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"General Angular Structure Method of Inverse Problems: Solutions in the Radiative Transfer Theory. Part I"

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Please note: These are preliminary notes intended for internal distribution only.

Abstract

Presented in these lectures are the results developed on the basis of the Sobolev scientific school. They have been obtained by author in the recent years in the field of the precise analytical and semi-analytical calibrating solutions of the direct-inverse problems connected with the classical radiative transfer theory and atmospheric optics. The mathematical basis of considered joint direct-inverse problems solutions is the presentation of the azimuth harmonics of atmospheric and underlying surface brightness coefficients on the basis of the forms given by V. Ambarzumyan, S. Chandrasekhar, V. Sobolev and Van de Hulst. Precise algorithms for vertically uniform slabs, bounded from below an arbitrary orthotropical reflecting bottom was elaborated. Taking into account the above-mentioned classical representations of angular distributions for radiation fields, it is possible to construct a system of linear algebraic equations for the retrieval of primary optical parameters of "atmosphere-underlying surface" system. It should be remarked that specially that in a frame of the developed analytical approach, the high azimuth harmonics of system brightness coefficients allow to retrieve the atmospheric optical parameters only. Optical properties of underlying surfaces are retrieved making use of the zero harmonics only.

The numerical results obtained in the frame of the presented analytical structural method allow to estimate strictly the following problems:

- the stability solution;
- the influence of chosen grid points, errors and discretization levels;
- information content compression.

GENERAL ANGULAR STRUCTURE METHOD OF INVERSE PROBLEMS SOLUTIONS IN THE RADIATIVE TRANSFER THEORY

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BASIC TOPICS

Part I. Finite optical thickness ($\tau_0 < \infty$) planet atmosphere bounded from below arbitrary orthotropic underlying surface

1. The statement of conciliated direct-inverse problems solutions on the basis of radiative transfer theory analytical representations.
2. Using of precise analytical angular structures given by V.Ambarzumyan for reflected radiation fields on the upper boundary of atmosphere.
3. Precise analytical angular structure method of joint direct-inverse problems solutions in the radiative transfer theory.
4. Inverse operators $\left(\hat{L}^{-1}\right)$ precise constructions:

- a slab without reflecting bottom;
- a slab bounded from below by horizontally uniform orthotropic underlying surface;
- a slab bounded from below by non-horizontally uniform orthotropic underlying surface;
- a slab with placed a small size object at the lowest atmospheric boundary.

5. Numerical results: stability of inverse problems solutions, compression of informational contents, errors (ε) and discretization (δ) levels influence.

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BASIC TOPICS

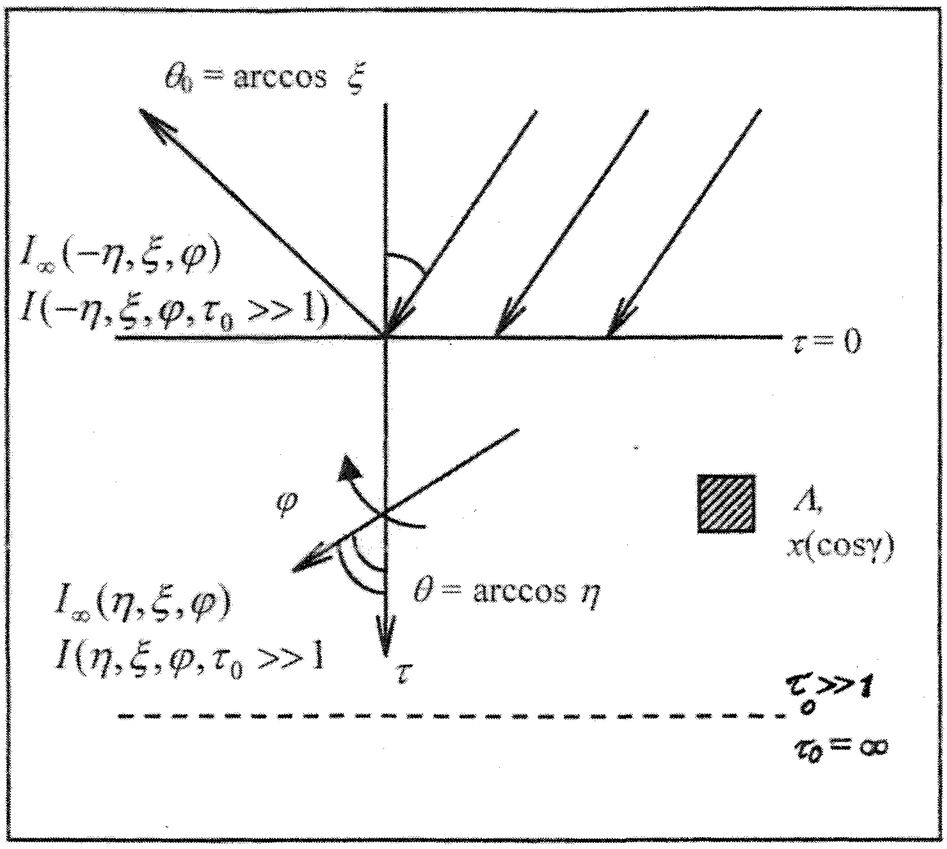
Part II. Semi-infinite (τ_0=∞) and optically thick (τ_0>>1) planet atmospheres

1. The statement of conciliated direct-inverse problems solutions on the basis of radiative transfer theory analytical representations.
2. Using of precise angular structures given by V.Ambarzumian for reflected radiation in the case of semi-infinite atmosphere (**τ_0=∞**)
3. Using of precise angular structures given by S.Chandrasechar for reflected radiation on the basis of linear singular integral equations in the case of semi-infinite atmosphere (**τ_0=∞**).
4. Using of precise angular structures given by V.Sobolev and van de Hulst for reflected radiation in the case of optically thick atmosphere (**τ_0>>1**).
5. Precise analytical angular structure method of joint direct-inverse problems solutions in the radiative transfer theory.
6. Construction of precise inverse operator $\left(\hat{L}^{-1}\right)$ on the basis of reflected radiation angular structure given by V. Ambarzumian in the case of semi-infinite atmosphere (**τ_0=∞**).
7. Construction of precise inverse operator $\left(\hat{L}^{-1}\right)$ on the basis of linear singular integral equations given by S.Chandrasekhar in the case of semi-infinite atmosphere (**τ_0=∞**).
8. Using of V.Sobolev-van de Hulst's exact asymptotic representations of reflected radiation fields for precise construction of inverse operators $\left(\hat{L}^{-1}\right)$ in the case of optically thick atmosphere (**τ_0>>1**):

- the case of conservative (pure) light scattering (Λ=1),
- the case of almost conservative (pure) light scattering (1-Λ<<1).

9. Numerical results: stability of inverse problems solutions, compression of informational contents, errors (ε) and discretization (δ) levels influence.

The statement of concilated direct-inverse problems solutions in the case of semi-infinite ($\tau_0 = \infty$) and optically thick ($\tau_0 >> 1$) planet atmosphere



Semi-infinite planet atmosphere
$\tau_0 = \infty,\ \Lambda,\ x(\cos\gamma)$

Optically thick planet atmosphere
$\tau_0 >> 1,\ \Lambda = 1,\ x(\cos\gamma)$

An optically thick planet atmosphere
$\tau_0 >> 1,\ 1-\Lambda << 1,\ x(\cos\gamma)$

Direct problems:

$$\hat{L}_\infty\{G\} = \{I_\infty\}, \qquad \hat{L}_\infty^m\{G^m\} = \{I_\infty^m\}$$

$$m = 0,1,\ldots,N$$

Inverse problems:

$$\{G\} = \hat{L}_{\infty,m}^{-1}\{I_\infty\}, \quad \{G^m\} = L_{\infty,m}^{-1}\{I_\infty^m\}$$

$$m = 0,1,\ldots,N$$

Classical representation for reflected radiation angular structure given by V.Ambarzumyan in the case $\tau_0 = \infty$

Azimuthal harmonics of reflection coefficient:

$$\rho_\infty^m(\eta,\zeta) = \frac{\Lambda}{2}\sum_{i=m}^{N}(-1)^{i+m}x_i\,\frac{\overline{\varphi}_i^m(\eta)\overline{\varphi}_i^m(\zeta)}{\eta+\zeta}, \qquad m = 0,1,2,\ldots,N \tag{1}$$

where

$$\overline{\varphi}_i^m(\zeta) = \overline{\mathrm{P}}_i^m(\zeta) + 2\zeta\int_0^1 \overline{\mathrm{P}}_i^m(-\eta')\rho_\infty^m(\eta',\zeta)d\eta', \qquad m = 0,1,2,\ldots,N \tag{2}$$

$$x(\cos\gamma) = \sum_{i=0}^{N}x_iP_i(\cos\gamma), \quad \overline{x}_i = \frac{x_i}{2i+1} \tag{3}$$

Values ρ_∞^m, $\overline{\varphi}_i^m$ are known and $\overline{\psi}_i^m \equiv 0$.

