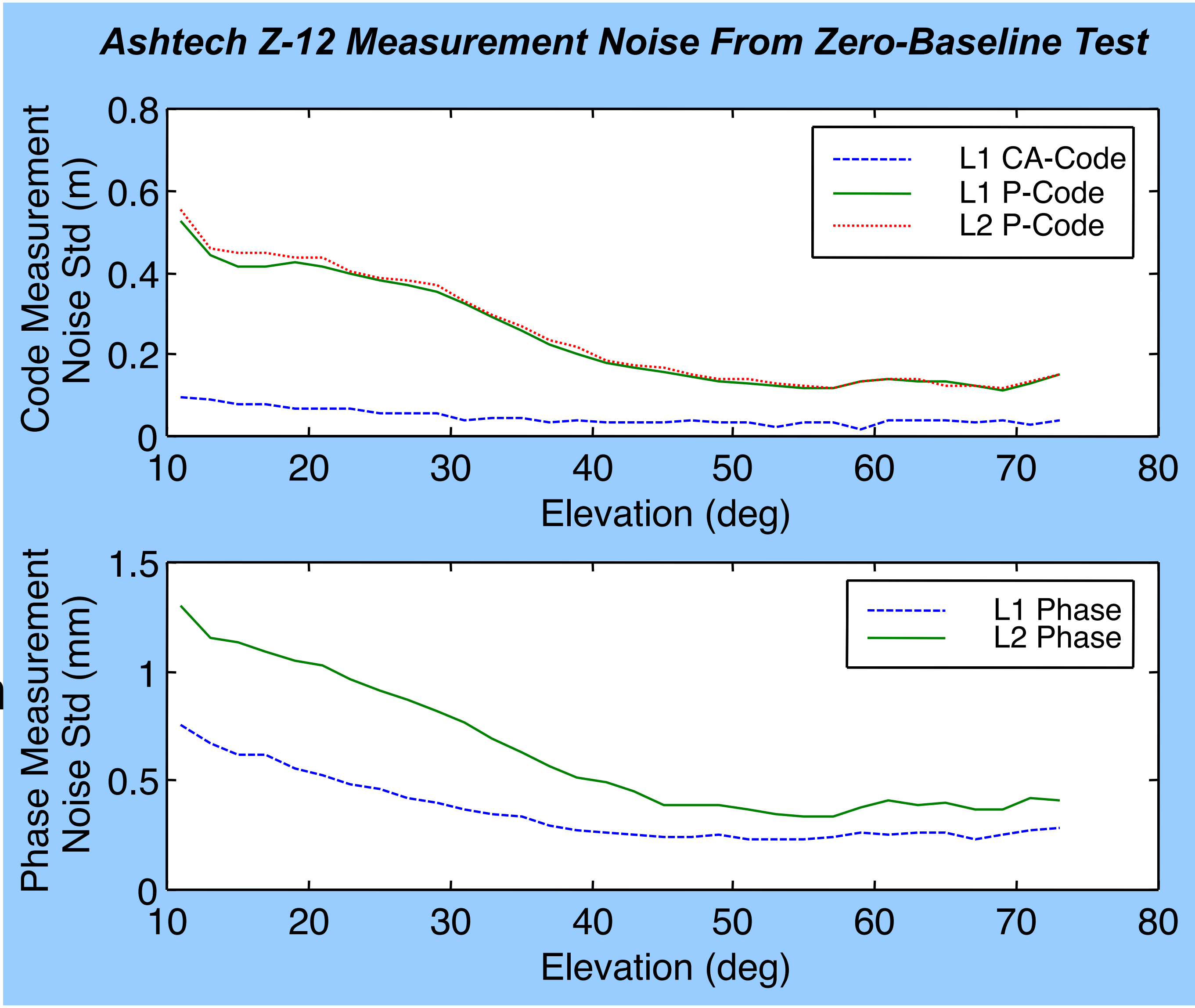
Broadcast ephemeris error

Measurement Noise

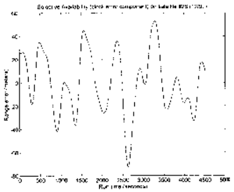
- **As with any measuring device, measurements from a GPS receiver exhibit measurement noise**
  - Uncorrelated in time at typical sampling intervals (i.e., white noise)
  - Gaussian probability density function
  - Zero-mean
  - Correlated with C/N_0 (lower $C/N_0 \rightarrow$ more noise)
- **Quantization error also lumped with noise**
  - Error due to LSB roundoff
  - Depends upon receiver implementation and the way data is reported from the receiver
- **Normally, measurement noise is not a significant problem**
  - Easy to remove by filtering
  - No bias

Sample of Measurement Noise vs. Satellite Elevation

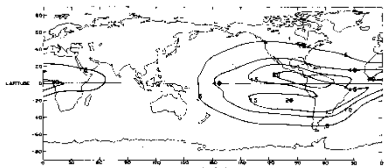
- **Measurement noise vs. elevation**
  - Based upon “zero-baseline” test (single antenna, two receivers)
  - Low elevation satellites typically have lower C/N_0 values



