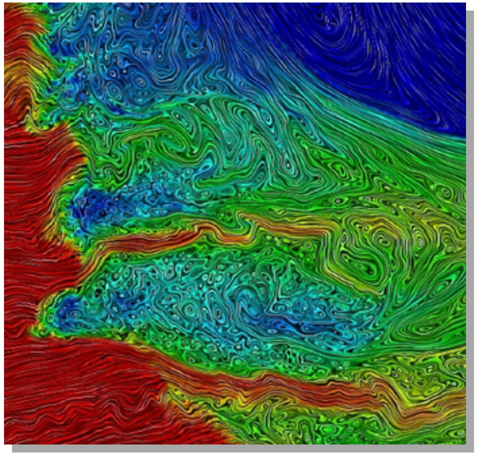
Joint ICTP-IAEA College on Plasma Physics

Kinetic Models of Space Plasma Turbulence



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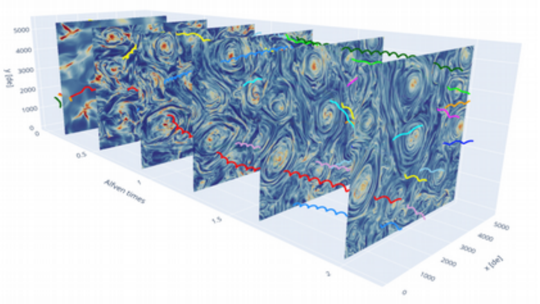
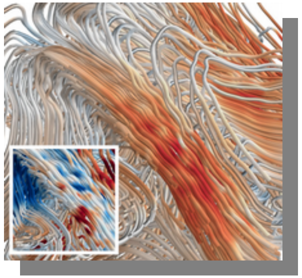




Structure of the course

Part I

- Introduction to astrophysical plasmas
- Numerical techniques for collisionless plasmas
- From global to local simulations of homogeneous turbulence

Part II

- The role of magnetic reconnection in plasma turbulence
- Intermittency in plasmas turbulence: beyond MHD
- A new paradigm: phase space turbulence

Part III

- Particles in plasma turbulence
- Turbulence and shocks
- Beyond the heliosphere, far away in the Universe

Part I

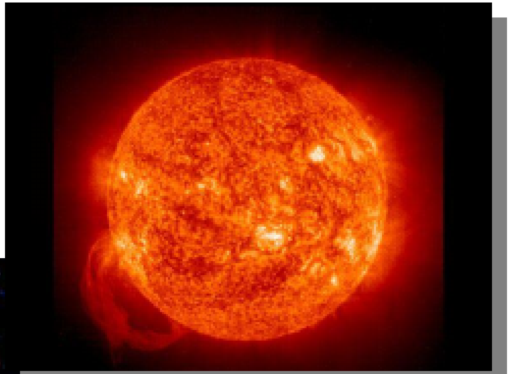
- **Introduction on astrophysical plasmas**
  Main properties of plasmas from spacecraft measurements

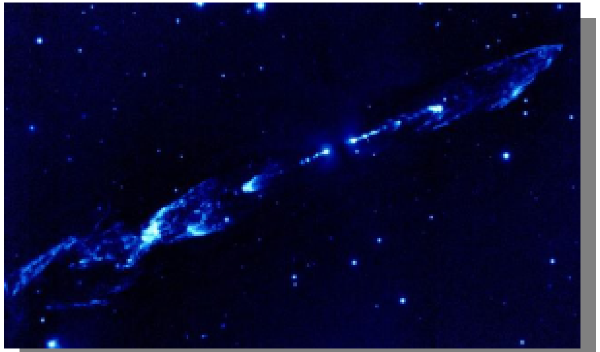
- **Numerical techniques for collisionless plasmas**
  The Eulerian and Lagrangian approaches
  Advanced numerical schemes for simulations of plasma turbulence

- **From global to local simulations of homogeneous turbulence**
  Homogeneous simulations of Vlasov turbulence
  Comparisons with data

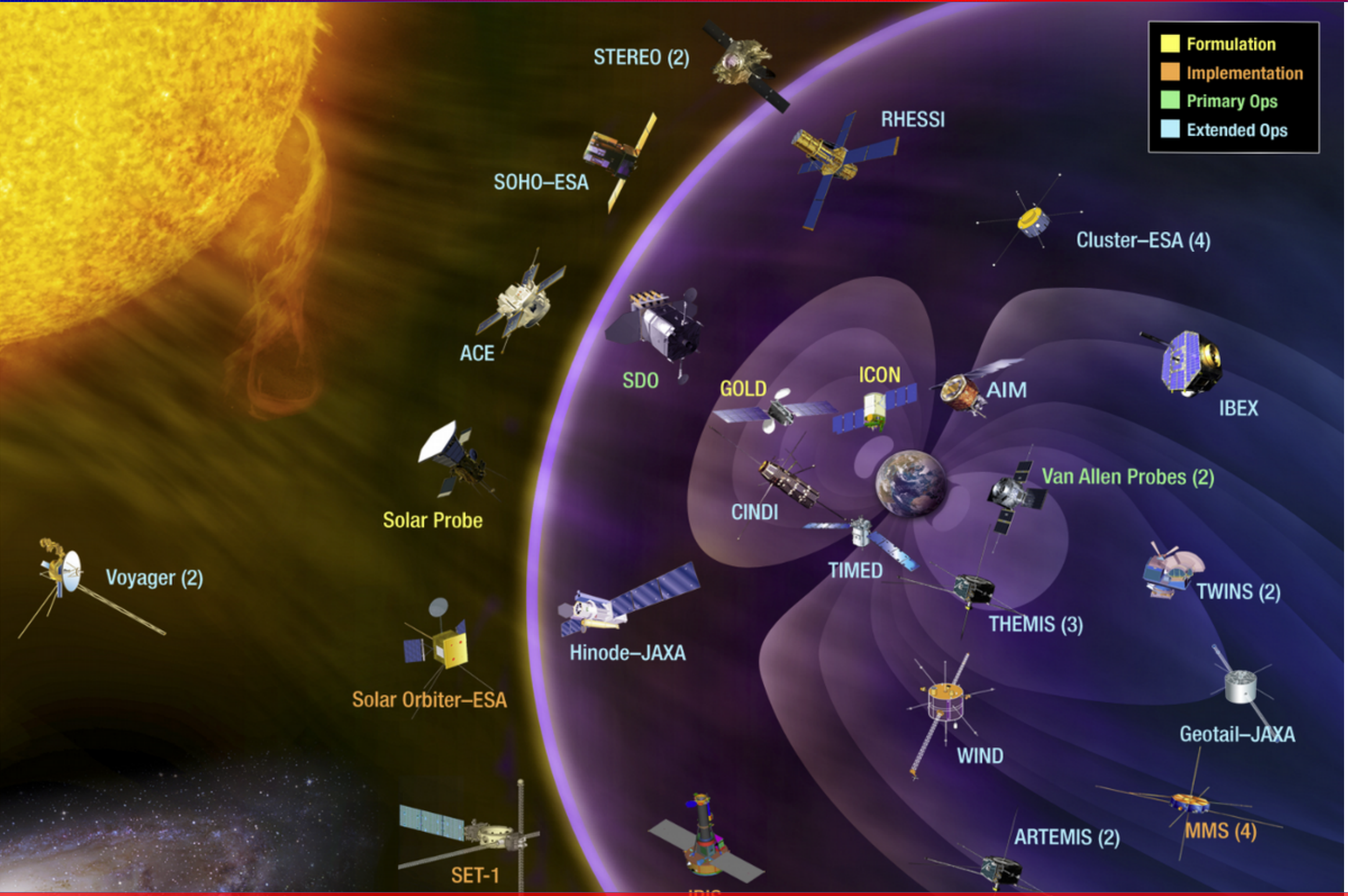
Plasma in the Universe

- A plasma is a ionized gas where charged particles interact via electromagnetic forces
- More than 99.9 % of matter in the Universe can be considered as a plasma
- Observations are somehow limited
- Plasma is mostly collisionless




The "new era" of space missions

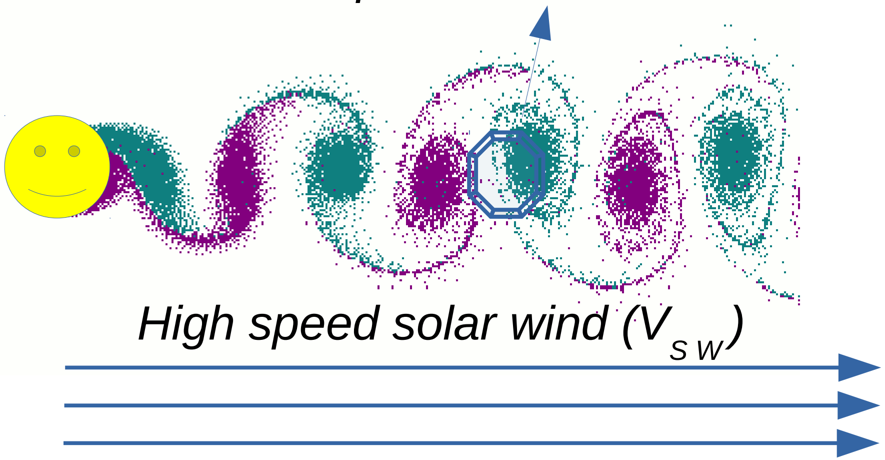


Tremendous technology improvement: a *golden age* for space missions

The heliosphere is a laboratory for the comprehension of the Universe

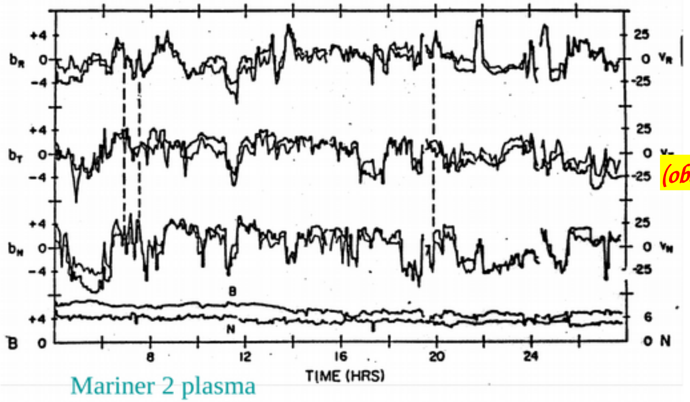
Pioneering measurements in the solar wind

Spacecraft that measures f(t)



High speed solar wind (V_{sw})

Alfvènic fluctuations

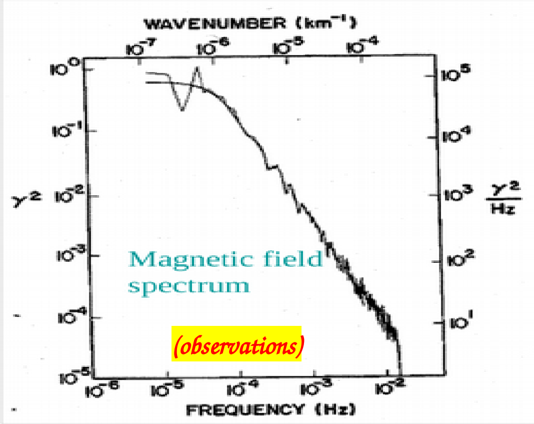


(observations)

Mariner 2 plasma and magnetic field data

Belcher and Davis, JGR, 1972

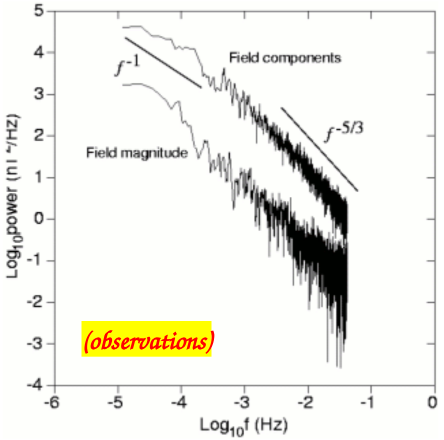
Power-law spectra



Magnetic field spectrum

(observations)

