Emission properties of an optical cavity-quantum dot system obtained by a deformed algebra

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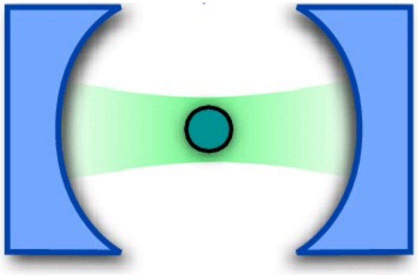
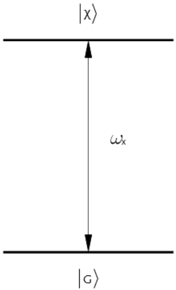


(a) Optical cavity



(b) Quantum dot

Figure: Representation of an optical cavity quantum dot system [5]

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Closed System: JC Hamiltonian

The optical cavity-quantum dot system can be modeled using the Jaynes-Cummings Hamiltonian:

$$\hat{H}_{JC} = \frac{\omega_c}{2}\left(\hat{a}^\dagger \hat{a} + \hat{a}\hat{a}^\dagger\right) + \omega_x \hat{\sigma}_+ \hat{\sigma}_- + g(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-). \tag{1}$$

Where $\hat{a}^\dagger$ and $\hat{a}$ are the ladder operators associated with the quantum harmonic oscillator and are associated with the cavity states while $\hat{\sigma}_+$ and $\hat{\sigma}_-$ are operators associated with the quantum dot and allow transitions between the quantum dot states.

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$
$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$\hat{\sigma}_+ |G\rangle = |\chi\rangle$$
$$\hat{\sigma}_- |\chi\rangle = |G\rangle$$

Closed system: Morse oscillator

The Morse potential is similar to the harmonic potential in certain value ranges:

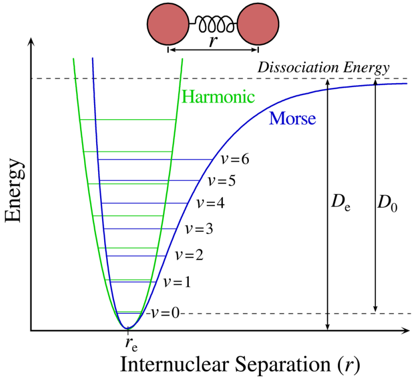


Figure: Morse potential and its eigenenergies compared to the harmonic potential

Closed system: Morse Hamiltonian

The Hamiltonian associated with the Morse oscillator can be written similarly to that associated with the harmonic oscillator by defining particular staircase operators:

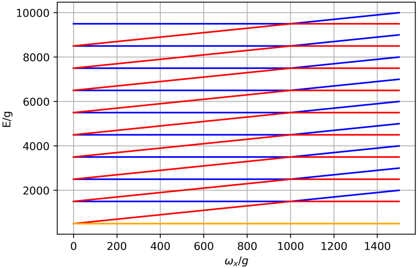
$$\hat{H}_M = \frac{\omega_c}{2}\left(\hat{A}^\dagger \hat{A} + \hat{A}\hat{A}^\dagger\right) + \omega_x \hat{\sigma}_+ \hat{\sigma}_- + g(\hat{A}\hat{\sigma}_+ + \hat{A}^\dagger \hat{\sigma}_-). \tag{2}$$

Where,

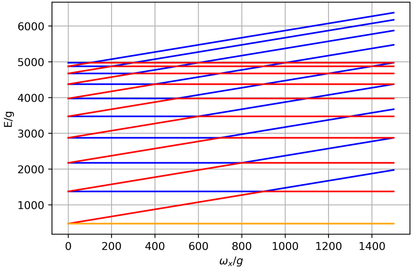
$$\hat{A}^\dagger \left|n\right\rangle = \sqrt{1-\chi_0(n+1)}\sqrt{n+1}\left|n+1\right\rangle \tag{3}$$

$$\hat{A}\left|n\right\rangle = \sqrt{1-\chi_0 n}\sqrt{n}\left|n-1\right\rangle .$$