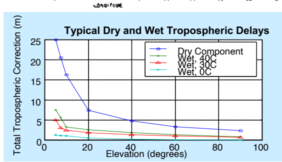
GPS Measurement Errors



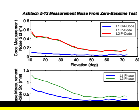
Selective Availability



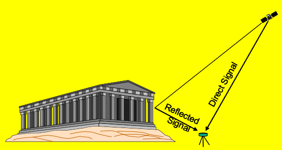
Ionosphere



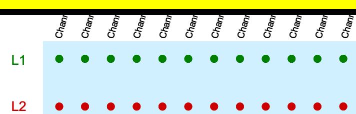
Troposphere



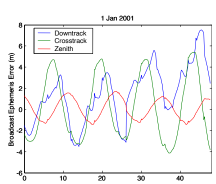
Receiver noise



Multipath



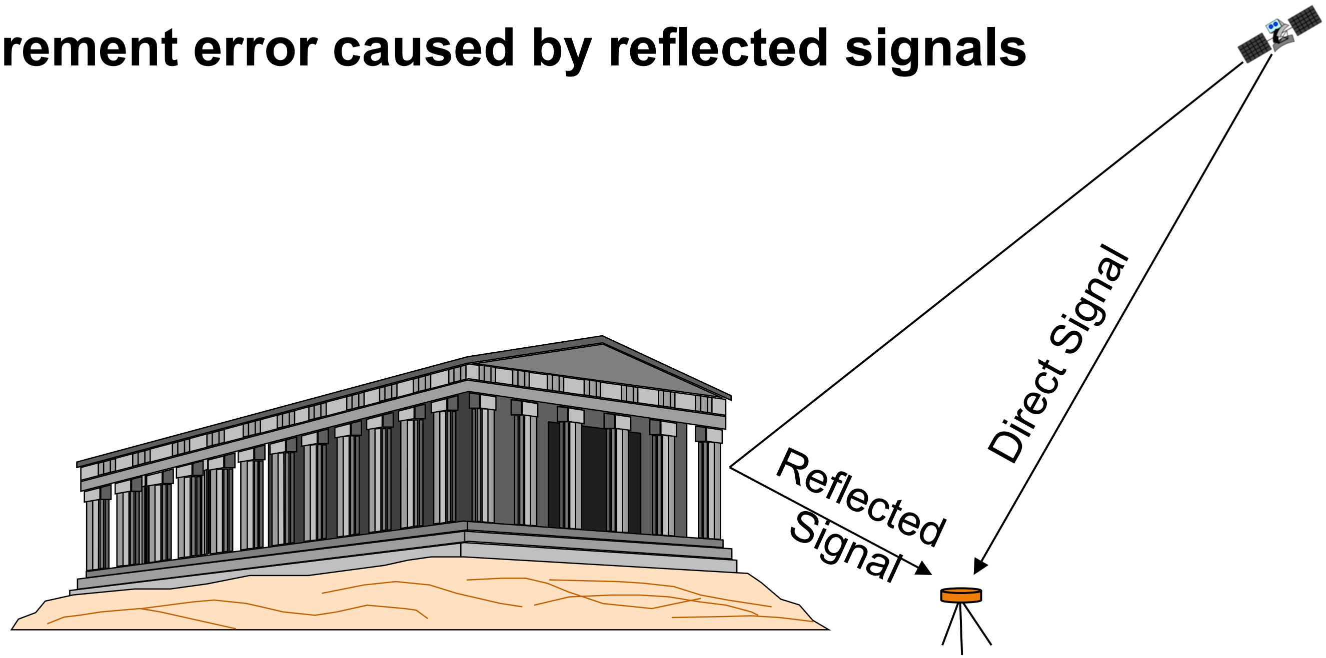
Hardware errors



Broadcast ephemeris error

Multipath

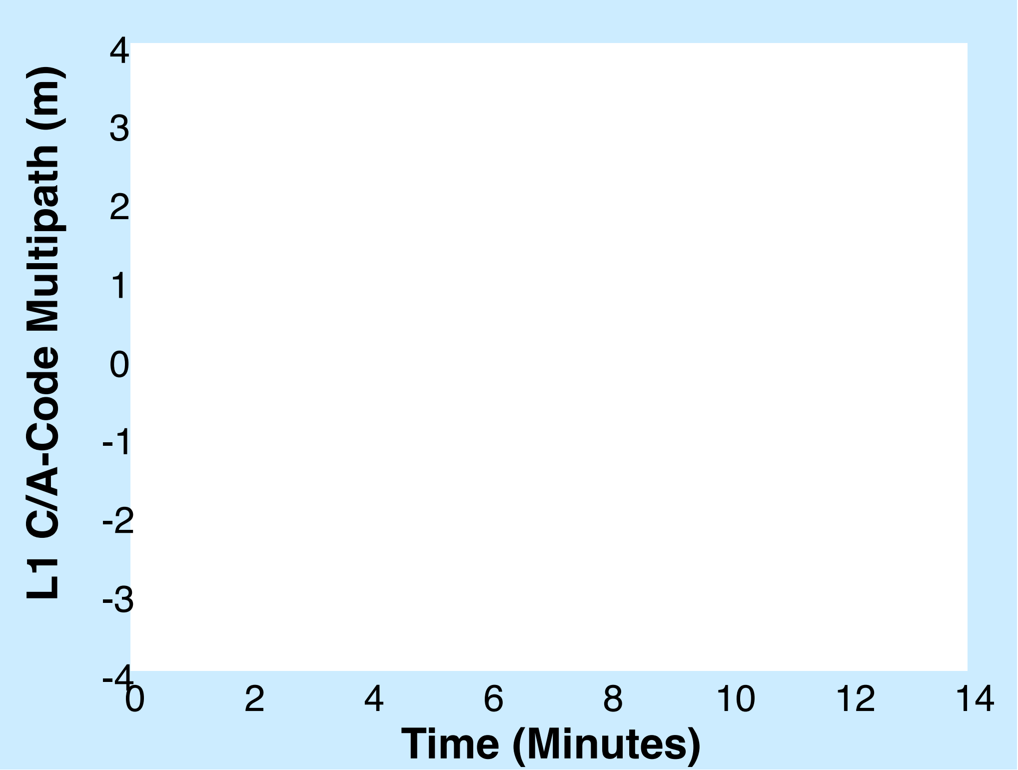
- **Measurement error caused by reflected signals**



  – Can affect both code and carrier-phase measurements
  – Potential multipath sources
    - Ground
    - Water
    - Buildings
    - Heating ducts

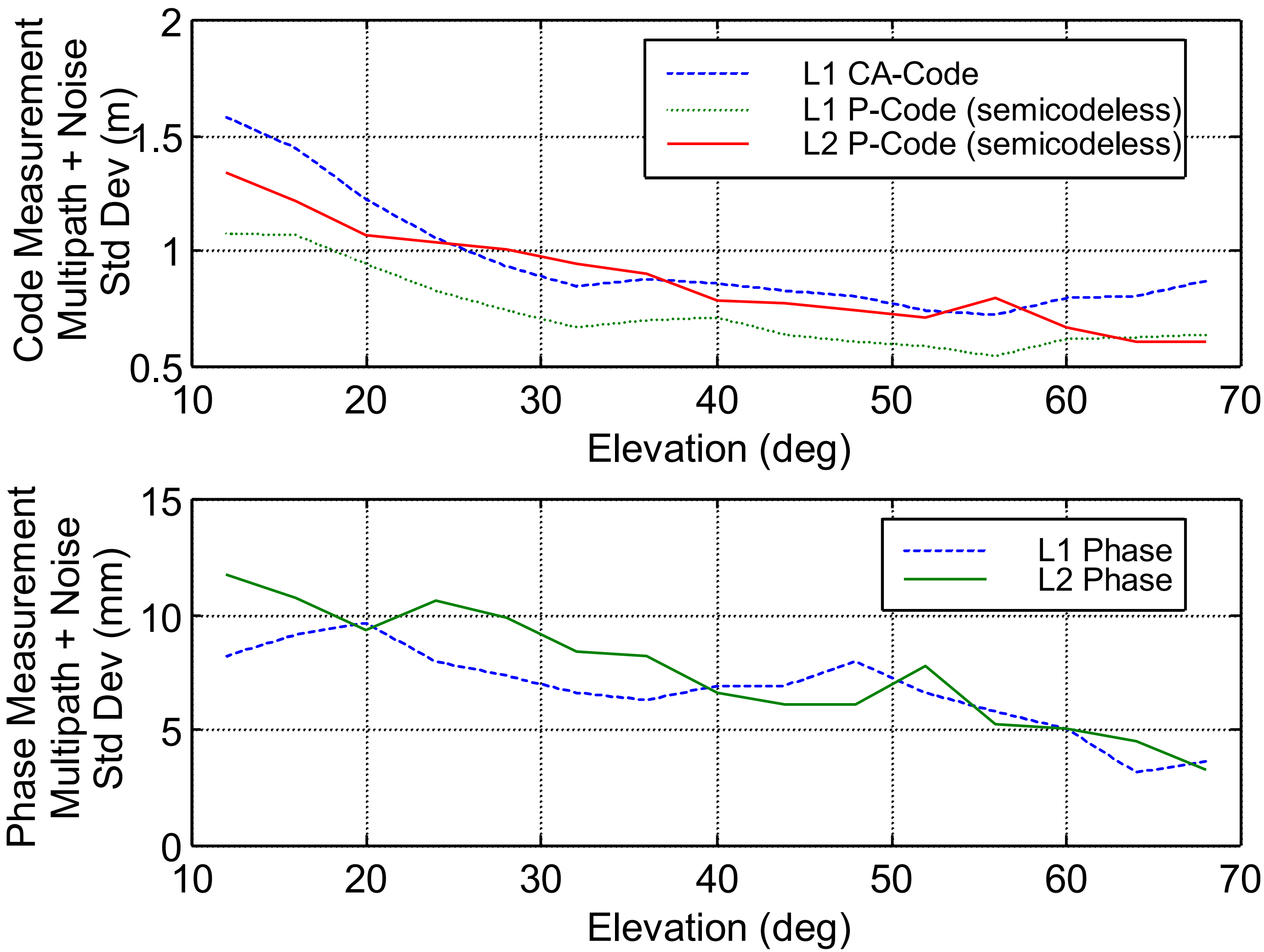
Multipath Example

- **Plot of multipath plus measurement noise**
  - Ashtech Z-Surveyor in T-38 on tarmac
  - Generated from code-minus-carrier observable
    - Absolute bias unknown
  - Code-carrier ionospheric divergence removed by fitting to 2nd order polynomial
  - Note periodic nature