SW at 2.8 AU: Matthaeus and Goldstein



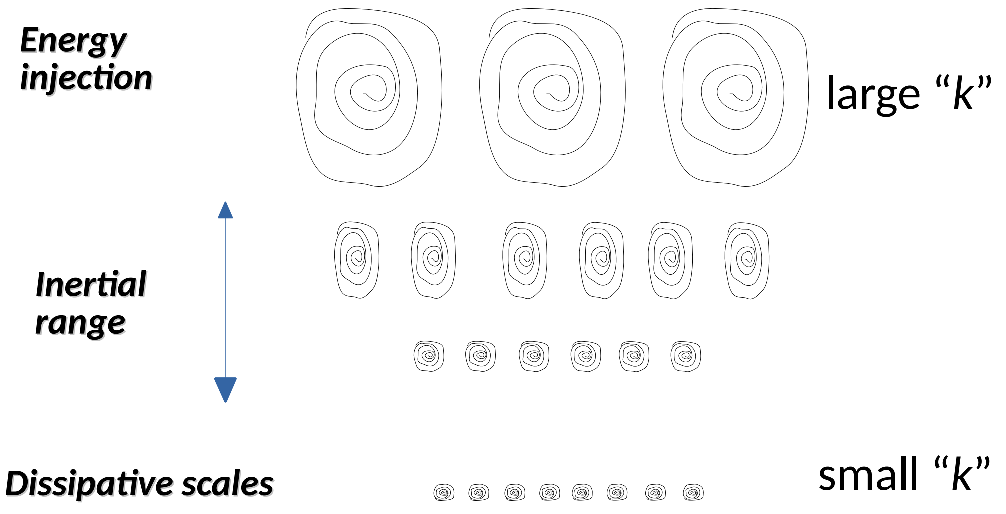
(observations)

Turbulence
Structures
Waves

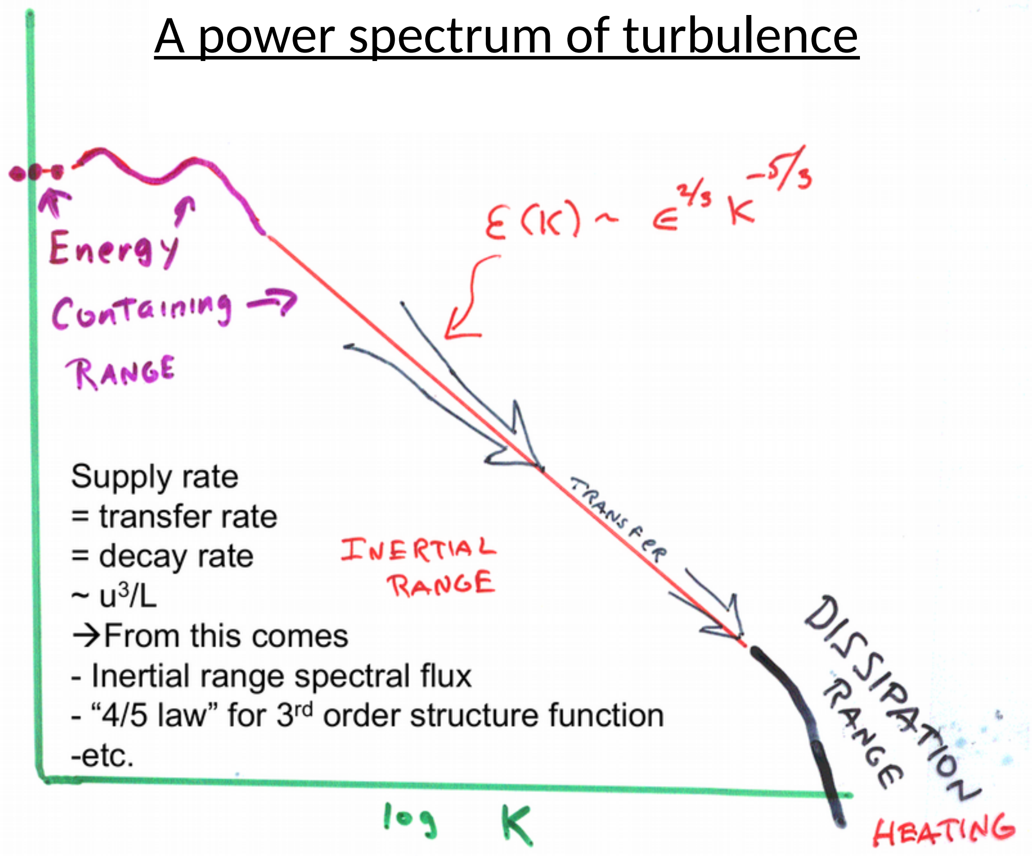
Suggestions from hydrodynamics: a cartoon

A simple picture of turbulence

The turbulent cascade

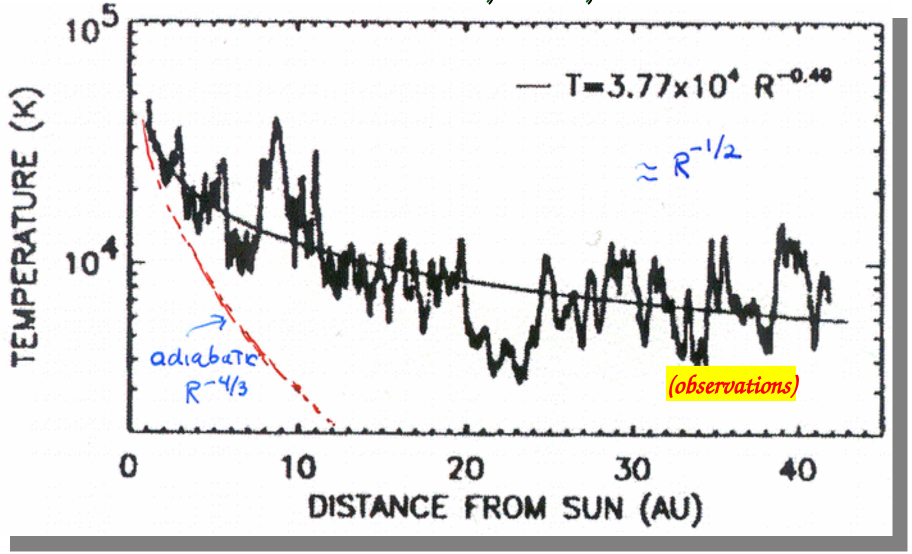


A power spectrum of turbulence

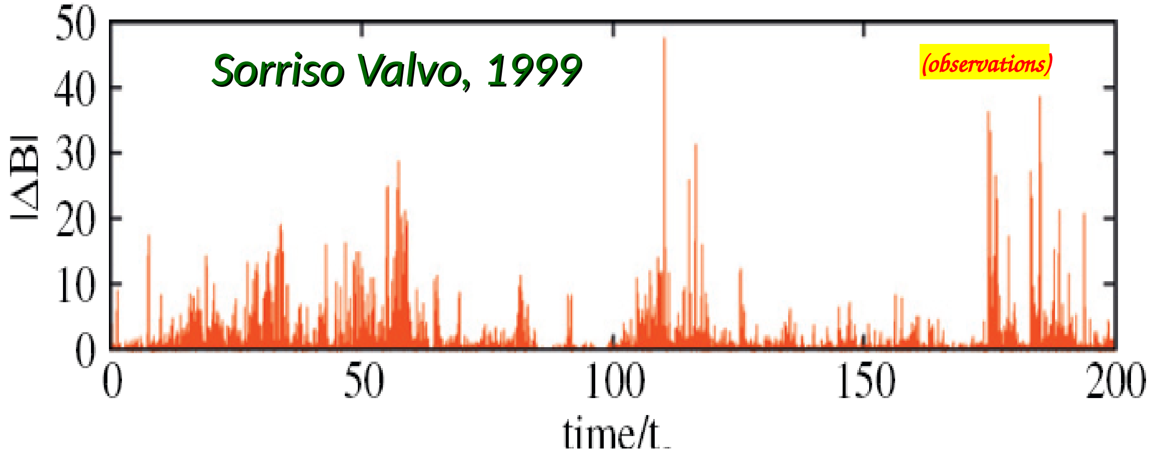


Turbulence in space plasmas

Richardson et al, GRL, 1995



Solar wind is turbulent and intermittent



Most of the plasma energization (plasma heating and particle acceleration) occurring in turbulent collisionless plasmas, such as those permeating the solar system is expected to occur at kinetic scales (scales ~ particle gyroradii and below)

How is the plasmas heated and how are particles accelerated?

Temperature “anomalies” in space plasmas

$$\Pi(\mathbf{x},t)=m\int\int\int[\mathbf{v}-\mathbf{V}_{bulk}(\mathbf{x},t)][\mathbf{v}-\mathbf{V}_{bulk}(\mathbf{x},t)]f(\mathbf{x},\mathbf{v},t)d^3v$$

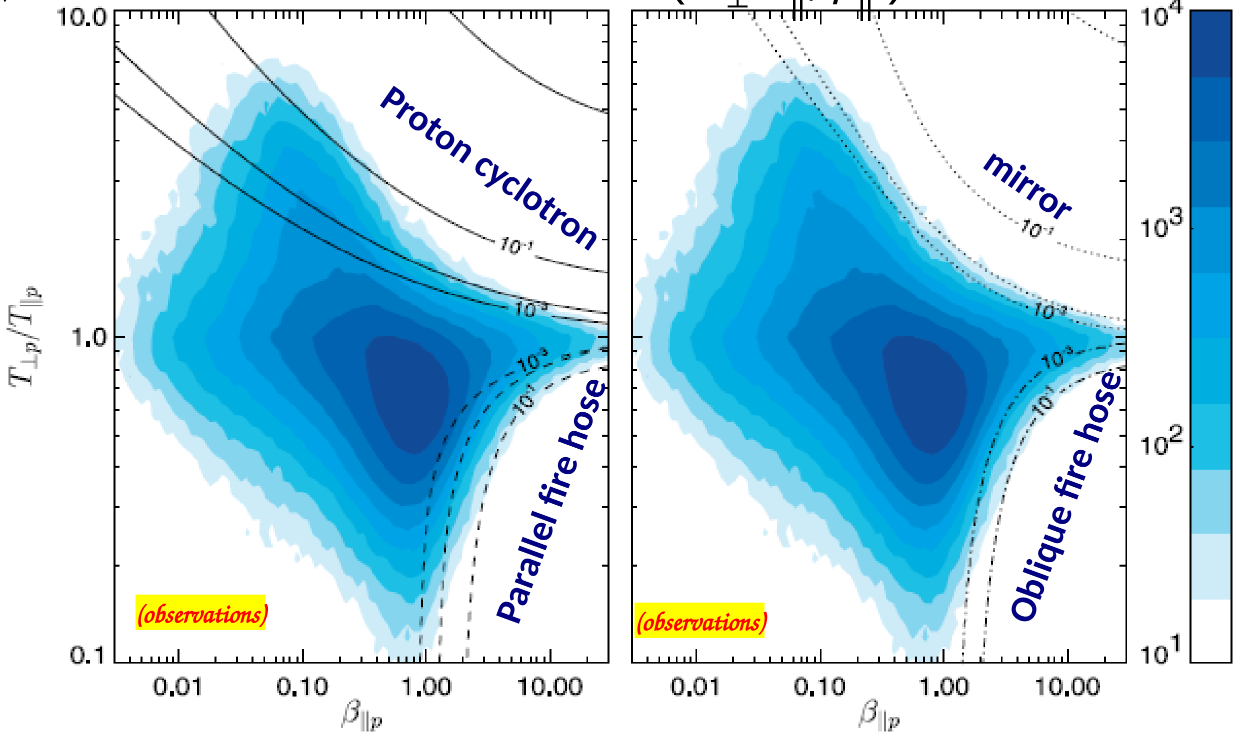
$$\Pi=p_\perp I+(p_\parallel-p_\perp)\mathbf{b}\mathbf{b}$$

$$\mathbf{b}=\frac{\mathbf{B}}{|\mathbf{B}|}$$

$T_{||}$ and $T_\perp$ ≡ parallel and perpendicular proton temperatures with respect to the ambient B