Closed system: Eigenenergies



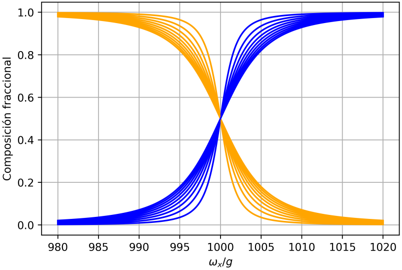
(a) Harmonic oscillator



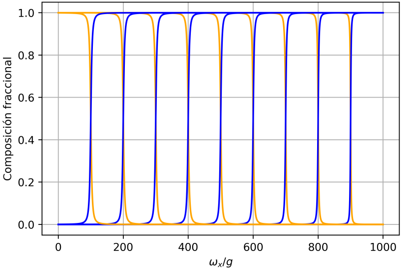
(b) Morse oscillator

Figure: Comparison of the energies of the first nine varieties. We take the following value for the parameters: $g = 0.1$, $\omega_c/g = 1000$ and $\chi_0 = 0.05$.

Closed system: Fractional composition

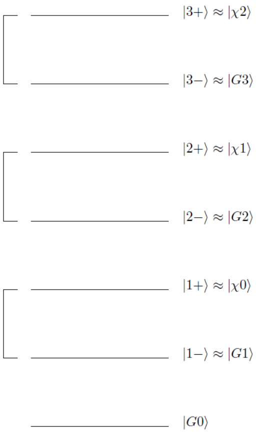


(a) Harmonic ocillator

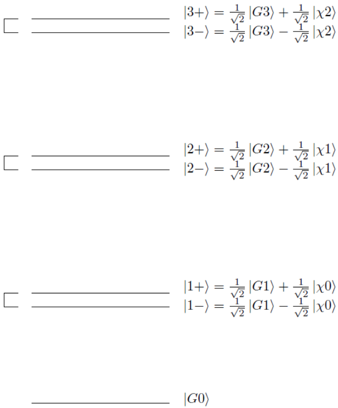


(b) Morse oscillator

Figure: Fractional composition of the polariton states of nine varieties. We take the following values for the parameters: $g = 0.1$ y $\omega_c/g = 1000$



(a) Energy levels off-resonance



(b) Energy levels in resonance

Closed system: Energy levels Morse case

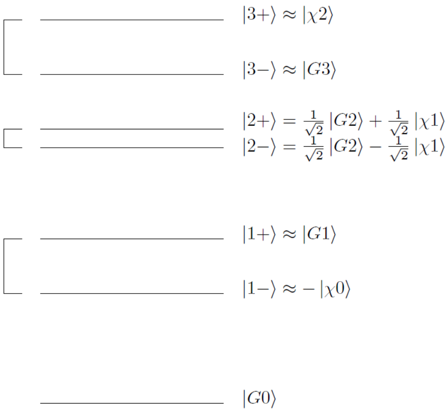


Figure: Energy levels

Contet

Open system: Master equation (harmonic case)

The hamiltonian takes the following form:

$$\hat{H}_T = \hat{H}_{JC} + \sum_i \omega_i \hat{b}_i^\dagger \hat{b}_i + \sum_i \lambda_i \hat{d}_i^\dagger \hat{d}_i + \sum_i g_{1i}(\hat{a}^\dagger \hat{b}_i + \hat{a}\hat{b}_i^\dagger) + \sum_i g_{2i}(\hat{d}_i^\dagger \hat{\sigma}_- + \hat{d}_i \hat{\sigma_+})$$

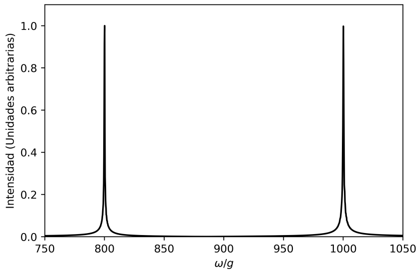
The Von - Neumman equation is:

$$\dot{\hat{\rho}} = [\hat{H}_T, \hat{\rho}]$$

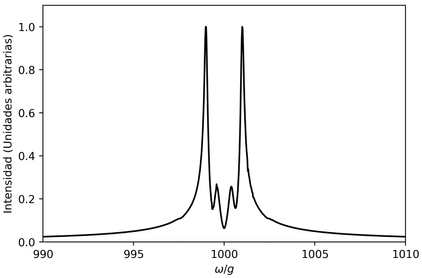
From which the following master equation can be derived:

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}_{JC}, \hat{\rho}] + \frac{\gamma}{2}\mathcal{L}_{\sigma_+}(\hat{\rho}) + \frac{P}{2}\mathcal{L}_{\sigma_-}(\hat{\rho}) + \frac{\kappa}{2}\mathcal{L}_{\hat{a}}(\hat{\rho}) \quad (4)$$

We calculate the following spectrum:



(a) Emission spectrum taking the following parameters: $g = 0.1$, $\gamma/g = 0.1$, $\kappa/g = 0.1$, $P/g = 0.05$, $\omega_c/g = 1000$ y $\omega_x/g = 800$



(b) Emission spectrum taking the following parameters: $g = 0.1$, $\gamma/g = 0.1$, $\kappa/g = 0.1$, $P/g = 0.05$, $\omega_c/g = 1000$ y $\omega_x/g = 1000$

Figure: Spectrum of the system (a) off-resonance y (b) in resonance.

Open system: Master equation (Morse case)

The hamiltonian is:

$$\hat{H}_T = \hat{H}_M + \sum_i \omega_i \hat{b}_i^\dagger \hat{b}_i + \sum_i \lambda_i \hat{d}_i^\dagger \hat{d}_i + \sum_i g_{2i}(\hat{d}_i^\dagger \hat{\sigma}_- + \hat{d}_i \hat{\sigma_+}) \quad (5)$$

$$+ \sum_i \left(\hat{A}^\dagger \hat{\kappa}_i(\hat{n}) \hat{b}_i + \hat{A} \hat{\kappa}_i^\dagger(\hat{n}) \hat{b}_i^\dagger \right). \quad (6)$$

The Von - Neumman equation takes the following form:

$$i\dot{\hat{\rho}}_T = [\hat{H}_T, \hat{\rho}_T].$$

From which the following master equation can be derived :

$$\dot{\hat{\rho}} = -i[\hat{H}_M, \hat{\rho}] - \{k_1(\hat{n})\hat{F}^\dagger \hat{F} + k_2(\hat{n})\hat{F}\hat{F}^\dagger, \hat{\rho}\} + \{\hat{F}^\dagger \hat{\rho} \hat{F}, k_3(\hat{n})\} \quad (7)$$

$$+\{\hat{F}\hat{\rho}\hat{F}^\dagger, k_4(\hat{n})\} + \frac{\gamma}{2}\mathcal{L}_{\hat{\sigma_+}}(\hat{\rho}) + \frac{P}{2}\mathcal{L}_{\hat{\sigma_-}}(\hat{\rho}), \quad (8)$$

Open system: Emission spectrum

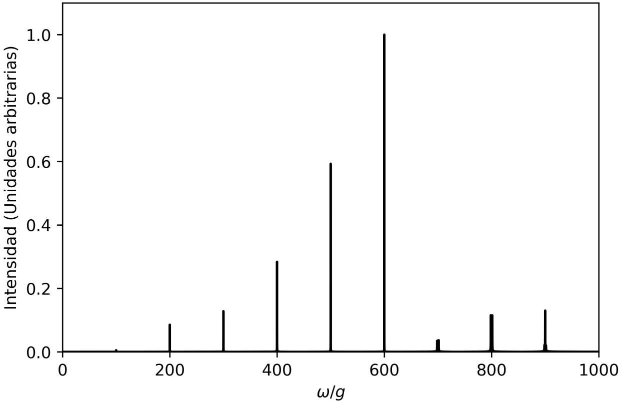
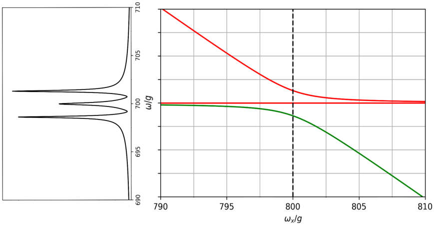
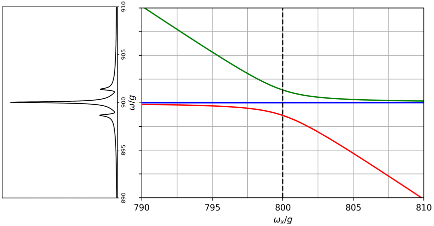


Figure: Emission spectrum. We take the following parameters $\gamma/g = 0.1$, $P/g = 0.05$, $\chi_0 = 0.05$, $\omega_c/g = 1000$, $g = 0.1$ y $\omega_x/g = 800$