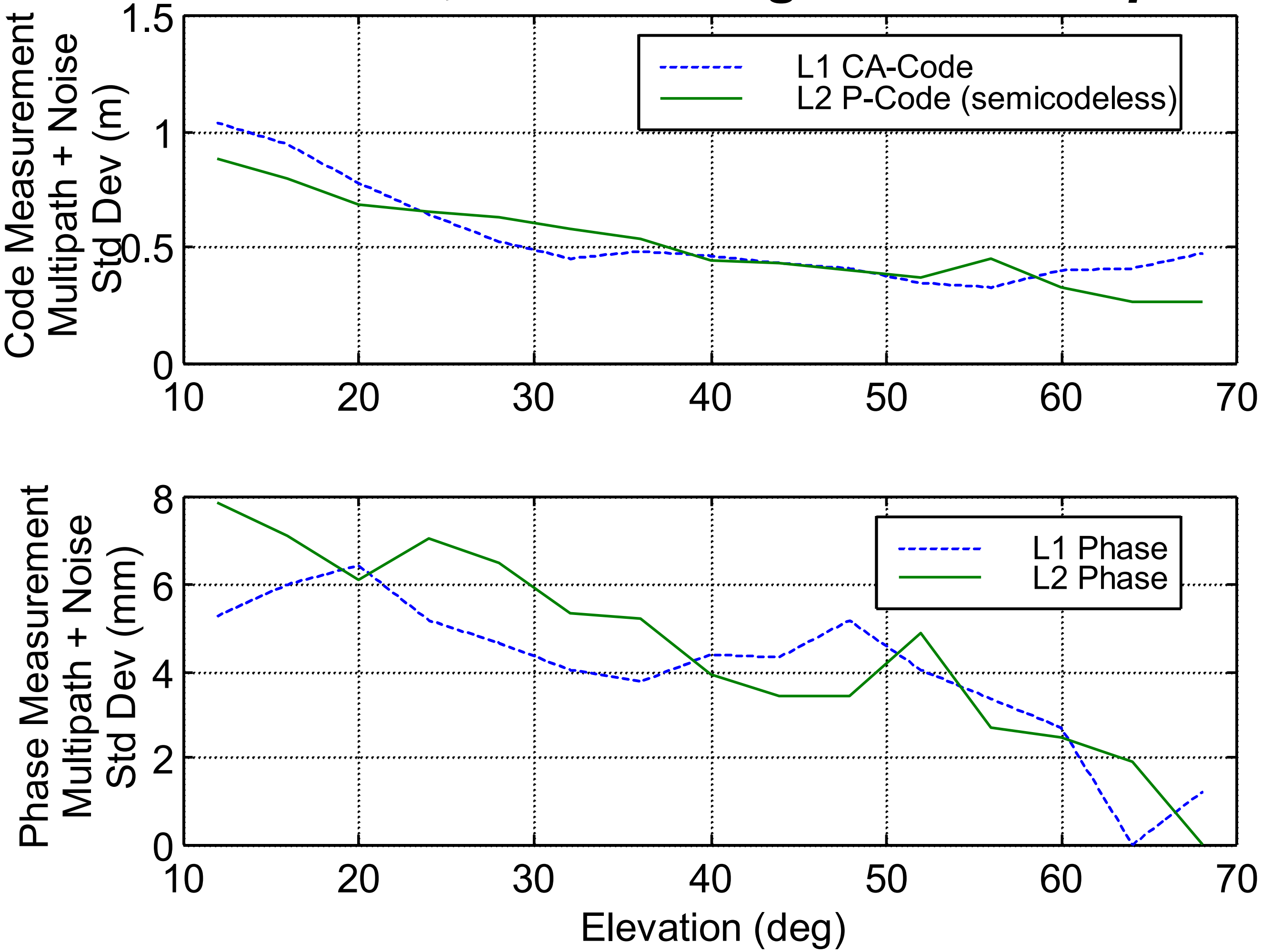
Examples of Multipath (plus Noise) vs. SV Elevation
Ashtech Z-12

Ashtech Z-12 Receiver, Dorne-Margolin Groundplane Antenna



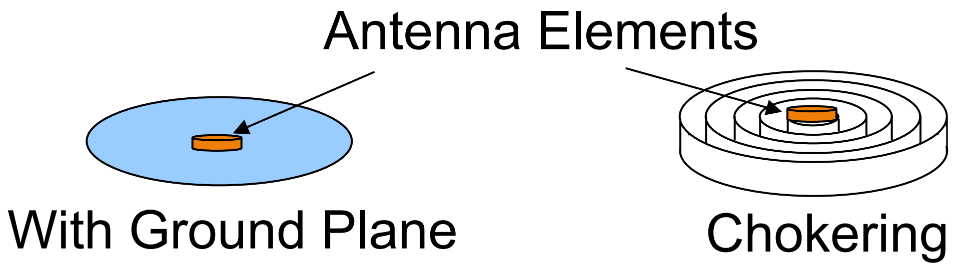
Examples of Multipath (plus Noise) vs. SV Elevation Trimble 4000ssi

Trimble 4000ssi Receiver, Dorne-Margolin Groundplane Antenna



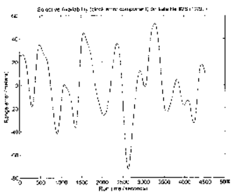
Multipath Mitigation Techniques

- **Antenna-based approaches**
  - Place antenna in low-multipath environment
  - Use groundplane or chokering antenna

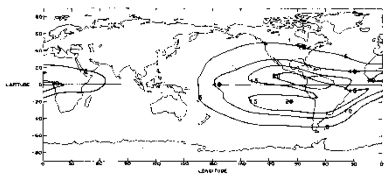


- **Measurement-based approaches**
  - Carrier-phase smoothing of the code
  - Use of C/N_0 and antenna gain pattern to estimate multipath
- **Receiver processing-based approaches**
  - Narrow correlator spacing
  - Advanced signal processing techniques
- **Other approaches**
  - Use of multiple receivers
  - Modeling of environment around antenna

