Multifluid description of astrophysical and space plasmas - Part 2 -

Daniel Gómez^{1,2}



Departamento de Física, Fac. Cs. Exactas y Naturales, UBA, Argentina
 Instituto de Astronomía y Física del Espacio, UBA-CONICET, Argentina

small scales: two-fluid equations

The dimensionless version, for a length scale L_0 , density n_0 and Alfven speed $v_A = B_0 / \sqrt{4\pi m_i n_0}$

$$\frac{d\vec{U}_i}{dt} = \frac{1}{\varepsilon} (\vec{E} + \vec{U}_i \times \vec{B}) - \frac{\beta}{n} \vec{\nabla} p_i - \frac{\eta}{\varepsilon n} \vec{J}$$

$$\frac{m_e}{m_i} \frac{d\vec{U}_e}{dt} = -\frac{1}{\varepsilon} (\vec{E} + \vec{U}_e \times \vec{B}) - \frac{\beta}{n} \vec{\nabla} p_e + \frac{\eta}{\varepsilon n} \vec{J} \qquad \text{where} \qquad \vec{J} = \vec{\nabla} \times \vec{B} = \frac{n}{\varepsilon} (\vec{U}_i - \vec{U}_e)$$

 We define the Hall parameter ε = c/ω_{pi}L₀ as well as the plasma *beta* β = p₀/m_in₀v_A² and the electric resistivity η = c²ν_{ie}/ω_{pi}L₀v_A
 Adding these two equations yields: where $\vec{U} = \frac{m_i\vec{U}_i + m_e\vec{U}_e}{m_i + m_e}$ and $p = p_i + p_e$ $\vec{U} = \frac{m_i\vec{U}_i + m_e\vec{U}_e}{m_i + m_e}$ $\vec{U} = \frac{m_i\vec{U}_i + m_e\vec{U}_e}{m_i + m_e}$ $\vec{v} = \frac{m_e}{m_i} ε = \frac{c}{\omega_{pe}L_0}$

Generalized Ohm's Law

Note that the equation of motion for electrons is also Ohm's law

$$\mu \frac{d\vec{U_e}}{dt} = -\frac{1}{\epsilon} (\vec{E} + \vec{U_e} \times \vec{B}) - \frac{\beta}{n} \vec{\nabla} p_e + \frac{\eta}{n\epsilon} \vec{J} \qquad \text{where} \qquad \mu = \frac{m_e}{m_i} \ll 1$$

Considering that

$$\vec{U} = \frac{m_e \vec{U}_e + m_i \vec{U}_i}{m_e + m_i}$$
$$\vec{J} = n(\vec{U}_i - \vec{U}_e)$$
$$\vec{U}_i = \vec{U} + \mu \epsilon \vec{J}$$
$$\vec{U}_e = \vec{U} - (1 - \mu)\epsilon \vec{J}$$
$$n = 1 \quad (\text{incompressible})$$

• In the limit of massless electrons (i.e. $\mu \rightarrow 0$)

 $0 = -(\vec{E} + (\vec{U} - \epsilon \vec{J}) \times \vec{B}) - \beta \epsilon \vec{\nabla} p_e + \eta \vec{J} \quad \text{also known as the generalized Ohm's law}$

• At large scales, much larger than the ion inertial length (i.e. $\epsilon \to 0$), it reduces to

$$E + \overrightarrow{U} \times \overrightarrow{B} = \eta \overrightarrow{J}$$

Ideal invariants in multi-fluid plasmas

➡ For each species s in the incompressible and ideal limit

$$m_{s}n_{s}\left(\partial_{t}\vec{U}_{s}-\vec{U}_{s}\times\vec{W}_{s}\right)=q_{s}n_{s}\left(\vec{E}+\frac{1}{c}\vec{U}_{s}\times\vec{B}\right)-\vec{\nabla}\left(p_{s}+m_{s}n_{s}\frac{U_{s}^{2}}{2}\right)$$

• Using that
$$\vec{J} = \frac{c}{4\pi} \vec{\nabla} \times \vec{B} = \sum_{s} q_{s} n_{s} \vec{U}_{s}$$
 and $E = -\frac{1}{c} \partial_{t} \vec{A} - \vec{\nabla} \phi$

we can readily show that energy is an ideal invariant, where

$$E = \int d^3 r \left(\sum_s m_s n_s \frac{U_s^2}{2} + \frac{B^2}{8\Pi} \right)$$

➡ We also have a helicity per species which is conserved, where

$$H_{s} = \int d^{3}r \left(\vec{A} + \frac{cm_{s}}{q_{s}} \vec{U}_{s} \right) \bullet \left(\vec{B} + \frac{cm_{s}}{q_{s}} \vec{W}_{s} \right)$$

Normal modes in 2F-HMHD

If we linearize our equations around an equilibrium characterized by a uniform magnetic field we obtain, in the incompresible case, the following dispersion relation:

$$\left(\frac{\omega}{\vec{k} \cdot \vec{B}_0}\right)^2 \pm \frac{k\varepsilon}{1 + \varepsilon_e^2 k^2} \left(\frac{\omega}{\vec{k} \cdot \vec{B}_0}\right) - \frac{1}{1 + \varepsilon_e^2 k^2} = 0$$