Hellinger et al. GRL (2006); Kasper et al. JGR (2006); Kasper et al., (2002)

Distribution PDF($T_\perp/T_{||}$, $\beta_{||}$)



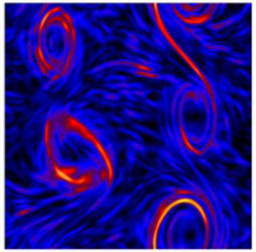
Kinetic instabilities influence the solar wind ➡ **Astrophysical plasmas are turbulent, intermittent and show many kinetic (non-fluid) phenomena** ➡ **We need simulations!**

Plasma models

Information loss

Liouville

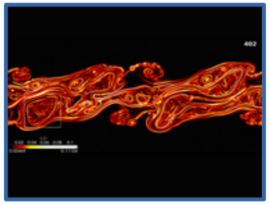
⇩



Vlasov-Maxwell

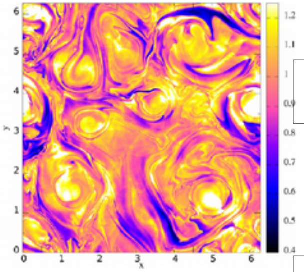
$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left[\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right]\cdot\nabla_v f_\alpha = 0 \quad + \quad Maxwell$$

⇩ *integrating*



Two fluid equations

⇩ *approximating*



Magnetohydrodynamics (MHD)

⇩ *neglecting **B***

Hydrodynamics (Navier-Stokes)

Complexity

Kinetic equations for a collisionless plasma

The Vlasov-Maxwell system

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left[\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right]\cdot\nabla_v f_\alpha = 0 \longrightarrow f_\alpha = f_\alpha(\mathbf{x},\mathbf{v},t)$$

$$\int f_\alpha(\mathbf{x},\mathbf{v},t)d\mathbf{x}d\mathbf{v} = N_\alpha$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\cdot\mathbf{E} = 4\pi\sum_\alpha q_\alpha\int f_\alpha(\mathbf{x},\mathbf{v},t)d\mathbf{v}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\sum_\alpha q_\alpha\int \mathbf{v}f_\alpha(\mathbf{x},\mathbf{v},t)d\mathbf{v} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}$$

6D + time!

Two numerical "philosophies"

Method 1: the Eulerian approach

The Vlasov equation is solved directly for the particle distribution function, on a phase space grid. Moments (density and current) are evaluated by direct integration of the distribution function

- Zero noise
- Very computationally demanding because of memory limitations

Method 2: the Lagrangian approach

Vlasov is solved via a *montecarlo* technique. The equations of motion of a large number of (macro) particles are solved and the distribution function is reconstructed. Maxwell equations are evaluated on a grid, through interpolation

- Very cheap from the computational point of view
- Numerical noise

We will use both methods, depending on the problem that we want to study

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Method 1: Eulerian

Full Vlasov–Maxwell

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left[\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right]\cdot\nabla_v f_\alpha = 0$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\cdot\mathbf{E} = 4\pi\sum_\alpha q_\alpha \int f_\alpha(\mathbf{x},\mathbf{v},t)d\mathbf{v}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\sum_\alpha q_\alpha\int \mathbf{v} f_\alpha(\mathbf{x},\mathbf{v},t)d\mathbf{v} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}$$

- Nonlinear integro-differential equation in 6D phase space + time
- Very hard and time demanding to solve numerically!
- To date, numerical solutions are available for approximated, reduced systems

easier

Hybrid Vlasov–Maxwell

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \frac{e}{m}\left[\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right]\cdot\nabla_v f = 0$$

$$\mathbf{E} = -\frac{\mathbf{u}\times\mathbf{B}}{c} + \frac{1}{nec}(\mathbf{j}\times\mathbf{B}) - \frac{1}{ne}\nabla P_e$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \frac{4\pi\mathbf{j}}{c}$$

$$n_e \simeq n_i \simeq n$$

$$P_e = P_e(n)$$

easier

1D-1V Vlasov–Poisson

$$\frac{\partial f_e}{\partial t} + v\frac{\partial f_e}{\partial x} - \frac{eE}{m}\frac{\partial f_e}{\partial v} = 0$$

$$\frac{\partial E}{\partial x} = 4\pi e\left[n_0 - \int f_e(x,v,t)dv\right]$$

Numerical technique for Eulerian-Vlasov

Vlasov equation is an advection equation in phase space

- Let us consider the 1D-1V case (we will discuss later the generalization to full phase space)

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + a\frac{\partial f}{\partial v} = \frac{\partial f}{\partial t} + \frac{\partial (vf)}{\partial x} + \frac{\partial (af)}{\partial v} = 0$$

$$f = f(x, v, t); \quad a = a(x)$$

- Let us focus on advection in *x* first (later we will discuss how to couple it to advection in *v*)

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = 0$$

$$f = f(x, t); \quad x \in [0, L]; \quad t \in [0, T]; \quad v = const.$$

$$f(x, 0) = f_0(x); \quad f(0, t) = f(L, t), \ \forall t \in [0, T]$$

For simplicity, periodic boundary conditions

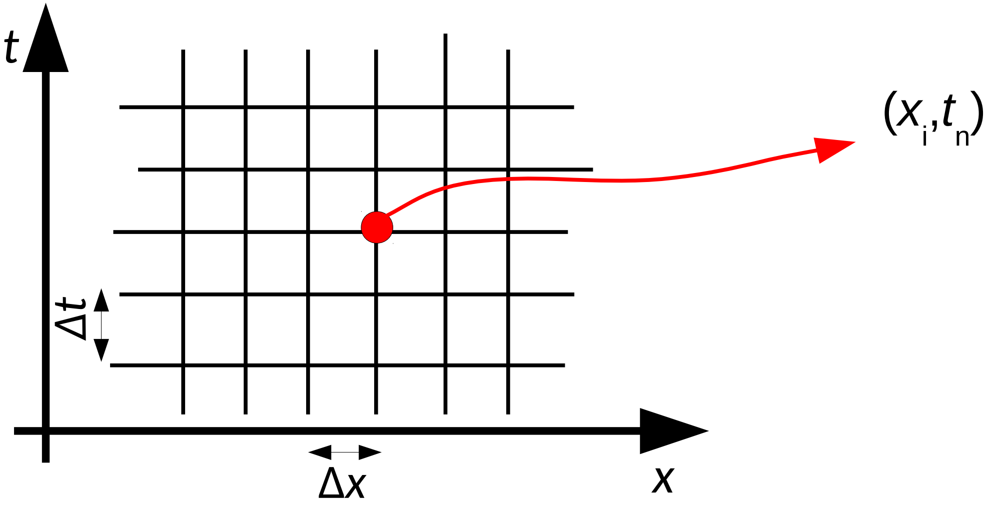
Three main steps:

1) Discretize (*x*,*t*) plane

2) Approximate derivatives in discretized plane (allowed operations are +,-,x,/)

3) Create algorithm to solve the equation

Spatial derivatives



Discretization:

$$x_i = (i-1)\Delta x; \quad i = 1, \cdots, N_x; \quad \Delta x = \frac{L}{N_x}$$

$$x_0 = 0, \quad x_{N_x} = L - \Delta x$$

$$t_n = n\Delta t; \quad n = 0, \cdots, N_t; \quad \Delta t = \frac{T}{N_t}$$

$$t_0 = 0, \quad t_{N_t} = T$$

Derivatives approximation (finite differences):

$$f(x_{i+1}) = f(x_i) + \Delta x \left(\frac{df}{dx}\right)_{x_i} + \frac{1}{2}\Delta x^2 \left(\frac{d^2 f}{dx^2}\right)_{x_i} + \frac{1}{3!}\left(\frac{d^3 f}{dx^3}\right)_{x_i} + o(\Delta x^4) \qquad (1)$$

$$f(x_{i-1}) = f(x_i) - \Delta x \left(\frac{df}{dx}\right)_{x_i} + \frac{1}{2}\Delta x^2 \left(\frac{d^2 f}{dx^2}\right)_{x_i} - \frac{1}{3!}\left(\frac{d^3 f}{dx^3}\right)_{x_i} + o(\Delta x^4) \qquad (2)$$

Euler forward

$$\left.\frac{df}{dx}\right|_{x_i} = \frac{f(x_{i+1}) - f(x_i)}{\Delta x} + o(\Delta x)$$

Euler backward

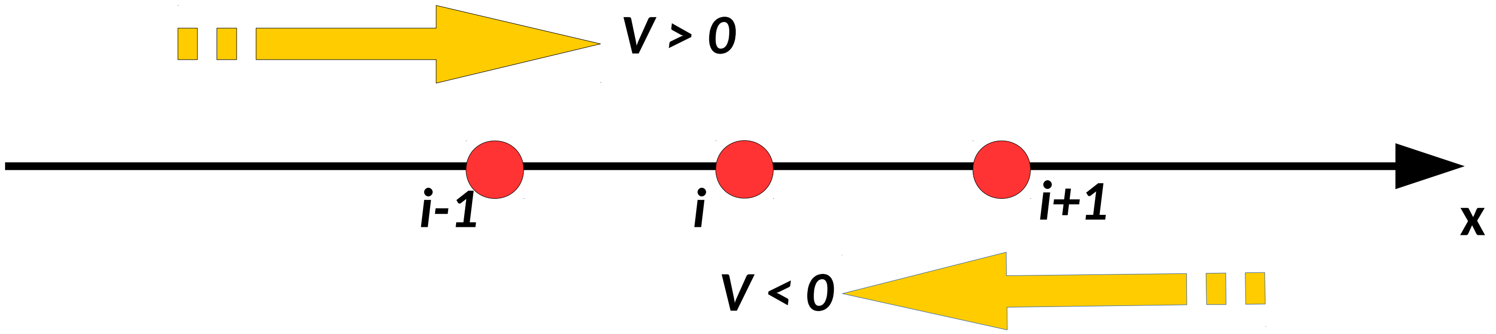
$$\left.\frac{df}{dx}\right|_{x_i} = \frac{f(x_i) - f(x_{i-1})}{\Delta x} + o(\Delta x)$$

$$(1) - (2) \Rightarrow \left(\frac{df}{dx}\right)_{x_i} = \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} + o(\Delta x^2)$$

$$(1) + (2) \Rightarrow \left(\frac{d^2 f}{dx^2}\right)_{x_i} = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{\Delta x^2} + o(\Delta x^2)$$

