united nations educational, scientific and cultural organization

international atomic energy agency

the
abdus salam
international centre for theoretical physics

Course on "Inverse Methods in Atmospheric Science"
1 - 12 October 2001

301/1332-18

"General Angular Structure Method of Inverse Problems: Solutions in the Radiative Transfer Theory. Part II"

Oleg I. SMOKTY
Institute for Informatics & Automation
Russian Academy of Sciences
St. Petersburg, Russia

Please note: These are preliminary notes intended for internal distribution only.

strada costiera, 11 - 34014 trieste italy - tel. +39 0402240111 fax +39 040224163 - sci_info@ictp.trieste.it - www.ictp.trieste.it

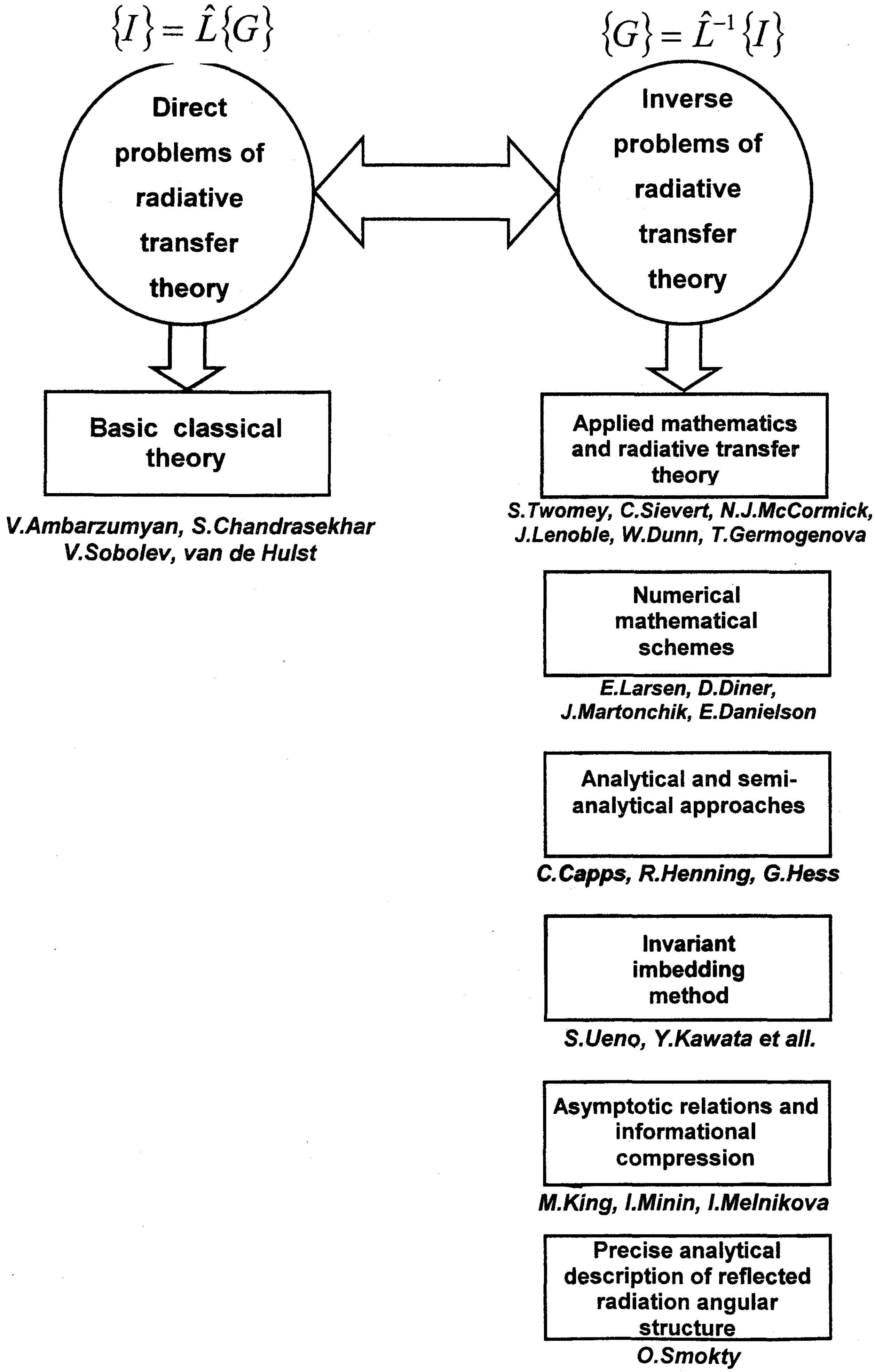
Abstract

Presented in these lectures are the results developed on the basis of the Sobolev scientific school. They have been obtained by author in the recent years in the field of the precise analytical and semi-analytical calibrating solutions of the direct-inverse problems connected with the classical radiative transfer theory and atmospheric optics. The mathematical basis of considered joint direct-inverse problems solutions is the presentation of the azimuth harmonics of atmospheric and underlying surface brightness coefficients on the basis of the forms given by V. Ambarzumyan, S. Chandrasekhar, V. Sobolev and Van de Hulst. Precise algorithms for vertically uniform slabs, bounded from below an arbitrary orthotropical reflecting bottom was elaborated. Taking into account the above-mentioned classical representations of angular distributions for radiation fields, it is possible to construct a system of linear algebraic equations for the retrieval of primary optical parameters of "atmosphere-underlying surface" system. It should be remarked that specially that in a frame of the developed analytical approach, the high azimuth harmonics of system brightness coefficients allow to retrieve the atmospheric optical parameters only. Optical properties of underlying surfaces are retrieved making use of the zero harmonics only.