Asymptotically, at very large k, we have two branches

$$\omega \xrightarrow{k \to \infty} \omega_{ce} \cos \theta$$
$$\omega \xrightarrow{k \to \infty} \omega_{ci} \cos \theta$$

while for very small k, both branches simply become Alfven modes, i.e.

$$\omega \xrightarrow[k \to 0]{} k \cos \theta$$



 Different approximations, just as one-fluid MHD, Hall-MHD and electron-inertia MHD can clearly be identified in this diagram.

some applications

MHD

RMHD heating of solar coronal loops (Dmitruk & Gomez 1997, 1999)

Kelvin-Helmholtz instability in the solar corona (Gomez, DeLuca & Mininni 2016)

Hall-MHD

3D HMHD turbulent dynamos. (Mininni, Gomez & Mahajan 2003, 2005; Gomez, Dmitruk & Mininni 2010)
2.5 D HMHD reconnection at Earth magnetopause (Morales, Dasso & Gomez 2005, 2006)
RHMHD turbulence in the solar wind (Martin, Dmitruk & Gomez 2010, 2012)
Hall MRI in accretion disks (Bejarano, Gomez & Brandenburg 2011)

Electron inertia



Two-fluid turbulence in the solar wind (Andres et al. 2014, 2016).

Fast reconnection in 2.5 D (Andres, Dmitruk & Gomez 2014, 2016).







Retaining electron inertia

➡ In the equation for electrons (assuming incompressibility)

$$\frac{m_e}{m_e}\frac{d\dot{U_e}}{dt} = -\frac{1}{\varepsilon}(\vec{E} + \vec{U_e} \times \vec{B}) - \beta_e \vec{\nabla}p_e + \frac{\eta}{\varepsilon}\vec{J} \qquad \qquad \vec{J} = \vec{\nabla} \times \vec{B} = \frac{1}{\varepsilon}(\vec{U_i} - \vec{U_e})$$

we replace

to obtain the following generalized induction equation (Andrés et al. 2014ab, PoP)

 $\vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \phi$ and $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\frac{\partial}{\partial t}\vec{B}' = \vec{\nabla} \times \left[(\vec{U} - \varepsilon \vec{J}) \times \vec{B}' \right] + \eta \nabla^2 \vec{B} \quad , \qquad \vec{B}' = \vec{B} - \varepsilon_e^2 \nabla^2 \vec{B} - \frac{\varepsilon_e^2}{\varepsilon} \vec{\omega}$$

- See also Abdelhamid, Lingam & Mahajan 2016 for extended MHD.
- Electron inertia is quantified by the dimensionless parameter

$$\varepsilon_e = \sqrt{\frac{m_e}{m_i}} \varepsilon = \frac{c}{\omega_{pe}L_0}$$

⇒ Just as the Hall effect introduces the new spatial scale $k_H = \frac{1}{\varepsilon}$ (the ion skin depth), electron inertia introduces the electron skin depth $k_e = \frac{1}{\varepsilon_e}$ which satisfies $k_e = \sqrt{\frac{m_i}{m_e}} k_H >> k_H$

Shocks in Space & in Astrophysics

- Shocks are an important source of heating and compression in space and astrophysical plasmas. Also, shocks are an important source of particle acceleration.
- For instance, shocks are formed during supernova explosions, when the stellar material propagates supersonically in the interstellar medium. Or when the solar wind impacts on each planet of the solar system.
- Because of the very low collisionality of most space and astrophysical plasmas, the shock thickness is determined by plasma processes.
- Even though the upstream and downstream regions can often be described using one-fluid MHD, the internal structure of the shock involves much smaller scales and therefore a fluidistic description like MHD cannot describe it properly.
- The image below shows the transverse structure of the Earth bow shock as observed by Cluster. The thickness is only a few electron inertial lengths (Schwartz et al 2011).







One dimensional two-fluid model

- We decided to use a two-fluid description to study the generation and propagation of perpendicular shocks.
- Perpendicular shocks correspond to the particular case where the magnetic field is tangential to the shock.
- We adopted a 1D version of the equations. All fields only vary in the direction across the shock (i.e. "x").
- Tidman & Krall 1971 derive stationary solutions (solitons) for these same equations.
- ➡ Our two species are ions and electrons.
- Compressibility is essential for shock formation and dissipation is neglected.

UP		Dou	Fru
T	Vy Vs Us	E_{y}	Eat
Z	î		→ E _× ★
Br	(,t)		

1D equations

- → Note that $U_i=U_e=U$.
- As a result, the electrostatic potential can be obtained from the // Euler eqs.
- The perpendicular Euler equations predict that m_e.V_e+m_i.V_i = const, and we choose it equal to zero.
- The only linear mode propagating in a homogeneous background are fast magnetosonic waves.
- We integrate this set of 1D eqs. using a pseudo spectral code with periodic boundary conditions and RK2.
- The initial profile is a finite amplitude train of fast magnetosonic waves.
- Dissipation in these simulations is set to zero.