} Centered differences

Upwind schemes (first-order Godunov method)



$$f_i^{n+1} = f_i^n - |v|\Delta t \left(\frac{f_i^n - f_{i-1}^n}{\Delta x} \right), \qquad v > 0$$

$$f_i^{n+1} = f_i^n + |v|\Delta t \left(\frac{f_{i+1}^n - f_i^n}{\Delta x} \right), \qquad v < 0$$

$$A \leq 1 \Rightarrow \Delta t \leq \frac{\Delta x}{|v|}$$

CFL stability condition

Phase space integration: the splitting scheme

Let's go back to our 1D-1V Vlasov equation

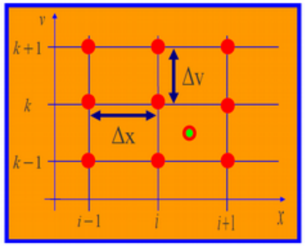
$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + a\frac{\partial f}{\partial v} = \frac{\partial f}{\partial t} + \frac{\partial (vf)}{\partial x} + \frac{\partial (af)}{\partial v} = 0$$

$$f = f(x,v,t); \quad a = a(x)$$

Now we know how to solve advection equations. Let's split the evolution in 2 parts:

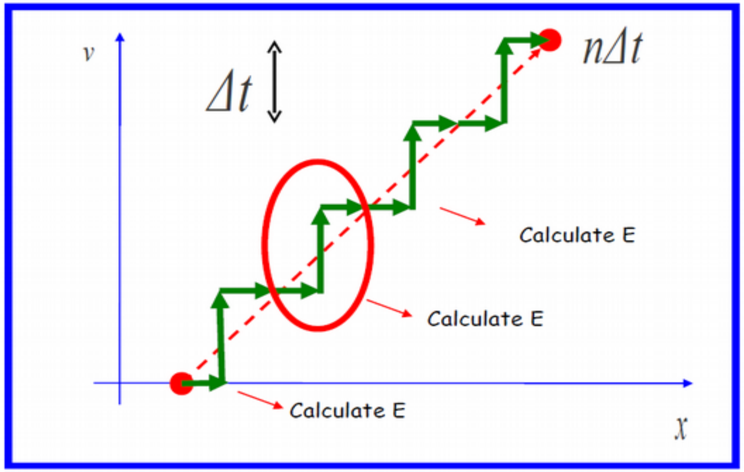
$$\frac{\partial f_x}{\partial t} + v\frac{\partial f_x}{\partial x} = 0$$
$$\frac{\partial f_v}{\partial t} + a\frac{\partial f_v}{\partial v} = 0$$

$$f_x(t+\Delta t) = \Lambda_x(\Delta t) f_x(t)$$
$$f_v(t+\Delta t) = \Lambda_v(\Delta t) f_v(t)$$



Couple f_x*(x,t)* and f_v*(x,t)* to get a solution for *f(x,v,t)*:

$$f(n\Delta t) = \{\Lambda_x(\Delta t/2)\Lambda_v(\Delta t)\Lambda_x(\Delta t/2)\}^n f_0 + o(\Delta t^3)$$



The splitting scheme

Cheng & Knorr, JCP, 1976;

Generalized to 6D in *Mangeney et al. JCP, 2000*

Method 2: Particle in Cell (PIC)

$$f_\alpha = f_\alpha(\mathbf{x}, \mathbf{v}, t) \quad \Rightarrow \quad \frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial t} + \frac{\partial f_\alpha}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{x}}{\partial t} + \frac{\partial f_\alpha}{\partial \mathbf{v}} \cdot \frac{\partial \mathbf{v}}{\partial t}$$

If one moves along a particle trajectory (characteristics):

$$\frac{\partial \mathbf{x}}{\partial t} = \mathbf{v}$$

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{q_\alpha}{m_\alpha}\left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right]$$

Therefore:

$$\left.\frac{df_\alpha}{dt}\right|_{orbit} = \frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right] \cdot \nabla_v f_\alpha = 0$$

Vlasov equation can be solved by integrating numerically the particle equations of motion (Runge-Kutta, Adams-Bashforth, Boris, etc.)

The distribution function can be reconstructed using the fact that it remains constant along particle trajectories

PIC: a global scheme

Individual particles are tracked in continuous phase space, whereas moments of the distribution are computed simultaneously on mesh points

$$\frac{\partial \boldsymbol{x}_p}{\partial t} = \boldsymbol{v}_p$$

$$\frac{\partial \boldsymbol{v}_p}{\partial t} = \boldsymbol{E} - \boldsymbol{v}_p \times \boldsymbol{B}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} = \nabla \times \left[\boldsymbol{u} \times \boldsymbol{B} - \frac{1}{\rho} \boldsymbol{j} \times \boldsymbol{B} - \frac{1}{\rho} \nabla P_e - \eta \boldsymbol{j} \right]$$

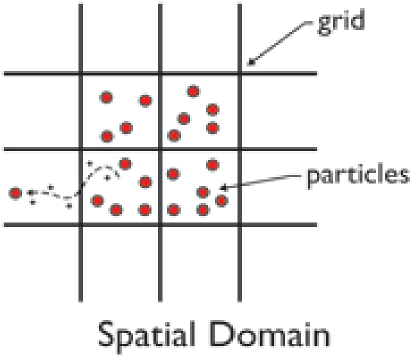
where

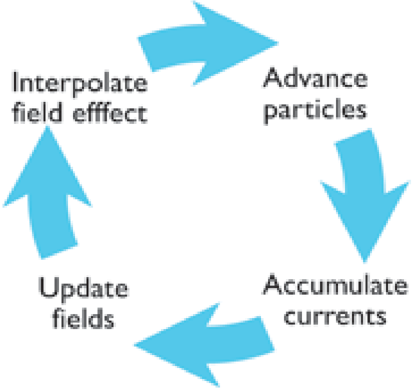
$$\boldsymbol{x}_p \equiv particle\ positions\,; \quad \boldsymbol{v}_p \equiv particle\ velocity\,;$$

$$\rho_e = \rho_i \equiv \rho = \int f_p(\boldsymbol{x}_p, \boldsymbol{v}_p) d^3 v_p\,; \quad \boldsymbol{u} = \frac{1}{\rho} \int \boldsymbol{v}_p f_p(\boldsymbol{x}_p, \boldsymbol{v}_p) d^3 v_p$$

$$P_e = \rho^\gamma\,; \quad \boldsymbol{j} = \nabla \times \boldsymbol{B}$$

- Approximate long range EM interactions
- Use macro-particles
- Track macro-particles continuously in velocity space
- Compute fields, densities and currents on a mesh (grid)
- Problem no longer scales as $O(n^2)$





PIC: a current-advance method

Current-advance method plus a leapfrog scheme. Based on:

$$\frac{\boldsymbol{x}_p^{[k+1]} - \boldsymbol{x}_p^{[k]}}{\Delta t} = \boldsymbol{v}_p^{[k+1/2]}$$

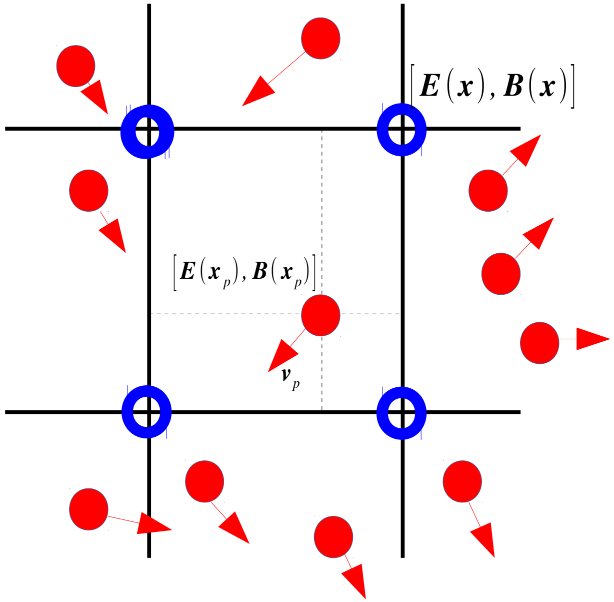
$$\frac{\boldsymbol{v}_p^{[k+1/2]} - \boldsymbol{v}_p^{[k-1/2]}}{\Delta t} = \boldsymbol{E} - \left(\frac{\boldsymbol{v}_p^{[k+1/2]} + \boldsymbol{v}_p^{[k-1/2]}}{2} \right) \times \boldsymbol{B}^{[k]}$$

+

Finite difference schemes for the fields:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \ldots$$

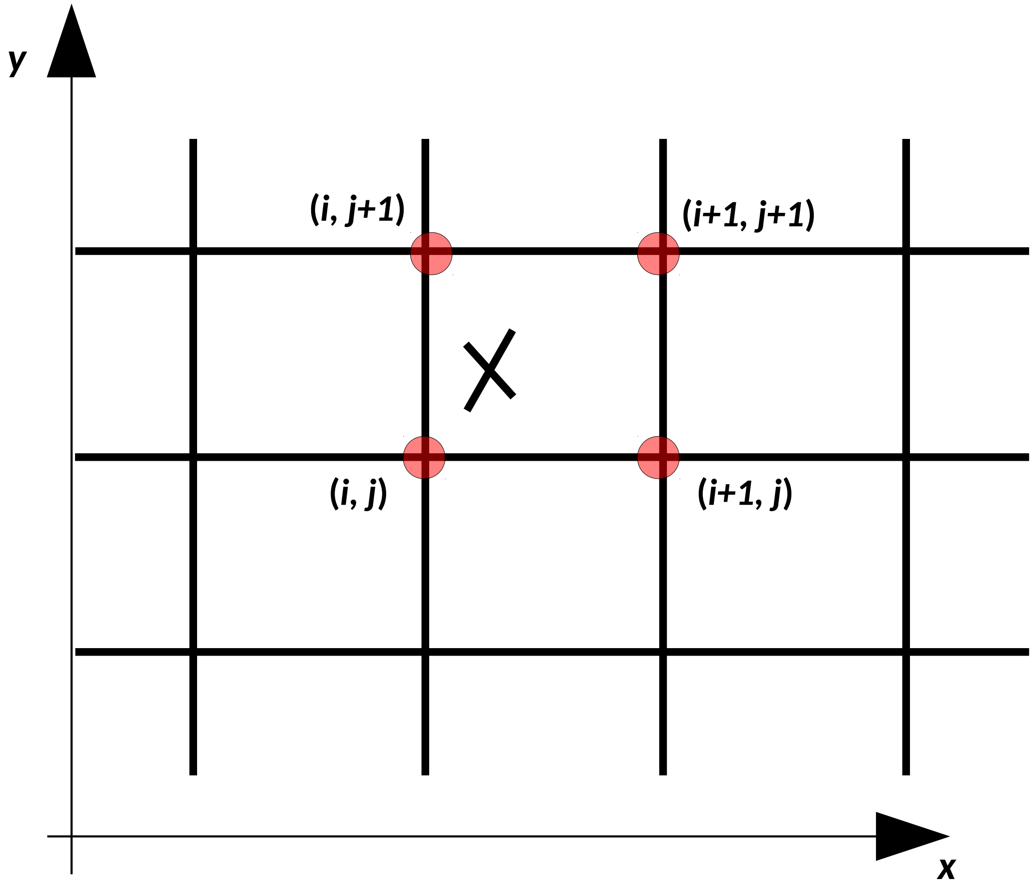
Interpolation is needed →

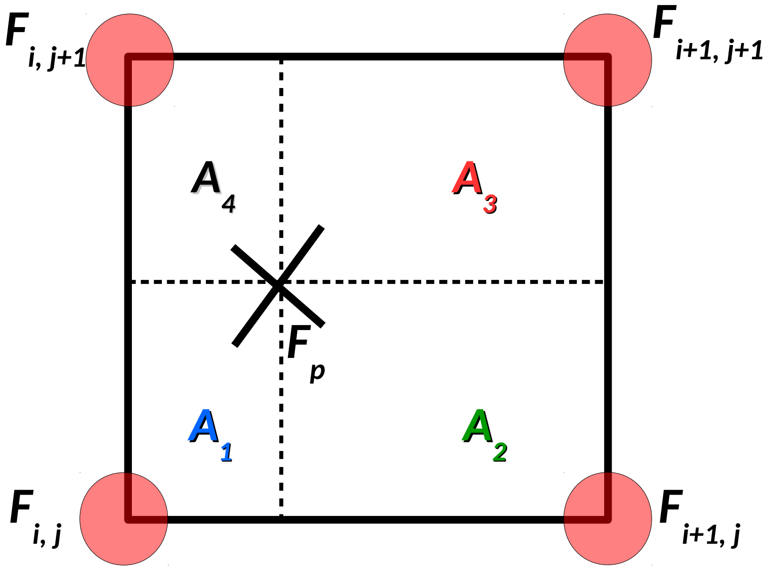


Matthews et al., JCP, 1994

PIC: Interpolation

Example of 2D interpolation





From bilinear approximation of the function, one gets:

$$F_p = \frac{F_{i,j} A_3 + F_{i+1,j} A_4 + F_{i+1,j+1} A_1 + F_{i,j+1} A_2}{A_1 + A_2 + A_3 + A_4}$$

