

JORGE MARIO ESCOBAR, BRAYAN FERNANDO DÍAZ, JESÚS MARÍA CALERO

# PHOTONIC CRYSTALS

WITH

# SUPERCONDUCTORS

A STUDY OF THEIR REFLECTANCE IN ANNULAR TERNARY SYSTEMS



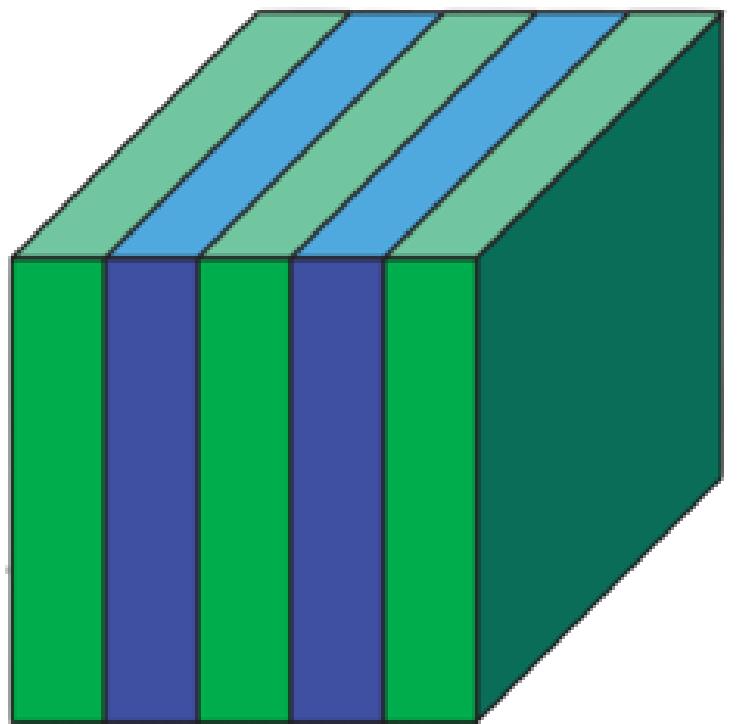
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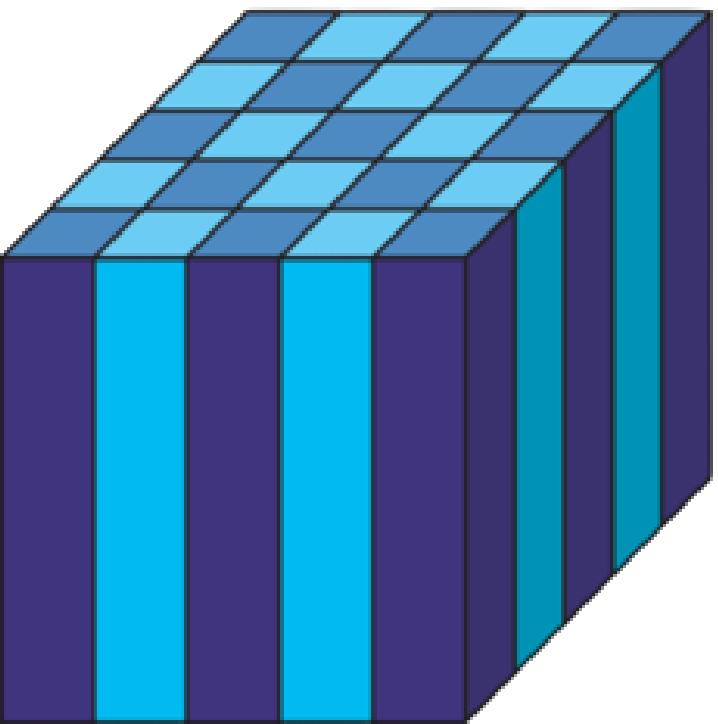
THEORETICAL SOLID STATE PHYSICS GROUP  
DEPARTMENT OF PHYSICS  
UNIVERSIDAD DEL VALLE  
COLOMBIA

