Magnetic reconnection

Ideal Ohm's law →

$$\vec{E}+\frac{\vec{v}\times\vec{B}}{c}=0 \Rightarrow \frac{\partial\vec{B}}{\partial t}+\nabla\times\left(\vec{B}\times\vec{v}\right)=0 \Rightarrow \frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{S}+\oint_C \vec{B}\cdot\left(\vec{v}\times d\vec{l}\right)=0$$

The total variation of the magnetic flux through a surface in motion with the fluid is conserved.

Frozen-in condition: two fluid elements connected by a field line at a given time stay connected at later times.

Magnetic reconnection

A 2D example

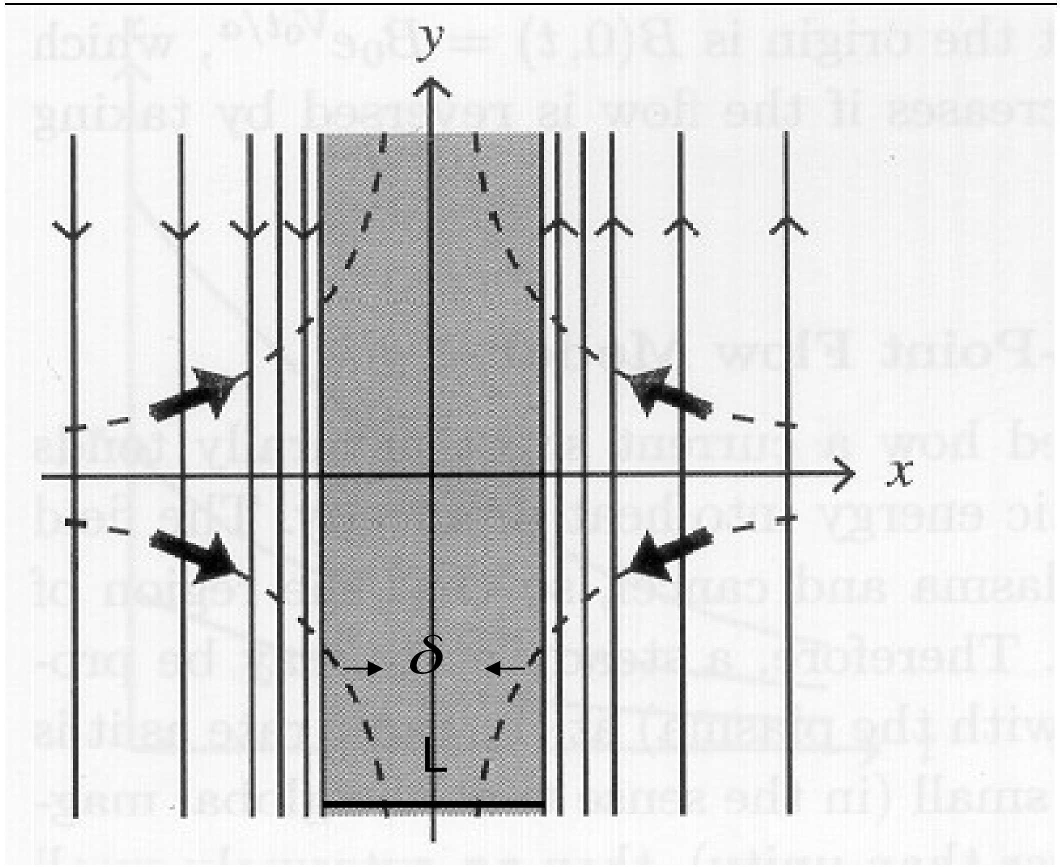
$$\vec{B} = B(x)\vec{e}_y \approx \frac{B_0 x}{L}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} \Rightarrow J_z \approx \frac{B_0}{L}$$

$$\vec{J} \times \vec{B} \approx \frac{B_0 x}{L^2}$$

$$L \rightarrow \delta \Rightarrow J_z \rightarrow B_0/\delta$$

$$\delta \rightarrow 0 \Rightarrow \vec{J} \times \vec{B} \rightarrow \infty$$



Magnetic reconnection

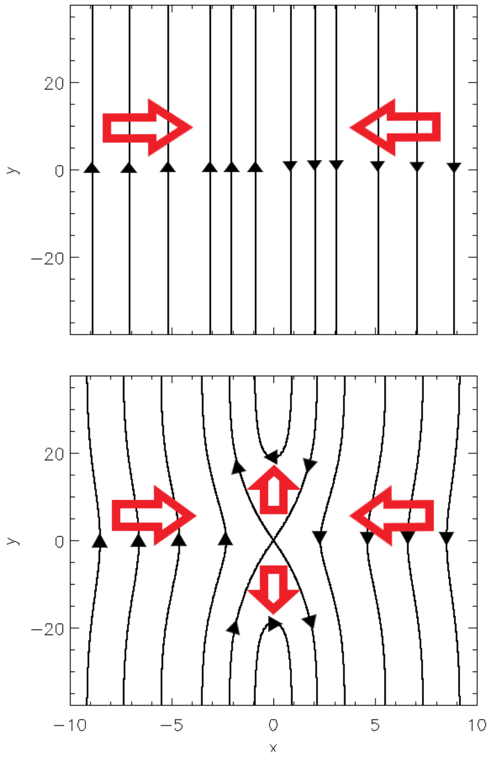
Non ideal Ohm's law

$$\vec{E}+\frac{\vec{v}\times\vec{B}}{c}\neq 0$$

$$\frac{\partial \vec{B}}{\partial t}+\nabla\times\left(\vec{B}\times\vec{v}\right)\neq 0$$

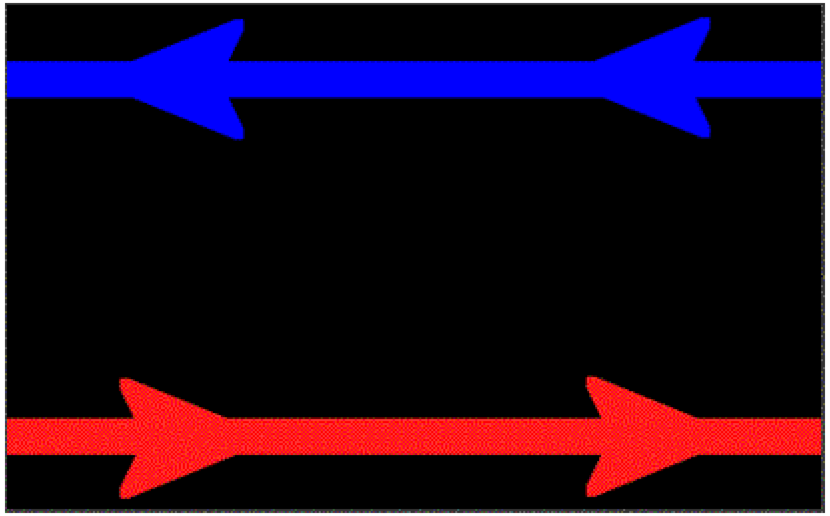
The magnetic flux is not conserved.

Magnetic field lines can now decouple from the plasma motion and reconnect



Change of magnetic field topology

Magnetic island formation



$$\frac{\partial \vec{B}}{\partial t}+\nabla\times\left(\vec{B}\times\vec{v}\right)=\frac{1}{S}\nabla^2\vec{B}$$

Local process causing global changes

Magnetic reconnection: Where?

