Outline

- Digital CMOS design
  - Boolean algebra
  - Basic digital CMOS gates
  - Combinational and sequential circuits
  - Coding - Representation of numbers

Representing Numbers

How values can be coded ?

In a digital circuit each signal can
take 2 values (0, 1) (Boolean world)

A vector of n bits can represent up to 2^n values

Representing Numbers

How values can be coded ?

What is the meaning of 0100 0110 ?

The character 'F'
The character 'Φ'
The number 46
The number 70
The number 123

Any symbol in a set where the $Card = 256$

Representing Numbers

How values can be coded ?

by itself a code has no signification

Representing Numbers

How values can be coded ?

arithmetic : dealing with numbers

How can I represent a number ?

Natural numbers
Relative numbers
Rational numbers
Real numbers

Representing Numbers

How can I represent a Natural number ?

I need at least n bits for a Natural ranging from 0 to $2^n - 1$

Standards

Representing Numbers

How can I represent a Natural number ?

Natural Binary Code :

The bits represent the successive powers of 2

$$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$$

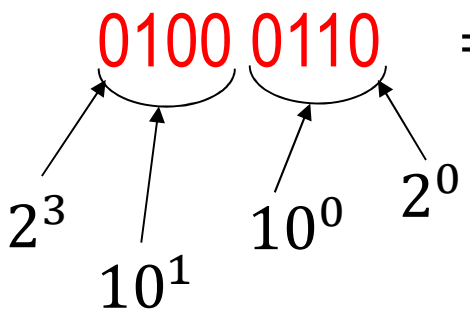
2^7 2^0

Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal :

The bits represent the successive powers of 2
The quartets represent the successive powers of 10



$= (2^1 + 2^2)\,10^0 + (2^2)10^1 = 46$

packed

Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal - Unpacked :

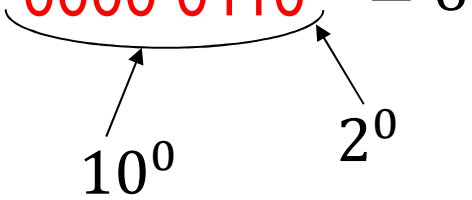
The bits represent the successive powers of 2

The bytes represent the successive powers of 10

In each byte the 4 Msb are 0

0100 0110 = Illegal

0000 0110 = 6



Representing Numbers

How can I represent a Relative number ?

Sign + Magnitude :

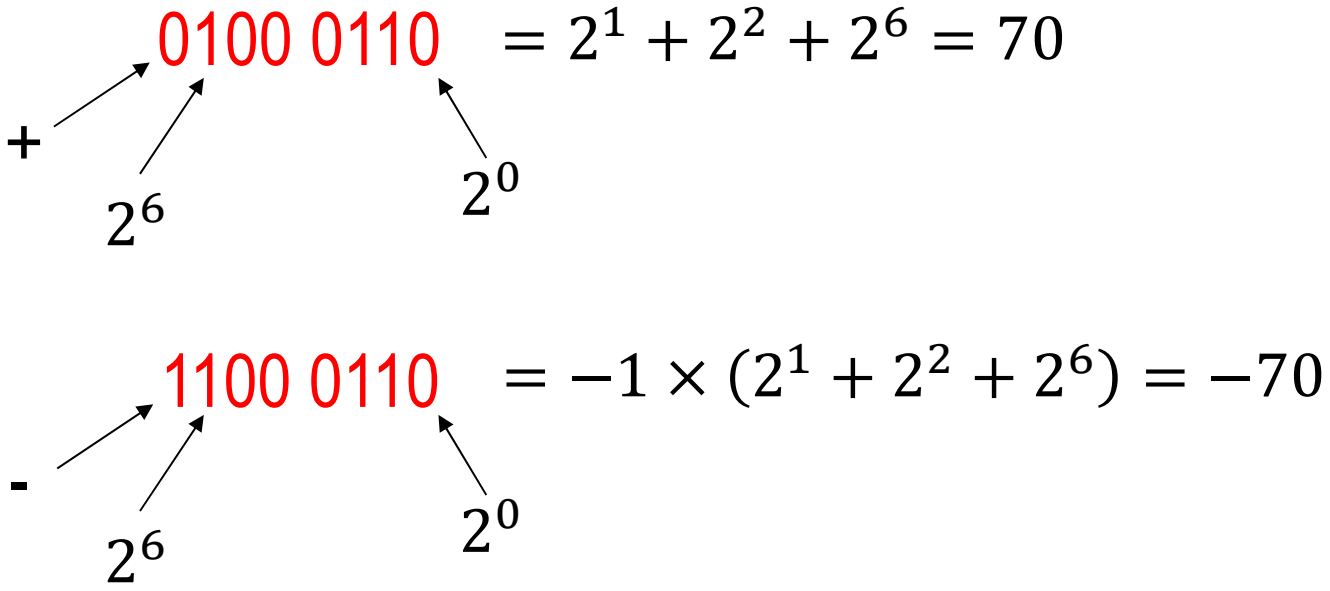
The bits represent the successive powers of 2