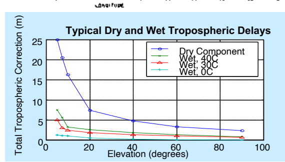
GPS Measurement Errors



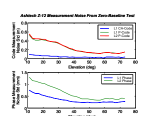
Selective Availability



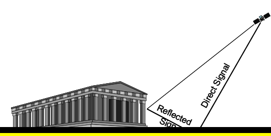
Ionosphere



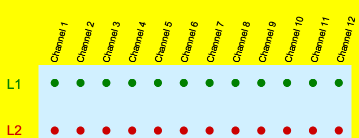
Troposphere



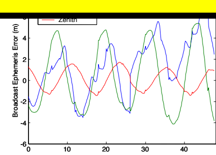
Receiver noise



Multipath

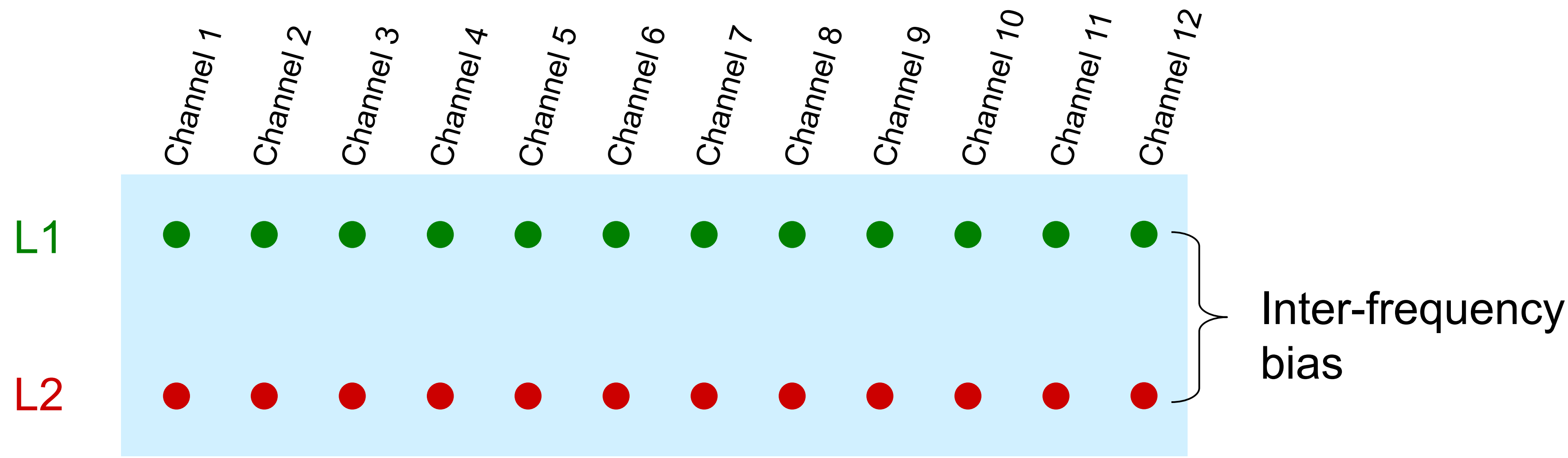


Hardware errors



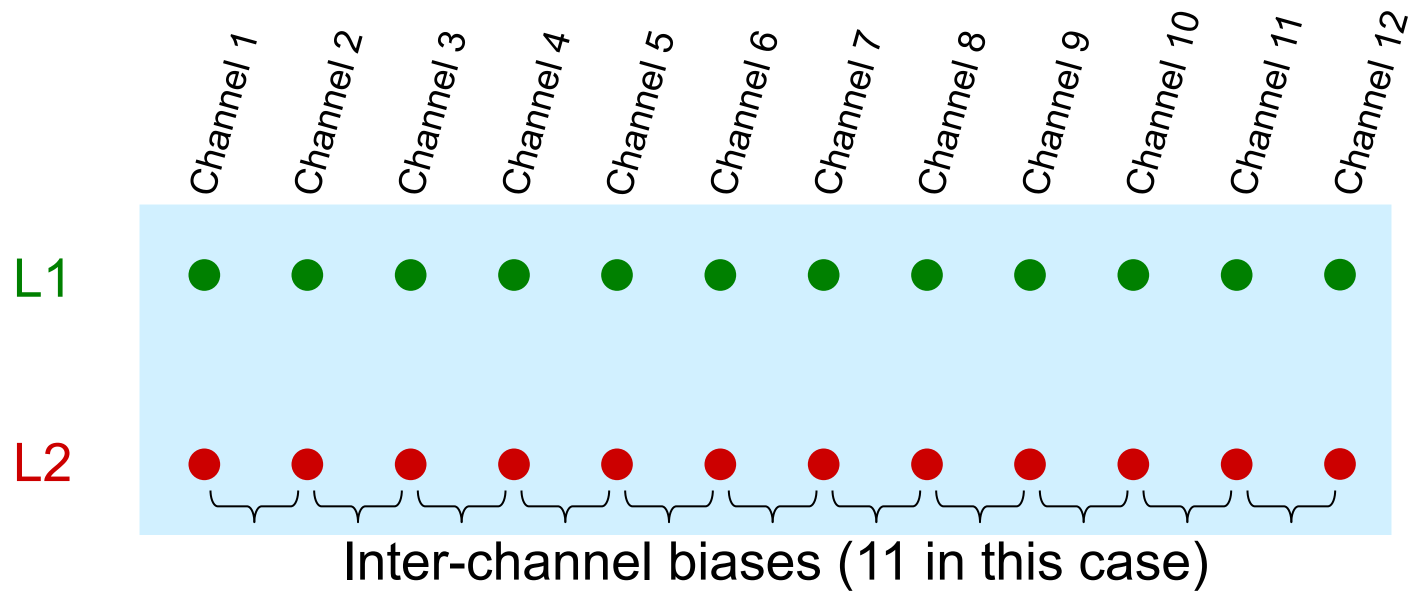
Broadcast ephemeris error

Hardware Errors – Inter-frequency Bias



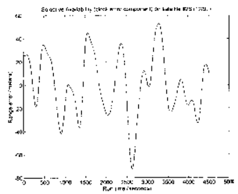
- **Measurement bias between L1 and L2 measurements**
  - Consistent across all receiver channels
- **Adverse effects**
  - Does not affect position solution (Why?)
  - Can affect ionospheric study
  - Can affect clock estimate

Hardware Errors – Inter-channel Bias

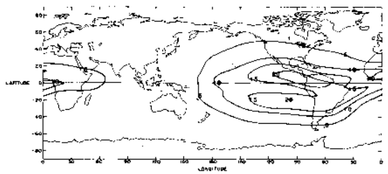


- **Measurement bias between channels in a receiver**
  - Typically very small (can be calibrated out in the factory)
  - Can be determined/removed by tracking a common satellite (like RCVR-3A)
- **Adverse effects**
  - Does not affect ionospheric study
  - Does affect position solution (although typically too little to worry about)
  - Does affect timing solution (although typically too little to worry about)

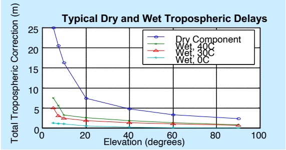
GPS Measurement Errors



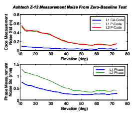
Selective Availability



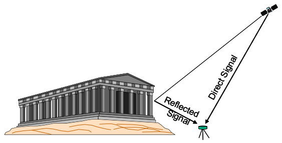
Ionosphere



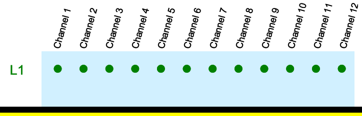
Troposphere



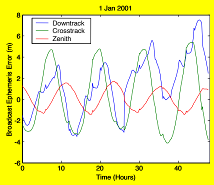
Receiver noise



Multipath



Hardware errors



Broadcast ephemeris error

Satellite Position Error

- **Perturbing forces and measurement error (by OCS) prevent perfect prediction of satellite positions**
  - Most can be modeled accurately
  - Even small errors can lead to significant position errors (due to integration)
    - Example: Venus gravity can result in

Force	Acceleration (m/s^2)
Earth gravity modeled as point mass	6.1×10^{-1}
Earth gravity oblateness modeled by the J2 coefficient	1.0×10^{-4}
Lunar gravity	3.9×10^{-6}
Solar gravity	1.0×10^{-6}
Summed effect of Earth gravity field, coefficients 2,1 to 4,4	2.2×10^{-7}
Solar radiation pressure	7.2×10^{-8}
Summed effect of Earth gravity field, coefficients 5,0 to 8,8	5.9×10^{-9}
Albedo (or Earthshine)[1]	1.5×10^{-9}
Thermal re-radiation[2]	1.4×10^{-9}
Solid Earth tide, raised by the Moon	1.3×10^{-9}
Solid Earth tide, raised by the Sun	4.5×10^{-10}
Venus gravity	1.1×10^{-10}

[1]Force due to electromagnetic radiation reflected by the Earth and thermal radiation emitted by the Earth
[2]Caused by anisotropic radiation of heat from the spacecraft

Ziebart, Cross, and Adhya, Modeling Photon Pressure: The Key to High-Precision GPS Satellite Orbits, *GPS World*, Jan 2002

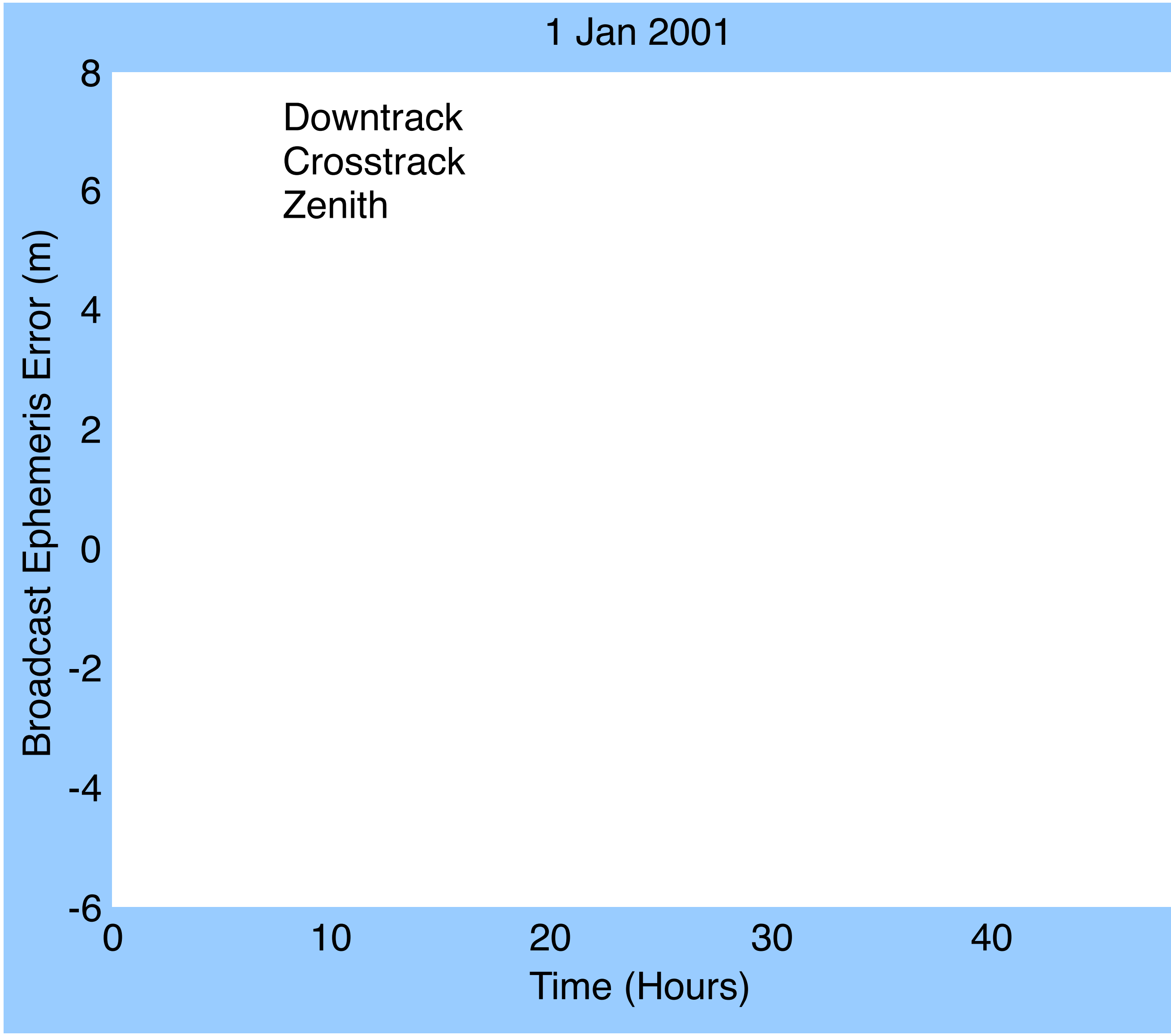
Satellite Position (Ephemeris) Error

- **The broadcast ephemeris are generated real-time by the control segment using data from 5 ground GPS receivers**
  - Typically has ~3 m accuracy[1]
- **Errors in ephemeris result in satellite position errors**
  - Not truly *measurement* errors
  - Do cause errors when the measurements are used, however
    - Cause error in expected measurement value
  - In terms of error analysis, can be treated as measurement errors
- **Precise orbits (~6 cm accuracy) can be obtained from National Geodetic Survey (and other organizations)**
  - Calculated using days of data from hundreds of reference stations
  - Can be obtained over internet (http://www.ngs.noaa.gov/GPS/GPS.html)
  - Serves as useful truth reference for broadcast ephemeris errors