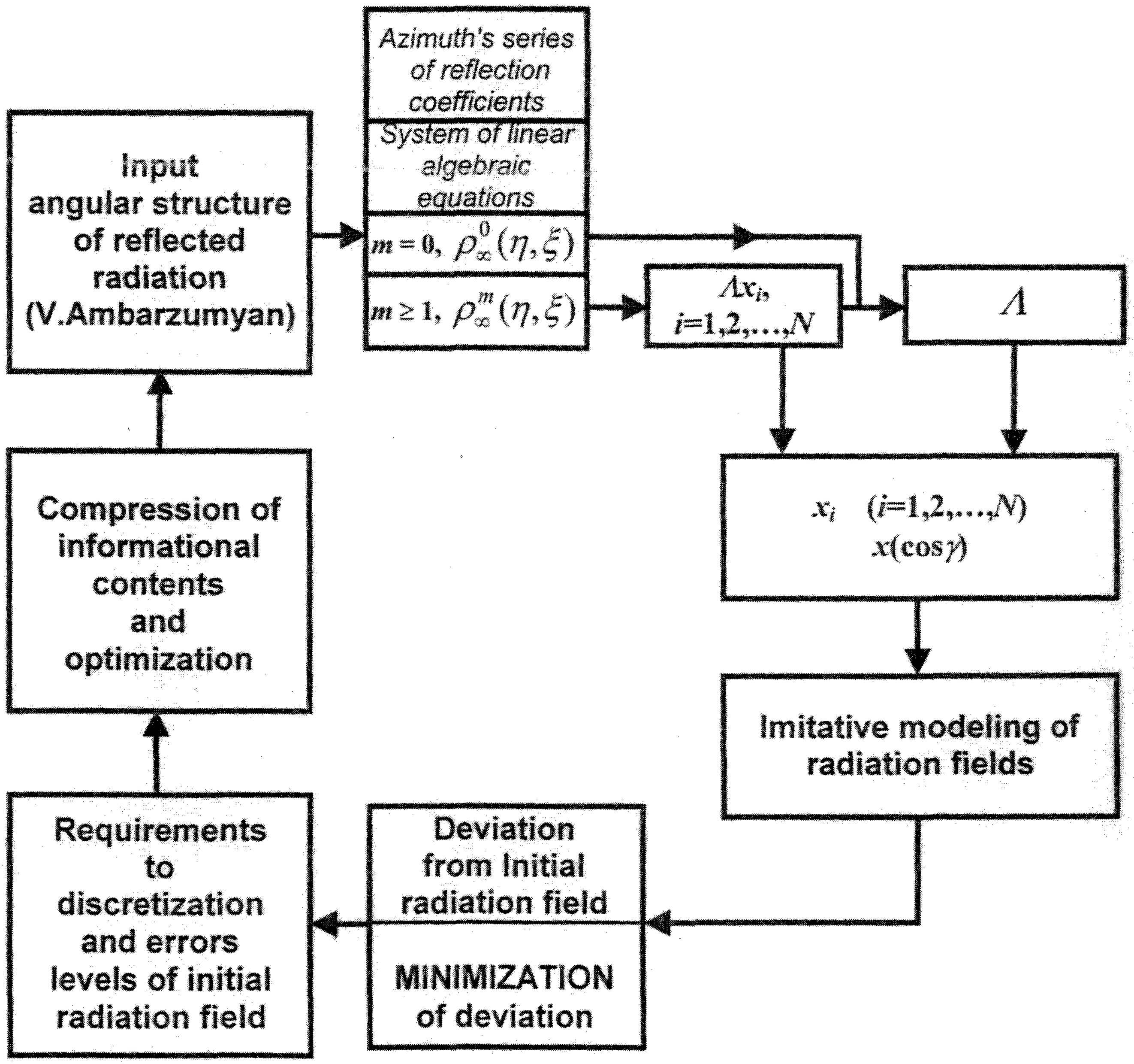
The polynomials $\overline{\mathrm{P}}_i^m(\zeta)$ are determined by following recurrent correlations:

$$\zeta\overline{\mathrm{P}}_i^m(\zeta) = \sqrt{\frac{(i-m+1)}{(2i+3)}\frac{(i+m+1)}{(2i+1)}}\,\overline{\mathrm{P}}_{i+1}^m(\zeta) + \sqrt{\frac{i^2-m^2}{4i^2-1}}\,\overline{\mathrm{P}}_{i-1}^m(\zeta) \tag{4}$$

Special conditions:

$$\overline{\mathrm{P}}_i^m(\zeta) = 0 \quad \text{for} \quad i < m, \quad \overline{\mathrm{P}}_i^m(\zeta) = \sqrt{\frac{(2m-1)!!}{2(2m)!!}}(1-\zeta^2)^m \tag{5}$$

General scheme of atmospheric optical parameters retrievement on the basis of angular structure for reflected radiation given by V.Ambarzumyan in the case semi-infinite atmosphere ($\tau_0 = \infty$)



Retrievement of primary atmospheric optical parameters in the case of semi-infinite atmosphere ($\tau_0 = \infty$)

1. The consideration of input radiation field according to V.Ambarzumyan 's angular structure for $\rho_\infty^m(\eta,\zeta)$:

The system of linear algebraic equations relatively to values $\Lambda\bar{x}_i (i = 1,2,...,N)$:

$$\rho_\infty^m(\eta,\zeta) = \frac{\Lambda}{2}\sum_{i=m}^{N}(-1)^{i+m} x_i \frac{\overline{\varphi}_i^m(\eta)\overline{\varphi}_i^m(\zeta)}{\eta+\zeta},$$

$m = 1,2,...,N, \qquad i = m,\ m+1,\ ...,\ N.$ (1)

The determinant of considered algebraic system:

$$\Delta_N = \prod_{m=1}^{N}\overline{\varphi}_m^m(\eta)\overline{\varphi}_m^m(\zeta) \neq 0\,. \qquad (2)$$

From considered algebraic system for $m \geq 1$ values $\Lambda x_i (i = 1,2,...,N)$ are calculated in a unique manner.

From considered algebraic system for $m=1$ values Λ are calculated in a unique manner:

$$\Lambda = \frac{2(\eta+\zeta)\rho_\infty^0(\eta,\zeta) - \sum_{i=1}^{N}(-1)^i\Lambda\bar{x}_i\overline{\varphi}_i^0(\eta)\overline{\varphi}_i^0(\zeta)}{\overline{\varphi}_0^0(\eta)\overline{\varphi}_0^0(\zeta)} \qquad (3)$$

Classical representation of reflected radiation angular structure given by S. Chandrasekhar on the basis of linear singular integral equations in the case $\tau_0 = \infty$.

Regularization's form of linear singular integral equation for azimuth's harmonics $\rho_\infty^m(\eta,\zeta,\zeta)$; $m = 0,1,2,\ldots N$:

$$g^m(\eta)\,\rho_\infty^m(\eta,\zeta) = \frac{\Lambda}{2}\int_0^1 A^m(\eta,\mu)\,\rho_\infty^m(\mu,\zeta)d\mu +$$

$$+\frac{\Lambda}{2}\eta\int_0^1 A^m(\eta,\mu)\frac{\rho_\infty^m(\mu,\zeta)-\rho_\infty^m(\eta,\zeta)}{\mu-\eta}d\mu + \frac{\Lambda}{4}\frac{A^m(\eta,-\zeta)}{\eta+\zeta}, \qquad (1)$$

where functions $g^m(\eta)$ are equal

$$g^m(\eta) = \begin{cases} 1-\dfrac{\Lambda}{2}\eta\displaystyle\int_0^1 A^m(\eta,-\mu)\frac{d\mu}{\mu}, & m=0 \\ 1-\dfrac{\Lambda}{2}\eta\displaystyle\int_0^1 A^m(\mu,\mu)\frac{d\mu}{\eta+\mu}+\frac{\Lambda}{2}\eta\int_0^1\frac{A^m(\mu,\mu)-A^m(\eta,\mu)}{\mu-\eta}d\mu, & m>0 \end{cases}$$

(2)

The kernel functions $A^m(\eta,\mu)$ are determined by following manner:

$$A^m(\eta,\mu) = \sum_{i=m}^{N} x_i\frac{(i-m)!}{(i+m)!}R_i^m(\eta)\mathrm{P}_i^m(\mu). \qquad (3)$$

The polynomials $R_i^m(\eta)$ are calculated from recursive relation:

$$(i-m+1)\,R_{i+1}^m(\eta)+(i+m)\,R_{i-1}^m(\eta)=(2i+1-\Lambda_i)\,\eta\,R_i^m(\eta) \qquad (4)$$

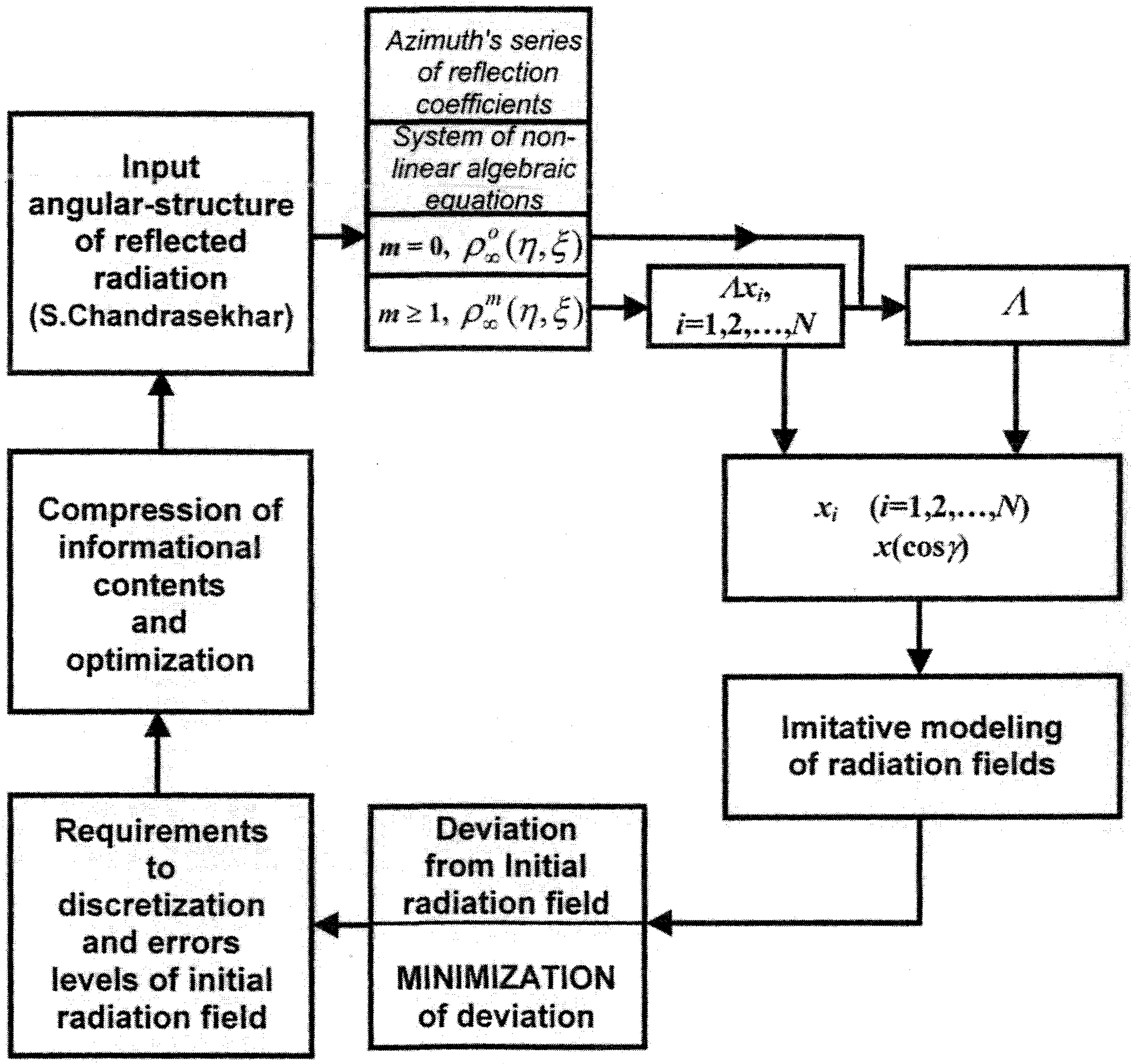
Special conditions:

$R_i^m(\eta)=0, \quad i<m;$

$$R_m^m(\eta)=\mathrm{P}_m^m(\eta). \qquad (5)$$

The polinomials $\Lambda A^m(\eta,\zeta)$ generate a non-linear algebraic form relatively to values Λx_i ($i = 1,2,\ldots N$).