The Msb represents the sign (1 means negative)

Representing Numbers

How can I represent a Relative number ?

Sign + Magnitude :



Representing Numbers

How can I represent a Relative number ?

2's complement :

The bits represent the successive powers of 2

The Msb represents -2^{n-1}

Representing Numbers

How can I represent a Relative number ?

2's complement :

$$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$$

-2^7 2^6 2^0

$$1100\ 0110 = 2^1 + 2^2 + 2^6 - 2^7 = -58$$

Representing Numbers

How can I represent a Relative number ?

2's complement :

$$0100\ 0110 \quad = 2^1 + 2^2 + 2^6 = 70$$

70

$$2^7 = 1 + 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

$$2^7 = 1 + 2^0 + \quad 2^3 + 2^4 + 2^5 \quad + 70$$

$$-70 = 1 + 2^0 + \quad 2^3 + 2^4 + 2^5 \quad - 2^7$$

$$-70 = \quad 2^1 + \quad 2^3 + 2^4 + 2^5 \quad - 2^7$$

$$1011\ 1010$$

Representing Numbers

How can I represent a Real number ?

Range

Precision

Representing Numbers

How can I represent a Real number ?

2's complement Fixed Point :

The bits represents the successive powers of 2

Including negative powers of 2

$$0100.0110 = 2^2 + 2^{-2} + 2^{-3} = 4.375$$

-2^3 2^0 2^{-1} 2^{-4}

Representing Numbers

How can I represent a Real number ?

Wide range
High precision

Floating Point

Representing Numbers

How can I represent a Real number ?

Normalized scientific representation

$$R = (-1)^S \times M \times 2^E$$

S : Sign (1 if negative)

M : Mantissa ($\in [1, 2[$)

E : Exponent (relative number)

Representing Numbers

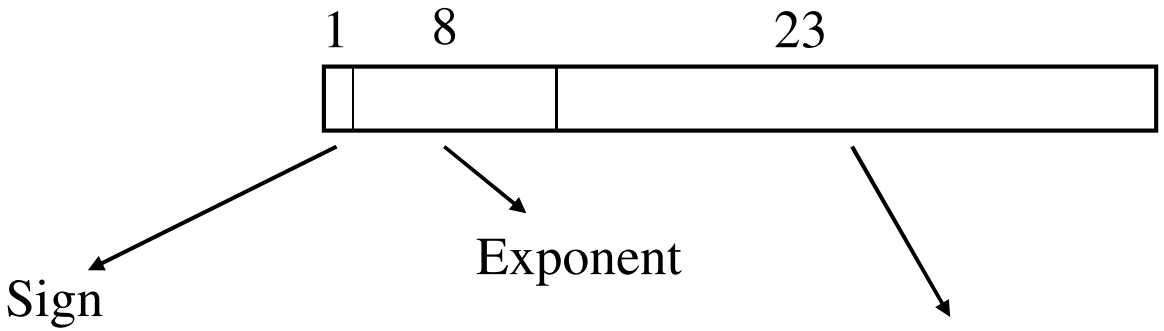
$$R = (-1)^S \times M \times 2^E$$

	Single Precision 32 bits	Double Precision 64 bits
S : Sign (1 if negative)	1 bit	1 bit
M : Mantissa ($\in [1, 2[$)	23 bits	52 bits
E : Exponent	8 bits	11 bits