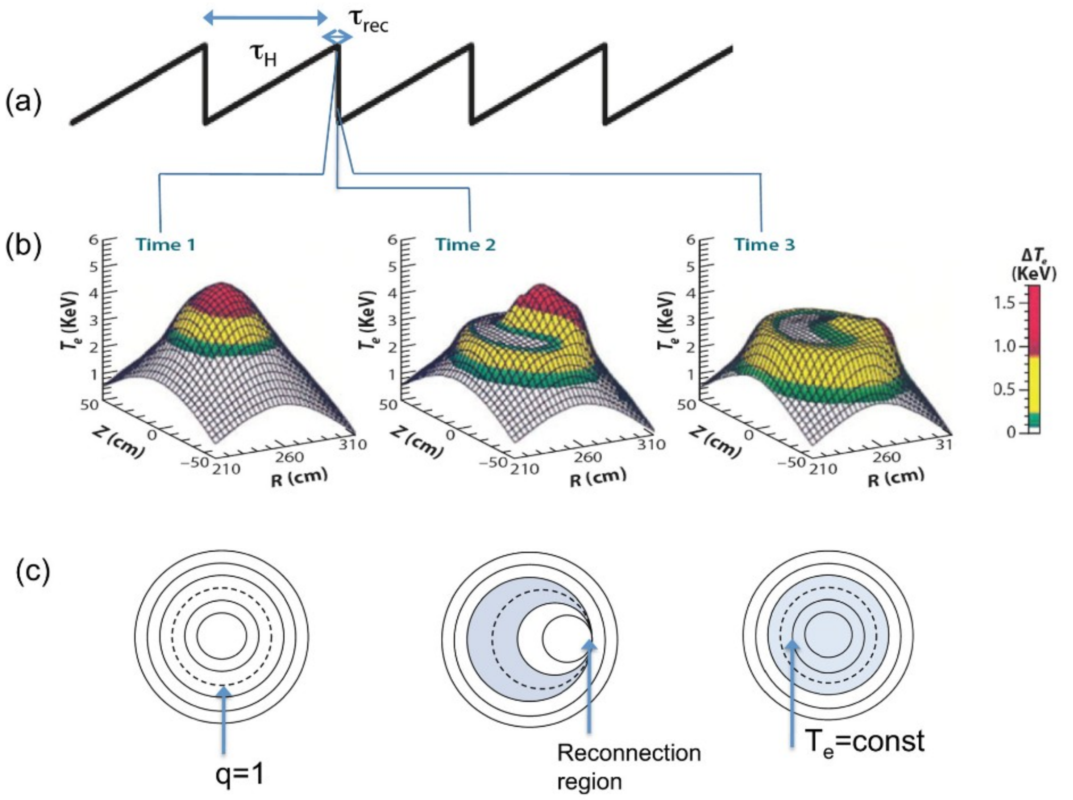
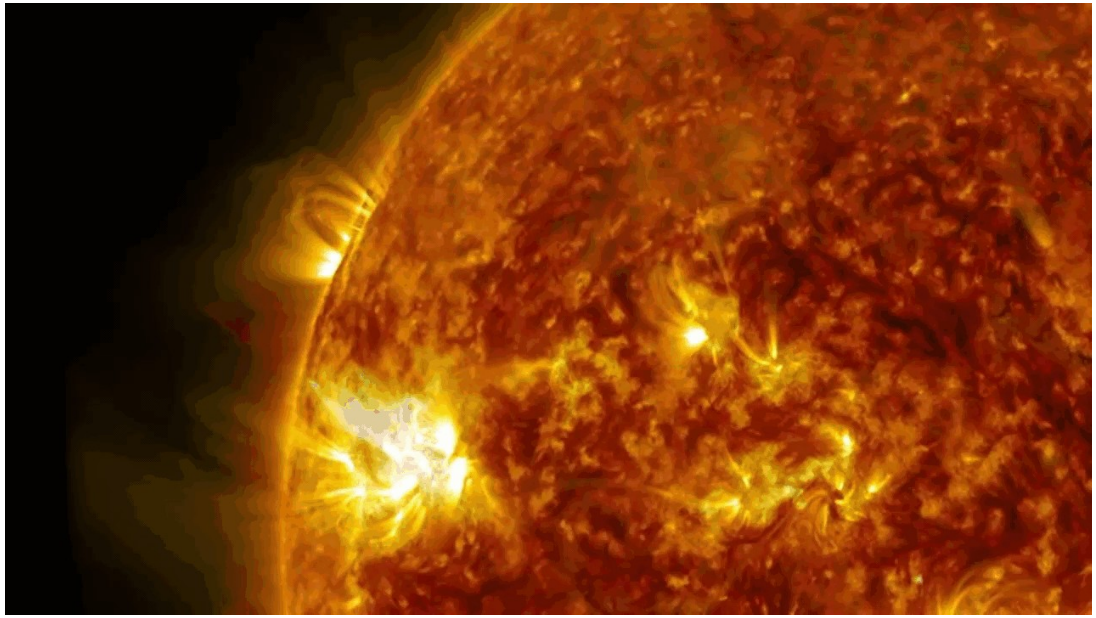
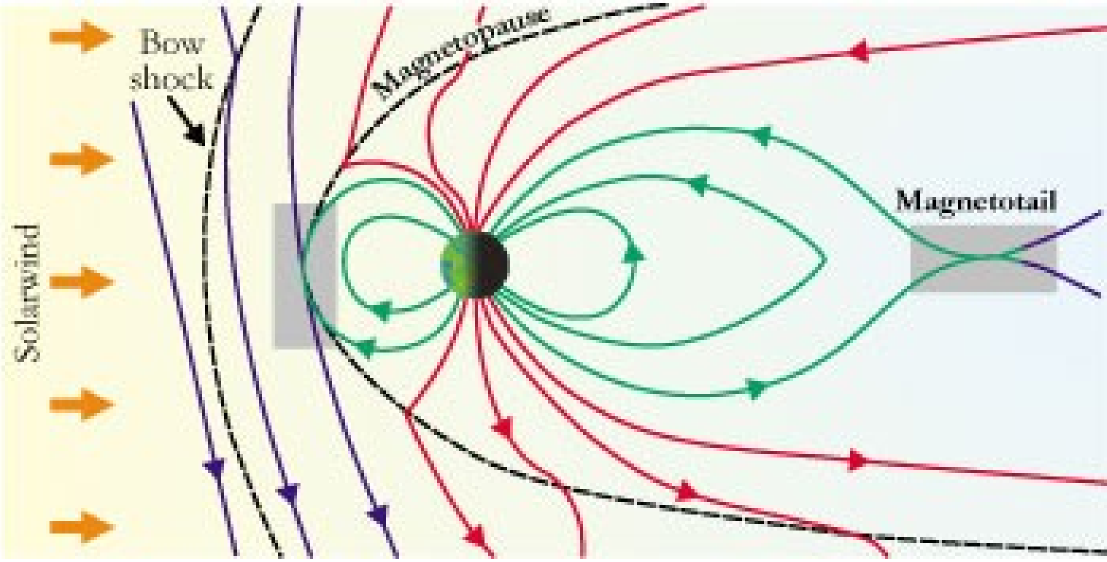


FIG. 1. (Color online) Evolution of measured central electron temperature (T_e) profile on a poloidal plane during a short crash phase, and expected flux surfaces during the same period based on MSE diagnostics. (a) Flux build up time (τ_H) is typically 100 ms and crash (reconnection) time (τ_{rec}) is 100–150 μs. (b) Crash phase evolution of T_e(R, Z) during 150 μs. (c) Shaded (gray) area shows constant T_e region indicating field lines that are reconnected through the reconnection region. Broken lines show the original radius of q = 1 flux surface.[8]

Yamada, PoP 2011





MR resistive or collisionless?

- At high temperature, such as in a fusion experiment, resistivity is very low.
- Discrepancies between theoretical predictions based on the resistive MHD and typical today's large experiments values brought the attention to electron inertia.
- From Sweet-Parker model (Wesson 1990) for the Jet parameters:

$$\tau_{res} \approx 3\,ms \;; \qquad \tau_{d_e} \approx 300\,\mu s$$

- Electron inertia drives the reconnection process providing the effective impedance to the parallel electric field, relaxing the frozen-in condition of the ideal MHD.

MR: linear results

- If we linearize the system of equations of the MHD model assuming an equilibrium with no flow equilibrium and:

$$\mathbf{B} = -\tanh(x)\mathbf{e}_y, \quad \psi = \log(\cosh(x)), \quad \phi = 0.$$

- and look for solution of the type:

$$\tilde{\psi} = \delta\psi(x)\exp(iky + \gamma t), \quad \tilde{\phi} = \delta\phi(x)\exp(iky + \gamma t)$$

- Adopting the standard matching asymptotic techniques for boundary layer problems we can find an expression for the free energy of the system for the instability to grow and for the dispersion relation for the growth rate of the reconnecting mode.

$$\Delta' \equiv \lim_{\delta\to 0}\left(\frac{d}{dx}\ln\psi_{out}\right)_{-\delta}^{+\delta}$$

- We report here the growth rates for the two regimes resistive and collisionless:

small Δ’ regime:

large Δ’ regime:

resistive

$$\gamma = 1.37\Delta'^{4/5}\epsilon_\eta^{3/5} \longleftrightarrow \gamma \sim \epsilon_\eta^{1/2}$$