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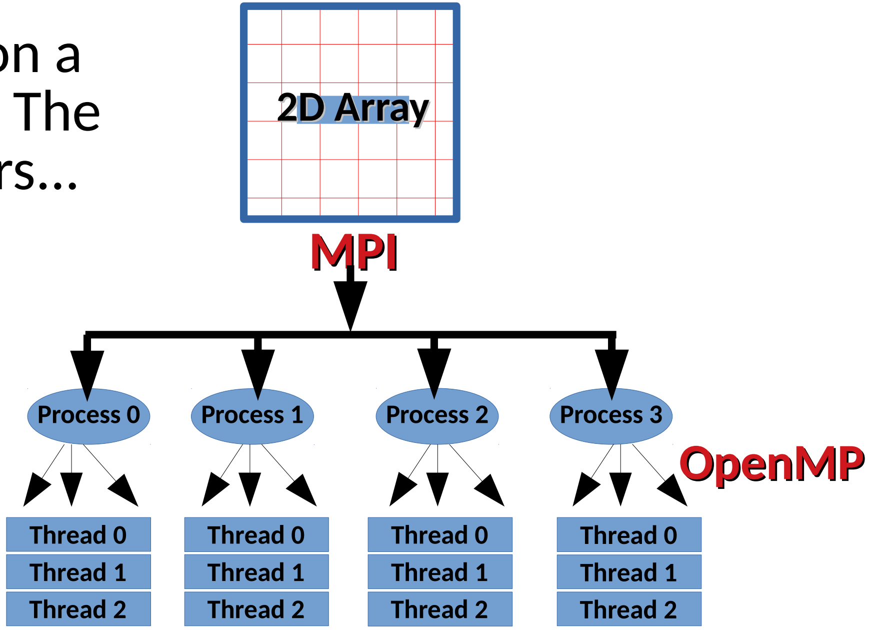
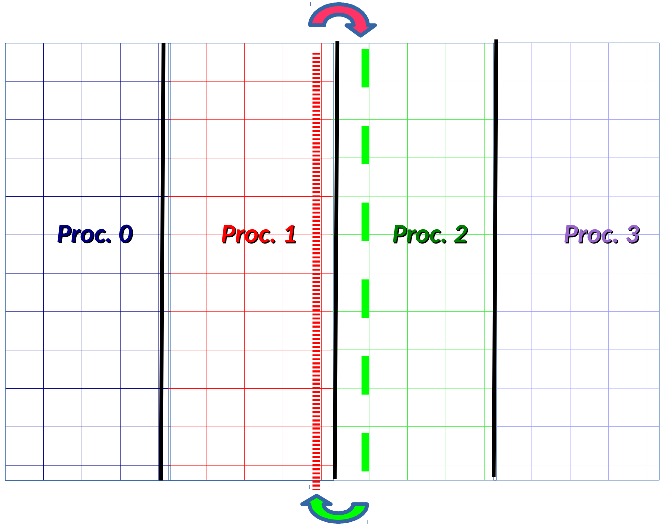
Parallel computing

Each processors works on a subspace of our domain. The problem is at the borders...

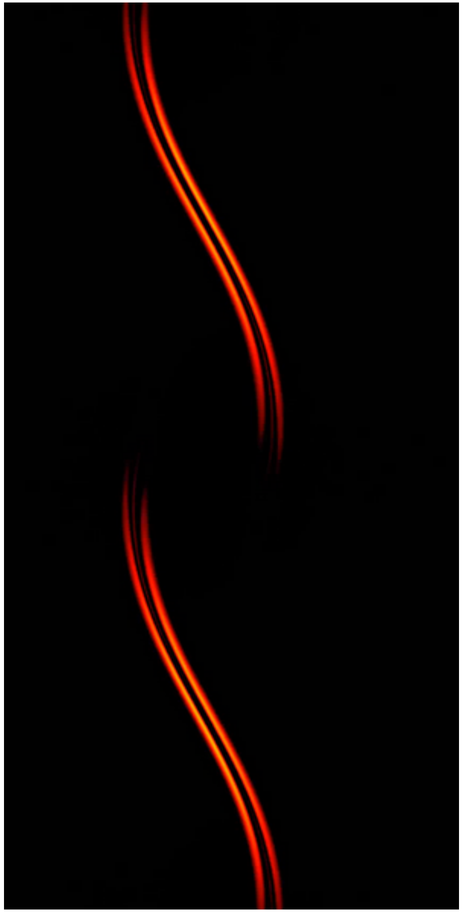




$$\left.\frac{\partial f}{\partial x}\right|_{x=x_i} = \frac{f(i+1,j,k)-f(i-1,j,k)}{2\Delta x}$$

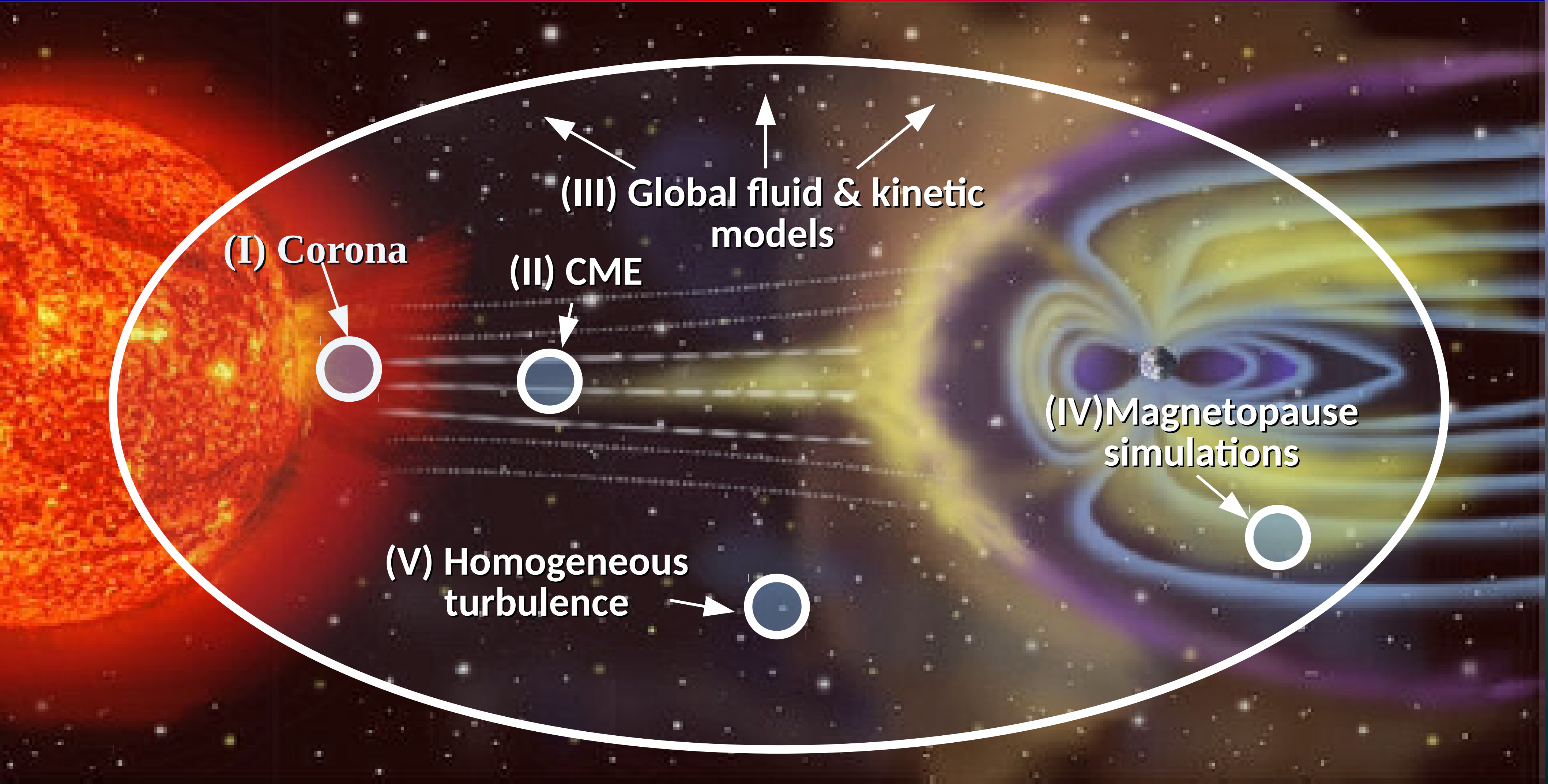
Proc. 1 “sends” the column *f*(*i*-1,*j*,*k*) to Proc. 2, and Proc. 2 sends *f*(*i*+1, *j*, *k*) to Proc. 1. This can be done whit simple calls like “mpi_isend” and “mpi_ireceive”. In the mean time, each processor computes derivatives and products that are far from the domain boundaries, taking advantage of OpenMP directives

