The Abdus Salam International Centre for Theoretical Physics

SISSA

Prof. Don Zagier

Will be giving a joint SISSA/ICTP/SUSTech course

Standard and less standard asymptotic methods

Venue:

ICTP, Budinich Lecture Hall

Tu/Th 4:00–5:30 p.m. from Feb. 15 to Mar. 10
Mo/We 2:00–3:30 p.m. from Mar. 14 to Mar. 23

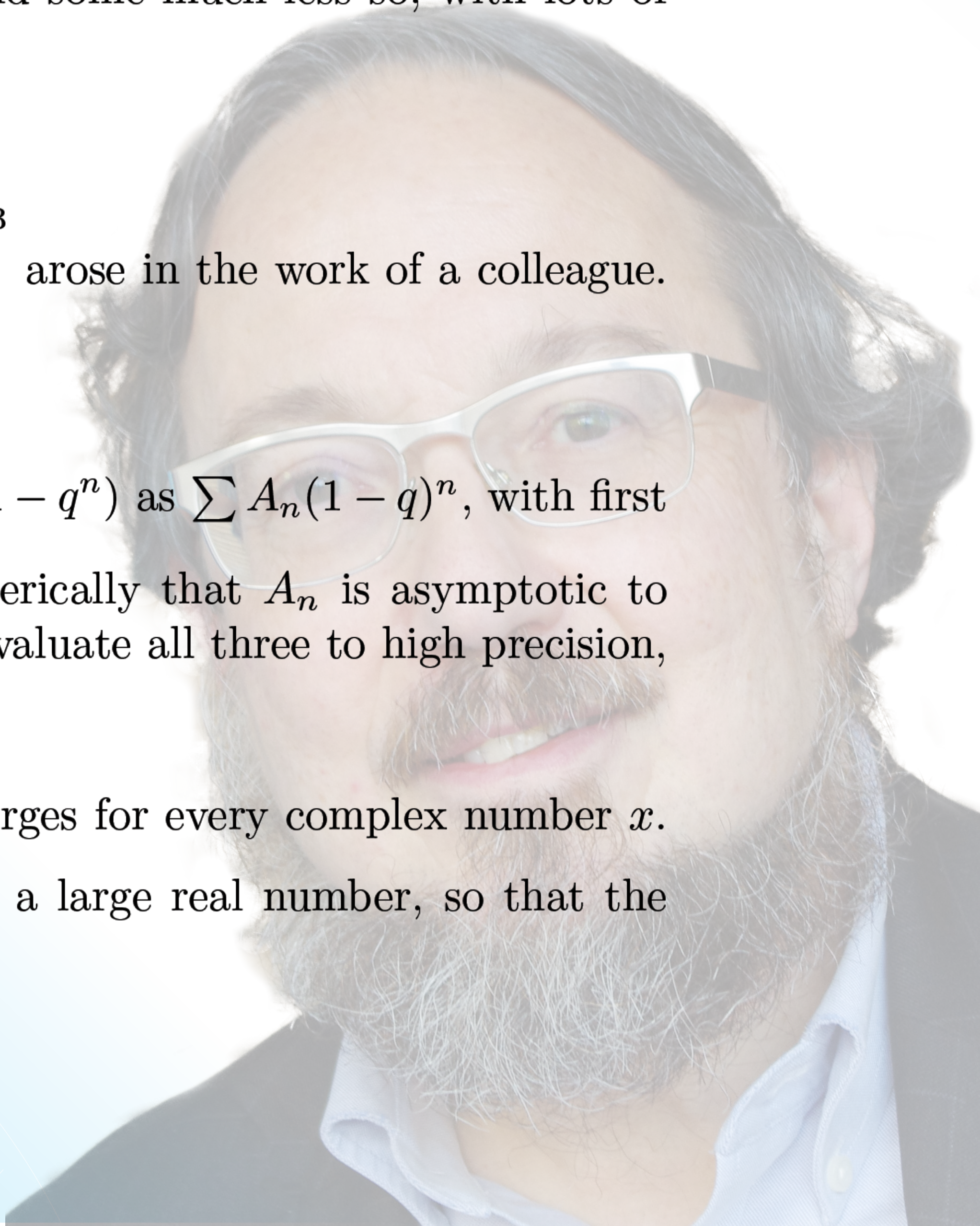
In every branch of mathematics, one is sometimes confronted with the problem of evaluating an infinite sum numerically and trying to guess its exact value, or of recognizing the precise asymptotic law of formation of a sequence of numbers $\{A_n\}$ of which one knows, for instance, the first couple of hundred values. The course will tell a number of ways to study both problems, some relatively standard (like the Euler-Maclaurin formula and its variants) and some much less so, with lots of examples.

Here are three typical examples:

1. The slowly convergent sum $\sum_{j=0}^{\infty} \binom{j+4/3}{j}^{-4/3}$ arose in the work of a colleague. Evaluate it to 250 decimal digits.

2. Expand the infinite sum $\sum_{n=0}^{\infty}(1-q)(1-q^2)\ldots(1-q^n)$ as $\sum A_n(1-q)^n$, with first coefficients 1, 1, 2, 5, 15, 53, ... Show numerically that A_n is asymptotic to $C\,n!\,\alpha^n\,n^\beta$ for some real constants α, β and C, evaluate all three to high precision, and recognize their exact values.

3. The infinite series $H(x) = \sum_{k=1}^{\infty} \frac{\sin(x/k)}{k}$ converges for every complex number x. Compute this series to high accuracy when x is a large real number, so that the series is highly oscillatory.



Registration link