collisonless

$$\gamma = 0.45\frac{d_e^3\Delta'^2}{\pi^2} \longleftrightarrow \gamma = d_e$$

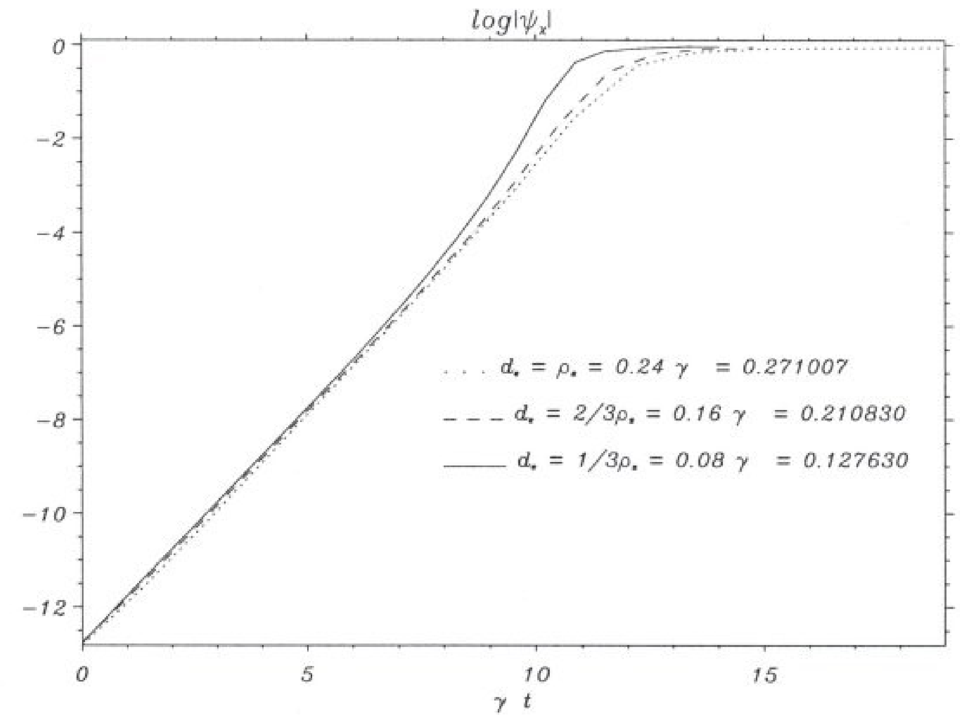
MR collisionless

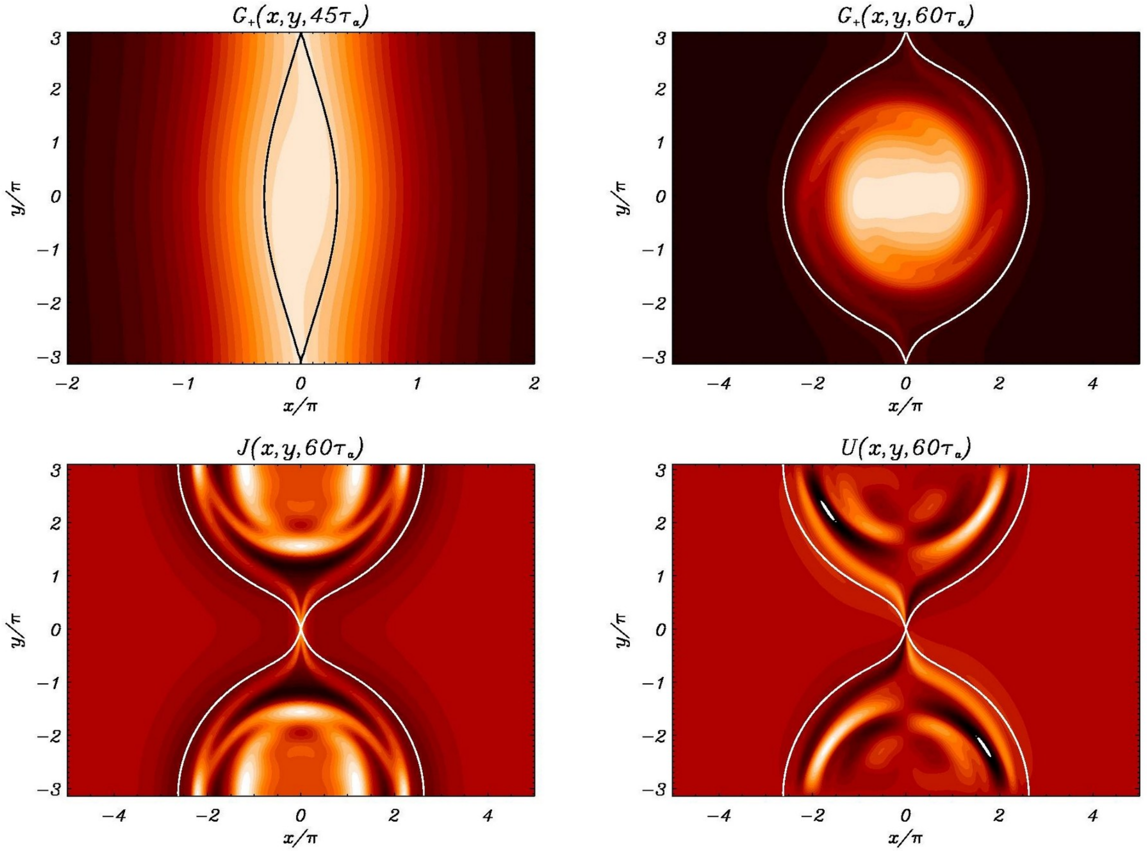
$$\frac{\partial \psi}{\partial t} + [\varphi, \psi] = -d_e^2 \frac{\partial J}{\partial t} - d_e^2 [\varphi, J] + \varrho_s^2 [U, \psi]$$

$$\frac{\partial U}{\partial t} + [\varphi, U] = [J, \psi]$$

$$J = -\nabla^2 \psi$$

$$U = \nabla^2 \varphi$$



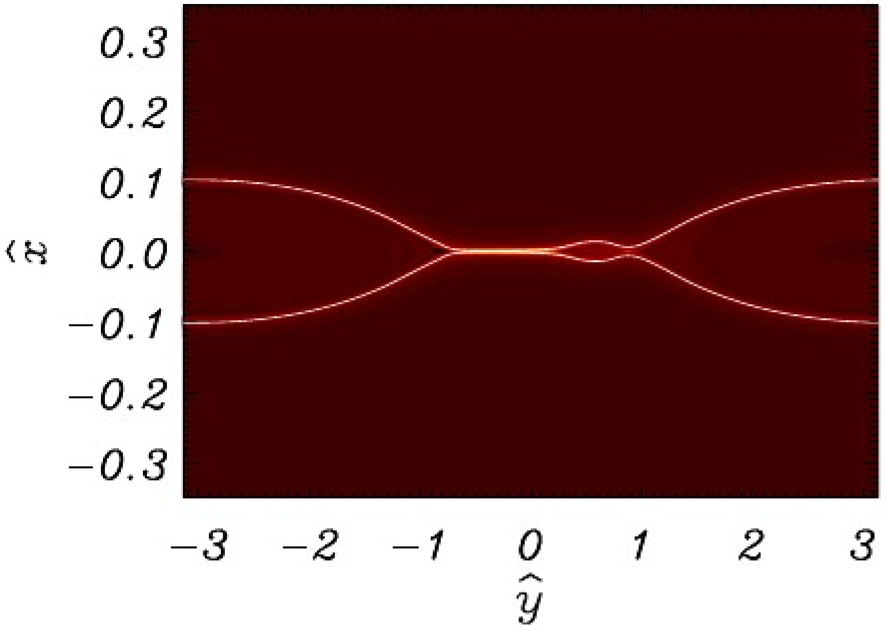


Past two decades results in 2D

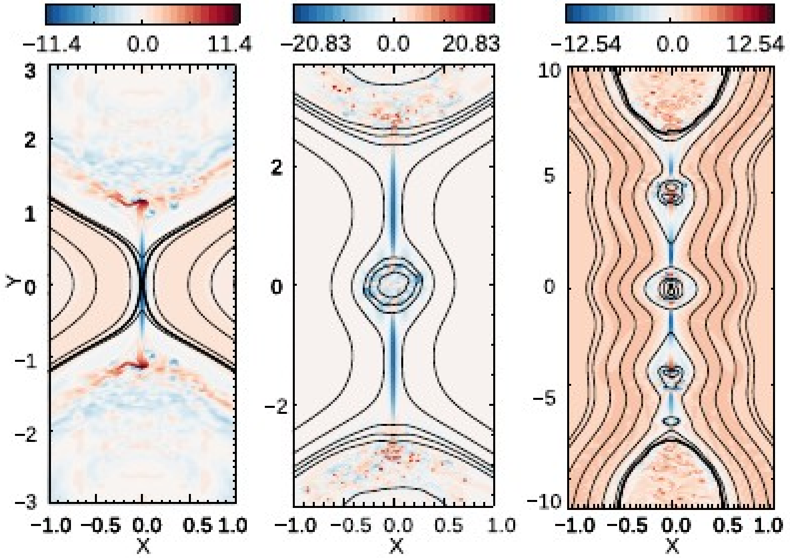
- Quasi-explosive behavior
- Phase mixing process
- Secondary instabilities

Secondary instabilities: Plasmoid

- Current sheets that form during the nonlinear development of spontaneous magnetic reconnection are characterized by a small thickness. They can become unstable to the formation of plasmoids, which allows the magnetic reconnection process to reach high reconnection rates.