General scheme of atmospheric optical parameters retrievement on the basis of angular structure for reflected radiation given by S.Chandrasekhar in the case semi-infinite atmosphere ($\tau_0 = \infty$)



Classical representation for angular structure of reflected radiation given by V.Sobolev and van de Hulst in the case τ_0 >>1

1. Conservative (pure) atmospheric light scattering ($\Lambda = 1$)

Representation of azimuthal harmonics $\rho^m(\eta,\zeta)$ according to asymptotic formula, given by V.Sobolev and van de Hulst

$$\bar{\rho}^m(\eta,\zeta,\tau_0) = \rho^m_\infty(\eta,\zeta) - \frac{4u_0(\eta)u_0(\zeta)}{(3-x_1)\tau_0+3\delta}, \qquad (1)$$

Special funtions u_0 and δ are equal:

$$u_0(\zeta) = \frac{3}{4}[\zeta+2\int_0^1\rho^0_\infty(\eta',\zeta)(\eta')^2 d\eta'],$$

$$\delta = 4\int_0^1 u_0(\zeta)\zeta^2 d\zeta. \qquad (2)$$

Accuracy (ε) of values $\rho^m(\eta,\xi,\varphi,\tau_0 >> 1)$ is given by M.King:

$$errors \quad \varepsilon \le 1\%, \qquad if \quad (1-\frac{x_1}{3})\tau_0 > 1.45 \qquad (3)$$

2. Almost conservative atmospheric light scattering ($1-\Lambda \ll 1$)

Representation of azimuthal harmonics $\rho^m(\eta,\zeta)$ according to asymptotic formula, given by V.Sobolev and van de Hulst

$$\bar{\rho}^m(\eta,\zeta,\tau_0) = \rho^m_\infty(\eta,\zeta) - h(\tau_0)u_0(\eta)u_0(\zeta) \tag{1}$$

Special functions $h(\tau_0)$ are equal

$$h(\tau_0) = \frac{4k}{3-x_1} + \frac{8k}{(3-x_1)(e^{2k\tau_0}-1)+6\delta k}\ . \tag{2}$$

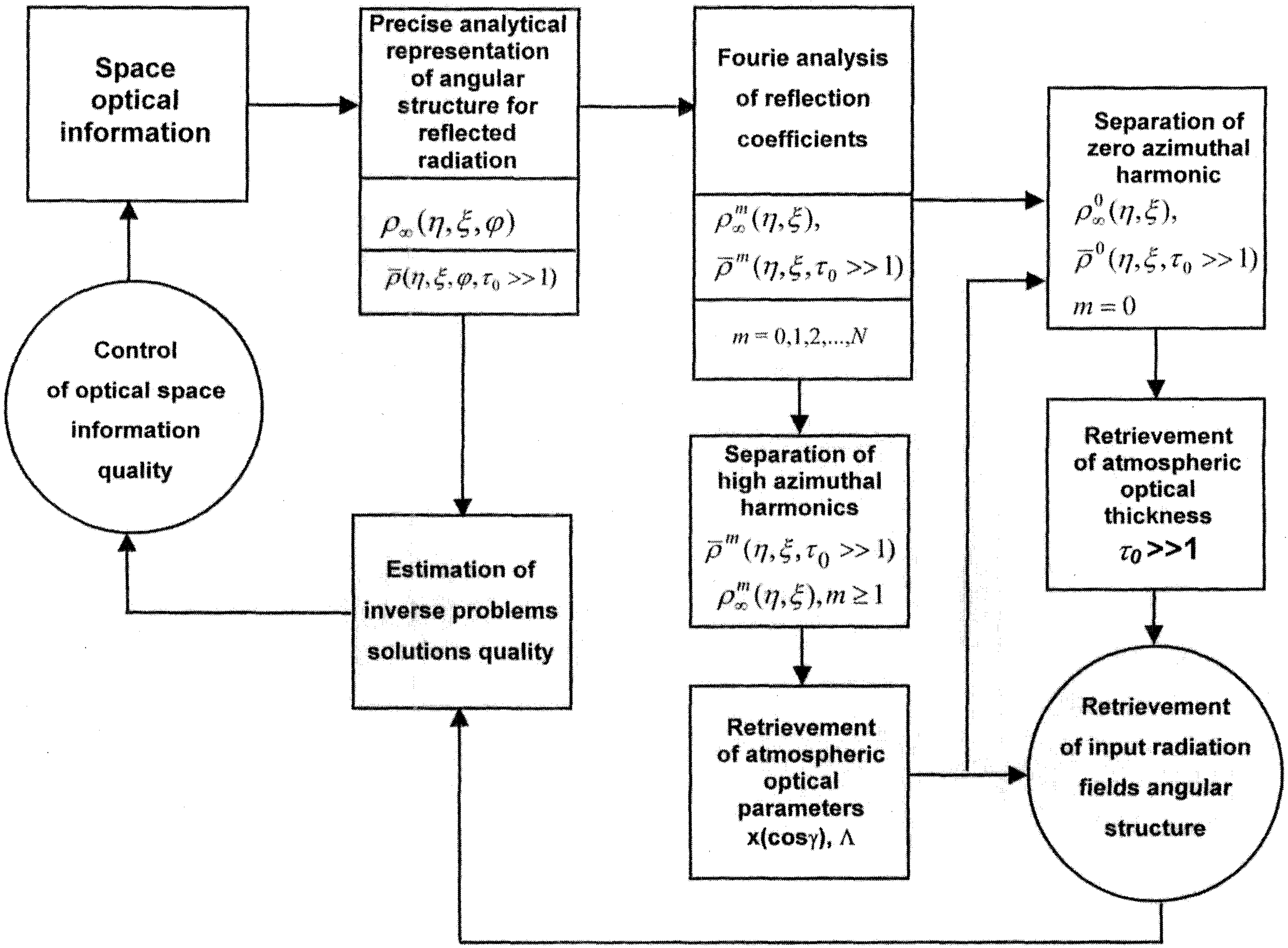
Accuracy (ε) of $\bar{\rho}^m(\eta,\zeta,\tau_0)$ given by M.King:

$$\text{errors } \varepsilon \le 1\%, \qquad \text{if} \quad (1-\frac{x_1}{3})\tau_0 > 1.45 \tag{3}$$

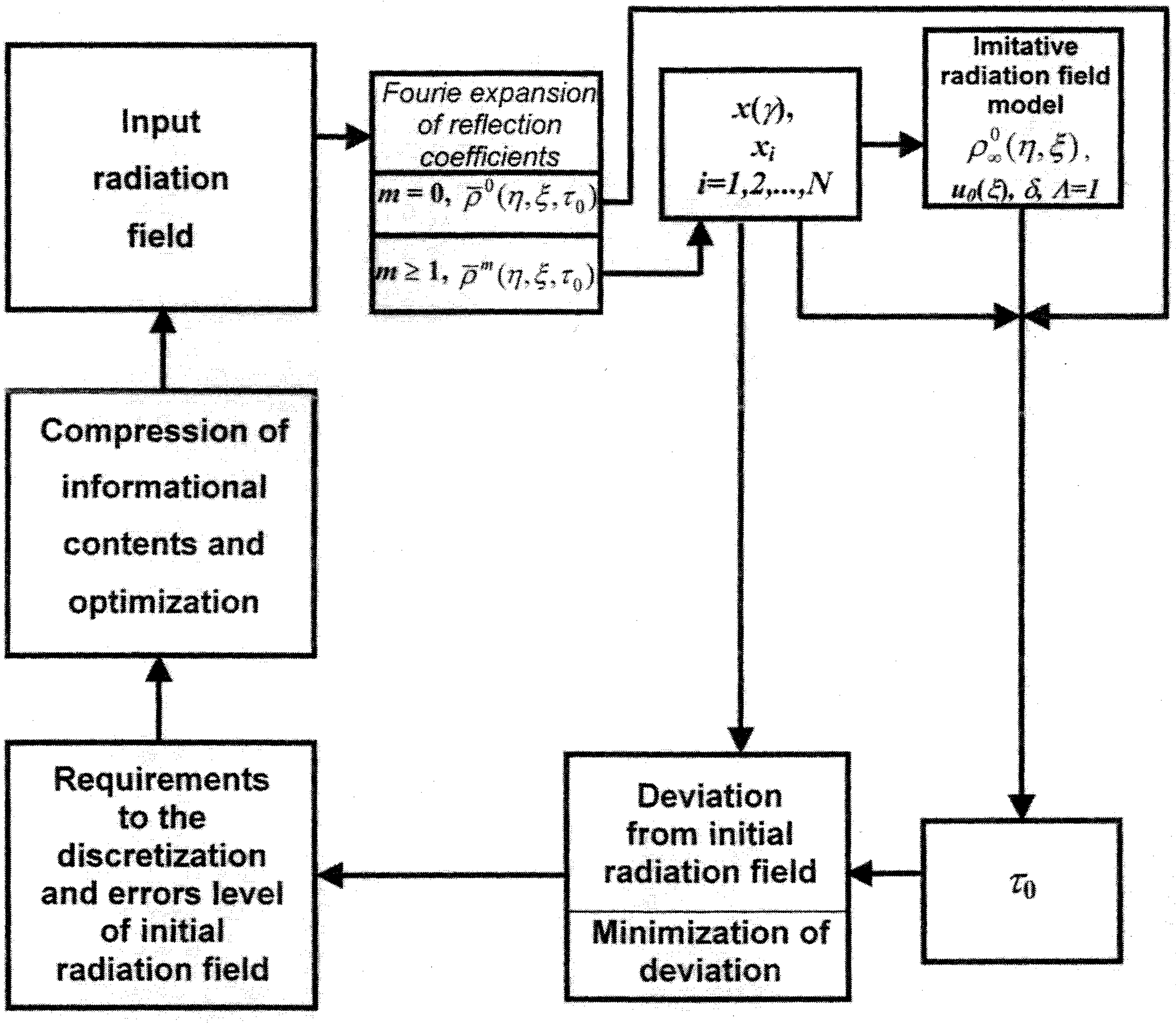
The root k is a smallest positive characteristic root which determines radiation field in deep layers of semi-infinite atmosphere ($\tau_0 = \infty$).