 [1]Rothacher, *GPS Satellite Orbits, Orbit Determination, and the IGS*, Nordic Research Course of the Nordic Geodetic Commission, 1996.

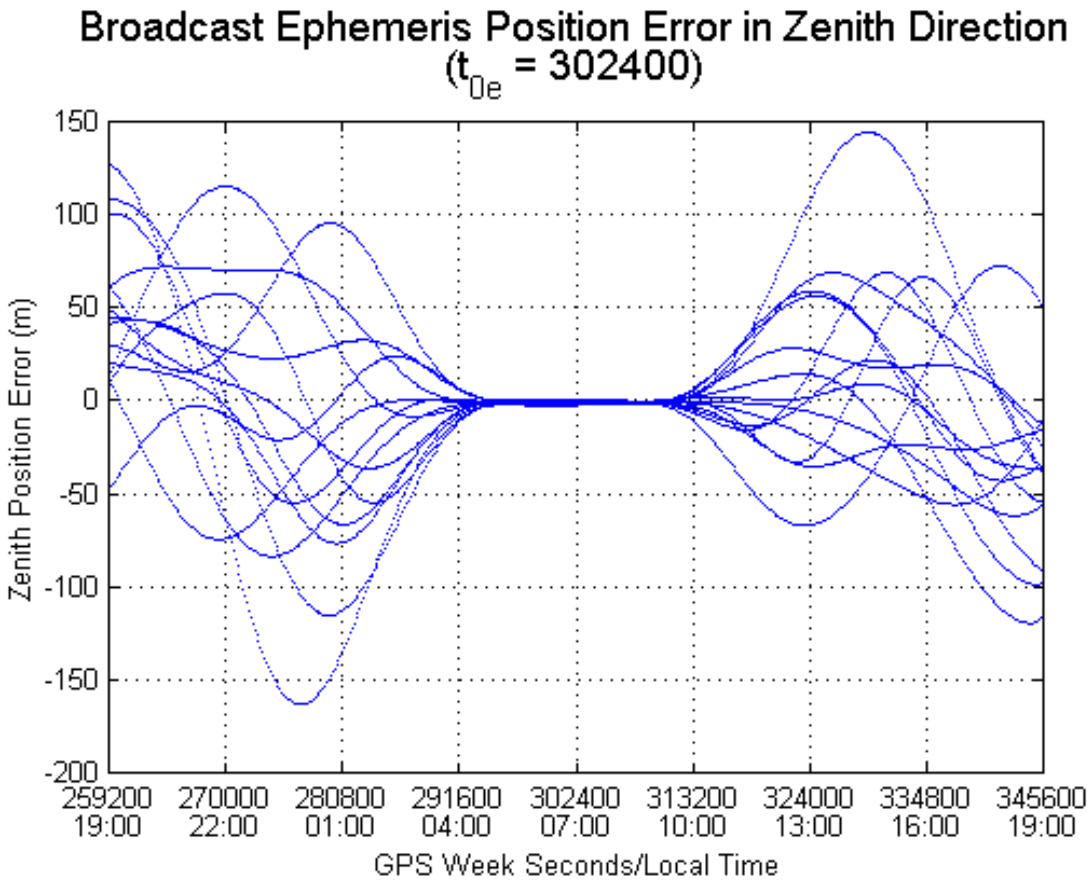
Broadcast Ephemeris Errors Over 2 Days

- **Difference between broadcast and ephemeris data for PRN 1**
  - Note: this includes many different broadcast ephemeris data sets
    - Transitions between sets can be seen as jumps in the plots



Example of Satellite Position Errors (1/3)

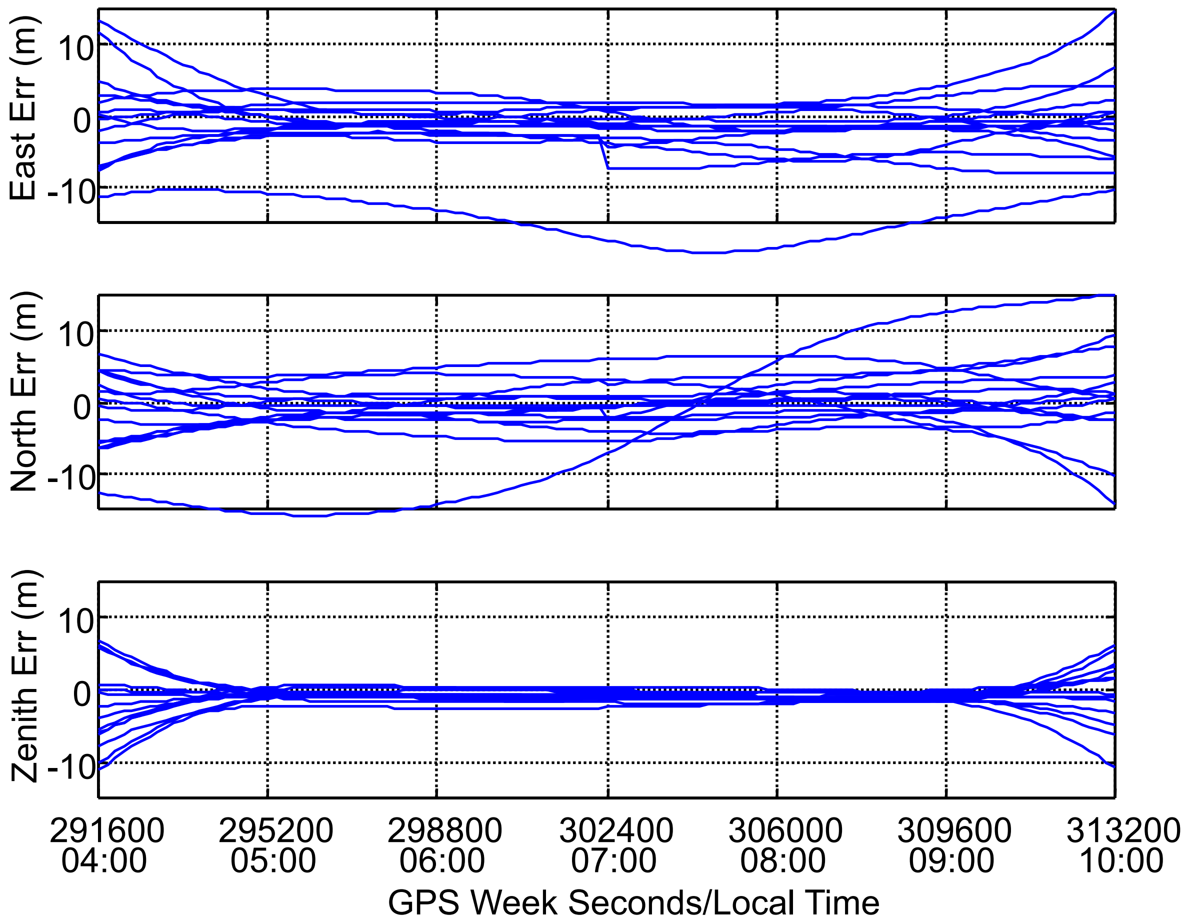
- Errors in broadcast ephemeris (compared to NGS precise ephemeris)
  - Traces shown for PRNs 1, 3, 5, 8, 9, 15, 17, 21, 23, 25, 26, 29, 30, and 31
  - Ephemeris for all these PRNs had t_{0e} within 100 seconds of 302400 GPS week seconds ($302300 < t_{0e} < 302500$) (20 Jan 99)