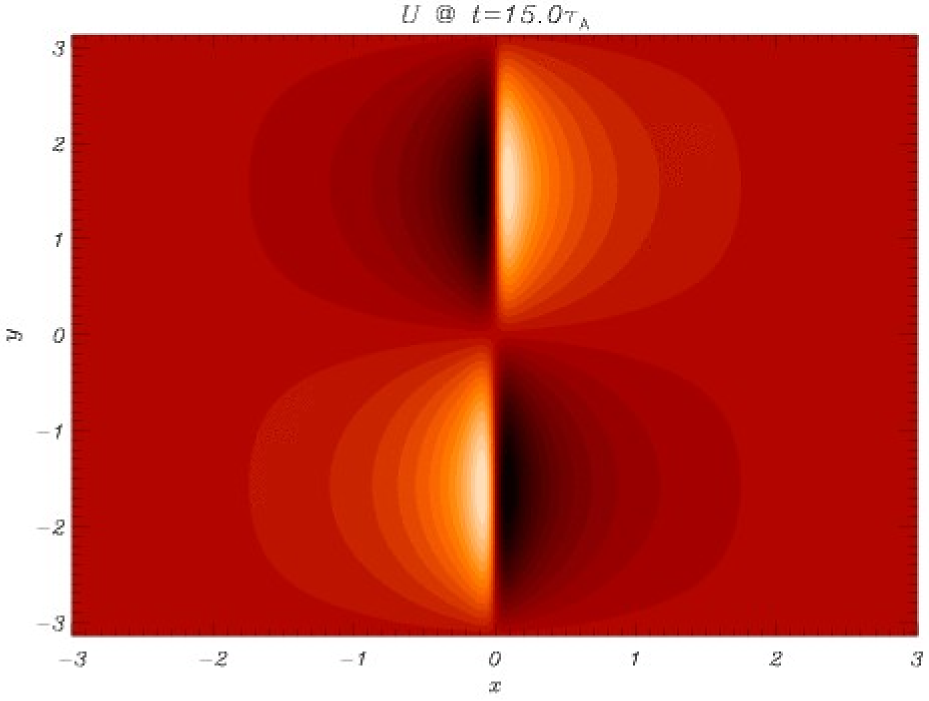


Resistive
Militello et al. PoP 2014



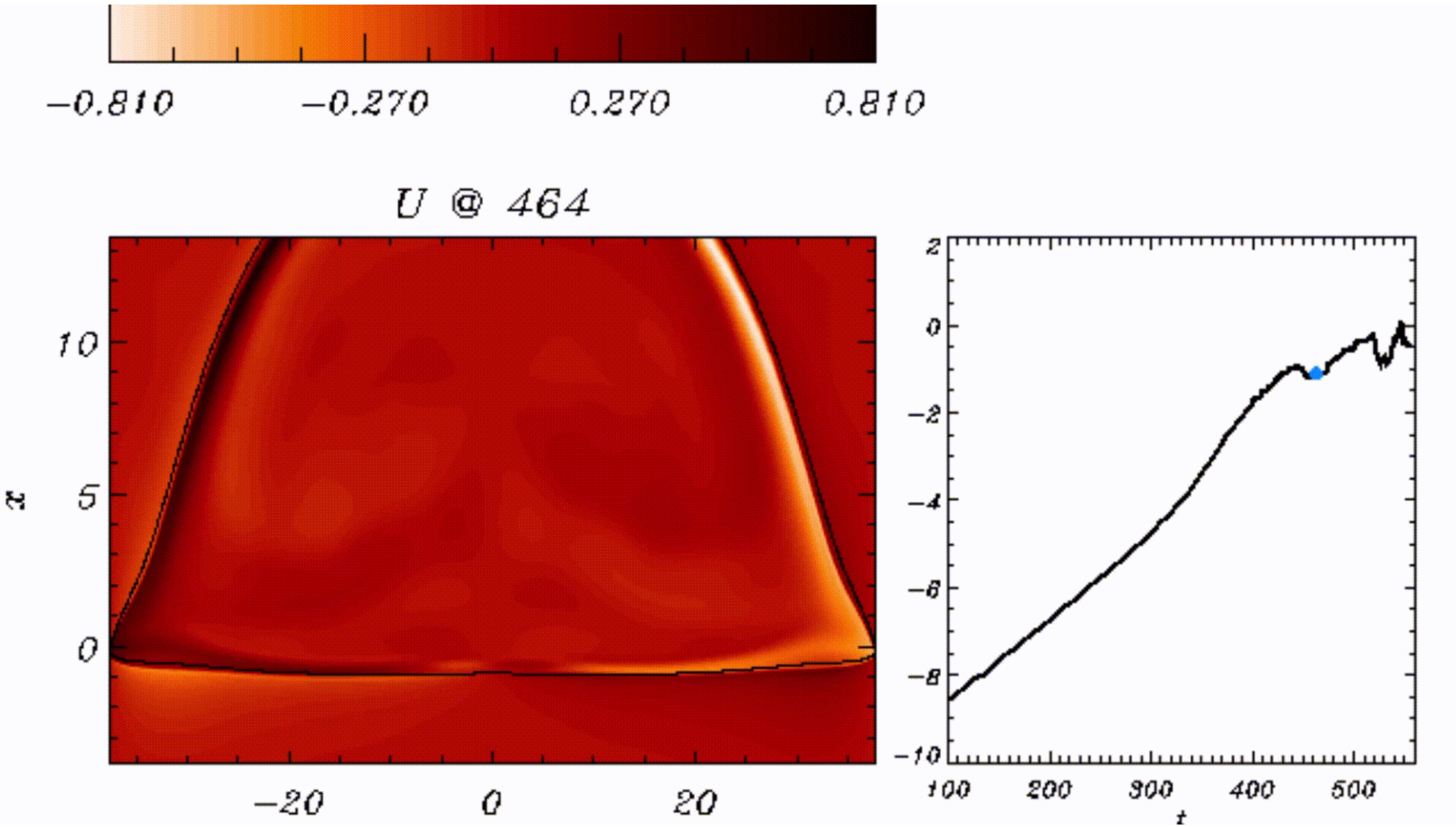
Collisionless
Granier et al. PRE 2022

Secondary instabilities: Kelvin-Helmholtz



Grasso et al. PoP 2007

Secondary instabilities: Kelvin-Helmholtz



Grasso et al. PoP 2020

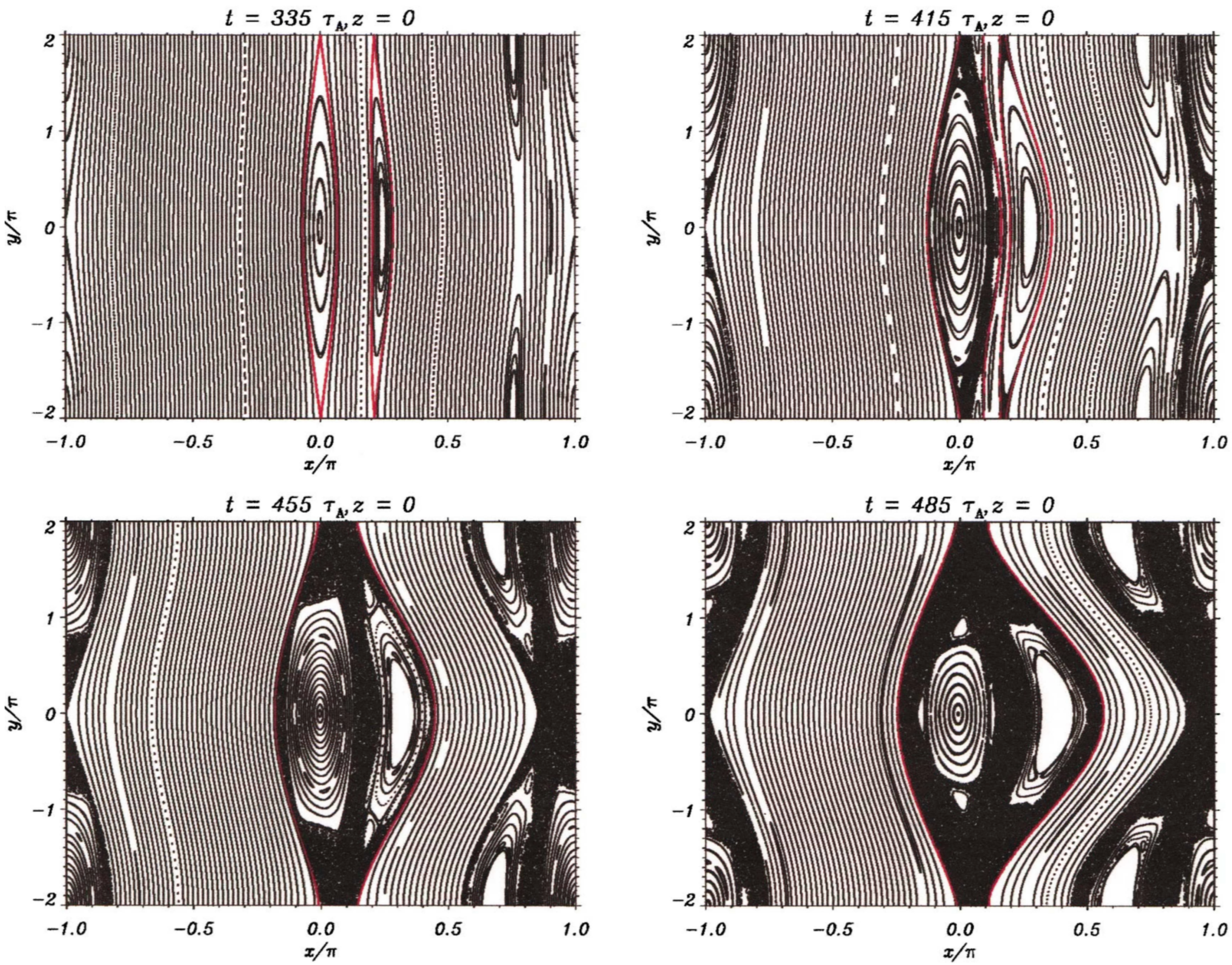
MR collisionless

Past two decades results in 3D

Magnetic chaos

Coherent structures influencing transport

Borgogno et al. PoP 2005



$$\vec{B} = B_0 \vec{e}_z + \nabla \psi\ (\vec{x}, t) \times \vec{e}_z$$

Magnetic field line equations:
for fixed time, *t*, *z* plays the role of field line time.

$$\frac{dx}{dz} = -\frac{\partial \psi}{\partial y}; \qquad \frac{dy}{dz} = \frac{\partial \psi}{\partial x}$$

- A reconnection event has been induced by a double helical perturbation resonating at two different surfaces.
- The island sizes increase until they start to influence each other leading to a chaotic setting initially enclosed between the islands
-

NO INFORMATION ON TRANSPORT!