Plasma simulations of turbulence

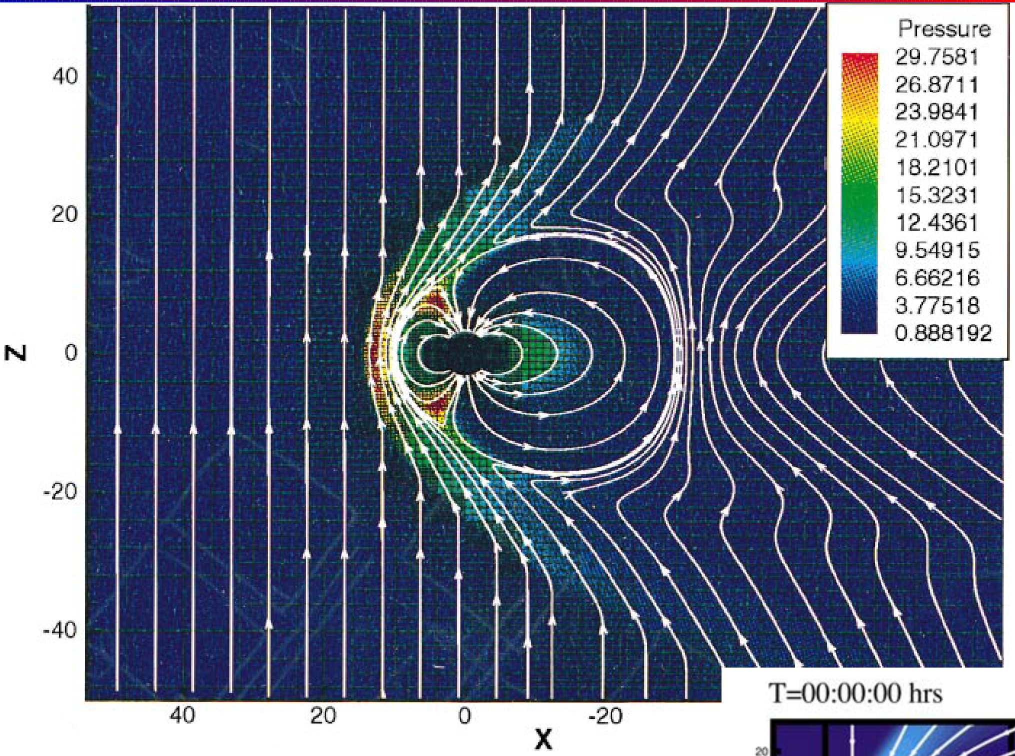


Karimabadi et al, PoP, 2013

Global & local simulations

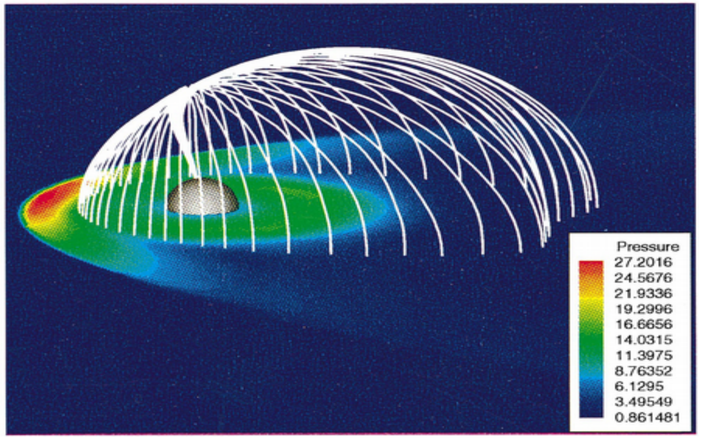


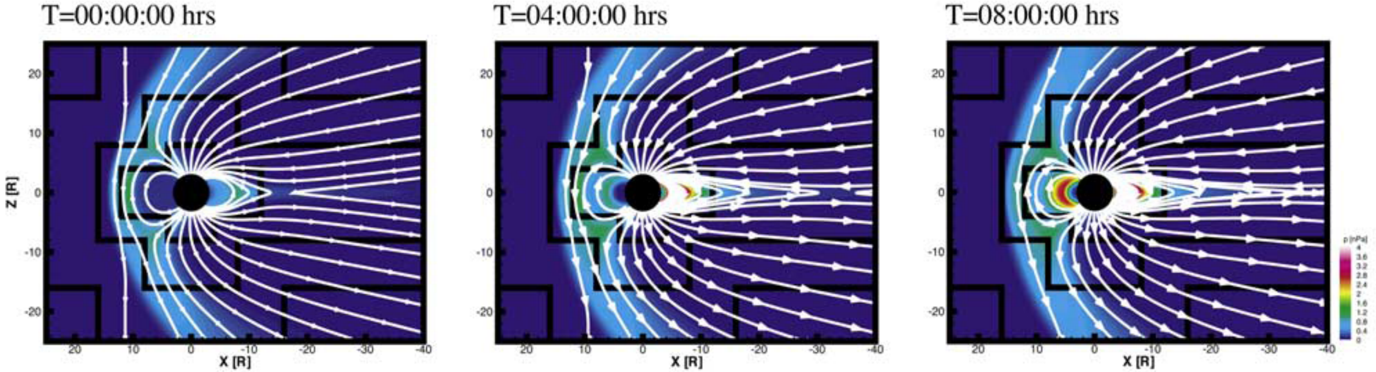
A “classic”: MHD global simulations



Powel et al. 1998

3D compressible MHD simulation, based on a upwind finite volume method, of the solar wind – earth system

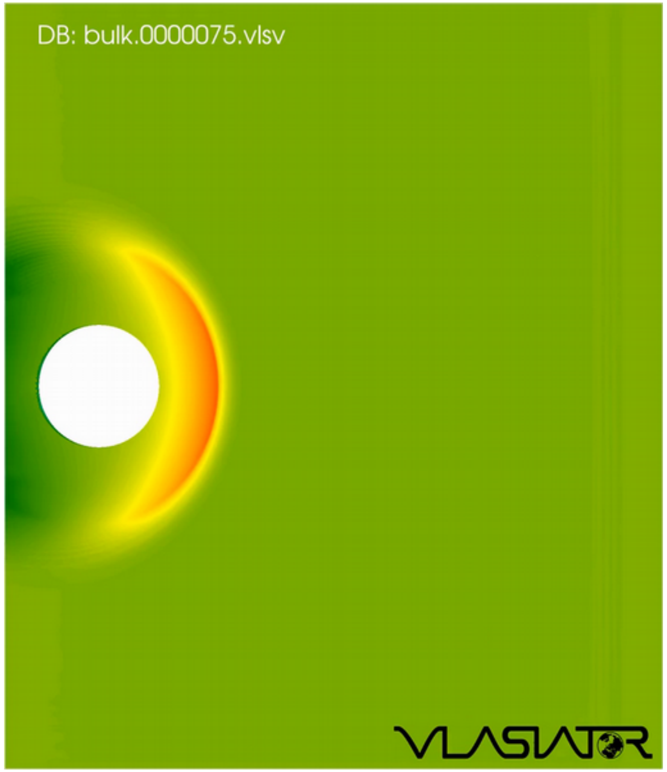




Can we model large systems with self-consistent kinetic simulations?

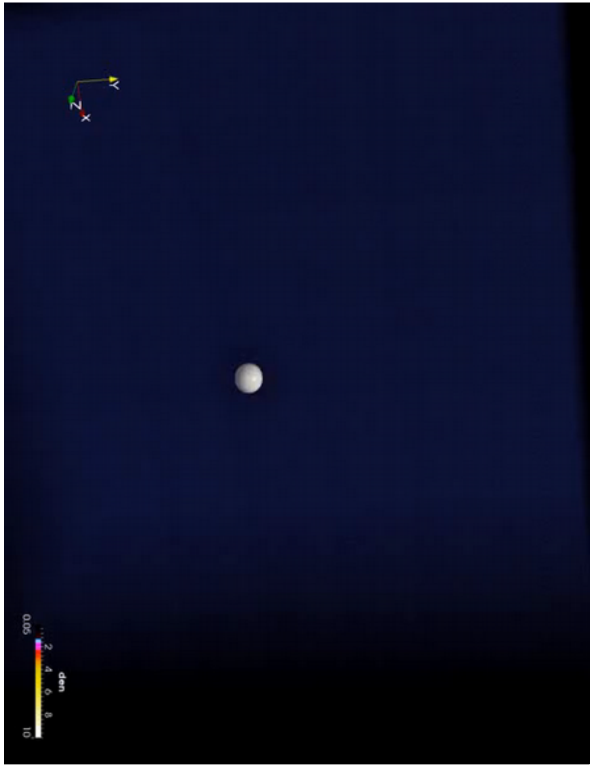
Global kinetic simulations

Hybrid Vlasov



M Palmroth et al., LRCA 4 (2018)

Full PIC



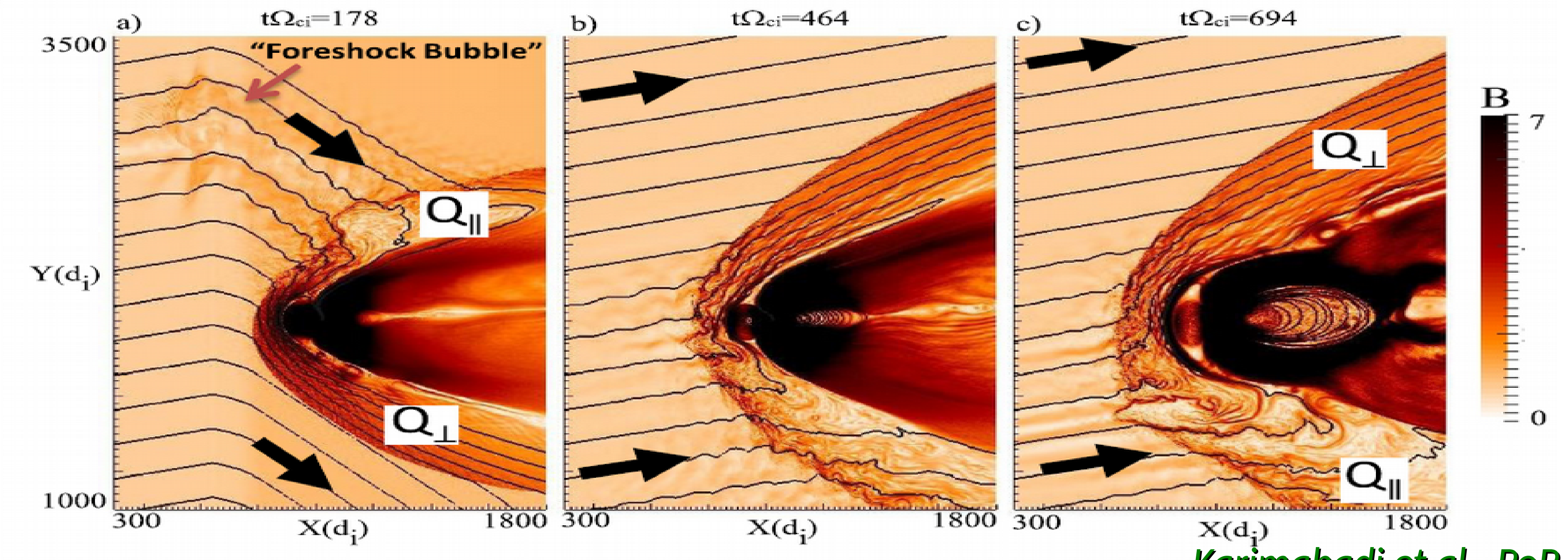
V. Roytershteyn et al, PoP (2014)

Global Vlasov simulations: kinetic effects

TABLE I. List of parameters for the global hybrid simulation runs shown in this paper. The number of particles per cell was 200. The spatial resolution was $0.5d_i$ except for runs 1, 5 ($1d_i$) and run 6 ($0.25d_i$).