# PHOTONIC CRYSTALS

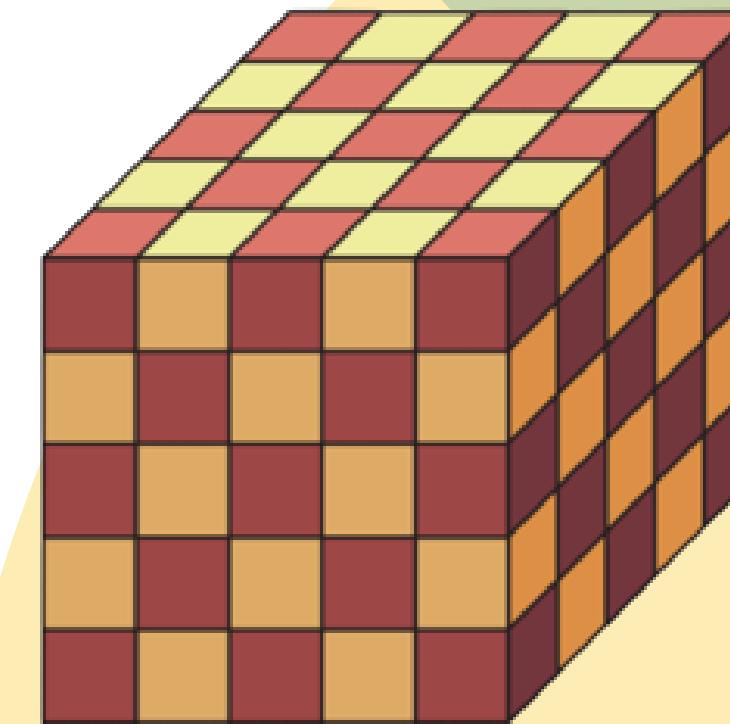
1-D



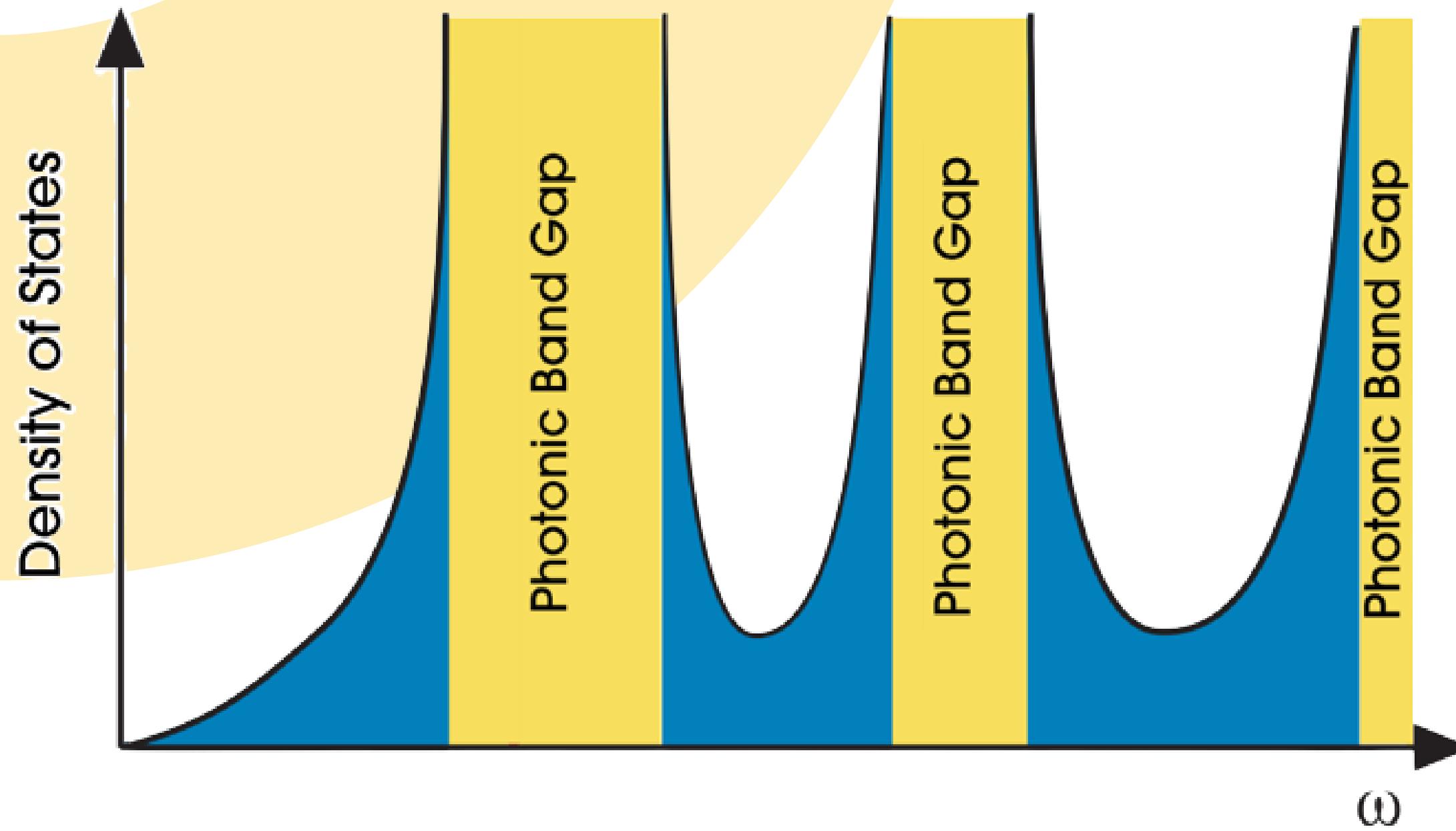
2-D



3-D



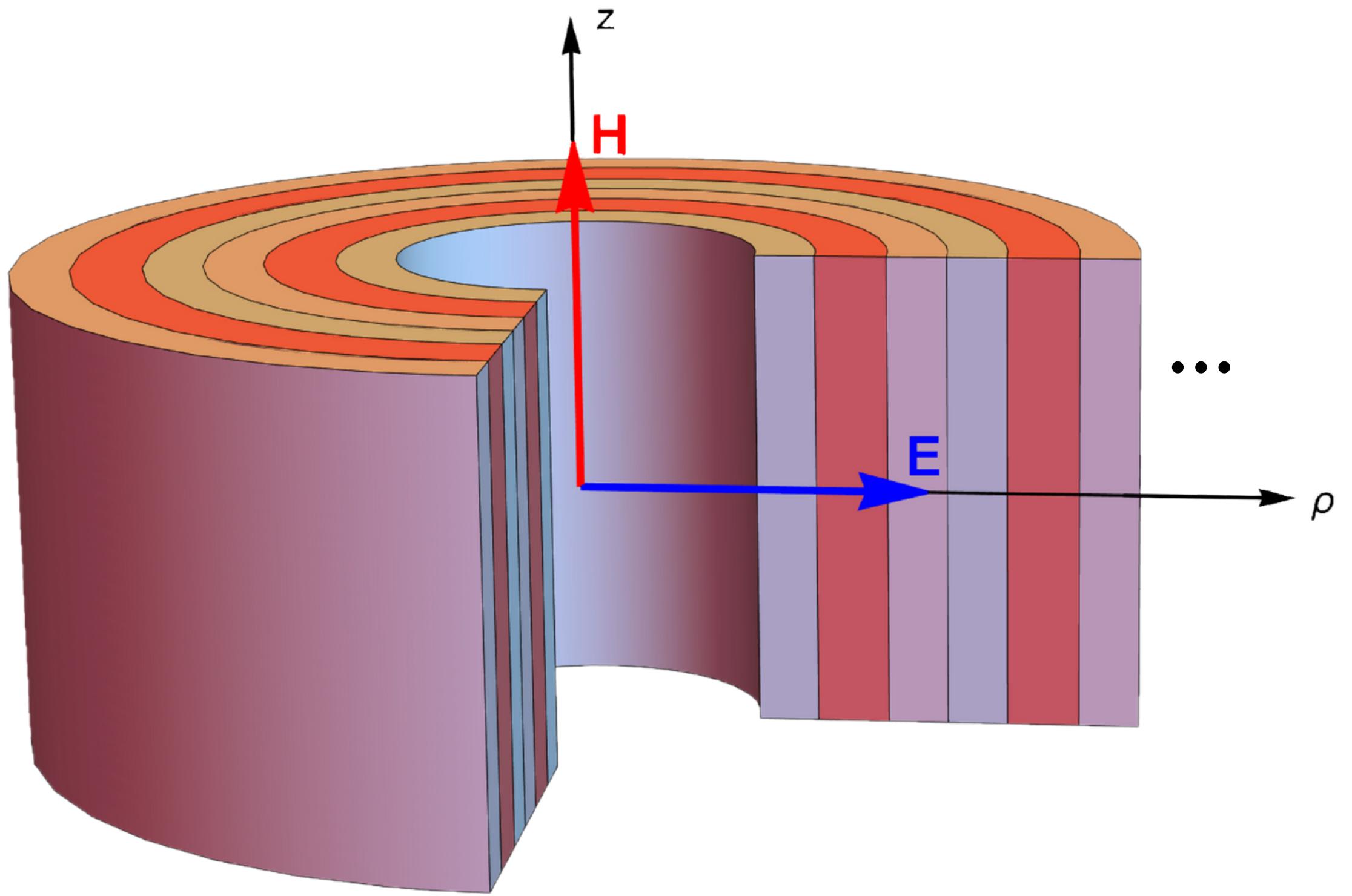
# PHOTONIC BAND GAPS



THESE PHOTONIC BAND GAPS  
ALLOW US TO DESIGN FILTERS  
AND REFLECTORS FOR  
ELECTROMAGNETIC WAVES.

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- J. D. Joannopoulos, *Photonic Crystals*, Princeton University Press, (2008).  
M. S. Chen et al., Solid State Communications, 149, 1888–1893 (2009).  
V. V. Nikolaev et al., Fiz. Tekh. Poluprovodn., 33, 174–179 (1999).