MR collisionless

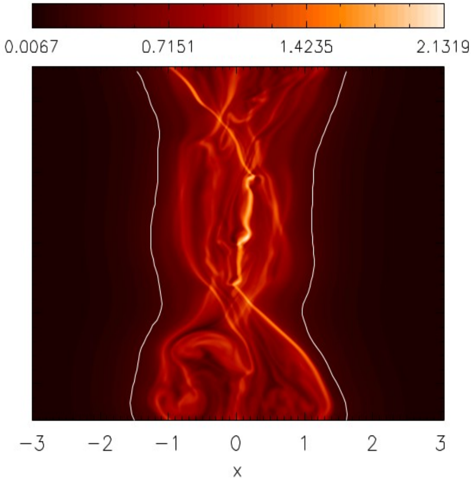
Past two decades results in 3D

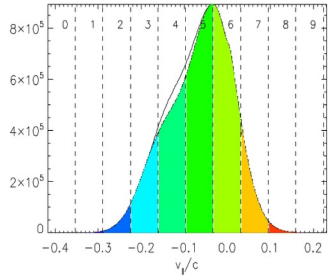
Magnetic chaos

Coherent structures influencing transport

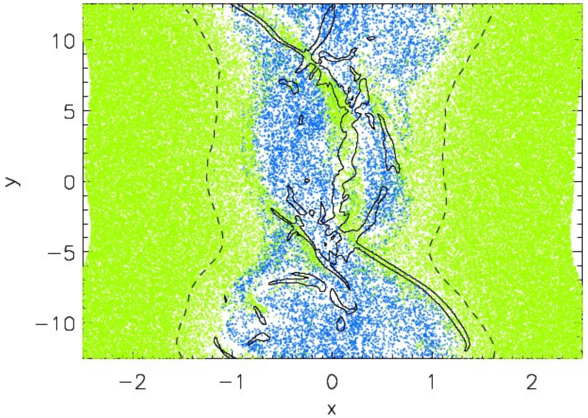
$$\frac{\partial(\psi - d_e^2\nabla_\perp^2\psi)}{\partial t} + [\varphi, \psi - d_e^2\nabla_\perp^2\psi] - \rho_s^2[\nabla_\perp^2\varphi, \psi] = \frac{\partial\varphi}{\partial z} - \rho_s^2\frac{\partial\nabla_\perp^2\varphi}{\partial z},$$

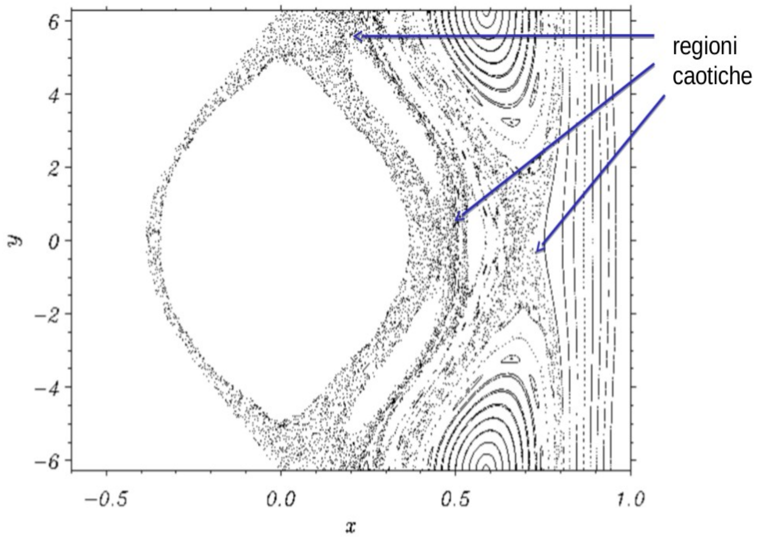
$$\frac{\partial\nabla_\perp^2\varphi}{\partial t} + [\varphi, \nabla_\perp^2\varphi] + [\nabla_\perp^2\psi, \psi] = \frac{\partial\nabla_\perp^2\psi}{\partial z},$$

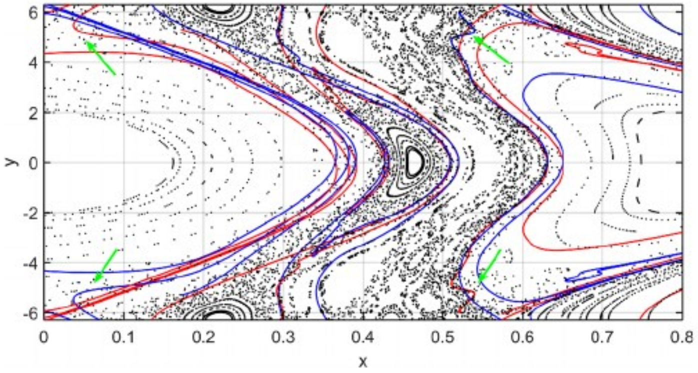




Perona et al. PoP 2017







Di Giannatale et al. PoP 2018

Applications

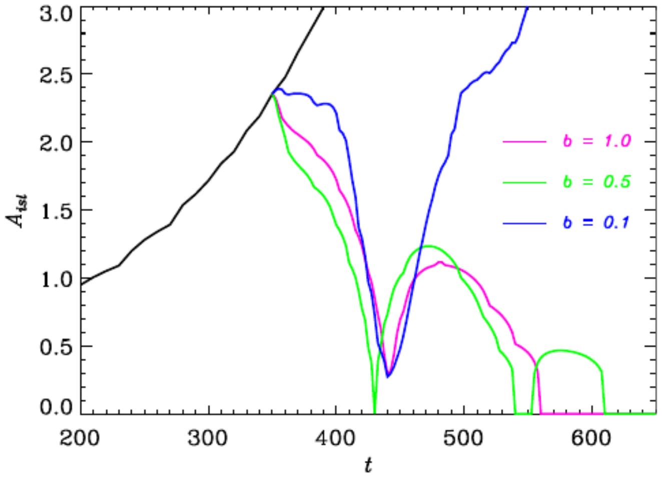
- Control of magnetic islands in tokamaks by ECCD injection

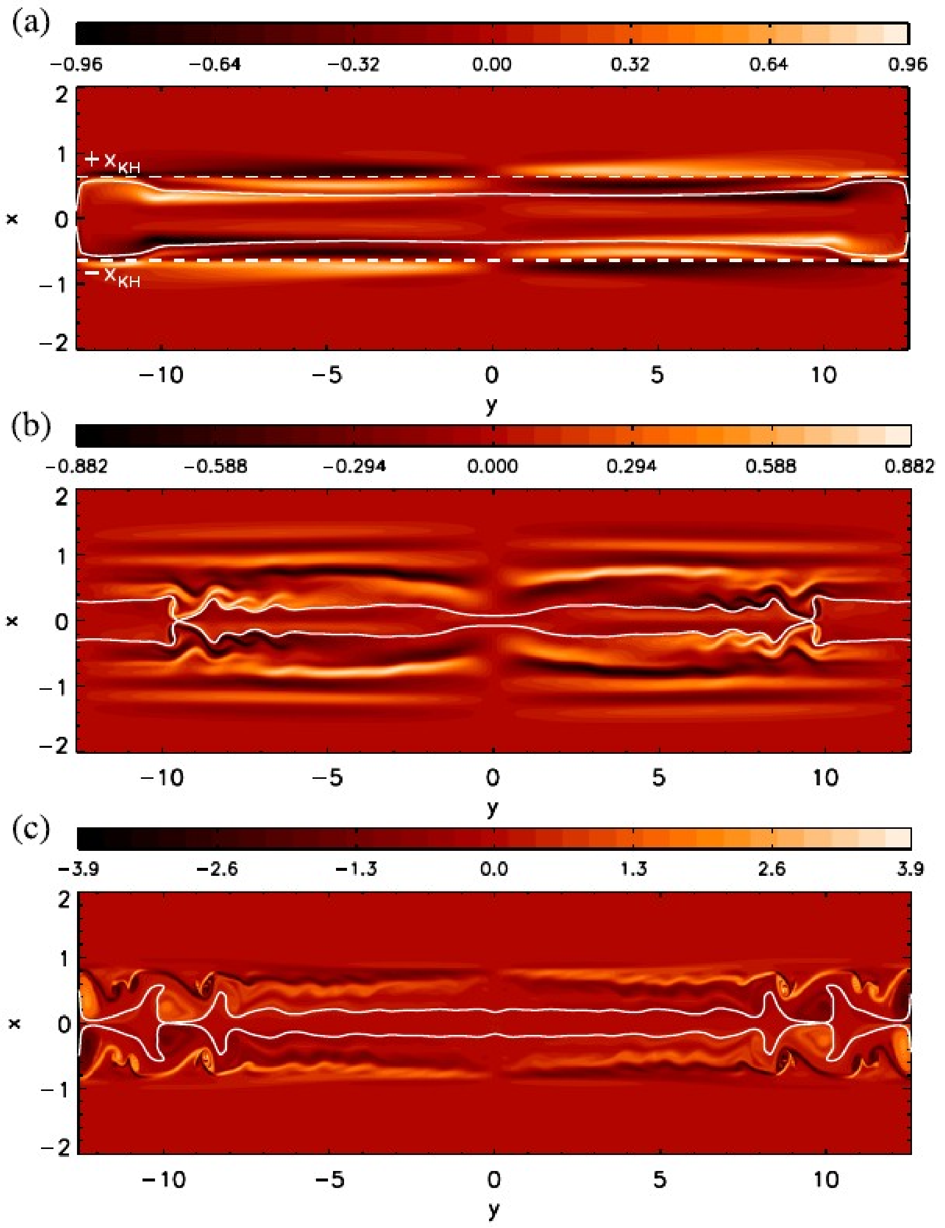
The magnetic island evolution under the action of a current generated externally by electron cyclotron wave beams is studied using a RRMHD plasma model. The use of a 2D reconnection model shows novel feature of the actual nonlinear evolution as compared to the zero-dimensional model of the generalized Rutherford equation. When the radiofrequency control is applied to a small magnetic island, the complete annihilation of the island width Is followed by a spatial phase shift of the island, referred as “flip” instability. On the other hand, a ECCD injection in a large nonlinear island can be accompanied by the occurrence of a KH instability. These effects need to be taken into account in designing tearing mode control systems based on radio frequency current-drive

$$\frac{\partial \psi}{\partial t} + \mathbf{v}_\perp \cdot \nabla \psi = -\eta (J - J^{(0)} - J_{ec}),$$

$$\frac{\partial U}{\partial t} + \mathbf{v}_\perp \cdot \nabla U = \mathbf{B}_\perp \cdot \nabla J.$$

$$J_{ec}(x,y,t) = J_m(t) \exp\left(- \frac{(\psi(x,y,t) - \psi_O(t))^2}{\delta^2} \right)$$