Example of Satellite Position Errors (2/3)

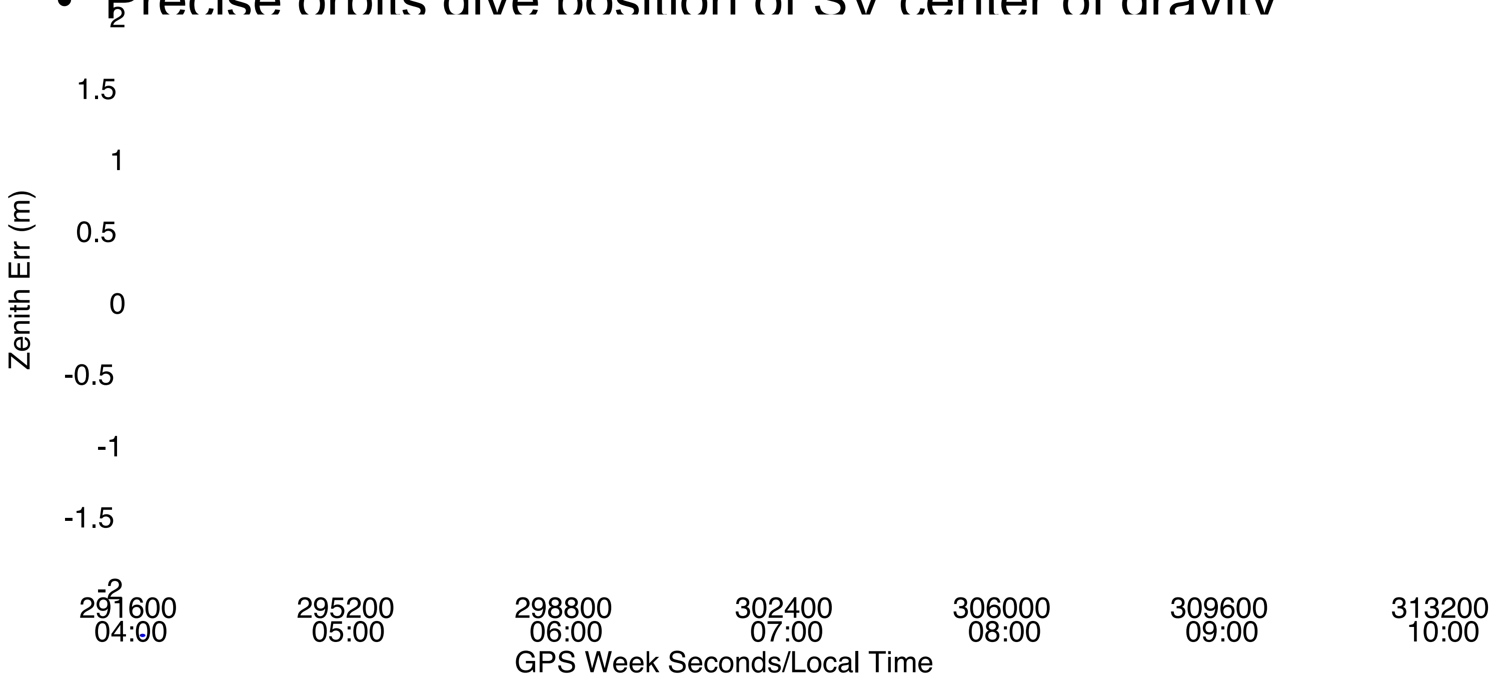
- **Breakdown of plot on previous page, showing +/- 3 hours from t_{0e} in each axis**
  - East/West
  - North/South
  - Zenith (Vertical)

Best accuracy in zenith direction. Why?



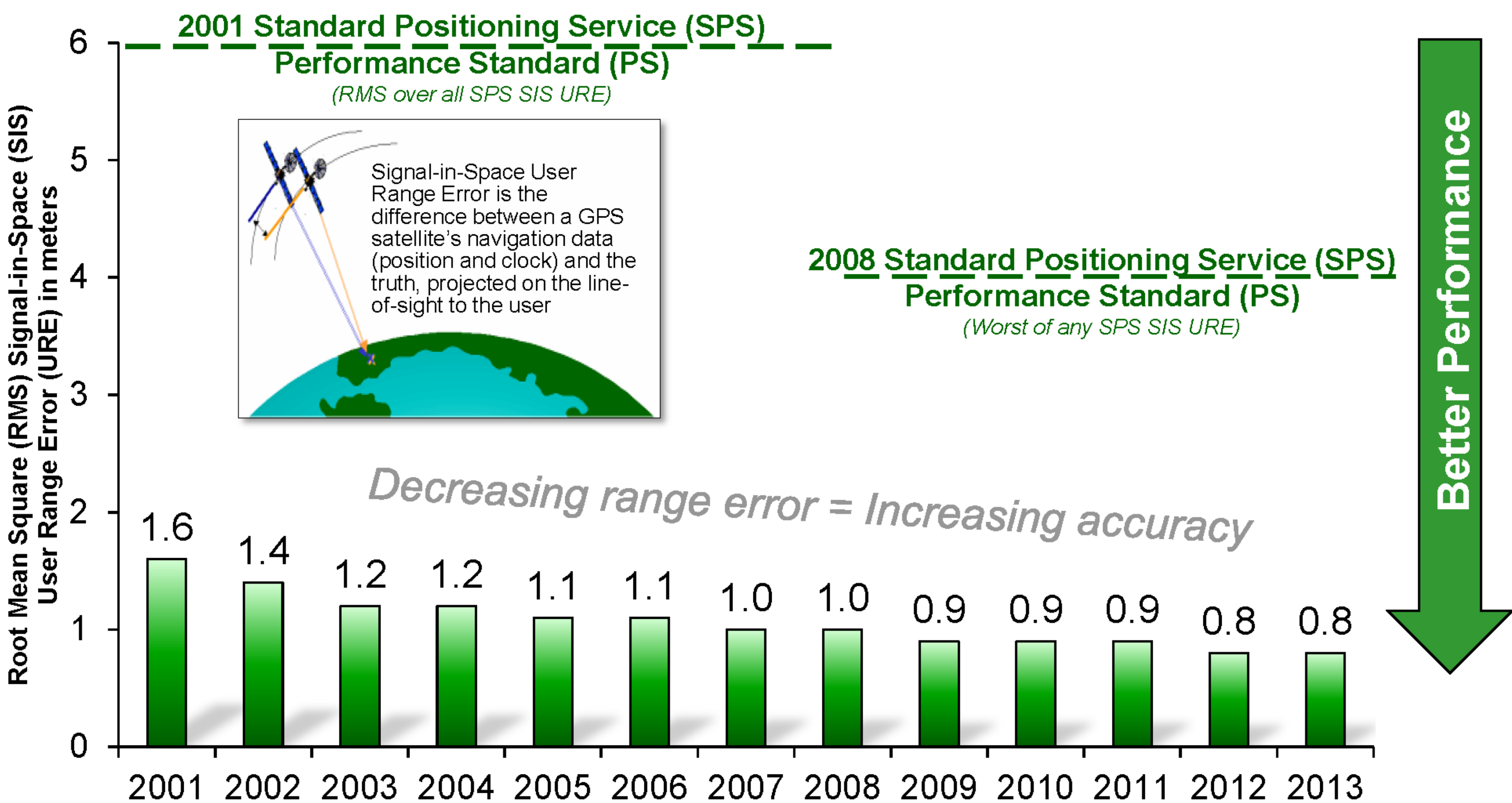
Example of Satellite Position Errors (3/3)

- **Highlight of zenith difference between broadcast ephemeris position and precise orbits**
  - Bias is due to 95.2 cm offset between SV center of gravity and antenna phase center
    - Broadcast ephemeris gives position of antenna phase center
    - Precise orbits give position of SV center of gravity



Signal in Space User Range Error

source: www.gps.gov



System accuracy better than published standard

Observed GPS Positioning Errors with Typical SPS and PPS Receivers

Error Source	Typical Range Error Magnitude (meters, 1σ)		
	SPS (w/ SA)	SPS (w/o SA)	PPS
Selective Availability	24.0	0.0	0.0
Ionosphere[a]	7.0	7.0	0.01
Troposphere[b]	0.7	0.7	0.7
SV Clock & Ephemeris	3.6	3.6	3.6
Receiver Noise	1.5	1.5	0.6
Multipath[c]	1.2	1.2	1.8
Total User Equivalent Range Error (UERE)	25.3	8.1	4.1

[a]For SPS: 7.0 is typical value of ionosphere after applying ionospheric model. Actual values can range between approximately 1-30 m.