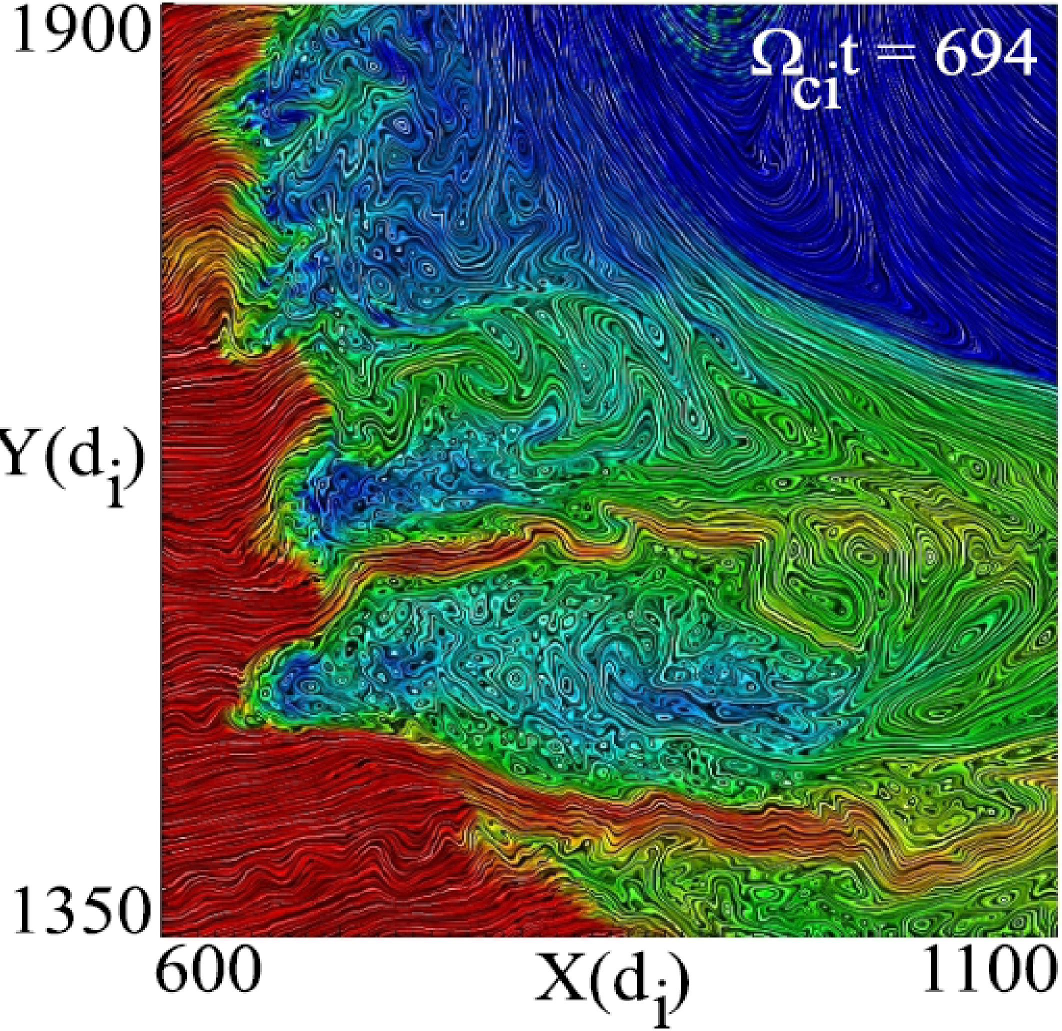
Run	x_{max}	y_{max}	D_p	$IMFB_z$	M_A	IMF_1	IMF_2	t_{flip}
1	2048	8192	100	0	8	−45°	10°	90
2	2048	4096	100	0	8	10°	10°	N/A
3	2048	4096	100	0	8	−45°	10°	90
4	8192	8192	300	0	10	−45°	10°	150
5	2048	4096	100	0.6	8	10°	10°	N/A
6	512	512	50	0.6	8	10°	10°	N/A
7	4096	2048	150	0.6	10	10°	10°	N/A

Time evolution of the magnetosphere where the IMF direction changes in time. The arrows indicate the direction of the magnetic field. A large foreshock bubble is evident



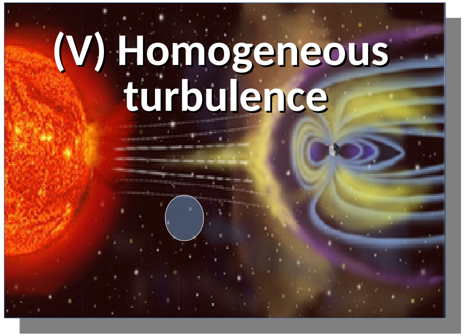
Karimabadi et al., PoP, 2014

Kinetic turbulence?



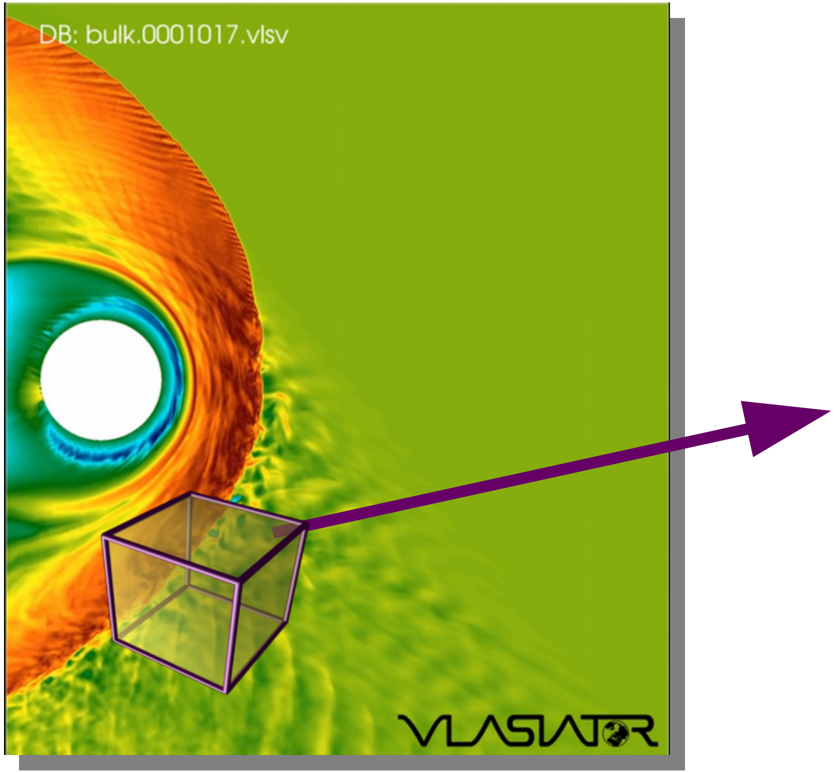
For all the systems we saw so far, high-resolution simulations suggest:

- Quasi-homogeneous turbulence
- Small scale structures
- Small scales reconnection events?

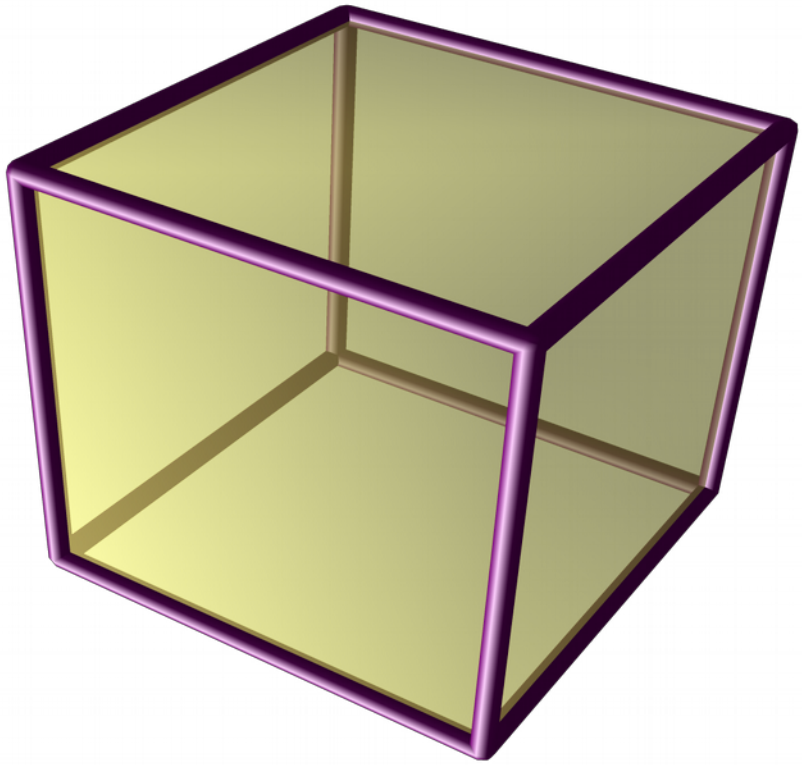


From global to local

Global simulations



Local simulations



Eulerian Hybrid Vlasov-Maxwell (HVM) solver

- Kinetic protons & alpha particles, while electrons are treated as a massless fluid
- *3[rd]* order splitting scheme

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right] \cdot \nabla_v f = 0$$

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \frac{1}{n}\mathbf{j} \times \mathbf{B} - \frac{1}{n}\nabla P_e + \eta \mathbf{j}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mathbf{j}$$

$$n_e \simeq n_i \simeq n$$

$$P_e = nT_e$$

$$V_A = \frac{B}{(4\pi\rho_p)^{1/2}}; \qquad \Omega_p = \frac{eB}{m_p c}; \qquad d_p = \frac{V_A}{\Omega_p}$$

NOISE-FREE! but →

Phase space resolution needed for turbulence studies:

$N_x = N_y = N_z = 512$

$N_{vx} = N_{vy} = N_{vz} = 71$

To save *f* → 1000 Tb !!

Too expensive, from any point of view!

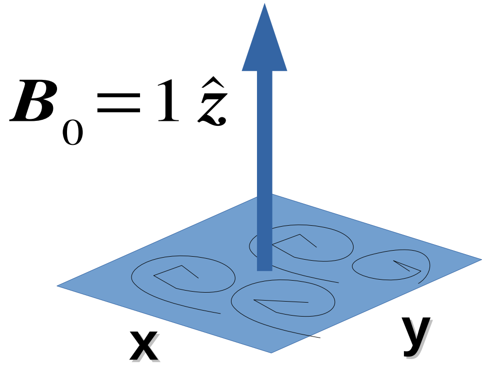
Valentini et al., JCP (2007), PRL (2010), PRL (2011)

HVM in a reduced geometry

Reduced geometry 2D-3V:

- 2D in space (fields have 3 components but depend only on 2 coordinates)
- 3V in the velocity space
- Turbulence is studied in a plane perpendicular to B_0, where its intensity is maximum (spectral anisotropy)

$$f \equiv f(x, y, v_x, v_y, v_z)$$



- Periodic boundary conditions in space
- $f(|v|) > v_{max} = 0$
- Parameters:

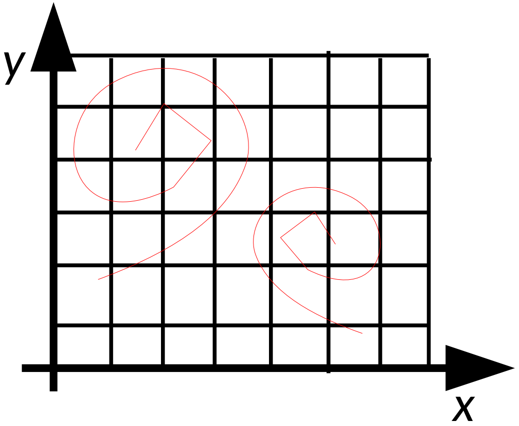
$$L_0 = 2\pi\alpha d_i,\ B_0 = 1\,\hat{e}_z,\ T_e/T_i = 1,$$

$$\eta = 1.7\times10^{-2},\ v_{max} = \pm 5 v_{ti},$$

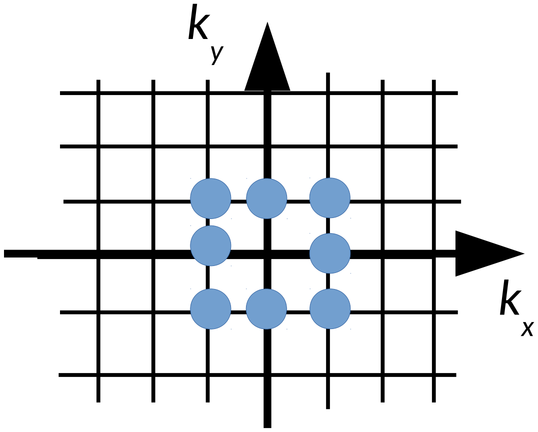
$$N_x = N_y = 512^2,\ N_v = 81^3 \rightarrow 3.5\times10^{10}\ points$$

A recipe to study turbulence: initial conditions

- Decay turbulence simulations
- $\delta b/B_0 \sim 1/3, 2/3$ (as in the solar wind)
- Plasma beta from $\beta \sim 0.2$ to 5
- Maxwellian distribution with uniform density and temperature
- $L_0 \sim 100\, d_i$ (large scale box)
- Incompressible initial conditions
- No correlation between velocity and magnetic field
- Large scale, uncorrelated (random) eddies, in order to mimic turbulent cascade in fluids: energy only in a box in k-space, for $|k|<|K_*|$, and with random Fourier phases ϕ_k