Two-fluid ID egs $\frac{\mathcal{E}u[er]}{m_{s}n(\partial_{t}u_{s}+u_{s}\partial_{x}u_{s})=\frac{q_{s}n}{F}(-\partial_{x}\rho+v_{s}B)-\beta_{s}\partial_{x}n^{s}$ Euler, $M_s n(\partial_t V_s + u_s \partial_x V_s) = \frac{q_s n}{s} \left(-\partial_t A - u_s B \right)$ $0 = n(u_i - u_e)$ $-\varepsilon \rangle_{x} B = n (V_{i} - V_{e})$ Ampère,

shock formation

- We numerically integrate the 1D twofluid equations to study the generation of the shock, internal structure and propagation properties (Gomez et al 2021).
- Our initial condition is a finite amplitude fast magnetosonic wave.
- The movie shows the evolutions of various profiles (parallel velocity, particle density and perpendicular magnetic field.
- Once the shock is formed, it propagates without distorsion.





- Once the shock is formed, we can study the transverse structure of the various relevant physical quantities.
- We can see the propagation of trailing waves in the downstream region, with wavelengths of a few electron inertial lenghts.
- Below we see the temporal evolution of the particle density profile. The initial profile is shown in light blue.











4

Ramp thickness

- SHOCK THICKNESS vs ELECTRON INERTIAL SCALE 3.1 2.5 2.0 1.5 1.0 0.5 0.0 0.00 0.05 0.10 0.15 0.20 0.25 Λ,
- 0.3 84.8 25.6 835.0 m,/m, 11.8 6.5 0.10 2.5 0, 9. 0.2 0. N(X) 0.0 -0.1 4.0 4.5 5.0 5.5 х



We estimate the ramp thickness for each run and plot it against the corresponding electron inertial length. We find a perfect correlation, with ramp thicknesses of about ten electron inertial lengths.

Magnetic reconnection

- The standard theoretical model for two-dimensional stationary reconnection is the so-called Sweet-Parker model (Parker 1958)
- It corresponds to a stationary solution of the MHD equations. The plasma inflow (from above and below) takes place over a wide region of linear size∆ and is much slower than the Alfven speed (i.e. U_{in} << V_A).





 $M = \frac{U_{in}}{U_{out}} \simeq S^{-1/2}$

➡ The dimensionless reconnection rate is

where $S = \frac{\Delta v_A}{\eta}$ is the Lundquist number.

Since for most astrophysical and space plasmas is S >> 1, the reconnection rate is exceedingly low.

Plasma Inflow

$$u << v_A$$

 δ_1^{\dagger}
 Δ
Plasma Outflow
 $v \sim v_A$

Two-fluid simulations

➡ We perform simulations of the 2F-MHD equations in 2.5D geometry to study magnetic reconnection. We force an external field with a double hyperbolic tangent profile to drive reconnection at two X points (Andres et al. 2014a, PoP).

→ We also study the turbulent regime of the 2F-MHD description, to look for changes at the electron skin-depth scale (Andres et al. 2014b, PoP).