[b]Residual error after using tropospheric model

[c]For PPS: includes increase in multipath that results from using L1 and L2 code measurements to remove ionospheric error.

National Research Council, *The Global Positioning System: A Shared National Asset*, National Academy Press, 1995.

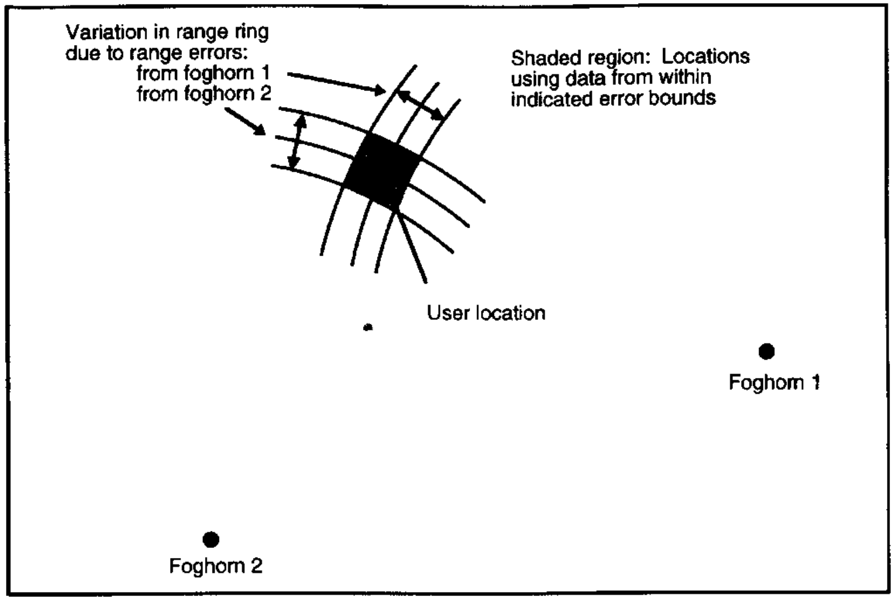
Measurement Domain vs. Position Domain

- **Pseudorange errors are errors in "measurement domain"**
  - Errors in the measurements themselves
  - UERE is one example
- **Ultimately, we'd like to know errors in "position domain"**
  - The position errors that result when using the measurements
  - Errors in position domain are different than measurement errors!
    - Can be larger
    - Can be smaller
  - Dependent on measurement geometry
- **Mathematical representation**
  - We have covariance matrix of measurements ($\mathbf{C}_\rho$).
  - We want covariance matrix of calculated position and clock error ($\mathbf{C}_\mathbf{x}$)
- **In GPS applications, this problem is approached using concept called Dilution of Precision (DOP)**

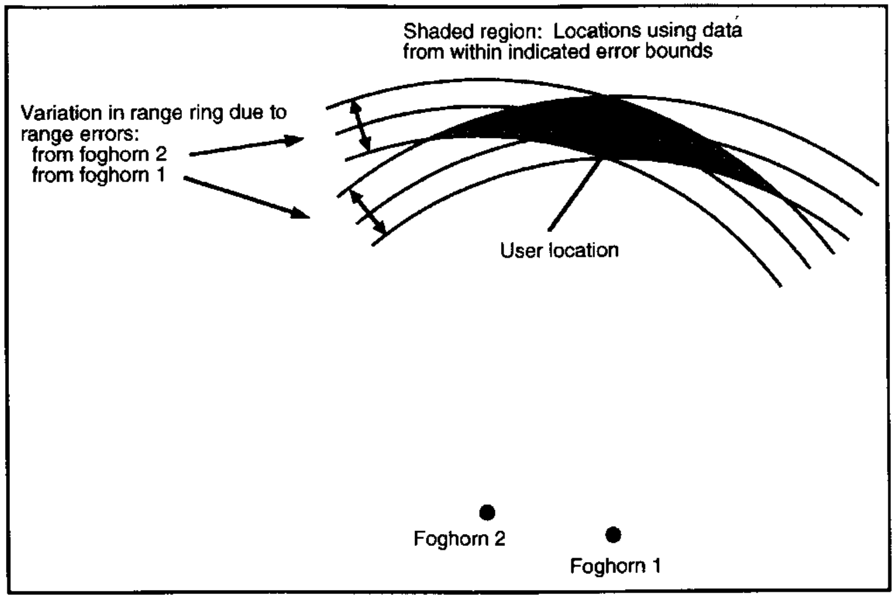
Effect of Geometry on Positioning Accuracy (Foghorn Example)

Consider the foghorn example, except allow for a measurement error

Good Geometry Example



Poor Geometry Example



Obtaining $\mathbf{C_x}$ from Least-Squares Analysis (1/2)

- **Definition of $\mathbf{C_x}$**

$$C_x = \begin{bmatrix} \sigma^2_{x_u} & \sigma_{x_u y_u} & \sigma_{x_u z_u} & \sigma_{x_u \delta t_u} \\ \sigma_{x_u y_u} & \sigma^2_{y_u} & \sigma_{y_u z_u} & \sigma_{y_u \delta t_u} \\ \sigma_{x_u z_u} & \sigma_{y_u z_u} & \sigma^2_{z_u} & \sigma_{z_u \delta t_u} \\ \sigma_{x_u \delta t_u} & \sigma_{y_u \delta t_u} & \sigma_{z_u \delta t_u} & \sigma^2_{\delta t_u} \end{bmatrix}$$

where, for example,

$$\sigma^2_{x_u} = E\left[\left(x_u - E[x_u]\right)^2\right]$$

$$= \text{variance of } x_u$$

$$\sigma_{x_u y_u} = E\left[\left(x_u - E[x_u]\right)\left(y_u - E[y_u]\right)\right]$$

$$= \text{covariance of } x_u \text{ and } y_u$$

- **Definition of $\mathbf{C_\rho}$**

$$\mathbf{C}_\rho = \begin{bmatrix} \sigma^2_{\rho_1} & \sigma_{\rho_1\rho_2} & \cdots & \sigma_{\rho_1\rho_n} \\ \sigma_{\rho_1\rho_2} & \sigma^2_{\rho_2} & \cdots & \sigma_{\rho_2\rho_n} \\ \vdots & \vdots & \ddots & \sigma_{\rho_3\rho_n} \\ \sigma_{\rho_1\rho_n} & \sigma_{\rho_2\rho_n} & \sigma_{\rho_3\rho_n} & \sigma^2_{\rho_n} \end{bmatrix}$$

Obtaining $\mathbf{C_x}$ from Least-Squares Analysis (2/2)

- **According to least-squares theory:**

$$\mathbf{C_x} = \left(\mathbf{H}^T \mathbf{C}_\rho^{-1} \mathbf{H}\right)^{-1}$$

  – Basic assumptions
    - Measurement errors are zero-mean
    - Measurement errors have a Gaussian distribution
- **Recall that the least-squares solution with measurement weighting was**

$$\Delta\mathbf{x} = \left(\mathbf{H}^T \mathbf{C}_\rho^{-1} \mathbf{H}\right)^{-1} \mathbf{H}^T \mathbf{C}_\rho^{-1} \Delta\rho$$
$$= \mathbf{C_x} \mathbf{H}^T \mathbf{C}_\rho^{-1} \Delta\rho$$

  – Consider case where the nominal position and clock error (used to calculate $\Delta\rho$) are actually the true position and clock error
    - The $\Delta\rho$ represents the measurement *errors*
    - The $\Delta\mathbf{x}$ represents the position and clock *errors*
    - The $\mathbf{C_x}$ matrix is a multiplier for the measurement errors ($\Delta\rho$)
      – "Large" $\mathbf{C_x}$ values $\rightarrow$ large position errors
      – "Small" $\mathbf{C_x}$ values $\rightarrow$ small position errors

Dilution of Precision (DOP)

- **In GPS, the concept of Dilution of Precision (DOP) is used**
  - Based upon covariance matrix of position and clock errors ($\mathbf{C_x}$)
  - Additional assumptions
    - All measurements have the same variance

$$\sigma_{\rho_1}^2 = \sigma_{\rho_2}^2 = \ldots = \sigma_{\rho_n}^2 = \sigma_{\rho}^2$$

    - Measurement errors are uncorrelated (i.e.,covariance values are zero)

$$\sigma_{\rho_j \rho_k} = 0, \quad j \neq k$$

  - Using these assumptions

$$\mathbf{C}_\rho = \mathbf{I}\sigma_\rho^2$$

and

$$\mathbf{C_x} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\sigma_\rho^2$$