Special functions $u_0(\eta)$ and δ are determined according to previous case ($\tau_0 \gg 1$, $\Lambda = 1$).

Precise angular structure method of joint direct-inverse problems solutions ($\tau_0=\infty$ and $\tau>>1$)



General scheme of atmospheric optical parameters retrievement in the case of optically thick atmosphere (τ_0>>1) and conservative (pure) light scattering (Λ=1)



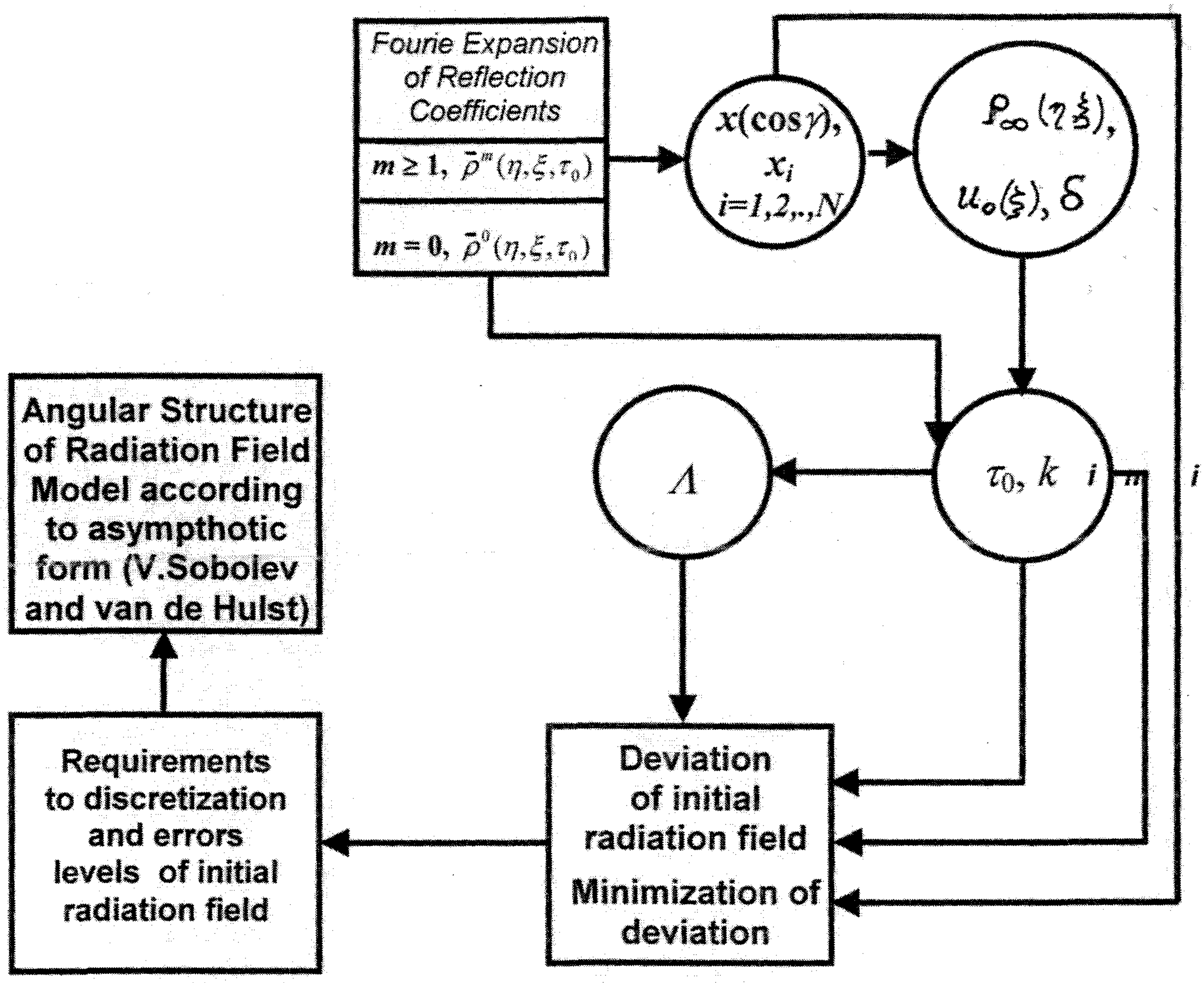
The consideration of input radiation field $\bar{\rho}^m(\eta,\zeta,\tau_0)$ according to V.Sobolev's - van de Hulst's angular structure

$$\bar{\rho}^m(\eta,\zeta,\tau_0) = \rho^m_\infty(\eta,\zeta,\varphi) - \frac{4u_0(\eta)u_0(\zeta)}{(3-x_1)\tau_0+3\delta} \qquad (1)$$

Finally the retrievement of values τ_0 and $x(\cos\gamma)$ are carried out according to relation

$$\tau_0 = \frac{1}{3-x_1}\left[\frac{4u_0(\eta)u_0(\zeta)}{\left|\rho^0_\infty(\eta,\zeta)-\rho^0(\eta,\zeta,\tau_0)\right|}-3\delta\right]. \qquad (2)$$

General scheme of optical parameters retrievement in the case of optically thick atmosphere ($\tau_0 \gg 1$) and *almost conservative (pure) light scattering* ($\Lambda \ll 1$)



The consideration of input radiation field $\bar{\rho}^m(\eta,\zeta,\tau_0)$ according to V.Sobolev's - van de Hulst's angular structure

$$\bar{\rho}^m(\eta,\varphi,\tau_0) = \rho^m_\infty(\eta,\zeta) - h(\tau_0)u_0(\eta)u_0(\xi). \qquad (1)$$

Special function $h(\tau_0)$ is equal

$$h(\tau_0) = \frac{4k}{3-x_1} + \frac{8k}{(3-x_1)(e^{2k\tau_0}-1)+8\delta k}, \qquad (2)$$

The root k is calculated from relation

$$1 - \Lambda = \cfrac{k^2}{3-\Lambda x_1 - \cfrac{4k^2}{5-\Lambda x_2 - \cfrac{9k^2}{7-\Lambda x_3 - \ldots}}} \qquad (3)$$

Finally the retrievement of values k, τ_0, $x(\cos\gamma)$ and Λ are carried out in a unique manner.

Quality control of inverse problems solutions

1. Stability of linear algebraic systems solutions for rank N >>1.

2. Accuracy of inverse problems solution (ε).

3. Angular discretization levels of inverse problems solutions (δ).

4. Informational contents of inverse problem solutions: correlation between input optical information (ε,δ) retrieved optical information ($\tilde{\varepsilon}$,$\tilde{\delta}$).

Basic parameter of inverse problems solutions quality

- Deviation values of radiation fields (initial and retrieved):

  values of radiation fields (initial and retrieved):
  $\Delta\{I\}$ and $\Delta\{\tilde{I}\}$ in a knots chosen grid points.

- Quality of inverse operators $\hat{L}^{-1}$:

  if $\sum\limits_{i=1}^{N}\left|\Delta\{I\}_i\right|^2 \le \varepsilon$, *then* $\sum\limits_{j=1}^{M}\left|\Delta\{G\}_j\right|^2 \le \tilde{\varepsilon}$ (1)

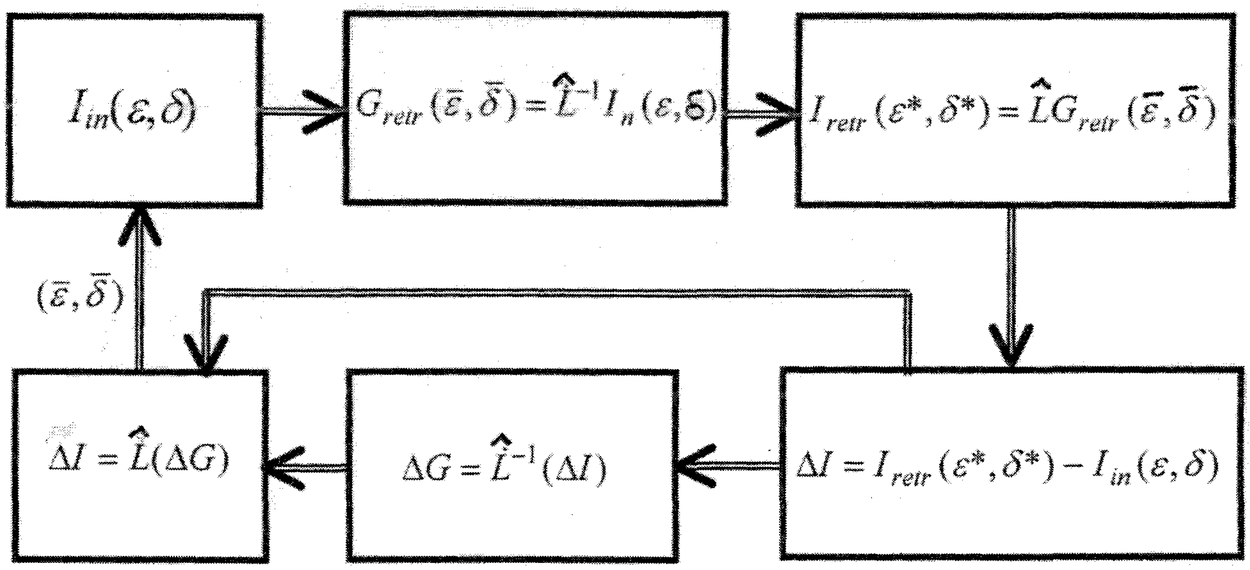
- Quality of inverse problem solutions:

$$\textbf{min} \sum_{i=1}^{N}\sum_{j=1}^{M}\left[\Delta\{I\}_i - \Delta\{\tilde{I}\}_j\right]^2 \le \varepsilon \qquad (2)$$

$$\textbf{min} \sum_{i=1}^{N}\sum_{j=1}^{M}\left[\Delta\{G\}_i - \Delta\{\tilde{G}\}_j\right]^2 \le \tilde{\varepsilon} \qquad (3)$$

Optimal levels of discretization (ε) and errors (δ) of initial (I_{in}) and retrieved (I_{retr}) radiation fields

$$\begin{cases} I_{in} = \hat{L} G_{in} \\ G_{retr} = \hat{L}^{-1} I_{in} \\ I_{retr} = \hat{L} G_{retr} \end{cases}$$



$$\Delta G = \left| G_{retr} - G_{in} \right| < \delta_\varepsilon$$

$$\underbrace{\Delta I = \left| I_{retr} - I_{in} \right| < \varepsilon}_{\text{min } \Delta I \text{ in a chosen grid points}}$$

The real quality of inverse problems solutions for the chosen grid points is determined by minimization of input radiation field deviation from appropriate value constructed on the basis of retrieved input optical parameters.