Two-fluid reconnection



Two-fluid reconnection



Two-fluid reconnection

Ľ,	1		1	I,	I,	Ų.			I,	ł,	ł,	ŗ,	ļ,	Ļ	Ļ	Ļ			. !	1	1	1	1	ł	1	1	1	ł	ļ	ļ	ł	ļ.	!		Ļ		ļ	ļ.	ŀ,	Ι.	١,	Ļ	Ņ	J.		1	I,	Ļ	ņ	Л.	١,	Ų.		ų,	1, t
F.	•	•	•	•	•	-	•	• •	•	•	•	-	•	•	÷	•	•	• •	• •	•	•	÷	•	÷	÷	-	÷	•	÷	÷	•	•	•	• •	•	÷	•	•	•	• •	• •	•	÷	•	• •	• •	•	•	•	• •		-		•	•
Ľ.	2			1	ŝ,	2					2	2	÷	÷	i.	2	1	2		1	4	÷		Ľ.	÷	2	j,	÷	÷	١,	1	1	2	h		1	1					÷	2	:	1		2	÷	1	2		1		÷	1
H:	:	•			-	5	•	: :		1	5	1	1	1	t	1	1	• •	•	1	1	t	1	Ŀ.	1	-	-	1	1	t	Ŀ,	- 1	D	U	1	1	1	1	•		1	-	1	:	: :		-	÷	5	: :		t.	• •	•	-
E.					5	-			-	÷	4	÷		1	-	-								÷	÷	÷		÷							-	-	-					-	1	÷			-	-	÷						
F:				1	÷	1		1	1	1	÷	÷		2	1	2		1			1	2	1	Ċ,	t	2									1	1	2	2	•	: :	1	1	5	5	1	1	1	1	1						-
Ε.									-	-	-	-										•	•	÷	•	•																-		-				-				-			
F.																					4																																		
-																																																							
-																																																							
E.																					-	-	-	-	-												1																		
				-	•	-			-		•	•		-	-	-					•	•	•	•	•	-									-	-	-	-	-				-	-	-			-	-						-
F:						-									-																											1													
E.	•	•	•	•	-	-	•	• •	•	-	-	-	•	•	•	-	-		• •	•	•	•	•	•	-	-	•	•	•	•	•	•	•	• •	-	-	•	•	•	• •	•	-	-	•	• •	• •	-	-	•	• •			• •	•	
F.				1	ŝ	2				5	2	Ę,	÷.	1	ļ,	1				1	÷	ŝ	ł	ł	î	1	ł	÷	ļ,	÷	÷	÷		2.2	1	1	1	1				÷	1	1			Ģ	÷	÷						
E:	•	•		1	•	-	•	• •	1	1	1	1	•	1	1	•	•	•	•	1	1	1	1	1	1	1	1	1	1	1	1	1	•		1	1	1	1	1			1	1	1			•	1	1	• •		1	• •	•	12
E.						2						2	÷	2	÷	÷						÷	÷	÷	÷	Ģ	÷	÷	÷	÷	÷	÷				÷	÷	2				÷	÷	2				÷	2			1			
E.	:			1	1	:	2	: :	1	1	1	1	1	1	1	:	1	:	1		1	1	1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	:	: :	1	1	1	1			1	1	1	: :		1	: :	1	
	•	•											-	-	÷	÷							•							÷			•								•	•	÷								-			•	
1	1			1	÷	1	2	1	. 1	1	1	1	2	2	2	1	1	: :			1	1	1	÷	1	1	2	2	2	2	2	2	2		1	1	1	2			1	1	2	2.3			1	1	1	1	1	1	1	1	1
	•	•	•	•	•	·			-	•	•	•	-	-	-	-	•	• •	• •		-	•	•	•	•	•	•	•	•	÷	÷	•		• •	•	•	•	-		• •	•	•	-		• •	• •	•	•	•		-	•	• •	•	•
=	:					:	2.			1	1	:	1	2	2	2	1				-	:	:	:	1	:	1	1	:	1	1	:	2			:	2	1			1	2	2	2.			1	1	2		1	1	: :	:	
F:	:				1	:				-	-	-	-	-	-	-	-		• •	•	-	-	•	•	•	•	-	•	•	•	•	-		• •	•	-	-	-		• •	-	-	-				-	-	-	• •	-	:		:	:-
F.													-	-	-	-	-	-			-	-					-					-				-	-	-			-	-													
H.	1	-		5		-	-			1	1	1	2	2	1	:	2	1		1	1			1	:	:	1	:	:	:	:	1	-	: :	1	1	1	1	1	: :	1	1	1	: :	1		1	2	2	: :		;	: :	1	+
F	•	•	•	•	•	•	-					•	-	-	÷	÷	÷						;	÷		÷		÷	÷	÷	÷	2			2		÷					÷	-				-	-	-		•			•	.=
E:			2	1	2		1		-	-	_	_	-	-	-				4	4	÷	_	1	1	1	1	1	1	;	:	1	1	2		-	÷	÷	:	-				1						÷			í.	40	.:	12
F.		-	-	-	+-	++	•	+	•	-	-	-					-	-	-	-	-	•	•	•	-	•	·	•	÷	•	-	-	•						-	-,	-			•	•	-	-		•		•	•••			
Ľ.	: '	1		5	7	7				7	7	7	2	2	2					1	1	1	:		2	:	:	:	:	:	:				:	2	2	2.			2	2	2			;	2	2	2	22	2	1	11	÷	12
F.	•	•	• •	•	•	•	•	• •	• •	•	-	-	-	-	•	-	-			-	-	-	·	•	•	•	-	·	·	÷	·	-	- 1	• •	-	-	-			• •	-	-	- 1			•	•	•	-		-	•	• •	•	•
E	:						-			-	-	-	-	-	-	-	-	-		-	-	-					-	•		•	•	-	-			-	-	-			-	-	-				-	-	2		÷	÷	: :	;	-
1:	-	-		:	:	:	-			-	-	-	-	-	-	-	-			-	-	-	•	•	:	:	-	•	•	•	:	-				-	-	-			-	-	-				-	-	-		-	:		:	:-
F-	•	•								-																											•						-					-	-		-				
H:	;			-	÷	1			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2		1	1	1	2	1		1	1	2	1	1	1	1	:	1		1	1	: :	1	1-
F	•	•	•	•	•	•			• •	•	•	•	•	-	-	•	•	•		-	•	•	•	•	•	•	•	•	•	•	•	•		• •	•	•	•	-	-	• •	•	•	•		• •	• •	•	•	•	• •	-	•	• •	•	· [
E.				1	1	1	-				÷	÷	1	2	i.	1	1					-	ł	1	ł	ł	ł	÷	1	1	1	1				1	1					-	1					1	1		1	1	1		1
E.	•	•		•	•	•			•	•	•	•	•	•	•	•	•		• •		•	•	•	•	•	•	1	•	•	•	•	•	•	• •	•	1	•	•	• •	•	•	•	•	•	1			•	•	• •		•		•	1
Η÷ι	1		T	T.	Ľ	Ľ.			1	1	É	É	Í.	Í.	Í.	Í.	Í.	[]	i	i	i	i	i	i	i	i	1	j.	i	i.	i.	i.	1				i.	1	T	T	Ľ	Ľ	Ľ.	1		T	Ľ	Ľ.	Ľ.	1	T.	i.		1	H