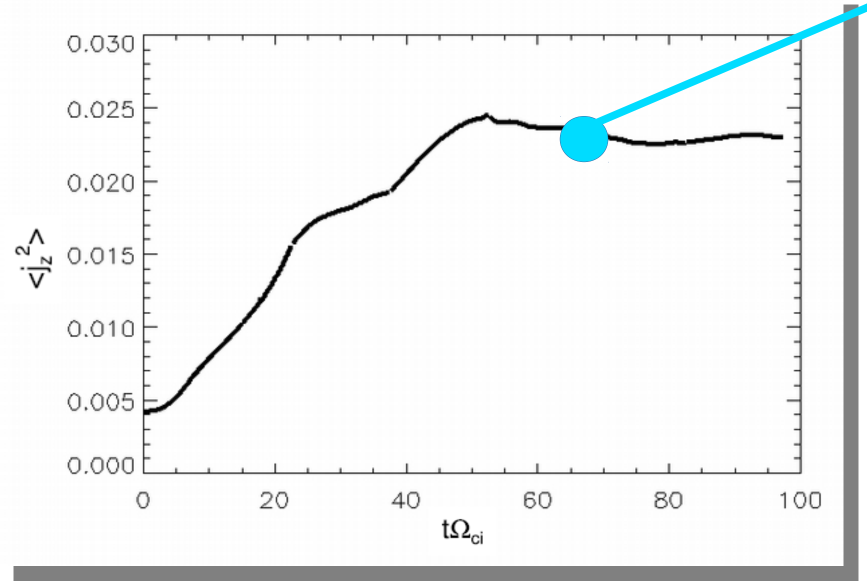
$$b_j(x,y)=\sum \left|\hat{b}_j(k_x,k_y)\right| e^{i(\boldsymbol{r}\cdot\boldsymbol{k}+\phi_k)}$$

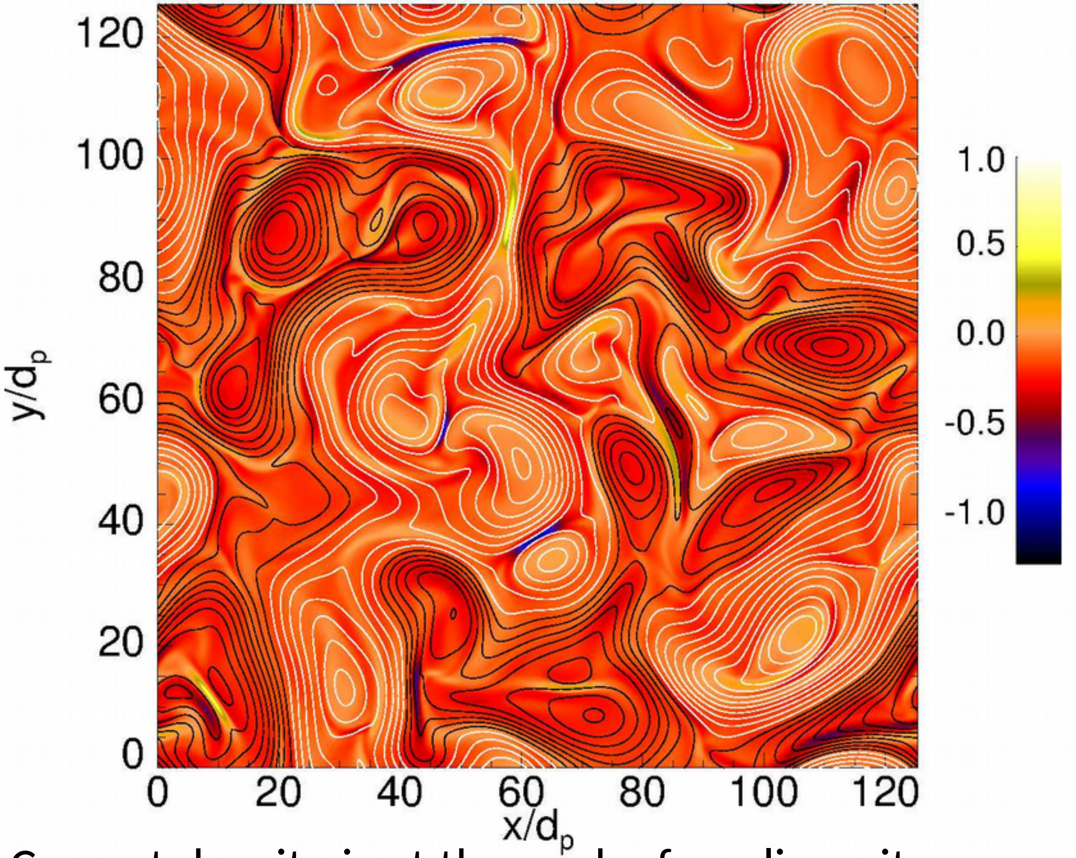


Decaying turbulence

The cascade is fully developed when small scale activity reaches its maximum. This time can be quantified as the maximum of the mean square current density:

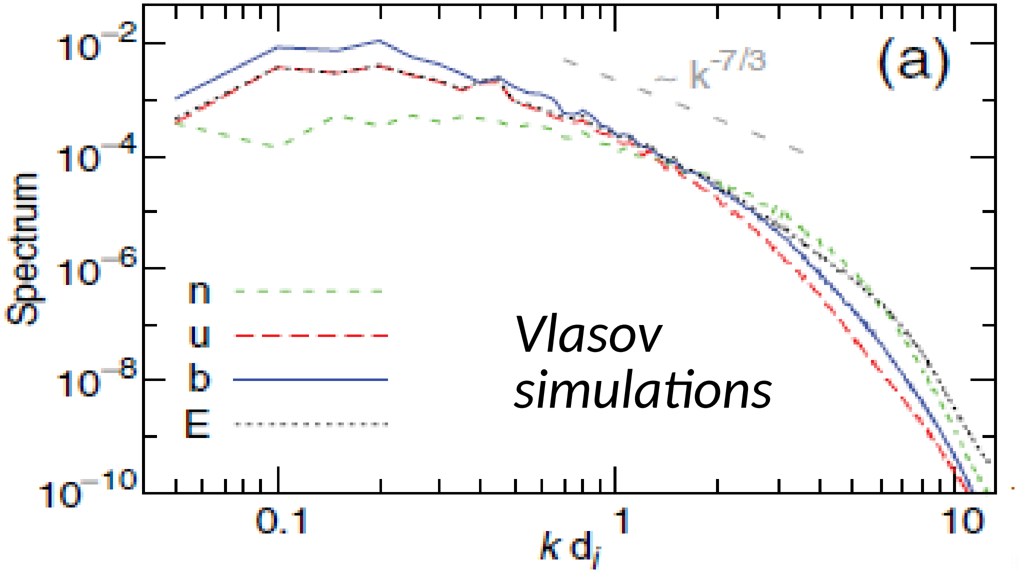
$$\langle j_z^2 \rangle = \frac{1}{L_0^2} \int [(\nabla \times \boldsymbol{B}) \cdot \hat{\boldsymbol{z}}]^2 \, d^2 x$$

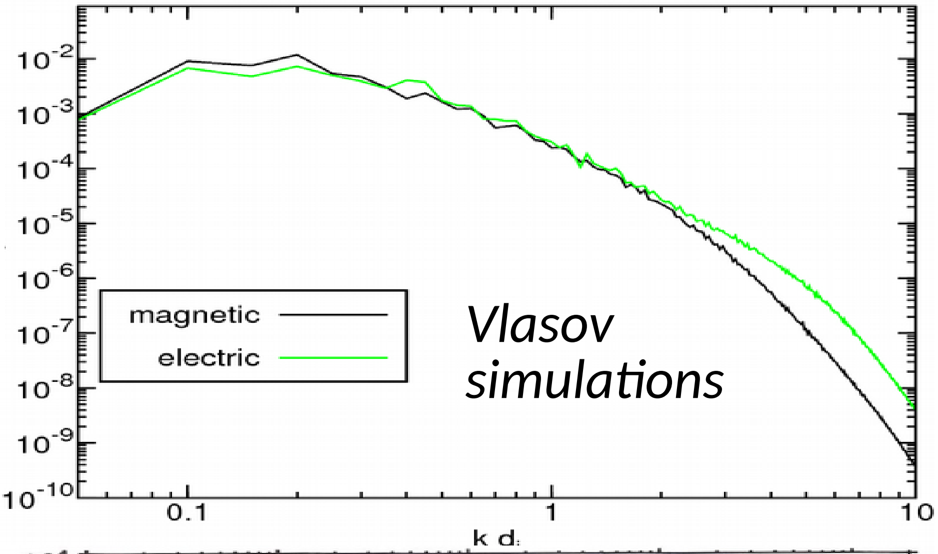


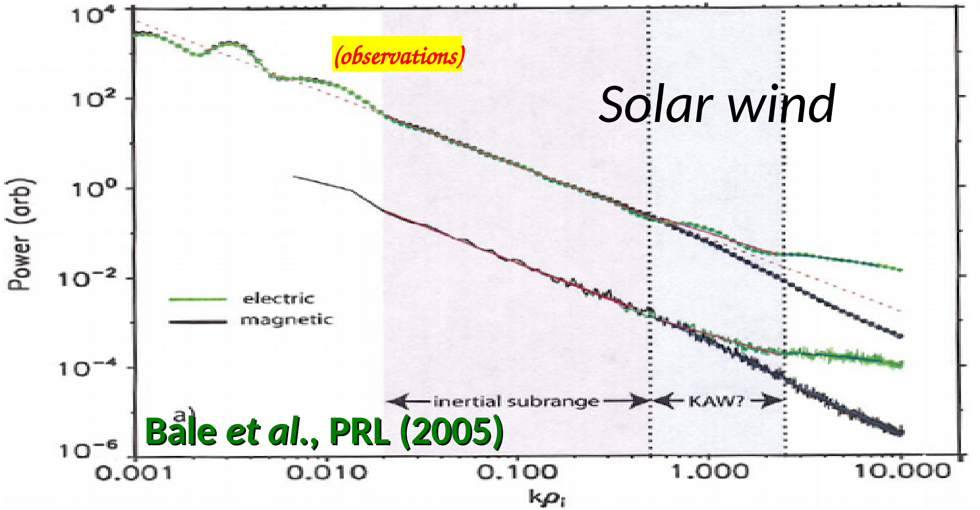


Current density j_z, at the peak of nonlinearity (shaded colors), together with the magnetic potential a_z, with black (a_z>0) and white (a_z<0) lines

Spectral features of turbulence







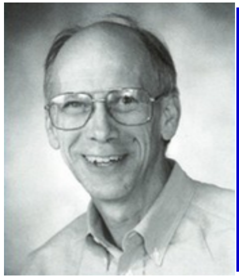
- Large scale Alfvenic correlations
- Intense electric activity at small scales
- Steepening of the magnetic spectrum at $kd_p \sim 1$

Several features commonly observed in space plasmas!

Take-home messages from Part I

- Equations that describe the plasma in a self-consistent way are very complex and computationally demanding
- There are two approaches to study plasma dynamics: the Eulerian (evolve the distribution function) and the Lagrangian approach (solve equations for particles)
- Global simulations suggest that turbulence is triggered throughout the heliosphere: local homogeneous simulations of turbulence represent an excellent strategy
- Numerical simulations are complementary to observational data. Understanding the reality cannot rely on simulations or observation alone, comprehension is given by a right balance among the two

Vlasov turbulence:
Servidio et al.,
PRL 2012, PRL 2016, PRL 2017



"Calculators can only calculate - they cannot do mathematics."

J. A. Van de Walle