# ANNULAR TERNARY SYSTEM

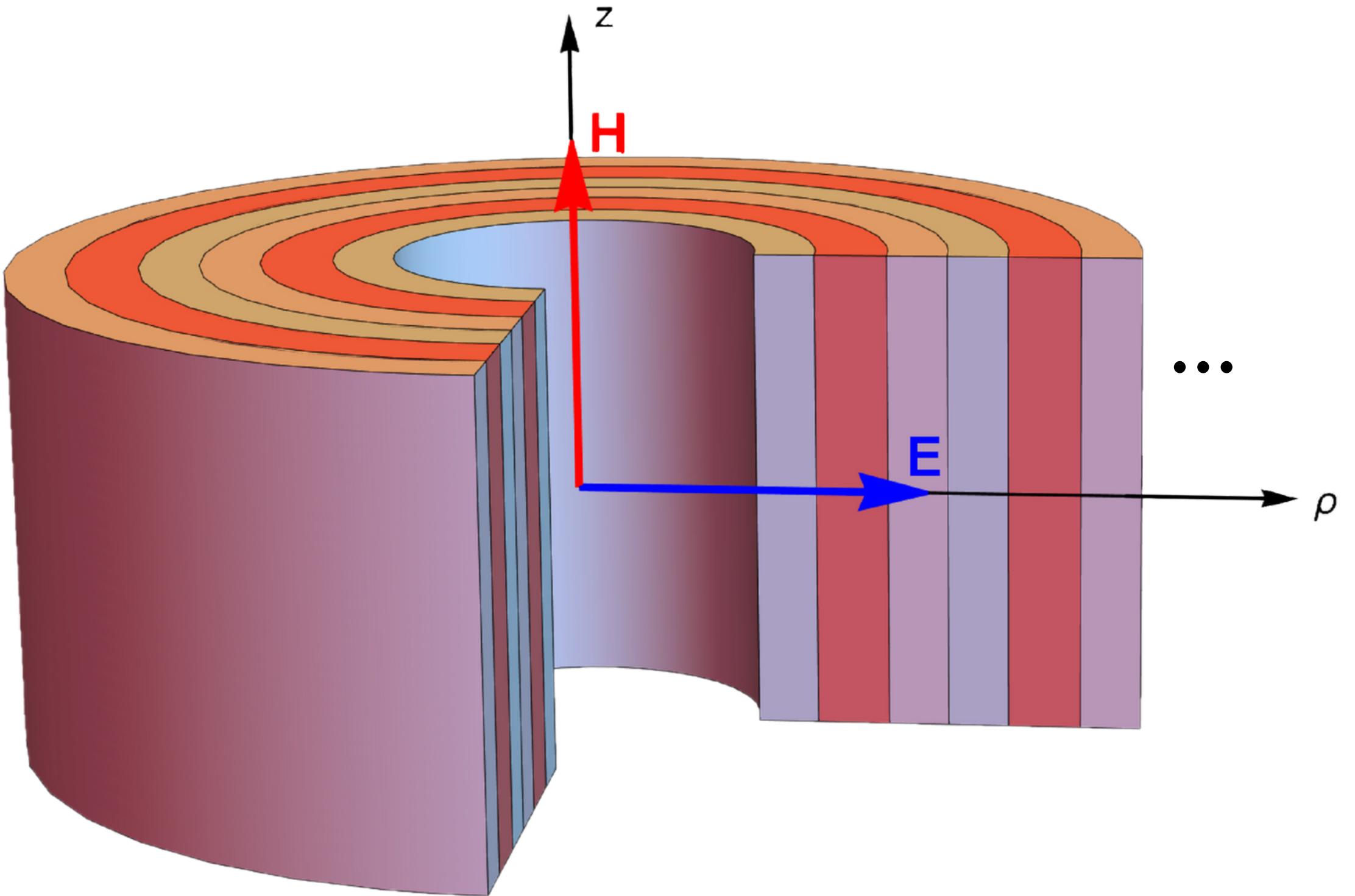


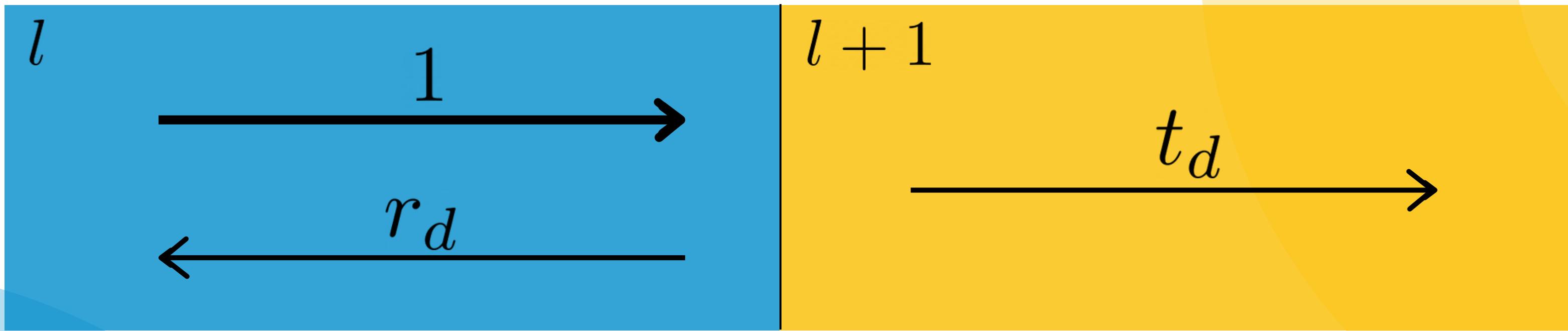
# ANNULAR TERNARY SYSTEM

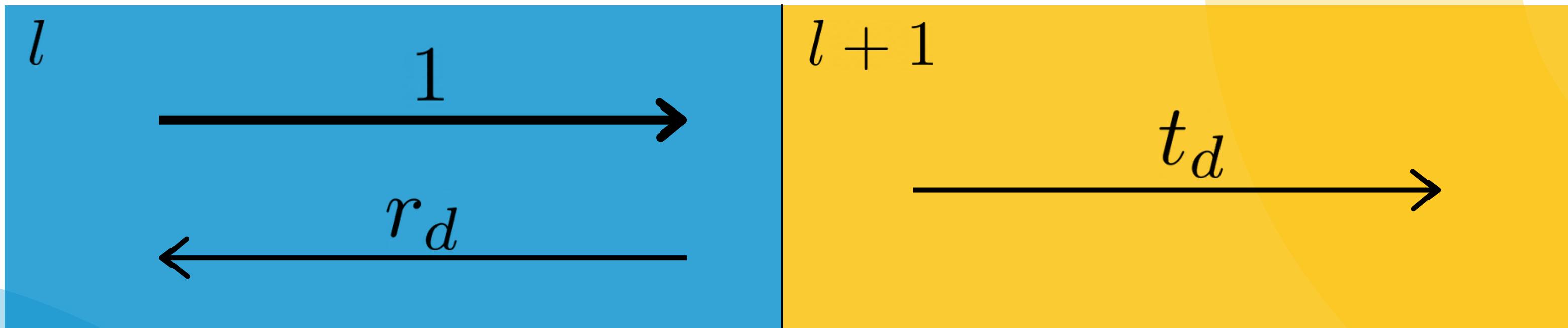
WE WILL ANALYZE THE  
OPTICAL RESPONSE OF THE  
SYSTEM IN TM  
POLARIZATION.