Two-fluid reconnection

H				1	1	1	1			1	1		11	1	1		1				1					1					11	1		1		1			1			11			11
H:		1	1.1	1	1	11									2	11					1		1	11		1.1				1												1.1			1-
		0	1												2					1	÷ .		2																			1.1			1
																				-						<u> </u>	. ,	10							-										
Ξ.					-			-			-									1		=	Ξ.	1.1	4	Δ.	4	HC.	J .																
Ξ.															-		-																-					• •	~						1
-	• •					e 1						•		-	-		-	- 1	•	*	•	• •	٠.	• •	•	• •	• •		• •				-	• •	•	• •		• •	•	8.1					1
-	• •		• •					•		• •	*	•	• •	-	-			• •	• •	1	1	• •	•			• •	•	•	• •	•	• -		-		•	• •	•			8.1	• •			• •	14
-	• •	•	• •	*	•	• •		•	• •		٠	•		-	-		•	• •			•		1			11		•	• •	•	• •		-		•	• •		* *	•	* 1	• •	• •		• •	1
- °	• •	•	• •				•	•	• •		*	•	• •	-	- 1						2.2										• •		-		•	• •	•	• •			• •	• •		• •	1-
-			• •			••	•	•	• •	• •	•	•								2																• •		• •		•				• •	
- *											1																																		
												-																																	
															•	• •		۰.							1	5	1	1				-						•							
									•			-		-				* •			4.1			1.1		1.1	1		• •		• •													• •	
Ľ		1									-				-	• •		* *		4	4	1.1	1	1.1					• •			* *			-			-				1			
		1						*	• •		-				-			• •		1	*		1				• •		• •		• •	* *			*									1.1	
								1						-						1	1	1		1.1	1	1			• •										1	1					
		1	1		2	1		2						-	-		-	11	٠.		1	1.1	1	1.1	1	1.1	٠.	1	• •		• •		-					11	1			1.1		11	
		1			2	1		1			1				2	1.1				1	1	1	1	1.1	1	1.1						1.1						11	1	1		1.1			
					2			2			2		1		2	11				1	11		1	11		1.1						2.2						1							
		2						0			2				2	2.2				1	1		2	1		2.2						11	2					2.2	2			0.0			
L.																	1						2										-												
L.																				4	1		1			1																			
F.																							1																				1.		
L.												-																								-	- 1						1.		-
F.					τ.			1				-					-	-					-						•											• •	• •	\$ 1	1.		-
-		-		•	•	• •	-	-					• •													1.1				1.1	1.					-	-		-				•		1-
F.	• •	۰.	• •		٠.	• •	-	-			-	-	• •				~		• •		۰.					1.4					1.0					-		• •	-			× 1	1	• •	1-
F.	• •	۰.	• •	•	۰.	5.5	-	-	- 1		-	-		• •			~	\$	• •		۰.					1.4					1.0					-			-	- 1	• •	* *	1.	1.1	1-
	•	۰.	• •	•	۰.	• •	1	-	- 1		-	-		•		• •	•	•	• •		۰.		۰.	• •		1.1	1.1			• •				-		-			•	• •	• •	× 1	1	• •	1-
-	1	*	11	1	1	1.1		*	+ -		-	-		• •	•	• •	-	•	• •	•	•	• •	*	• •		• •				• •	•	* *		•		-	- '	• •	•	• •	• •	* *	1		1-
1		2	11	1	1	11	1	-	•	• -	-	-			•	• •	-	•	• •	•	•	• •	*	• •					•	• •	•	* *		-		-		• •	•			2.1			1-
E		1	11	1	4	11		-			-	-			•	• •		-	• •	•	•	• •	•	• •		• •	• •	• •			-			-		+			•			2.2	1	11	1-
F?		;	; ;	1	1			-	-		-	-		-	-		-	-					*											-		-		-	2	2.2	-				1-
E -	1	1		,					-			2			1	11				2	2		3		!				*		1			1		-	21						1		1
		i	1 1	1				-							2			2			2	1			1	1	1		1	11	1			1	1	-			-			11	i	ii	1
Ei	1	11			-			-	-	-	<u> </u>			-			-		1	3	έ.	1	1	: :	1	1	1	11		1		11			1	-	-			-	1		11	1	
Ľ,	~ •	-		-	-	44	~	é	÷	-	-	~	÷	-	-	-			<u> </u>	-		-	2.	23		-	_			-			-	-	-	-	÷	53	2	20	-	**	-		
F.		~	**	-			e e	~	-	è-è		-	-	+	-	-	+	÷.,		+	-	· ~	•	• 1	٦,	1	, ,	1	1.	1.		+	÷	-	5	Ś.	÷÷	-	Ś.	÷,	-				-
F				~	~+			-+-			+	~*		~	• •	~	•	• •	•	۰.	× 1	• •		11	1		1					, ,	1	1.1	•		• •		٠,	~	5		~~	11	+-
H		I.	14	4	1			•	•	• •	•	-	• •	•	•	• •	•	•	• •	•	•	• •	1		1	• •		• •	•	• •	• •	••	•	• •	• •	-	• •	• •			1	7.1	1	11	1-
H	4	5	5.4	- 5	۰.	• •	~	~	•			-		• •	•	• •		•	• •	•	•	• •	•	• •	•	• •	• •	• •	`	• •	•	• •	••	•		-			-			11	4	11	1-
F.	• •	۸.	1.1		*	1		-	-			-		• •	-	• •	-	-	• •	1		• •		• •					`				•••	-		-			-			.,		, ,	4-
H	- 1	,			1			-	-		-	-	-		1	11	-	-			1		1						-					-		-							4.	4 4	+
Η,		1	, ,		`			-			-	-			1		1	-					1						1					-		-			*	· ·			• •	4 1	1-
F'		*			1			-				1	-		1	1	-	1		-			1	: :												-			*	• •			• •	• •	'-
F,		1			1			-	-						1						1		1													-	2					11			-
F:		1			1						-	1			1		1	-			1		1							1					1	-			2			11		1	17
E		1									-	-			-		-	-																								1			17
								-	-			1					-	-		1																-	-								
Ε.				1			-	-				-					-								1											-	-								
Ε,	1			1			-	-				-													1											-	-								
Π-								Ŀ.				1			1	1.1	1	1.1		1	1	1.1	1	1.1		1.1			1.1.	1.1		1.1					1						11		1.
											1.1							_																											_