Borgogno et al. PoP 2014

Applications

- MR under effect of a runaway current in tokamaks

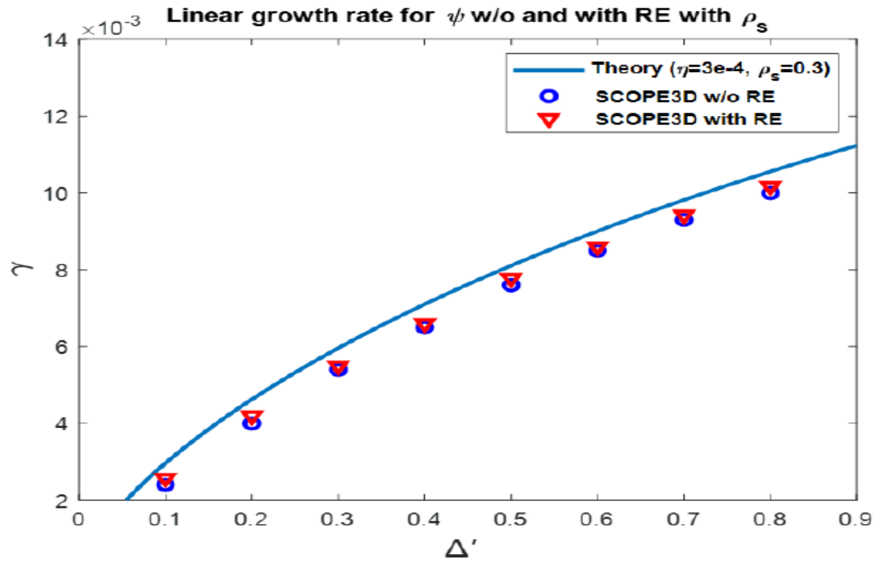
The goal of this work is to investigate the problem of the tearing stability of a post-disruption weakly collisional plasma where the current is completely carried by runaway electron, adopting a two fluid model in the description of the reconnection process.

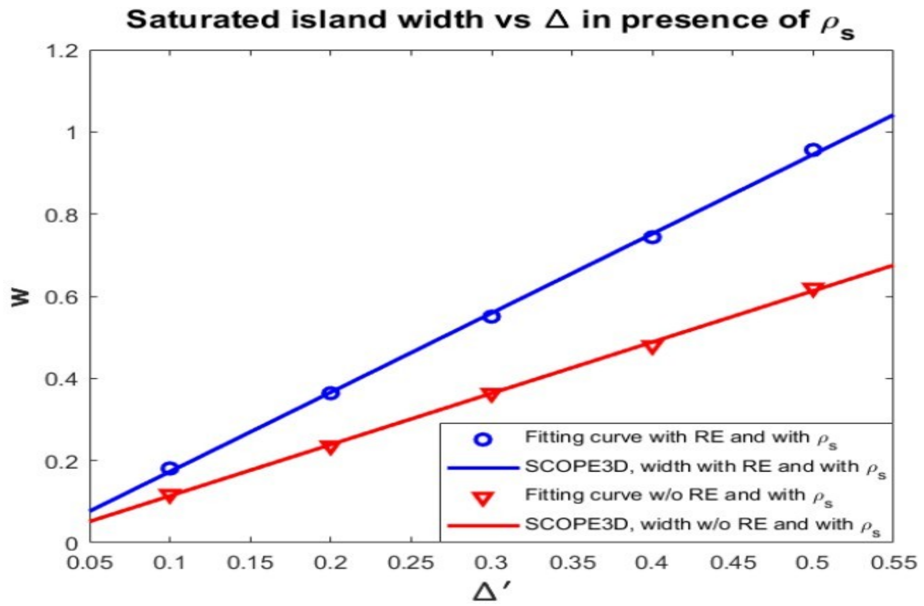
$$\frac{\partial(\psi - d_e^2\nabla_\perp^2\psi)}{\partial t} + [\varphi, \psi - d_e^2\nabla_\perp^2\psi] - \rho_s^2[\nabla_\perp^2\varphi, \psi]$$

$$= \frac{\partial \varphi}{\partial z} - \rho_s^2 \frac{\partial \nabla_\perp^2 \varphi}{\partial z},$$

$$\frac{\partial \nabla_\perp^2 \varphi}{\partial t} + [\varphi, \nabla_\perp^2\varphi] + [\nabla_\perp^2\psi, \psi] = \frac{\partial \nabla_\perp^2 \psi}{\partial z},$$

$$\frac{\partial J_r}{\partial t} + [\varphi, J_r] + \frac{c}{v_A}\left([\psi, J_r] - \frac{\partial J_r}{\partial z}\right) = 0$$





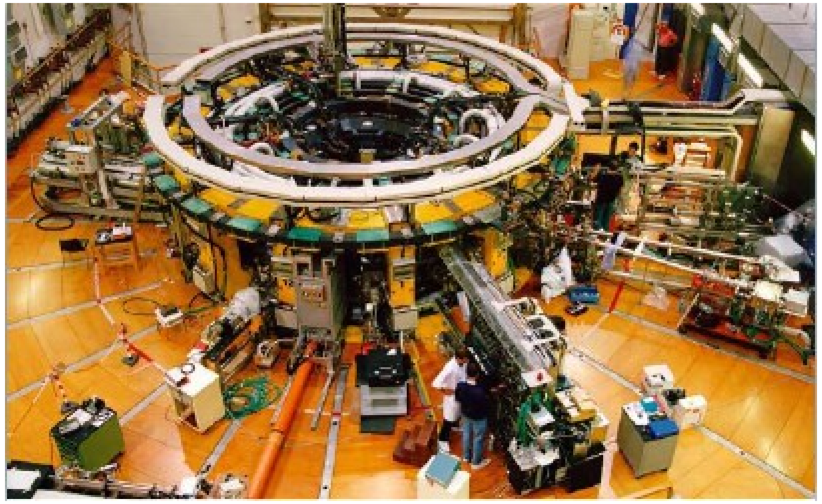
Grasso et al. JCS 2022

Applications

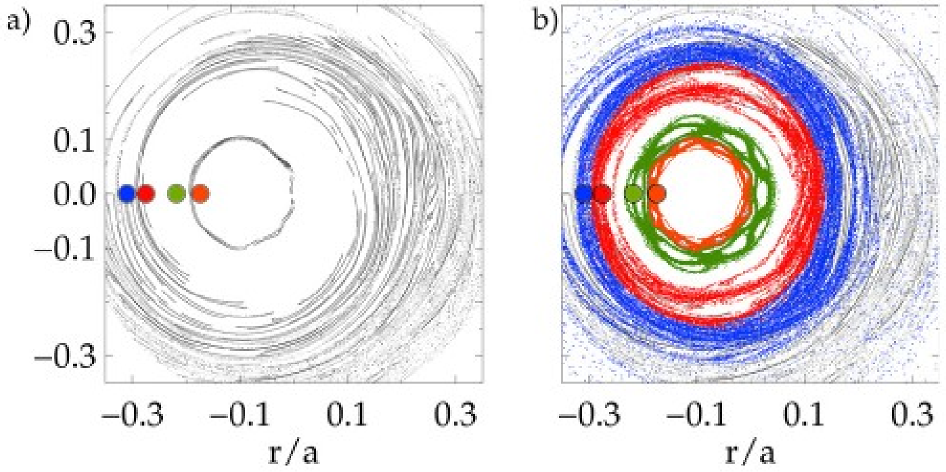
- Temperature transport barriers in RFX

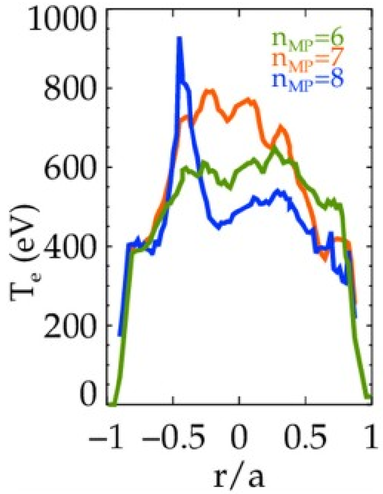
Increasing the plasma current RFX-mod experiments show (RFX group, Nature 2009):

- transition from chaotic configurations to ordered ones, named Quasi Single Helicity (QSH) states;
- consequent formation of a strong transport barrier enclosing high temperature zone.