NUMERICAL RESULTS

Models of optical atmospheric parameters

1) Atmospheric phase function $x(\cos\gamma) = \dfrac{1-g^2}{(1+g^2-2g\cos\gamma)^{3/2}}$

   $x_i = (2i+1)g^i$; $g = 0{,}5 \div 0{,}9$; $i = 0,1,\ldots,N$; $N \in \{5(5)170\}$.

2) Atmospheric single scattering albedo $\Lambda = 1$ and $1-\Lambda \ll 1$.

3) Atmospheric optical thickness $\tau_0 = \infty$ and $\tau_0 \in \{5(5)30\}$.

4) Interval of angular variables $\eta, \zeta \in \{0{,}1(0{,}1)0{,}9\}$.

5) Accuracy $\varepsilon \sim 10^{-4}$, angular discretization $\delta \sim 1/50$.

Region of investigations

- accuracy (ε) in dependence on total number N of azimuthal harmonics $\bar{\rho}^m(\eta,\zeta,\tau_0)$ and $\bar{\rho}^m_\infty(\eta,\zeta)$
- stability of inverse problem solutions $\rho^m_{perturb}(\eta,\zeta) = \rho^m_{exact}(\eta,\zeta)(1+\alpha)$, $\alpha \in [0{,}01;\, 0{,}07]$
- accuracy (ε) in dependence on angular variables η and ζ
- compression of informational contents (ε, δ)

Stability of inverse problems solutions

The perturbation of input (retrieved) radiation field

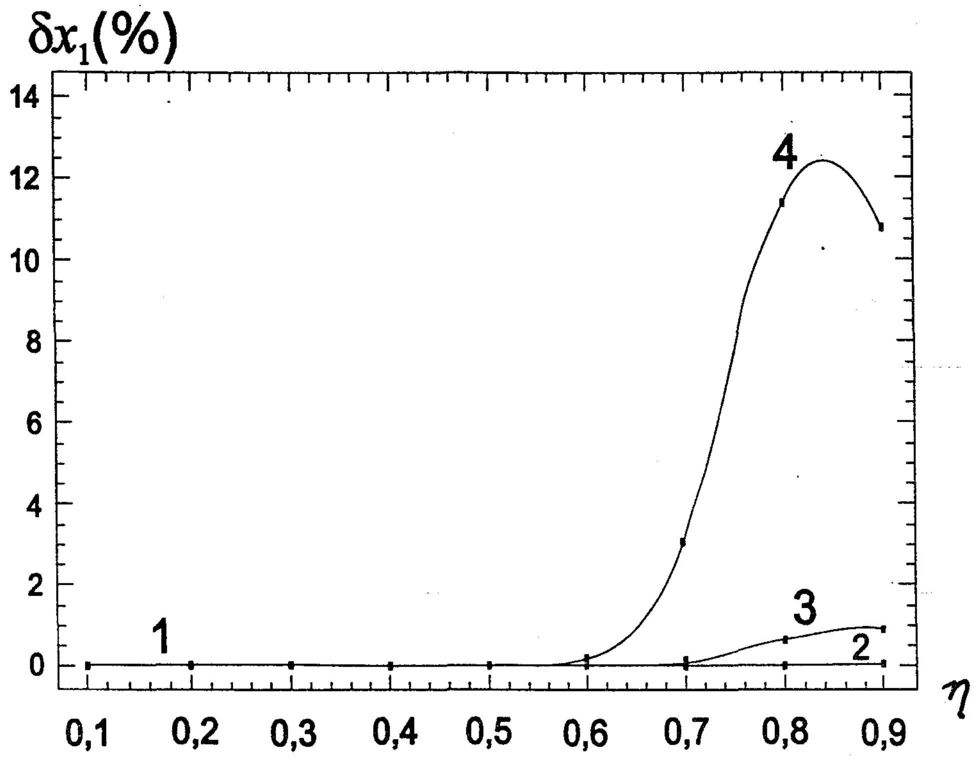
$$\{I^m\}_{pert} = \{I^m\}_{exact}(1+\alpha), \quad \alpha \in [-0.1;+0.1]$$

1. The retrievement of x_i (i=1,....N) and τ_0 are stable: small perturbations α generate small errors of retrieved optical parameters (x_i , τ_0).

2. The errors of retrieved coefficient x_i and τ_0 are equal approximately to errors of input optical information for η, $\xi < 0.8$.

For case ($\eta,\xi \to 1$) these errors grow.

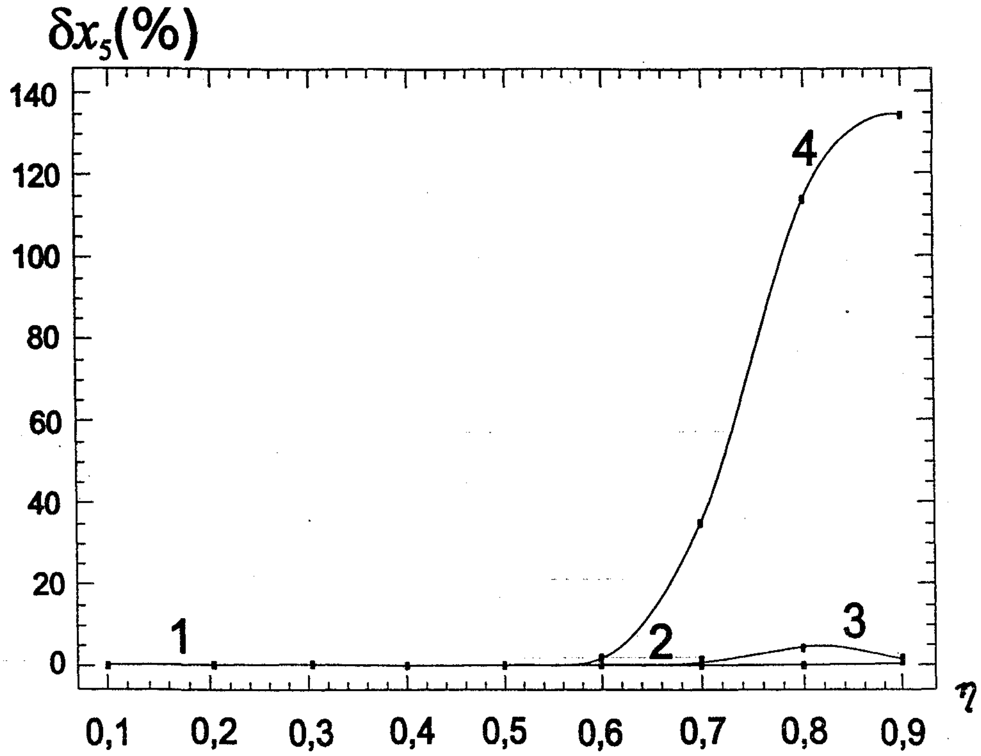
This conclusions are illustrated by Figures.



Dependence of retrievement accuracy for values x_1 on η:

1. $N = 5$, **2.** $N = 10$, **3.** $N = 15$, **4.** $N = 20$

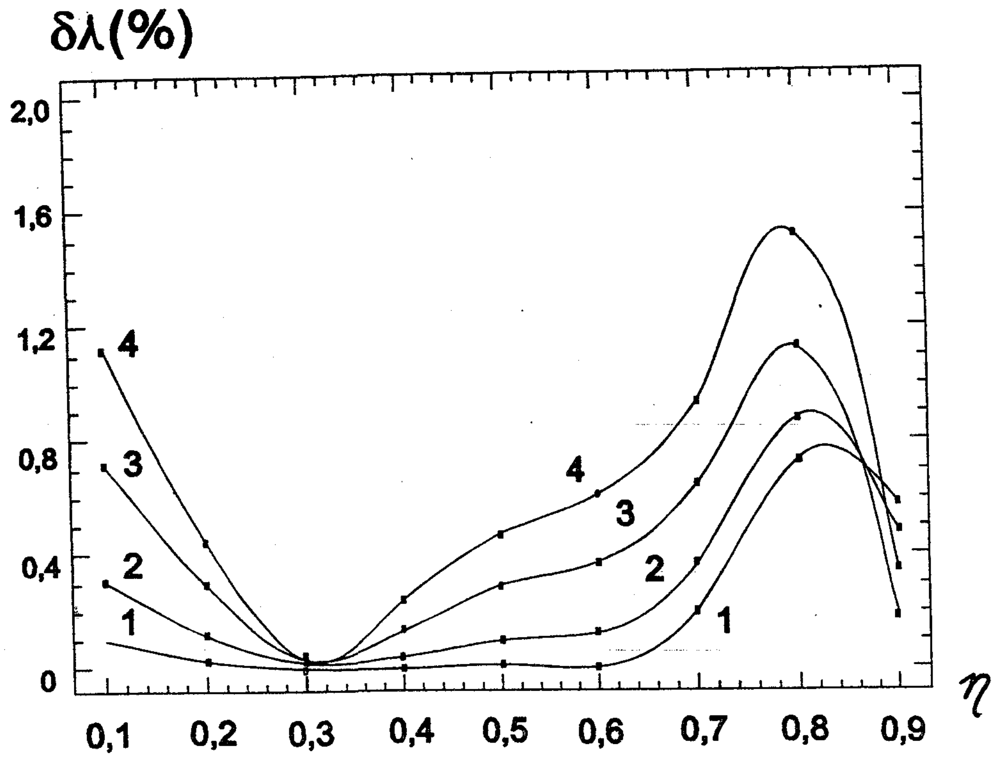
$\xi = 0.7$, $\tau_0 = \infty$



Dependence of retrievement accuracy for values x_5 on η:

1. $N = 5$, **2.** $N = 10$, **3.** $N = 15$, **4.** $N = 20$

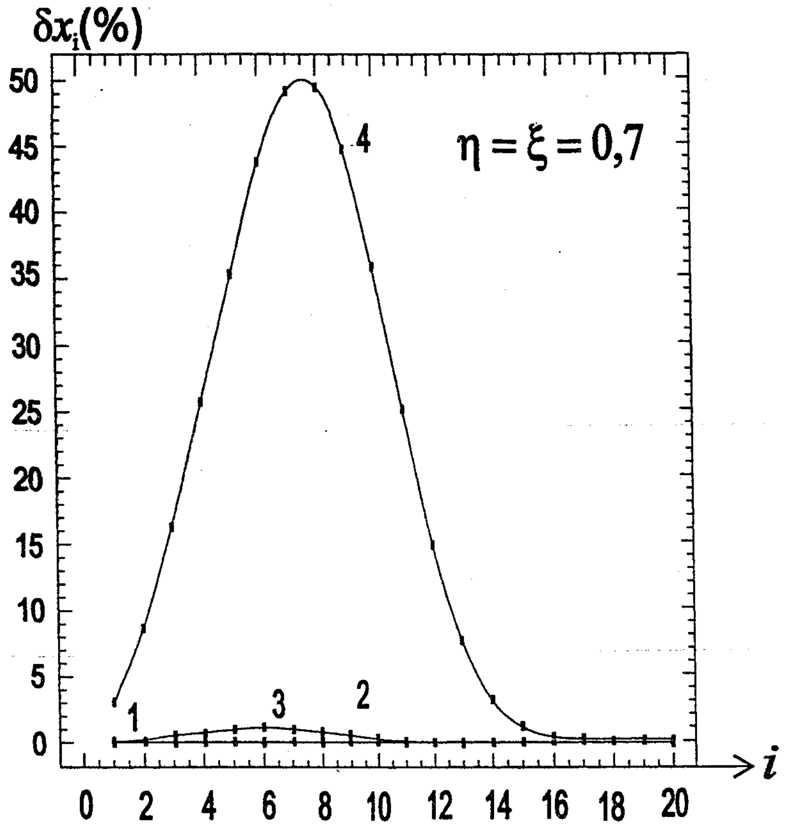
$\xi = 0.7$, $\tau_0 = \infty$



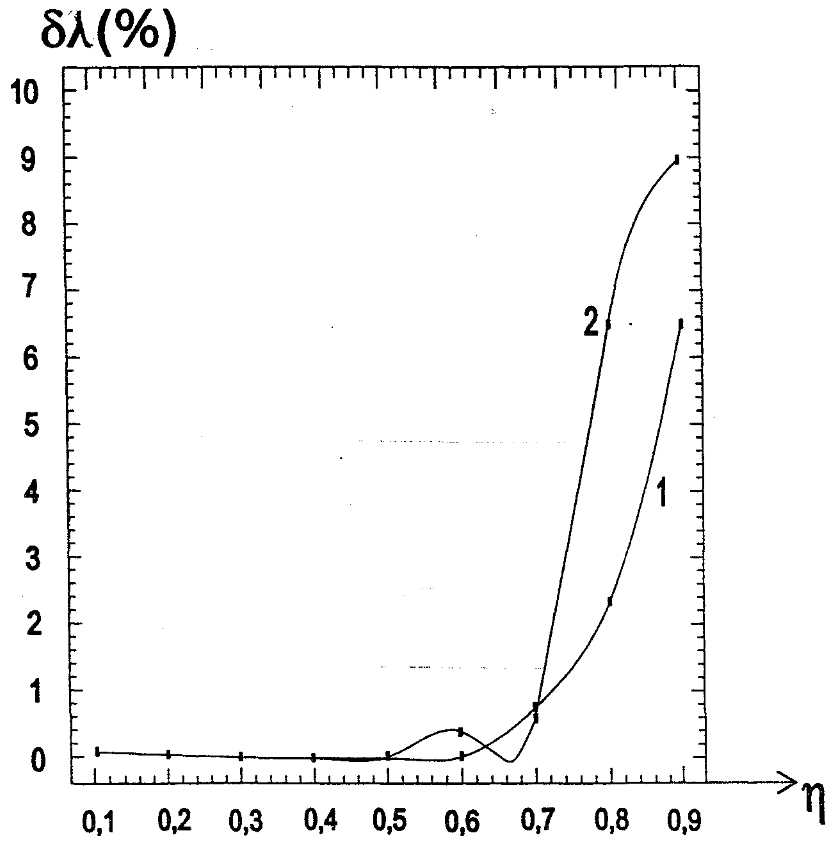
Retrievement stability of single scattering albedo Λ and x_1 in dependence on perturbation parameter α:

1. $\alpha = 0\%$, **2.** $\alpha = 1\%$, **3.** $\alpha = 3\%$, **4.** $\alpha = 5\%$,

$\xi = 0.7$, $N = 20$, $\tau_0 = \infty$

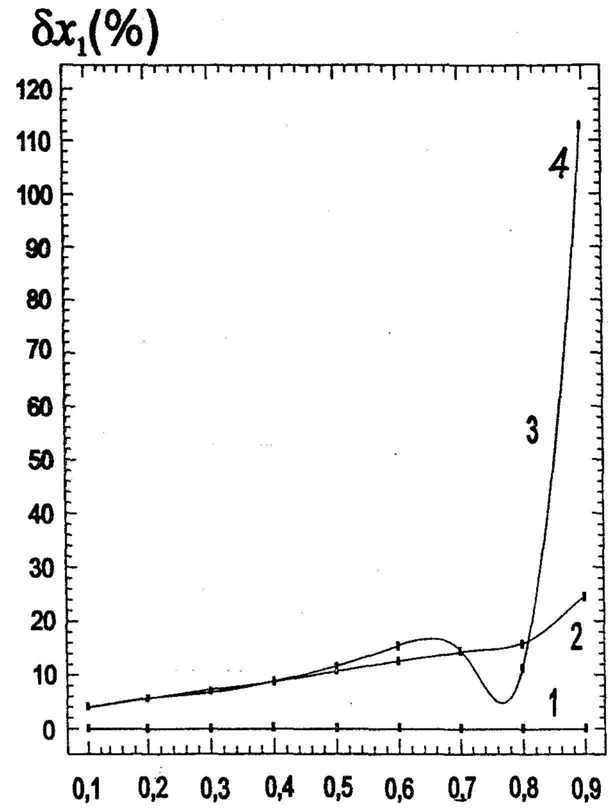


Dependence of values δx_{i} on current discretization's number $i = 1,2,\ldots N$: (**1.** $N = 5$, **2.** $N = 10$, **3.** $N = 15$, **4**. $N = 20$, $\eta = \xi = 0.7$; $\tau_0 = \infty$)



Dependence of values $\delta\Lambda$ on η:

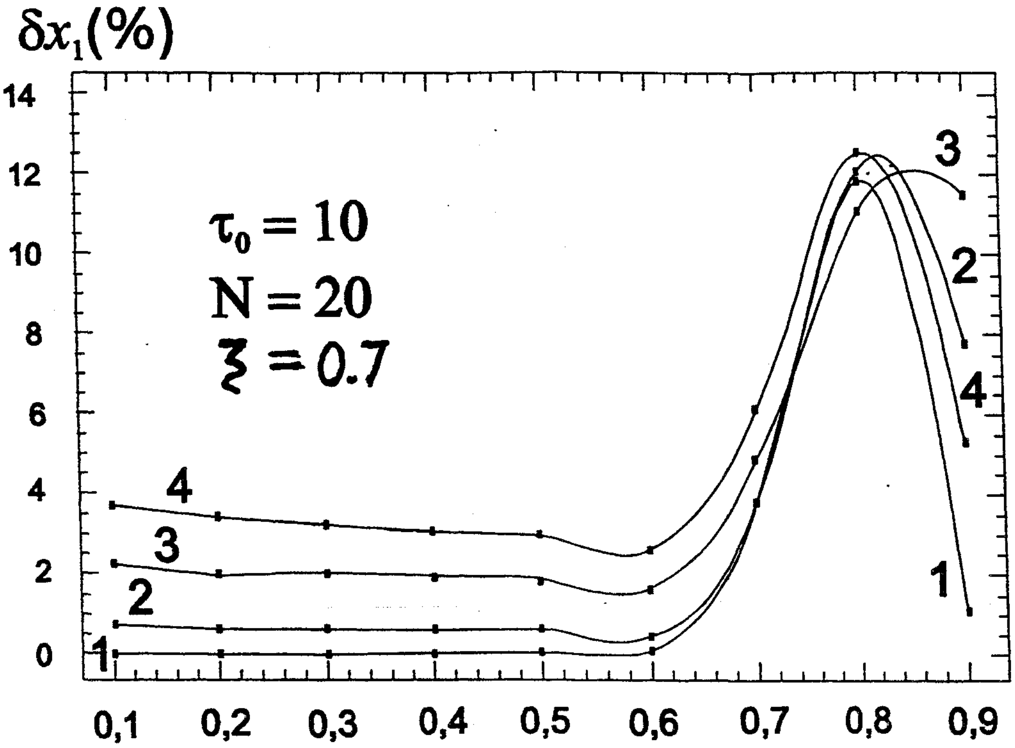
1. $\xi = 0.8$ **2 .** $\xi = 0.9$, $N = 20$