Two-fluid reconnection

Lateration		1		1.2.6.6				e e e d'e e e		1.1.1.1.1.1.1.1	 S 1 1	1.14
						1.1.1.1.1.1					 	
								~				
H					· · · + ·	· · · · ·	- 5 7	α \sim			 	1.1-
L				1.2.1.1	10 A 10 E	· · · ·	- U.Z	0			 	1.1.1.
					1.1.1.4						 5 6 1	1.1.
- · · · · ·					1							
											 	· · -
A 1 1 1 1											 	1 1
					* * * *	****					 	
					1.1							
L												5 has
											1.1.4	A 1.
												5 . 5
COLUMN STATE												1 1 -
the second second												1 1-
					1111							
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1				* * * *	* * * *							
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1							COLUMN STATE				 2.2.2	1 1 1
		Constant States	1								 	
	and the second second											
T R R R R R R R R R R	6.8.8.8.8.1	1.1.1.1.1.1.1								1.1.1.1.1.1.1	 1.00	1.17
- P. L. L. L. L.	CONTRACTOR OF A	1. 1. 1. 1. 1.	1			****				1.1.1.1.1.1	 	1.1-1
				1.1.1.1.1.1	1.1.1.1						 	
				1.0.0					1. 1. 1. 1. 1. 1.		 A 41.4	1.1.1
- · · · ·												-
L											 111	
					::::	1111					 111	11-
1111	::::::	*****				1111						1
<u>-</u>		:::::										
												1111
												1111

Two-fluid reconnection

HILLI				111111111	
He 1 1 1 1 4	e sur de la sur e			a second second	which is a state where the state where
- CONTRACTOR	******				a for a second second second second
- I I I I I I	********			1	a de la construction de la constru
			1 t = 1	4 DO	
F1 1 1 1 1 1				1.00	
Fi i i i i i i					

				A REPORT OF A REPORT	

1111111			*******		**********************
A A A A A A A A A A A A A A A A A A A					
EL LA SAN					**********************
	*******				*******************
	********		********	**********	********
				A 4 4 4 4 4 4 4 4 4 4 4 4	******
	*******				*****

E	********				****
				The second second	
Li i i i i i					
Les sur sur					
LETTIC					
-******					
		1	*******		
Le			*******	14111444444	
L			*******	11111111111	
L	• • · · · • • · · ·	********	*******	111111111111	
+· · · · · · · · ·		*********	********	111/000000	and the second
		*******	teres ()	1-1-1-1-1-1-A	and a second and a second
++	***********	******	1111111	1 2 2 2 2 2 2 2 2	Contraction of the second s
	*********	********	*******	ITTA AREAS	
+••••		********			
+++++++		*********			*******************
Here and the second sec					
			///////////////////////////////////////		

E			· · · · · · · · · · · · · · · · · · ·	111111111111	
			22221144 22221144	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
			2 2 2 2 2 1 1 4 4 1 2 2 2 2 2 1 1 4 4 4 2 2 2 2 1 1 1 4 4 4 2 2 2 1 1 1 4 4 4 2 2 2 1 1 1 4 4 4	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	· · · · · · · · · · · · · · · · · · ·

Two-fluid reconnection

	n e a la santa la santa sa	s Is is it is
the second second second second second		5 F F F
A REPORT OF A REPORT OF A REPORT		6 A A 4
Les a la l		
		1 1 1 1 L
		F 1 1 1
THE REPORT OF A PARTY		
	and a second s	
		5. 5. 5. 5. 6 .
FI1111111111444444		8.5.5 L
TELLEVENERGE		
FILLS A MARKED AND A MARKED		
THE FEFT NY VALUE AND AND	and the second	
PETERS SANSARS SAME		P. P. I. I.
THE R P. LEWIS CO., NAMES AND ADDRESS OF		8 C 8 E
THE R. P. LEWIS CO., NAMES IN CO., NAMES INC.		
THE R P. LEWIS CO., NAMES OF TAXABLE		F 1 1 1mm
THE R P. LEWIS CO., NAMES OF TAXABLE PARTY.		