Veranda et al. NF 2017



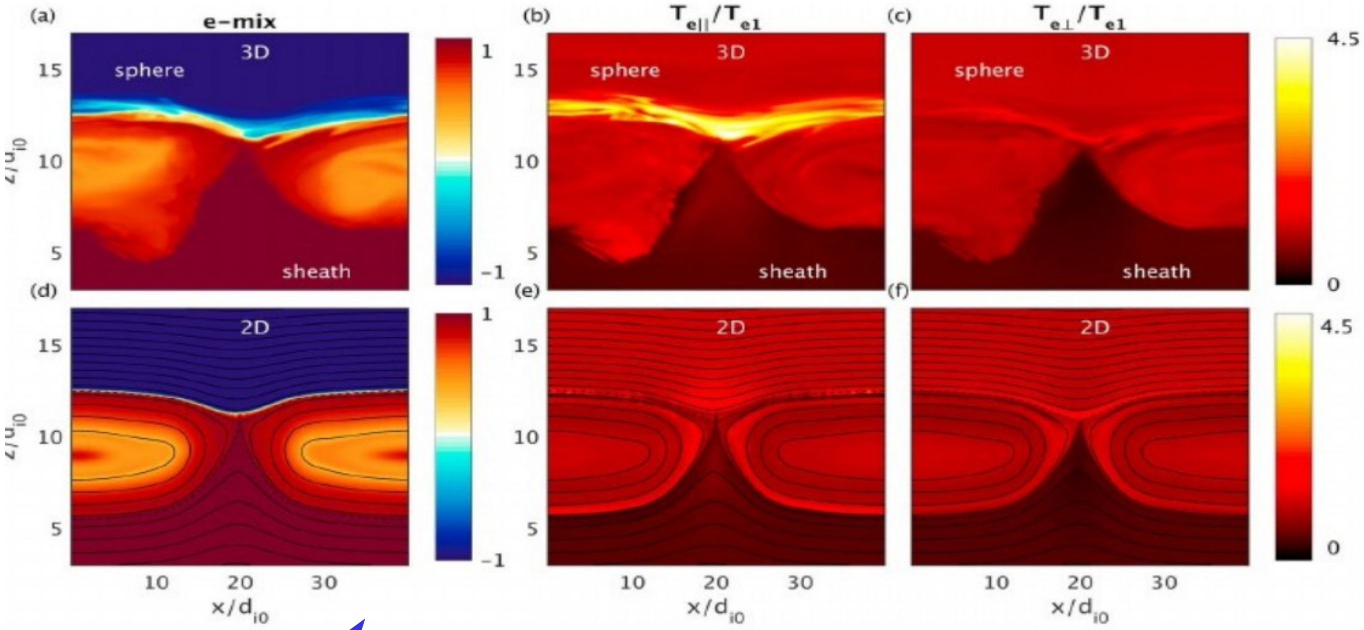


It exist a correlation between the transport and the magnetic barriers.

Applications

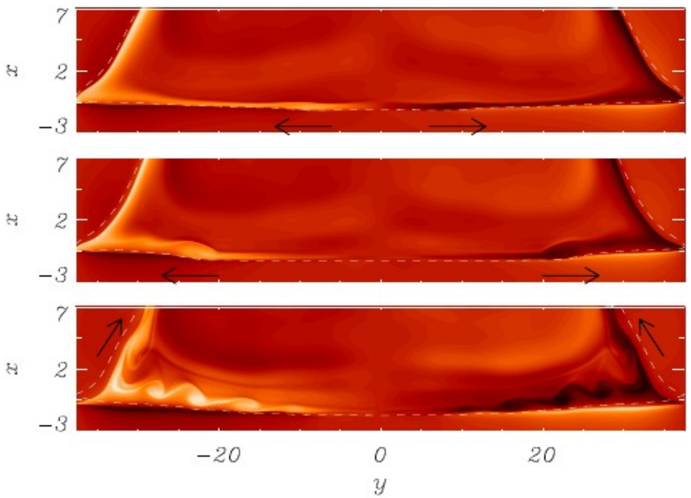
- Asymmetric reconnection occurring at the Earth's dayside

Literature analyzing data from the MMS mission explains the particle emission in the out of page direction by the passage of the spacecraftinside the asymmetric magnetic island.

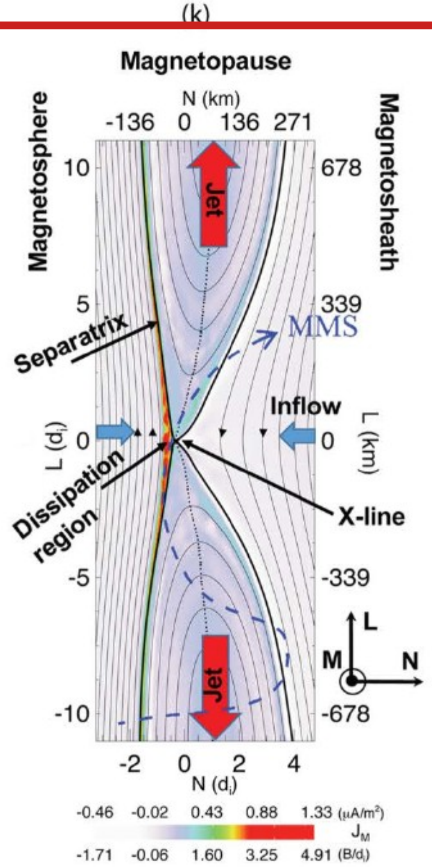


Le et al. GRL 44, 2096-2104 (2018)

Literature analyzing data from the MMS mission PIC-simulate MR in presence of asymmetry if both the magnetic field and the density profile and invoke LHDI to explain the destabilization of the shear flows



Grasso et al. JCS 2022



Burh et al. Science 2016

Kelvin-Helmholtz instability

- Magnetic reconnection processes always lead to the formation of regions with high velocity shear confined to narrow sheets.

- **Neutral gas:**

Vortex sheet →

$$V(x) = \begin{cases} V_1 & x < 0 \\ V_2 & x > 0. \end{cases}$$

Linear equation for the vorticity:

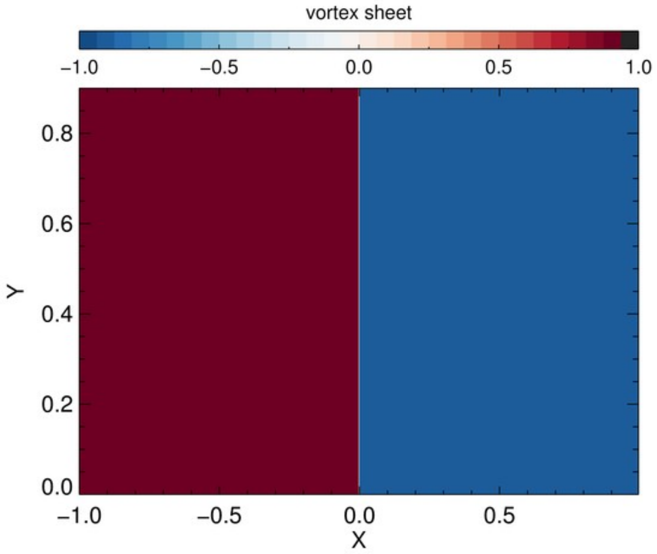
$$\partial_t w_1 + \mathbf{v}_0 \cdot \nabla w_1 + \mathbf{v}_1 \cdot \nabla w_0 = 0$$

$$w_1 = \nabla^2 \phi_1, \quad \mathbf{v}_1 = \mathbf{e}_z \times \nabla \phi_1.$$

A vortex sheet is always unstable to perturbations:

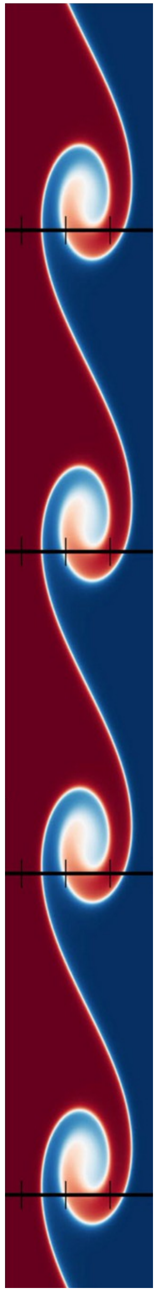
$$\phi_1 = \phi(x) exp\{ik(y - ct)\}$$

$$c = \omega / k = c_r + i c_i$$



Dispersion relation: $$c = \frac{V_1 + V_2}{2} \pm i \frac{|V_1 - V_2|}{2}$$

D. Biskamp, Magnetic Reconnectionin Plasmas, Cambridge University Press 2000



Kelvin-Helmholtz instability

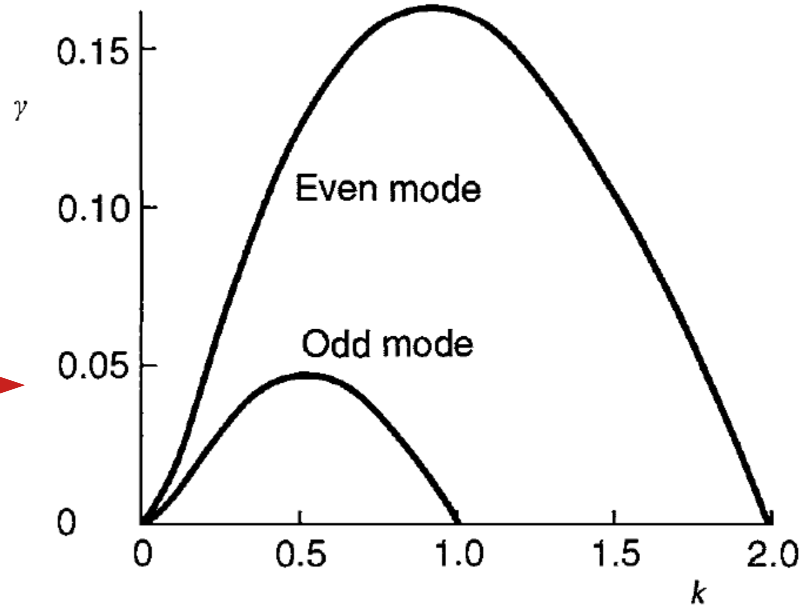
- The term KH is used in a broader sense for general shear-flow instabilities
- **Neutral gas:**