$$\mathbf{H} = H_z \hat{\mathbf{e}}_z$$

$$\mathbf{E} = E_\rho \hat{\mathbf{e}}_\rho + E_\phi \hat{\mathbf{e}}_\phi$$

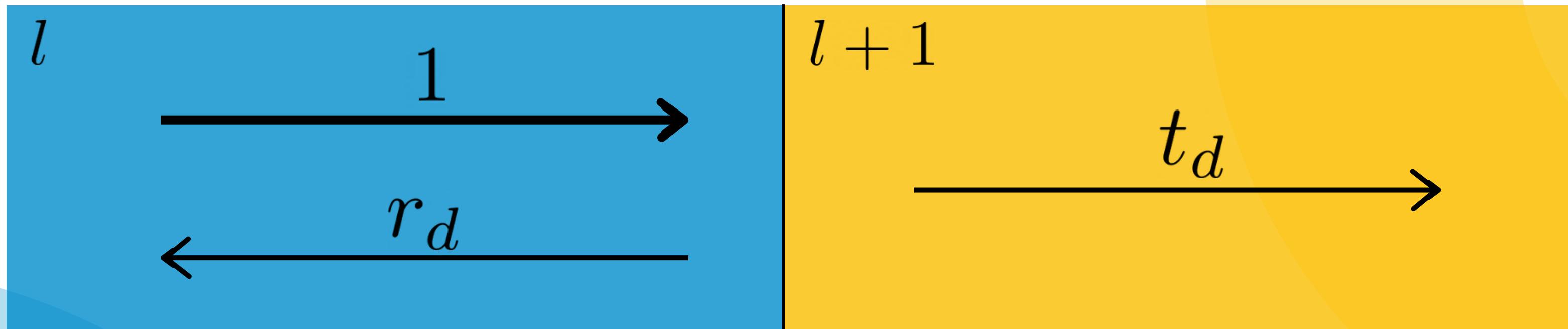






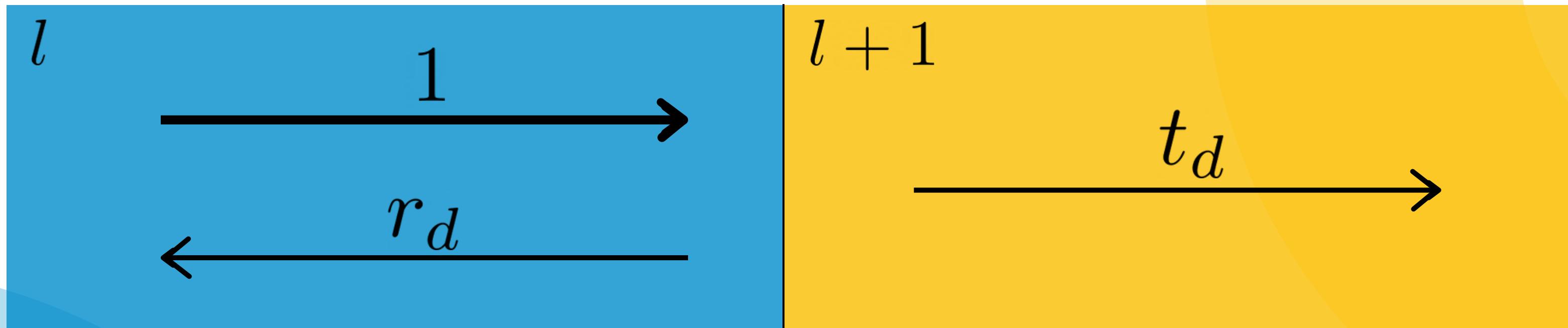
$$\begin{pmatrix} \text{EM} \\ \text{wave} \end{pmatrix}_l \rightarrow \begin{pmatrix} \text{EM} \\ \text{wave} \end{pmatrix}_{l+1}$$

# TRANSFER MATRIX METHOD



$$\hat{\mathcal{M}}_l \begin{pmatrix} \text{EM} \\ \text{wave} \end{pmatrix}_l = \begin{pmatrix} \text{EM} \\ \text{wave} \end{pmatrix}_{l+1}$$

# TRANSFER MATRIX METHOD



$$\hat{\mathfrak{M}}_l \begin{pmatrix} \text{EM} \\ \text{wave} \end{pmatrix}_l = \begin{pmatrix} \text{EM} \\ \text{wave} \end{pmatrix}_{l+1}$$

$$t_d = \frac{|\hat{\mathfrak{M}}_l|}{\mathfrak{M}_l^{22}} \quad \text{y} \quad r_d = -\frac{\mathfrak{M}_l^{21}}{\mathfrak{M}_l^{22}} \quad \rightarrow \quad R_d = \left| \frac{\mathfrak{M}_l^{21}}{\mathfrak{M}_l^{22}} \right|^2, \quad \text{con} \quad R_d = \boxed{R_d(\lambda)}$$