E. B. B. B. B. B. Statistics and the state of the stat		
HERE AND A REPORT OF A DESCRIPTION OF A		F 1 1 1-
		1111

Two-fluid reconnection

H	
Terrary and a second second second second second second	
the state of a state of the sta	a da
A REAL PROPERTY OF A REAL PROPER	
	560
The second	
Let be be a set of a	***************************************
	The second s
The second s	
The state of the s	

The second se	I I I I I I I I I I I I I I I I I I I
	A REAL PROPERTY OF A REA
Test 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
A TALALA TALALA TALALA A TALALA A TALALA A TALALA A TALALA A TALALA A TALAL	THE REPORT OF THE PARTY OF THE
The second s	and the second
	and a set of the set o
A PARTICULAR CONTRACTOR OF THE OWNER	
	1 - 1 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 -
THE LOCK AND ADDRESS OF A DECEMBER	a series a series of the serie
and the second	The second s
	TAXA SANANA ANA MANAMATAN AND AND AND AND AND AND AND AND AND A
THE REPORT OF A REPORT	A REPORT OF A R
The Lands and the second s	
	THE FERNAL AND ADDRESS OF A DREAM AND AND A DREAM AND
The both the both the both the second s	A REAL AND A
The second s	
	The state of the s
Here and the second second second second second second second	
The state of the s	
E F. S.	and a second
- First Contraction of the second state of the	
the second s	
E. E. E. B. B. B. B. B. S.	化化化物 医血液的 化苯基苯基 化分子 法法公共 化化化物 化合金 化合金化物
Free for the formation of the second se	
The state of the s	
http://www.weighttp://www.wei	
Let blit blit a second a secon	
TTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTT	
PERFECTION AND AND AND AND AND AND AND AND AND AN	
http:///////////////////////////////////	
	A REAL PROPERTY AND A REAL PROPERTY A REAL PROPERTY AND A REAL PROPERTY A REAL PRO
H	ISTILLY YING & The Concert and a series
L'IL CONTRACTOR STRATE PROVIDENT	مر و از بار و بار و مرجو المربو المربو المحصص مر المربو الروا و المرا ا
and the second	the second s
	Prizza A. A. A.
	Contraction of the second s
here a superson and the second	I I I I I I WANT I THE THE PARTY AND A THE AT A THE AT
La 1 2 4 2 4 2 2 2 2 2 2 2 2 2 2 4 4 4 4 4	
	I I I I I I I I I I I I I I I I I I I
FUTUTION 11/1/1/1/1/1/1/1/1/1/1/1/1/1/1/1/1/1/1	
LEFT 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
P	
Ferrers and service a service	
Lease and a second s	

Reconnected flux in 2F-MHD

The total reconnected flux at the X-point is the magnetic flux through the perpendicular surface that extends from the O-point to the X-point.

→ We compare the total reconnected flux between a run that includes electron inertia and another one that does not.





The reconnection rate is the time derivative of these two curves.

The apparent saturation is just a spurious effect stemming from the dynamical destruction of the X-point.

Reconnection rate in 2F-MHD

➡ For the 2D configuration and assuming incompressibility, we run several simulations with different values of the Hall parameter, which is the dimensionless ion inertial length.

➡ We compare the corresponding reconnected flux (above) and the reconnection rate (below) vs. time.

The reconnection rate is E_z at the X-point. From the equation for electrons, under stationary conditions $E_z = -\frac{m_e}{a} \hat{z} \cdot \vec{u}_e \, x \, \vec{w}_e$



FIG. 3. Reconnected flux Φ (upper panel) and reconnection rate r (lower panel) as a function of time for $\lambda = 0.07, ..., 0.16$ (from bottom to top). For all runs, the electron to ion mass ratio is $m_e/m_i = 0.015$.

At electron scales

$$\vec{u_e} \sim -\frac{1}{e n} \vec{j}$$

from where we obtain the following estimate for the dimensionless reconnection rate

$$R = \frac{c E_z}{B_0 v_A} \sim \frac{c}{w_{pi} L_0}$$



FIG. 1. Schematic 2.5D reconnection region.



FIG. 6. Quasi-stationary reconnection rate r (gray circles) as a function of the Hall parameter λ . The best linear-fit for $\log \lambda - \log r$ is shown in graydashed line. Inset: Ratio between quasi-stationary reconnection rates and the Hall parameter (gray squares) as a function of the Hall parameter.

Turbulence in the Solar Wind

The **solar wind** is a stream of plasma released from the upper atmosphere of the Sun, which impacts and affects the planetary magnetospheres.

Sahraoui et al. 2009 used magnetograms from the Cluster mission to derive power spectra of magnetic energy.





➡ They combine low-cadence data from FGM (parallel and perpendicular components) with high-cadence from STAFF-SC (also parallel and perpendicular).

At the largest scales, they obtain a K41 power spectrum ($k^{-1.62}$).

➡ As they go to smaller scales, they identify two breakups. An intermediate range with a power law k^{-2.50}, and an even steeper range at the smallest scales (k^{-3.82}).

Turbulence in 2F-MHD simulations

These breakups are a manifestation of physical effects beyond MHD.

➡ We performed incompressible 3072x3072 simulations of the full two-fluid equations. We excited a ring of large-scale Fourier modes and let the system relax while the turbulent energy cascade takes place (Andres et al. 2014b, PoP).