  There are two classes of solution:

Bickley jet → $V(x) = \mathrm{sech}^2 x$

- **Even mode:** corresponds to kinking of the jet (like KH for vortex sheet)
- **Odd mode:** corresponds to pinching of the jet

Kink (even) mode more unstable →



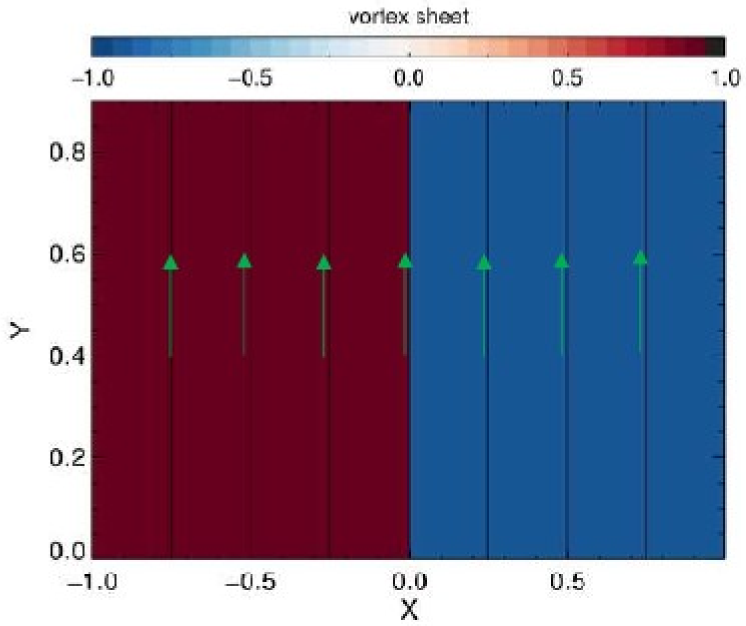
Magnetized Kelvin-Helmholtz

- A magnetic field parallel to the velocity field influences the instability

Vortex sheet $+B_y$ →

$$V(x) = \begin{cases} V_1 & x < 0 \\ V_2 & x > 0. \end{cases}$$

$$B_y(x) = B_0$$



- Dispersion relation: $c = \frac{V_1+V_2}{2} \pm \sqrt{v_A^2 - \frac{1}{4}(V_1 - V_2)^2}$

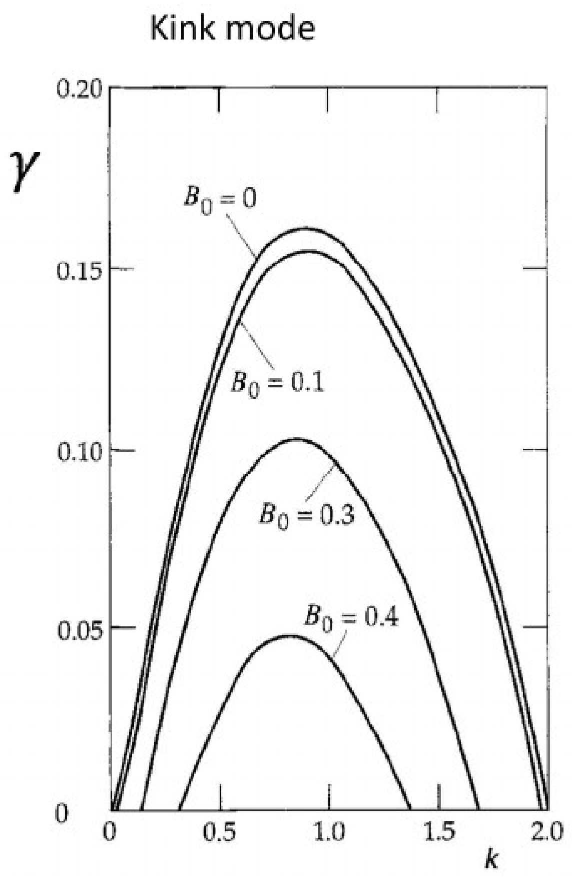
- **IF** $v_A > \frac{1}{2}(V_1 - V_2)$ ➡ **NO KH**

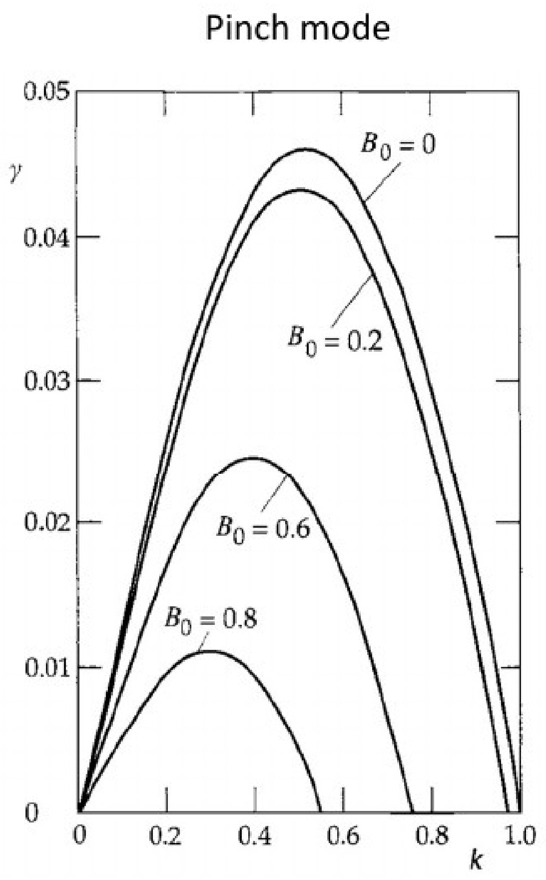
Magnetized Kelvin-Helmholtz

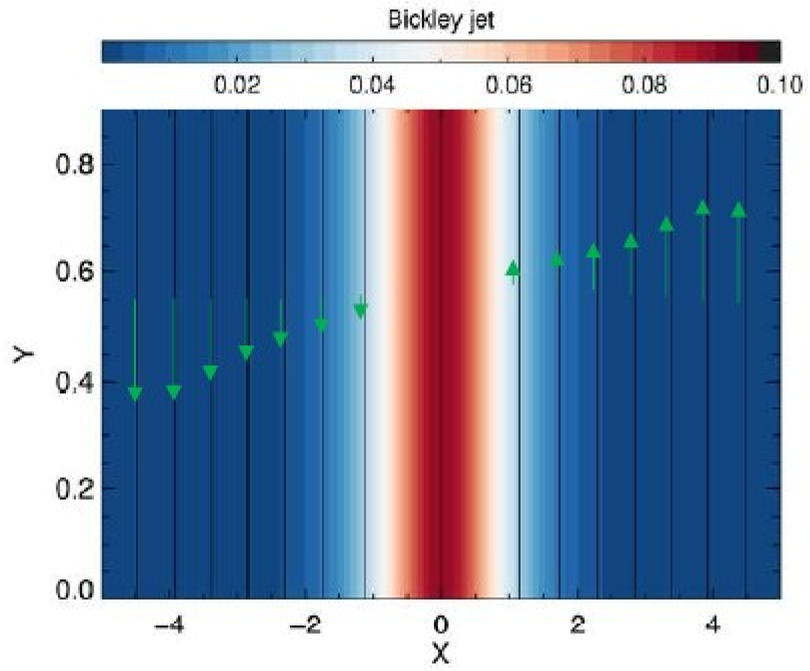
Bickley jet $+B_y$

$$V(x) = \mathrm{sech}^2 x$$

$$B_y(x) = B_0 \tanh x$$







- Stabilizing role of the magnetic field at small and large scales
- Pinch modes more unstable

Summary

- Fluid equations allow to simplify problems and to switch physical parameters one at time
- In plasma physics the problem of magnetic reconnection can be firstly addressed in the fluid framework
- Examples of studies carried out in the fluid framework addressing specific issues relevant in laboratory and space plasmas
- The process of magnetic reconnection is intimately linked to fluid instabilities
- Tomorrow we see how these two classes of instabilities coexist and compete in a turbulent context