Retrievement accuracy of values x_1 and Λ in dependence on η

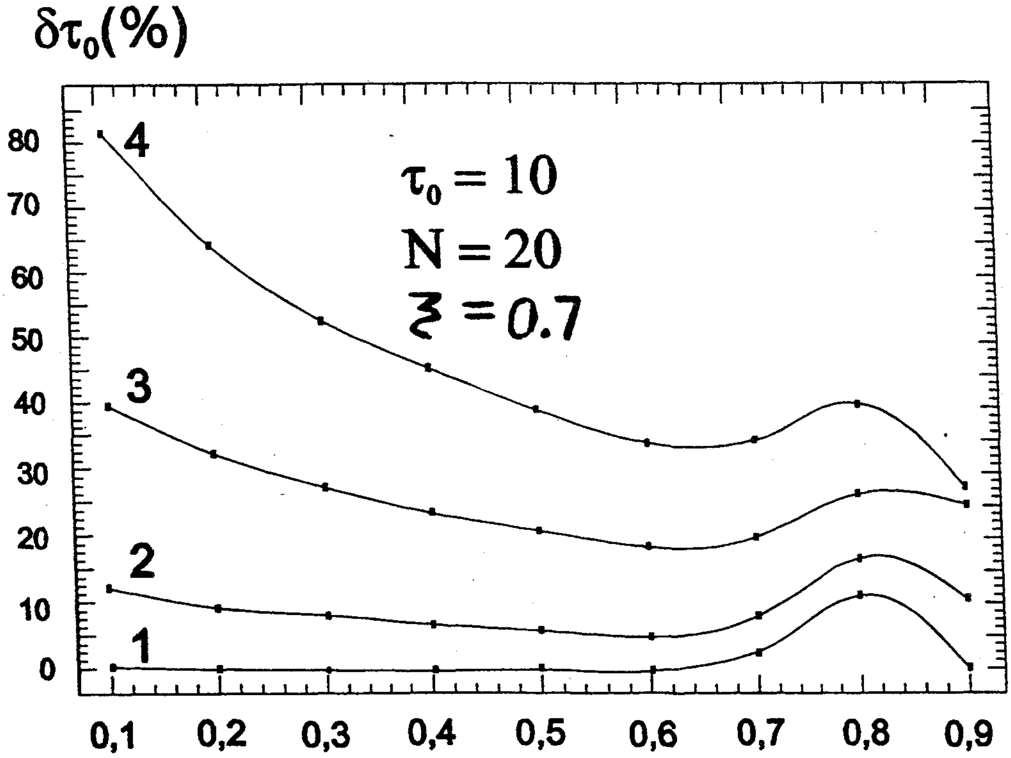
1. $N = 5$, **2.** $N = 10$, **3.** $N = 15$, **4.** $N = 20$,

$\xi = 0.5$; $\tau_0 = \infty$



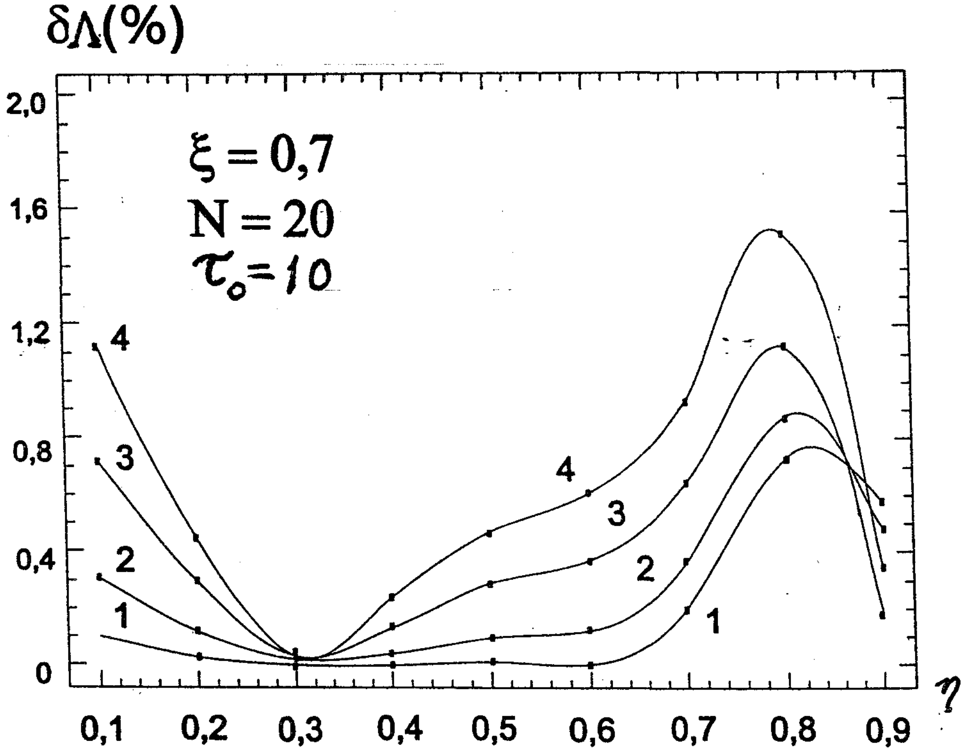
Stability of coefficient x_1 retrieving for ζ=0,7

$1 \to \alpha = 0\%$ $2 \to \alpha = 1\%$ $3 \to \alpha = 3\%$ $4 \to \alpha = 5\%$



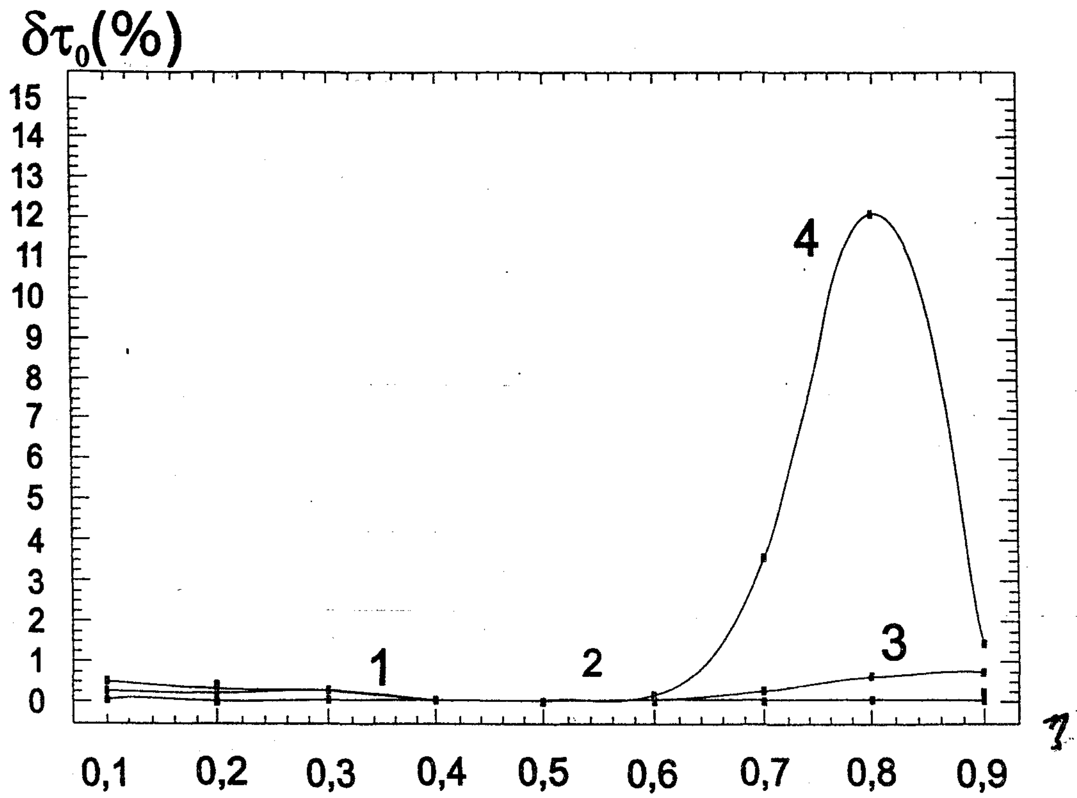
Stability of optical thickness τ_0 retrieving

$1 \to \alpha = 0\%$ $2 \to \alpha = 1\%$ $3 \to \alpha = 3\%$ $4 \to \alpha = 5\%$



Stability of single scattering albedo Λ retrieving

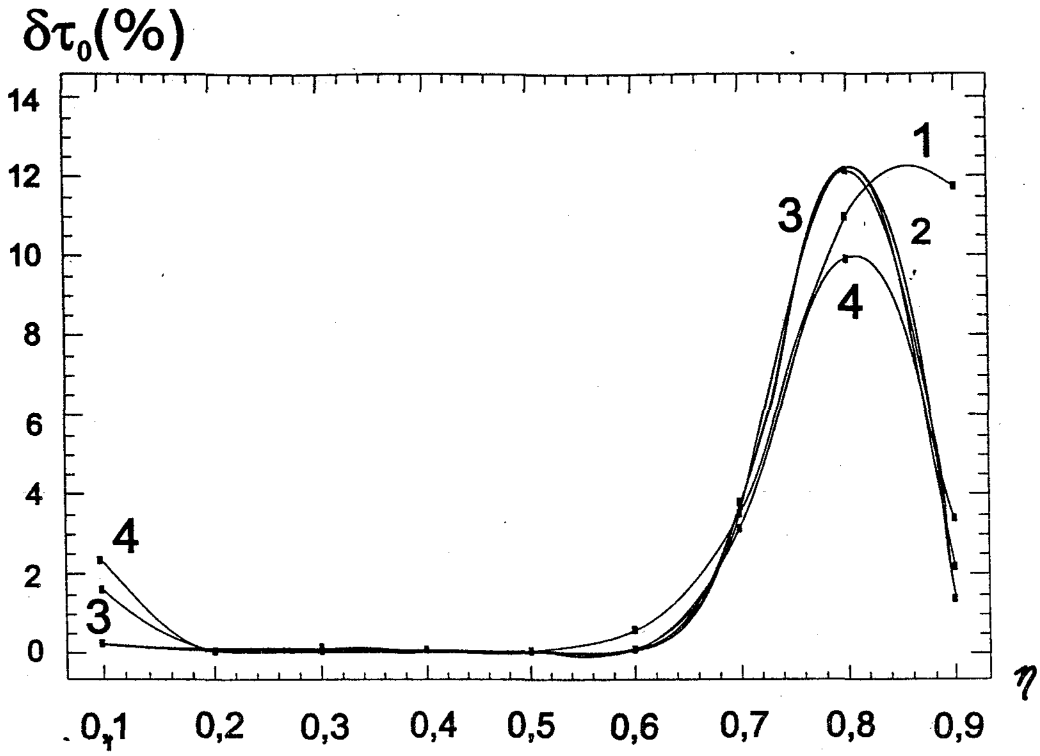
$1 \rightarrow \alpha = 0\%$ $2 \rightarrow \alpha = 1\%$ $3 \rightarrow \alpha = 3\%$ $4 \rightarrow \alpha = 5\%$



