

Outline

- ❑ Digital CMOS Design
- ❑ Arithmetic Operators
- ❑ Floating Point Arithmetic Operators
 - Square Root
 - Division



Division

Two real number y and z using floating point representation

Find a real number x such as

$$x + \varepsilon = \frac{y}{z}$$

- Calculation cannot be performed in one cycle
- Need iterative operation



Division

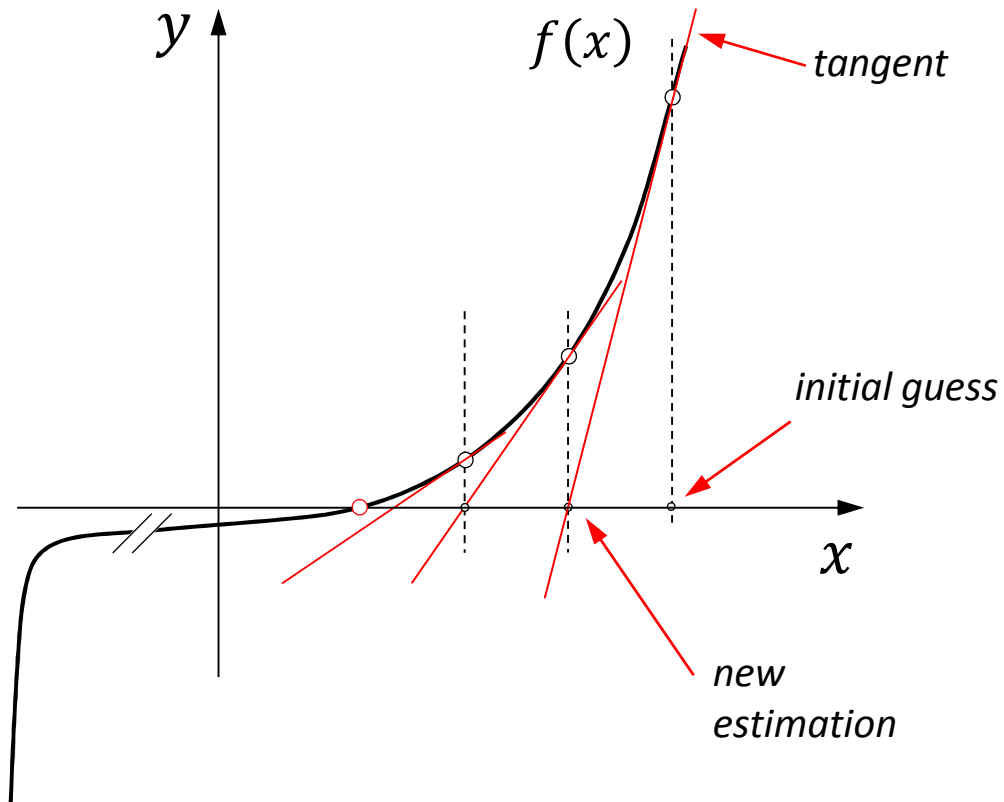
○ Direct method → digit-by-digit

○ Indirect method → resolve a non-linear equation

Division – Indirect Method

Resolving a non
linear equation

$$f(x) = 0$$



Division – Indirect Method

Resolving a non linear equation $f(x) = 0$

Taylor series in the neighborhood of x_0

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + \dots$$

1st order : $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$



Division – Indirect Method

Resolving a non linear equation $f(x) = 0$

Iterative resolution starting from an initial guess x_0

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$f(x) = 0$$

$$f(x_0) + f'(x_0)(x - x_0) = 0$$

$$x = \frac{-f(x_0)}{f'(x_0)} + x_0$$

Newton-Raphson method



Division – Indirect Method

Resolving $u = \frac{1}{z}$

Find a function f such as $f(u) = 0$ for $u = \frac{1}{z}$

$$f(u) = 1/u - z$$

$$f(u) \approx f(u_0) + f'(u_0)(u - u_0)$$

$$f(u) \approx \left(\frac{1}{u_0} - z \right) - u_0^{-2}(u - u_0)$$

$$f(u) = 0 \quad u = u_0 + \frac{(u_0^{-1} - z)}{u_0^{-2}}$$

$$u = u_0(2 - zu_0)$$



Division – Indirect Method

Resolving $u = \frac{1}{z}$

Each iteration $u_{i+1} = u_i(2 - zu_i)$

multiply !!

$$u = \frac{1}{z}$$

$$x = \frac{y}{z} = u \cdot y$$