M. Born, E. Wolf, *Principles of Optics*, P. 77, Moscow, (1970).

P. Markos, C. M. Soukoulis, *Wave Propagation*, Princeton University Press, (2008).

# THEORETICAL TREATMENT

## MASTER EQUATION

$$\nabla \times \left[ \frac{1}{\epsilon(\mathbf{r}, \omega)} \nabla \times \mathbf{H}(\mathbf{r}) \right] = \left( \frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r})$$

# THEORETICAL TREATMENT

## MASTER EQUATION

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Description in  
cylindrical coordinates

## BESSEL PARTIAL DIFFERENTIAL EQUATION

$$\left[ \frac{1}{\rho^2} \frac{\partial}{\partial \phi} \left( \frac{1}{\epsilon(\mathbf{r}, \omega)} \frac{\partial}{\partial \phi} \right) + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{1}{\epsilon(\mathbf{r}, \omega)} \frac{\partial}{\partial \rho} \right) \right] H_z(\mathbf{r}) = - \left( \frac{\omega}{c} \right)^2 H_z(\mathbf{r})$$

# THEORETICAL TREATMENT

## MASTER EQUATION

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Solutions in traveling  
wave basis

$$\begin{cases} H_z^\pm = A H_m^{(1)}(k\rho) e^{im\phi} \\ E_\phi^\pm = ipA \frac{\partial}{\partial(k\rho)} H_m^{(1)}(k\rho) e^{im\phi} \end{cases}$$

# THEORETICAL TREATMENT

## MASTER EQUATION

$$\nabla \times \left[ \frac{1}{\epsilon(\mathbf{r}, \omega)} \nabla \times \mathbf{H}(\mathbf{r}) \right] = \left( \frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r})$$

Description in  
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## BESSEL PARTIAL DIFFERENTIAL EQUATION

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$$\hat{\mathfrak{M}}_l = \begin{pmatrix} H_m^{(1)}(k_l \rho_l) & H_m^{(2)}(k_l \rho_l) \\ \frac{1}{k_l} H_m'^{(1)}(k_l \rho_l) & \frac{1}{k_l} H_m'^{(2)}(k_l \rho_l) \end{pmatrix}$$