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision :



Relative number between -127 and 128

The code 0000 0000 means -127

Natural Binary Code by Excess of 127

Fixed point positive real number

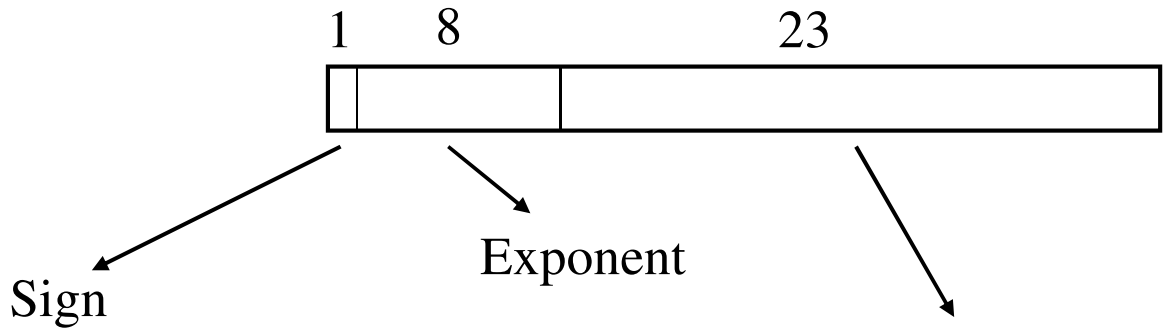
1. • • •

The 1 is not represented !!

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



The code 1111 1111 (128) means $\pm\infty$ or an error

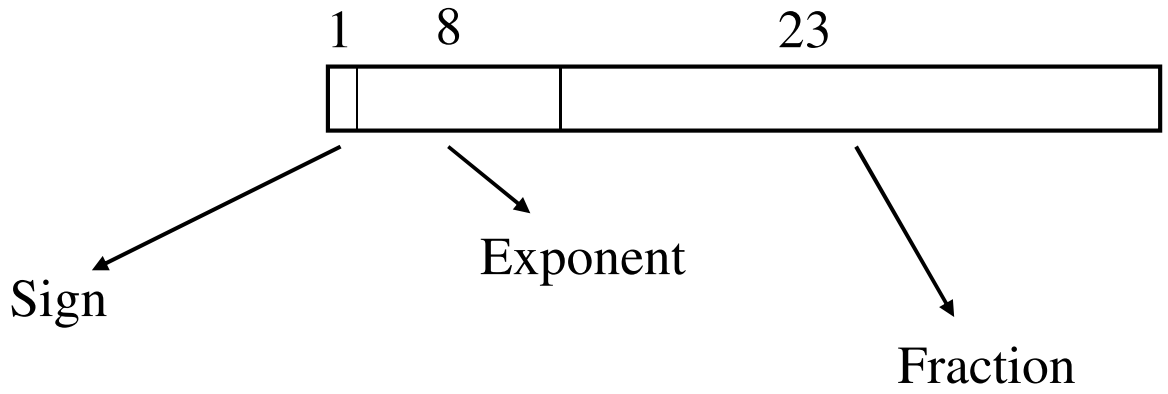
.000…000 means $\pm\infty$

other values mean error (NaN)

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Range and precision



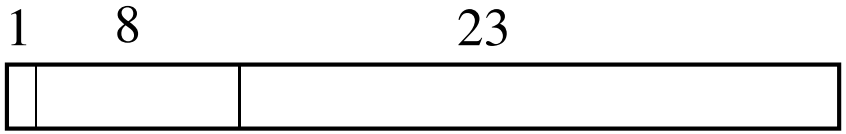
$$R \in]-2^{128}, 2^{128}[$$

$$\text{Precision} = 2^{-24+E}$$

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



Sign

Exponent

Mantissa

The code 0000 0000 (-127) indicates denormalized Mantissa

(Subnormal numbers)

0.00 … 000 means 0