(a) Triplet around $\omega/g = 700$



(b) Triplet around $\omega/g = 900$

Figure: Triplets around $\omega/g = 700$ y $\omega/g = 700$ due to the transitions $|4+\rangle \rightarrow |3+\rangle$ (straight red (a)), $|3-\rangle \rightarrow |2-\rangle$ (red curve (a)), $|3-\rangle \rightarrow |2+\rangle$ (green curve(a)), $|2+\rangle \rightarrow |1-\rangle$ (green (b)), $|2-\rangle \rightarrow |1-\rangle$ (red (b)) y $|1+\rangle \rightarrow |G0\rangle$ (blue (a)). We take the following parameters: $\omega_c/g = 1000$ y $g = 0.1$, $\omega_x/g = 800$.

Open system: Emission spectrum

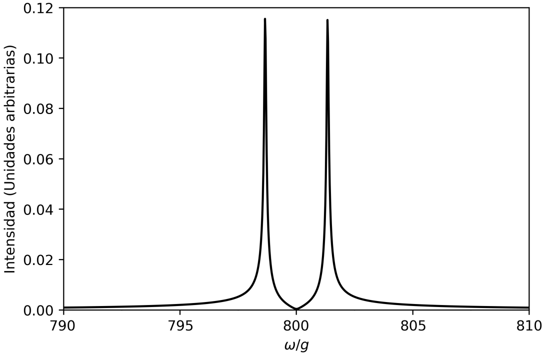
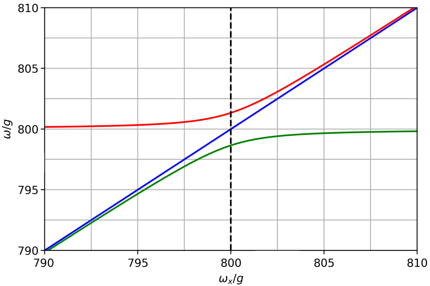


Figure: Triplet centered at $\omega/g = 800$ by taking $\omega_x/g = 800$.

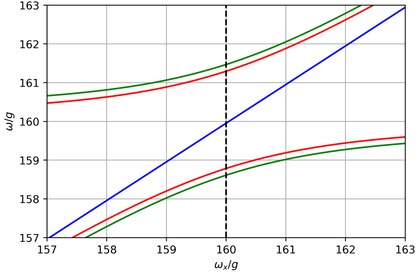
Open system: Emission spectrum



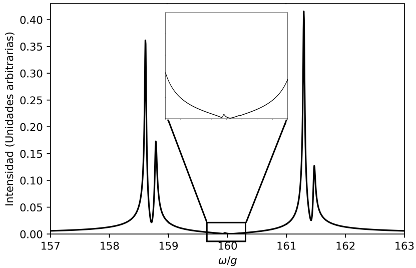
(a) Transitions varieties 2 and 1 (b) Transitions varieties 3 and 2

Figure: Transitions $|2+\rangle \rightarrow |1+\rangle$ (red (a)), $|2-\rangle \rightarrow |1+\rangle$ (green(a)), $|1-\rangle \rightarrow |G0\rangle$ (blues), $|3+\rangle \rightarrow |2-\rangle$(green (b)) y $|3+\rangle \rightarrow |2+\rangle$(roja (b)). We take the following parameters: $\omega_c/g = 1000$ y $g = 0.1$.

Open system: Emission spectrum



(a) Optical transitions



(b) Emission spectrum

Figure: The same transitions as in the 9 and their respective spectra using $g = 0.5$. We took $\omega_c/g = 200$, $\omega_x/g = 160$, $\gamma/g = 0.02$, and $P/g = 0.01$. The appearance of the central peak that completes the quintuplet is highlighted.

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Conclusions

- Considering the radiation as a Morse oscillator generates that the phenomenology of interest comes out of resonance, creating a set of frequencies of interest associated to the transitions of the Morse oscillator.
- When the quantum dot takes the value of any of the mentioned frequencies, polaritons are created in a single variety, while the others behave as the uncoupled basis.
- The emission spectrum of the system is composed of a series of equidistant peaks associated to the frequencies of the Morse oscillator. If any variety presents polariton states, a series of new peaks are created which, together with some of the peaks mentioned above, constitute a set of triplets centered on the frequency of the quantum dot.

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