Apply boundary conditions  
to build the matrix

# SUPERCONDUCTOR REFRACTIVE INDEX

$$n(\lambda, T) = \sqrt{1 - \left( \frac{\lambda}{2\pi\lambda_L(T)} \right)^2}$$

**TWO FLUIDS MODEL**



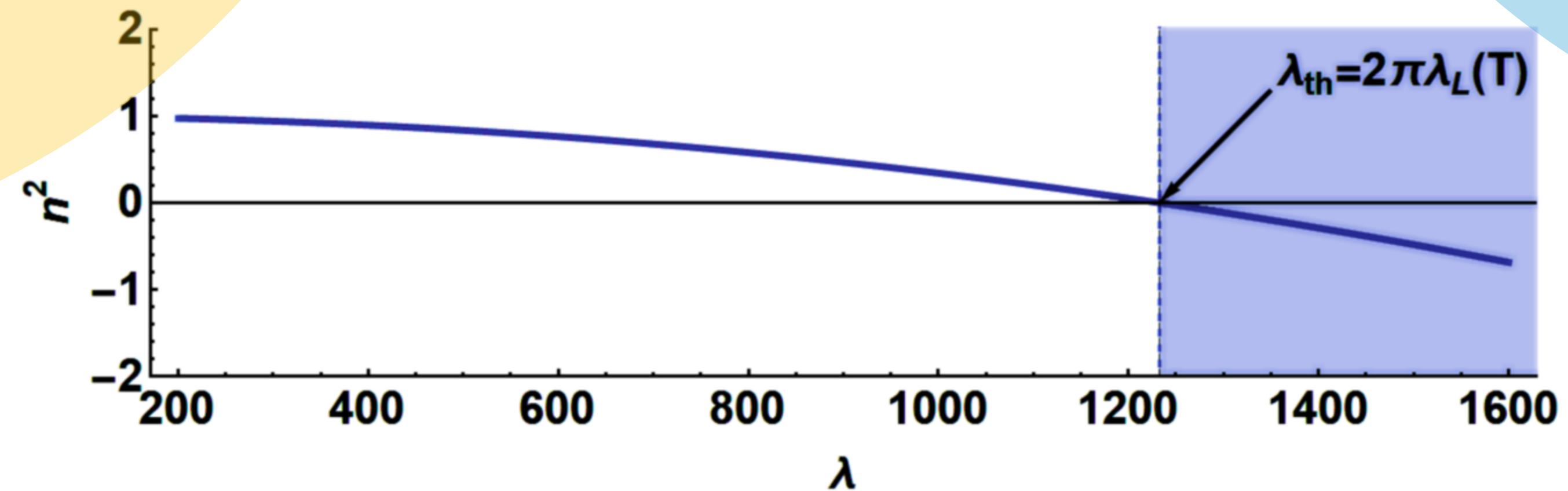
**LONDON PENETRATION DEPTH**

$$\lambda_L(T) = \frac{\lambda_0}{\sqrt{1 - \left( \frac{T}{T_c} \right)^4}}$$

# SUPERCONDUCTOR REFRACTIVE INDEX

$$n(\lambda, T) = \sqrt{1 - \left( \frac{\lambda}{2\pi\lambda_L(T)} \right)^2}$$

AT FIXED TEMPERATURE



# OUR SYSTEM

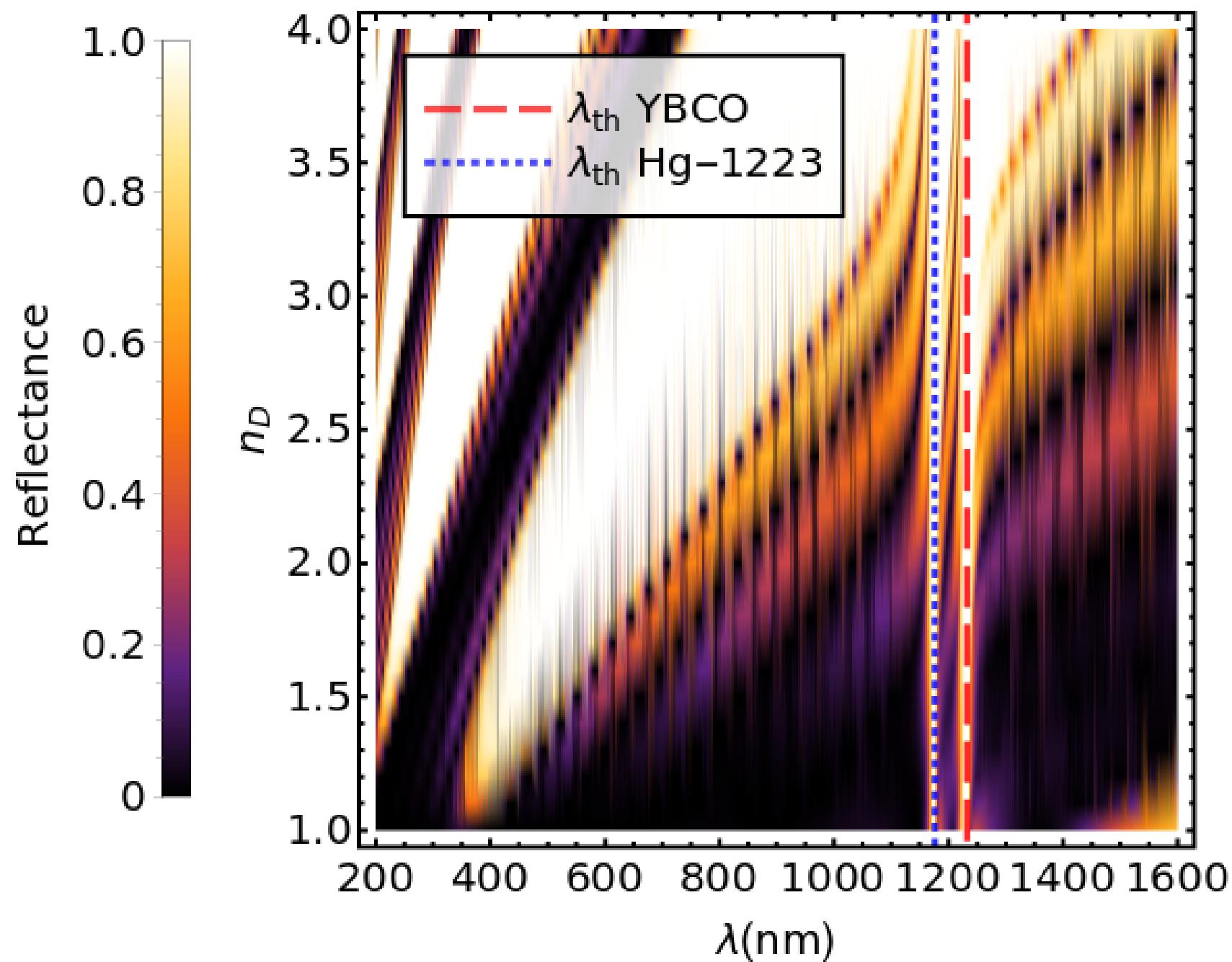
Superconductor  
 $\text{YBa}_2\text{Cu}_3\text{O}_7$   
(YBCO)  
 $T_c=92\text{K}$

Dielectric  
 $\text{SrTiO}$   
 $n_D=2.437$

Superconductor  
 $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$   
(Hg-1223)  
 $T_c=135\text{K}$