The magnetic energy power spectrum shows two breakups at the approximate locations of the proton (k_p) and electron (k_e) scales.





FIG. 1. Magnetic energy spectra for EIHMHD cases with $\lambda_i = 1/10$ and $m_e/m_p = 1/1836$ (black) and $m_e/m_p = 0.015$ (gray).

- The spectrum is K41 (i.e. $k^{-5/3}$) at $k \ll k_p$.
- At intermediate scales ($k_p \ll k \ll k_e$) is $k^{-7/3}$.
- \blacksquare Beyond the electron scale ($k_e \ll k$) a new range takes place $k^{-11/3}$.
- ➡ All these inertial ranges can be obtained using Kolmogorov-like arguments on the energy transfer rate given by

$$F_k \simeq k(u_k^3 + u_k B_k B'_k + (1 - \delta)\lambda J_k B_k B'_k + (1 - \delta)\delta\lambda^2 \partial_l J_k B_k).$$

Turbulence in 2F-MHD simulations





FIG. 2. Magnetic energy spectra for $m_e/m_p = 0.015$. Vertical dashed gray lines correspond to $k_{\lambda} \sim 10$, $k_{\lambda} \sim 82$, and $k_{\nu} \sim 550$. The compensated spectrum for the HMHD (solid line) and EIHMHD (dashed line) regions are shown in the lower panel.

FIG. 3. Magnetic energy spectra for $m_e/m_p = 1/1836$. Vertical dashed gray lines correspond to $k_{\lambda_e} \sim 10$, $k_{\lambda_e} \sim 430$, and $k_e \sim 650$. The compensated spectrum for the HMHD (gray line) and EIHMHD (green line) regions in the same format as Figure 2.

Conclusions

- One-fluid MHD is a reasonable theoretical framework to describe the large-scale dynamics of plasmas.
- Two-fluid MHD introduces new physics (Hall, electron pressure, electron inertia) and also new spatial scales, such as the proton and electron inertial lengths. We studied the role of these plasma effects on three relevant phenomena for astrophysical and space plasmas: <u>shocks</u>, <u>reconnection</u> and <u>turbulence</u>.

Shocks:

1D simulations show that a train of fast magnetosonic waves decay into a train of shocks. Thickness is a few times the electron inertial scale.

<u>Reconnection</u>:

Our 2F-MHD simulations show fast magnetic reconnection without energy dissipation (Andres et al. 2014a, PoP). The reconnection rate scales like the ion inertial scale and is independent from the electron mass.

• <u>Turbulence</u>:

Externally driven 2F-MHD runs show turbulent regimes. The magnetic energy spectrum displays breakups at the ion and electron inertial scales (Andres et al. 2014b, PoP). Spectral slopes can also be obtained by dimensional analysis.

Induced magnetospheres



Figure 1. Geometry of MAVEN orbit 2844 between 17:40 and 18:20 UTC in cylindrical MSO coordinates. The orbit is shown in black and the mean position of the bow shock and MPB (Vignes et al., 2000) in magenta and green respectively.

- Planetary magnetospheres form because of an equilibrium between the local magnetic field and the solar wind ram pressure.
- Planets like Mars or Venus do not have magnetic fields. The interaction is between the wind and the planetary ionospheres.
- A transition region or magnetic pileup boundary (MPB) is formed beyond the downstream region.



- We ran 3D multifluid simulations in collaboration with Dong (Dong et al. 2014, GRL).
- It includes protons, several ionospheric ions, and massless electrons.
- The picture shows the region where the plasma beta equals unity to identify the MPB.

Magnetic pileup boundary





A nice analytical exercise is to assume B=0 and look for 1D stationary solutions for a plasma made of three species:

- p: protons e: electrons h: heavy ions
- To the right, we have the solar wind made of a fully ionized hydrogen plasma.
- To the left, above of the planetary surface, we have a static and gravitationally stratified atmosphere, made of singly charged heavy ions (such as O+, O2+, CO+ and CO2+).
 - In between, electrons satisfy electric charge quasi-neutrality, as observed in the picture.
- In the picture below, the density profiles for different values of the Mach number are shown.
- To the right of this transition, we have a shocked, subsonic solar wind. To the left, we have a static ionosphere.
- When we include the magnetic field, fieldlines carried by the solar wind remain frozen to the electrons and pile-up in front of the much denser ionosphere.

Magnetic pileup boundary





Figure 6. Simulation results on the \hat{x} axis. From top to bottom: (a) Magnetic field strength. (b) Volume current density components and strength. (c) Magnitude of the convective, Hall and pressure gradient electric fields. (d) Hall electric field components. (e) Different pressures, as indicated. The bow shock (not depicted) is at approximately 1.5 \mathbb{R}_M .

Figure 5. Comparison of MAVEN data and the simulation results over the trajectory. From top to bottom: Magnetic field magnitude. Electron density (LPW values for densities greater than 1 cm^{-3}). Solar wind ion density (proton density only for simulation data). Ion density of different species (H+, O+, O2+, CO2+), STATIC data is represented by dots and simulation data is the continuous line. The MPB from simulation data is shaded in green and from MAVEN data is shaded in grey.