  - The matrix $\left(\mathbf{H}^T\mathbf{H}\right)^{-1}$ is called the DOP matrix
    - Directly relates measurement errors to position errors

Use of Local-Level Coordinate Frame (1/2)

- **Normally, DOPs describe errors in geodetic (local-level) coordinate frame (east, north, up), rather than the ECEF frame.**
  - Need to modify the H matrix so that the errors refer to the local-level frame
  - Original H matrix (used to calculate position)

$$\mathbf{H}^E = \begin{bmatrix} \mathbf{a}_1^{E^T} & 1 \\ \mathbf{a}_2^{E^T} & 1 \\ \vdots & \vdots \\ \mathbf{a}_n^{E^T} & 1 \end{bmatrix}$$

    - "$\mathbf{a}$" vectors are unit line-of-sight vectors between user and SV in *ECEF frame*
    - This will give the $\mathbf{C}_\mathbf{x}$ matrix described previously
  - New H matrix for DOP calculations

$$\mathbf{H}^G = \begin{bmatrix} \mathbf{a}_1^{G^T} & 1 \\ \mathbf{a}_2^{G^T} & 1 \\ \vdots & \vdots \\ \mathbf{a}_n^{G^T} & 1 \end{bmatrix}$$

    - "$\mathbf{a}$" vectors are now unit line-of-sight vectors between user and SV in *geodetic (ENU) frame*

Use of Local-Level Coordinate Frame (2/2)

- **Local-level "a" vectors can be calculated using direction cosine matrix (DCM)**

$$\mathbf{a}^G = \mathbf{C}_E^G \mathbf{a}^E$$

$\mathbf{C}_E^G =$ DCM that rotates from ECEF to geodetic (E,N,U) frame

$$\mathbf{C}_E^G = \left(\mathbf{C}_G^E\right)^{-1} = \left(\mathbf{C}_G^E\right)^T$$

- **When $\mathbf{H}^G$ is used to calculate the covariance $\mathbf{C}_\mathbf{x} = \left(\mathbf{H}^{G^T}\mathbf{H}^G\right)^{-1}\sigma_\rho^2$, then $\mathbf{C}_\mathbf{x}$ is defined as**

$$\mathbf{C}_\mathbf{x} = \begin{bmatrix} \sigma_e^2 & \sigma_{en} & \sigma_{eu} & \sigma_{e\delta t_u} \\ \sigma_{en} & \sigma_n^2 & \sigma_{nu} & \sigma_{n\delta t_u} \\ \sigma_{eu} & \sigma_{nu} & \sigma_u^2 & \sigma_{u\delta t_u} \\ \sigma_{e\delta t_u} & \sigma_{n\delta t_u} & \sigma_{u\delta t_u} & \sigma_{\delta t_u}^2 \end{bmatrix}$$

  – This is what we desire to describe using DOPs

DOP Values

- **Desirable to characterize the $\mathbf{C_x}$ matrix using a single number**
  - For DOPs
    - Cross-correlation terms ignored
    - Root-Sum-Square (RSS) value of variables of interest, normalized by σ_{UERE}
    - Example:

$$GDOP = \frac{\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_u^2 + \sigma_{\delta t_u}^2}}{\sigma_{UERE}}$$

    - GDOP can be calculated directly from DOP matrix

$$\left(\mathbf{H}^{G^T}\mathbf{H}^G\right)^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix} \qquad GDOP = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}}$$

    - Note that GDOP relates UERE with RSS of errors

Key relationship!

$$\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_u^2 + \sigma_{\delta t_u}^2} = GDOP \times \sigma_{UERE}$$

Types of DOPs

- **The "Big Three"**
  - GDOP (Geometric DOP)

$$GDOP = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}}$$

$$\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_u^2 + \sigma_{\delta t_u}^2} = GDOP \times \sigma_{UERE}$$

  - PDOP (Position DOP)

$$PDOP = \sqrt{D_{11} + D_{22} + D_{33}}$$

$$\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_u^2} = PDOP \times \sigma_{UERE}$$

  - HDOP (Horizontal DOP)

$$HDOP = \sqrt{D_{11} + D_{22}}$$

$$\sqrt{\sigma_e^2 + \sigma_n^2} = HDOP \times \sigma_{UERE}$$

- **Less common (for navigators, at least!)**
  - VDOP (Vertical DOP)

$$VDOP = \sqrt{D_{33}}$$

$$\sqrt{\sigma_u^2} = VDOP \times \sigma_{UERE}$$

  - TDOP (Time DOP)

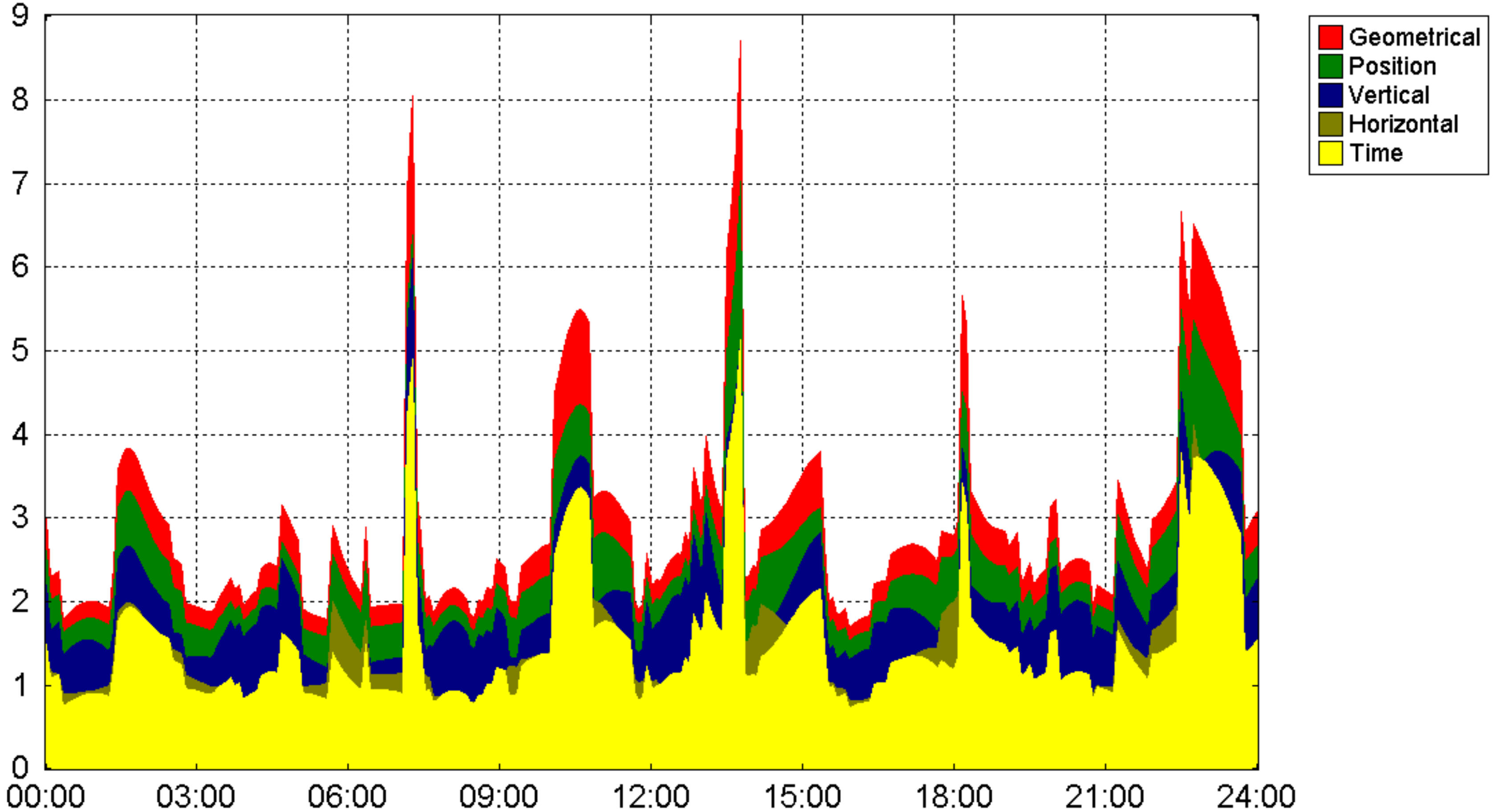
$$TDOP = \sqrt{D_{44}}$$

$$\sqrt{\sigma_{\delta t_u}^2} = TDOP \times \sigma_{UERE}$$

    - Note: time is in units of meters

Typical DOP Plot

Dayton Ohio – 24 Apr 2003 – All Visible SVs (above 10° elevation)



Questions?