# RESULTS

## Dielectric refractive index variation



$T = 77K$

## Superconductor refractive index

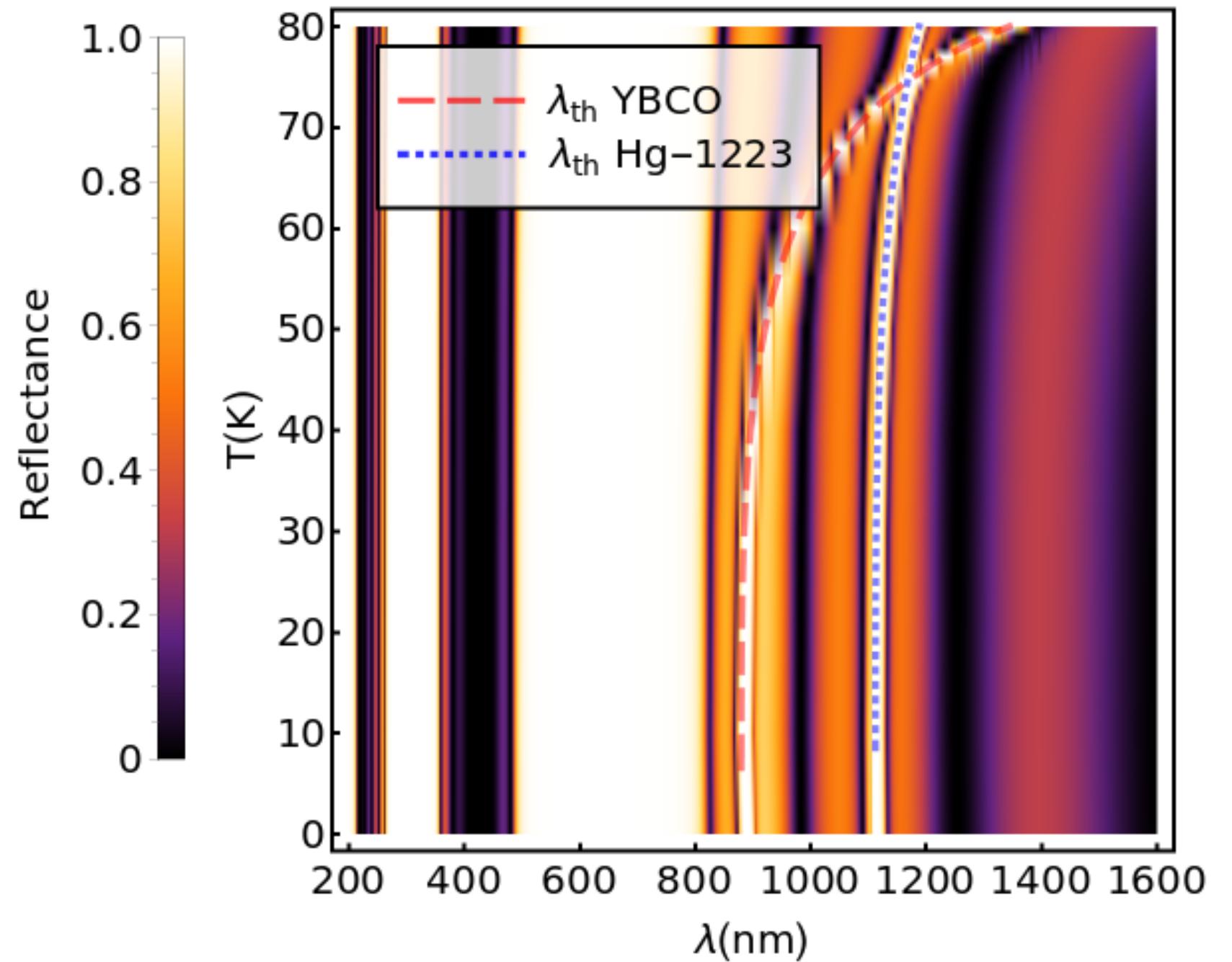
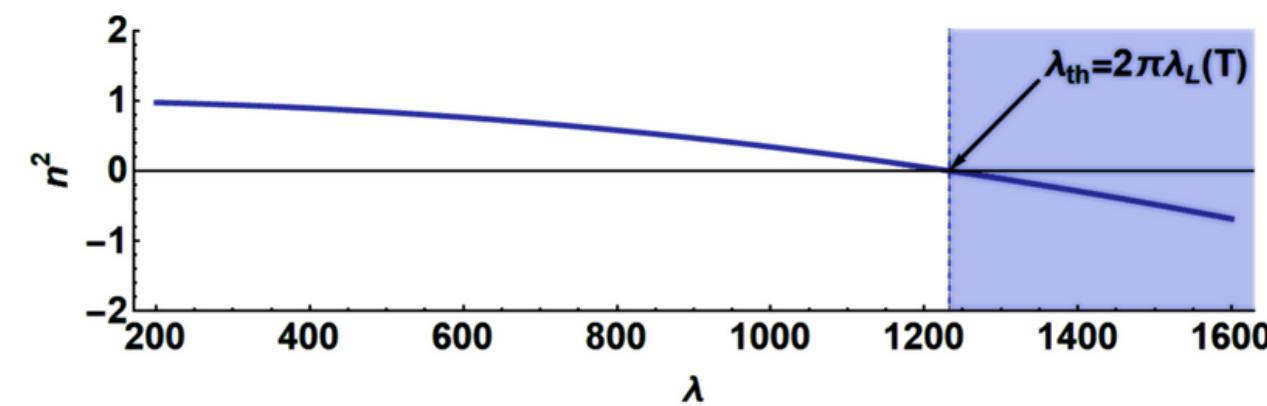
$$n(\lambda, T) = \sqrt{1 - \left( \frac{\lambda}{2\pi\lambda_L(T)} \right)^2}$$

# RESULTS

## Temperature variation

**Superconducting threshold**

$$\lambda_{th} = 2\pi\lambda_L(T) = 2\pi \frac{\lambda_0}{\sqrt{1 - \left(\frac{T}{T_c}\right)^4}}$$



$$n_D = 2.437$$

# CONCLUSIONS

- Characterizing the optical response allows us to control the propagation of electromagnetic waves in the crystal.
- Introducing superconductors into the system produces new gaps, presenting new technological applications such as thermal sensors.

# PERSPECTIVES

- It is possible to study the optical response of the system to variations of other parameters such as crystal radius or layer thickness.
- Other combinations of materials can be proposed and even alternate the order in which the superconductors are configured.

# THANK YOU

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