Retrievement accuracy of values τ_0 and x_1 in dependence on η:

1. $N=5$, **2.** $N=10$, **3.** $N=15$, **4.** $N=20$,

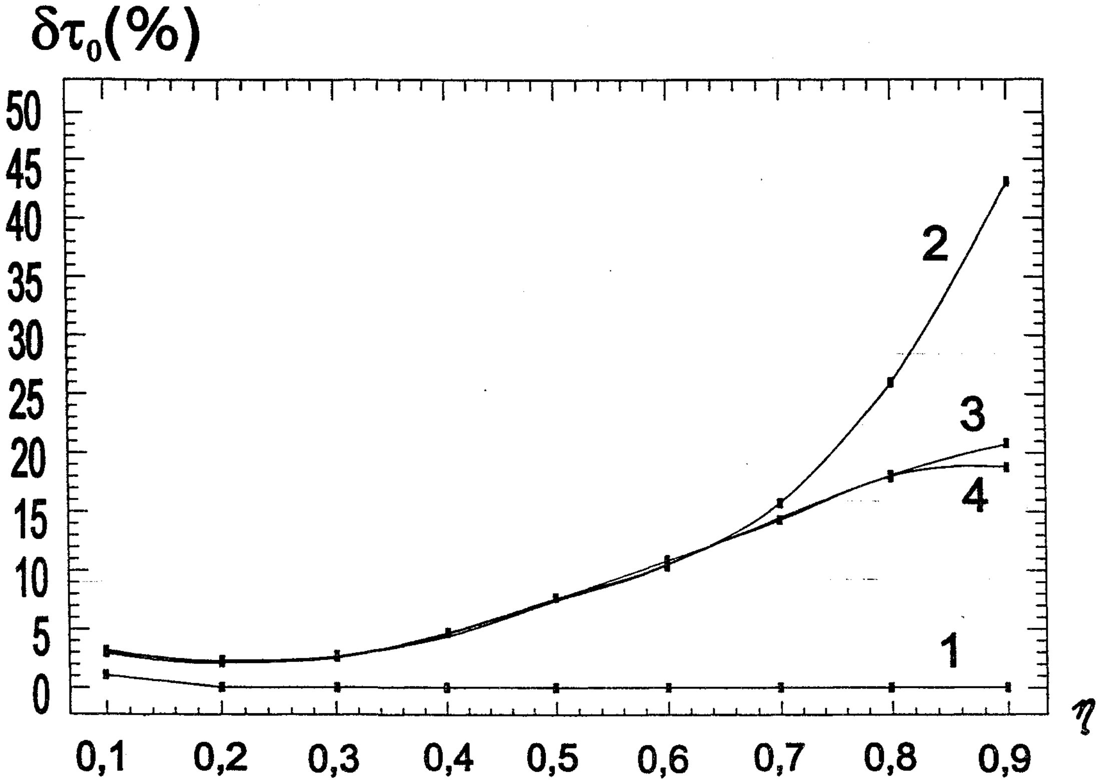
$\xi=0.7$; $\Lambda=1$, $\tau_0=10$



Retrievement accuracy of values τ_0 and x_1 in dependence on η:

1. $\tau_0 = 5$, **2 .** $\tau_0 = 10$, **3.** $\tau_0 = 15$, **4 .** $\tau_0 = 20$,

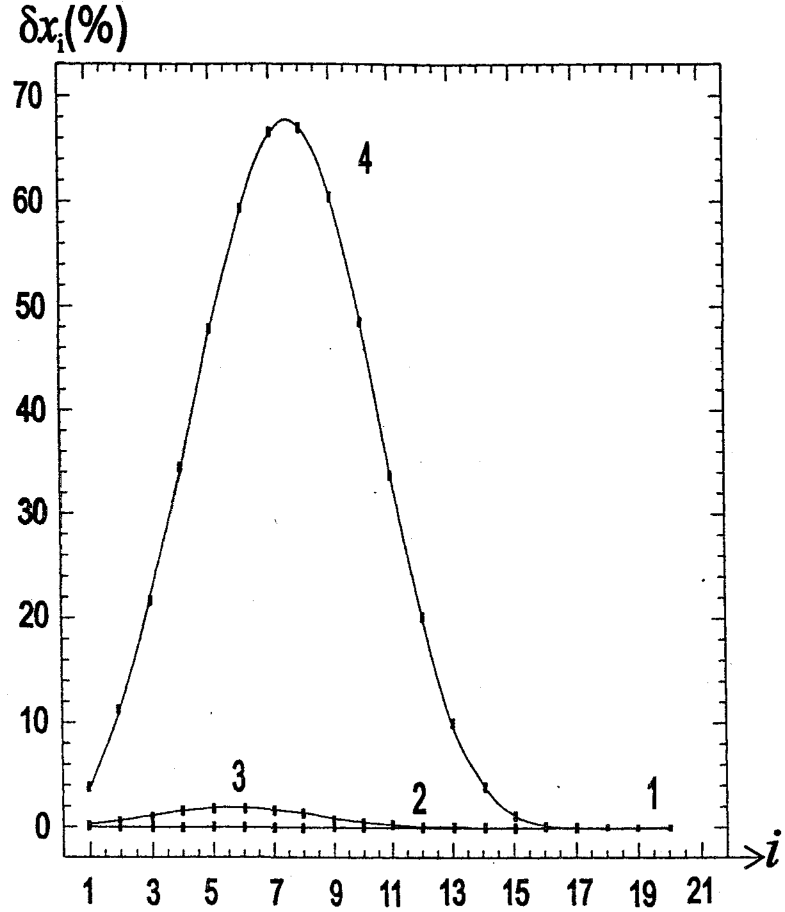
$\Lambda = 1$,



Dependence of values $\delta\tau_0$ and δx_i on η :

1. $N = 5$, **2 .** $N = 10$, **3.** $N = 15$, **4 .** $N = 20$,

$\tau_0 = 10$; $\Lambda = 1$



Dependence of values δx_i on current discretization's number i :

1. $N = 5$, **2 .** $N = 10$, **3.** $N = 15$, **4 .** $N = 20$,

$\eta = \xi = 0.7$; $\tau_0 = 10$; $\Lambda = 1$

Basic Results

1. Approach to conciliated construction of precise direct and inverse operators for radiative transfer equations solutions has been developed making use of Ambarzumyan's angular separation.

$$G_{g,\delta} \Rightarrow \hat{L}_{g,\tilde{\delta}}(G) \Rightarrow \hat{L}^{-1}_{\tilde{g},\tilde{\delta}}[\hat{L}(G)] \Rightarrow G^*_{g,\delta^*}$$

$$L\{Y\} = B$$

$$\Lambda x_m = y_m\,, \quad Y=\{y_0, y_1, \dots, y_n\}$$

$$4\rho^m(0,\eta,\tau_0)\zeta = b_m\,; \qquad B=\{b_0, b_1, \dots, b_m\}$$

$$a_m = \begin{cases} 0, & m > i \\ (-1)^{i+m}\dfrac{(i-m)!}{(i+m)!}P_i^m(0)\varphi_i^m(\zeta,\tau_0) & \end{cases}$$

$$i = m, m+1 \dots, N.$$

2. Exact inverse operator for semi-infinite vertically uniform atmosphere $\tau_0 = \infty$ has been constructed on the basis of exact direct operator determined by linear singular integral equation obtained by S.Chandrasekhar.

$$g^m(\eta)\rho_\infty^m(\eta,\zeta) = \frac{\Lambda}{2}\eta\int_0^1 A^m(\eta,\zeta)\rho_\infty^m(\eta,\zeta)d\eta +$$

$$\frac{\Lambda}{2}\eta\int_0^1 A^m(\eta,\eta')\frac{\rho_\infty^m(\eta',\zeta)-\rho_\infty^m(\eta,\zeta)}{\eta'-\eta}d\mu + \frac{\Lambda}{4}\frac{A^m(\eta,-\zeta)}{\eta+\zeta}$$

3. Application of linear integral equations, obtained S.Chandrasekhar and of developed earlier by author angle-structural method for radiative transfer theory inverse problems solutions in the case optically thick uniform slab ($\tau_0 >> 1$) has been given.

$$\rho(\eta,\zeta,\varphi,\tau_0) = \rho_\infty(\eta,\zeta,\varphi) - h(\tau_0)U_0(\eta)U(\zeta),$$

$$h(\tau_0) = \frac{4k}{3-x_1} + \frac{8k}{(3-x_1)(e^{2k\tau_0}-1)+6\delta k}$$

$$\rho^m(\eta,\zeta,\tau_0) = \rho^m_\infty(\eta,\zeta),\ m \geq 1$$

$$\rho(\eta,\zeta,\varphi,\tau_0) = \rho_\infty(\eta,\zeta,\varphi) - \frac{4\bar{U}_0(\eta)\bar{U}_0(\zeta)}{(3-x_1)\tau_0 + 3\delta}$$

4. Analysis of informational contents on the basis of azimutal harmonics compression, influence of discretization and errors levels, including inverse problems solutions stability in the case of optically thick ($\tau_0 >> 1$) media has been carried out.

$$\varepsilon \geq 2qN,\ q \approx 1/6$$

$$[L(G)]_{pert} = [L(G)]_{exact}(1+\alpha)$$

CONCLUSION

1. Precise conciliated construction of direct $\hat{L}$ and inverse operator $\hat{L}^{-1}$ for semi-infinite atmosphere ($\tau_0 = \infty$) has been carried out making use of classical Ambarzumyan's angular separation for reflected radiation fields.

2. Precise conciliated construction of direct $\hat{L}$ and inverse operator $\hat{L}^{-1}$ for semi-infinite atmosphere ($\tau_0 = \infty$) has been carried out making use of classical Chandrasekhar's linear singular integral equation for angular separation of reflected radiation fields.

3. In framework of inverse problem solution the application of asymptotic approximation of reflected radiation field in the case optically thick atmosphere ($\tau_0 >> 1$) is given in the case Λ=1 and 1 - Λ << 1.

4. Numerical calculations analysis for consideration stability problems, compression of informational contents and errors and discretization levels influence has been carried out.

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