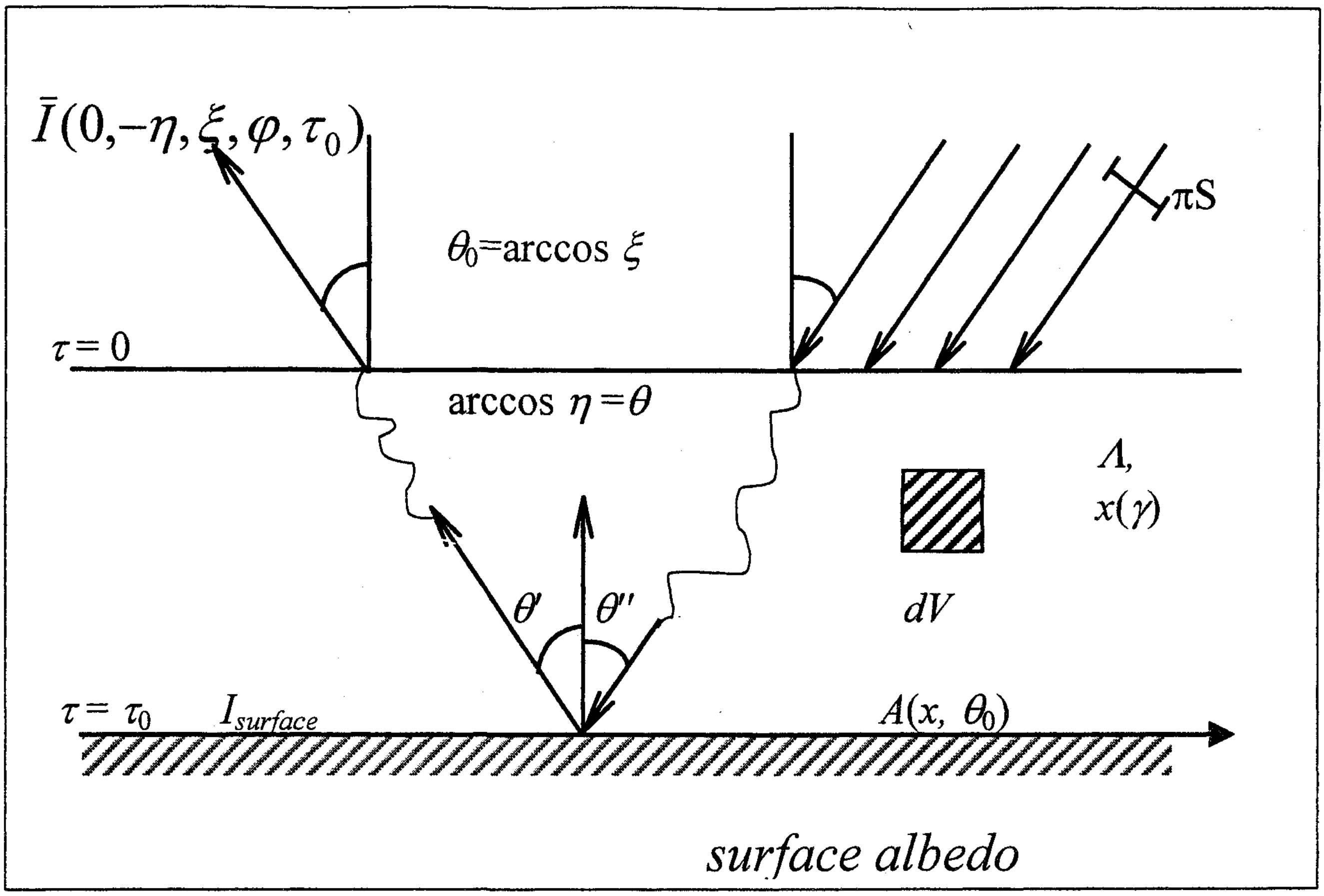
The numerical results obtained in the frame of the presented analytical structural method allow to estimate strictly the following problems:

- the stability solution;
- the influence of chosen grid points, errors and discretization levels;
- information content compression.

Introduction



The statement of inverse problems solutions



Basic input optical parameters

- Atmospheric optical thickness: $\tau_0 = \int_0^\infty \alpha(z)dz \geq 0$ (1)
- Surface albedo : $A(x,\xi) \leq 1$
- Single scattering albedo : $\Lambda \leq 1$
- Atmospheric phase function $x(\cos\gamma) = 1 + \sum_{i=1}^{N} x_i P_i(\cos\gamma)$,

Basic input radiative parameters

- Reflection coefficient $\bar{\rho}(\eta,\zeta,\varphi,\tau_0)$: $\bar{I}(0,-\eta,\zeta,\varphi,\tau_0) = S\zeta\bar{\rho}(\eta,\zeta,\varphi,\tau_0)$
- Fourie harmonics $\bar{\rho}^m(\eta,\zeta,\tau_0)$ of reflection coefficient $\bar{\rho}(\eta,\zeta,\varphi,\tau_0)$:

$$\bar{\rho}(\eta,\zeta,\varphi,\tau_0) = \bar{\rho}^0(\eta,\zeta,\tau_0) + \sum_{i=1}^{N} \bar{\rho}^m(\eta,\varsigma,\tau_0)\cos m\varphi,$$

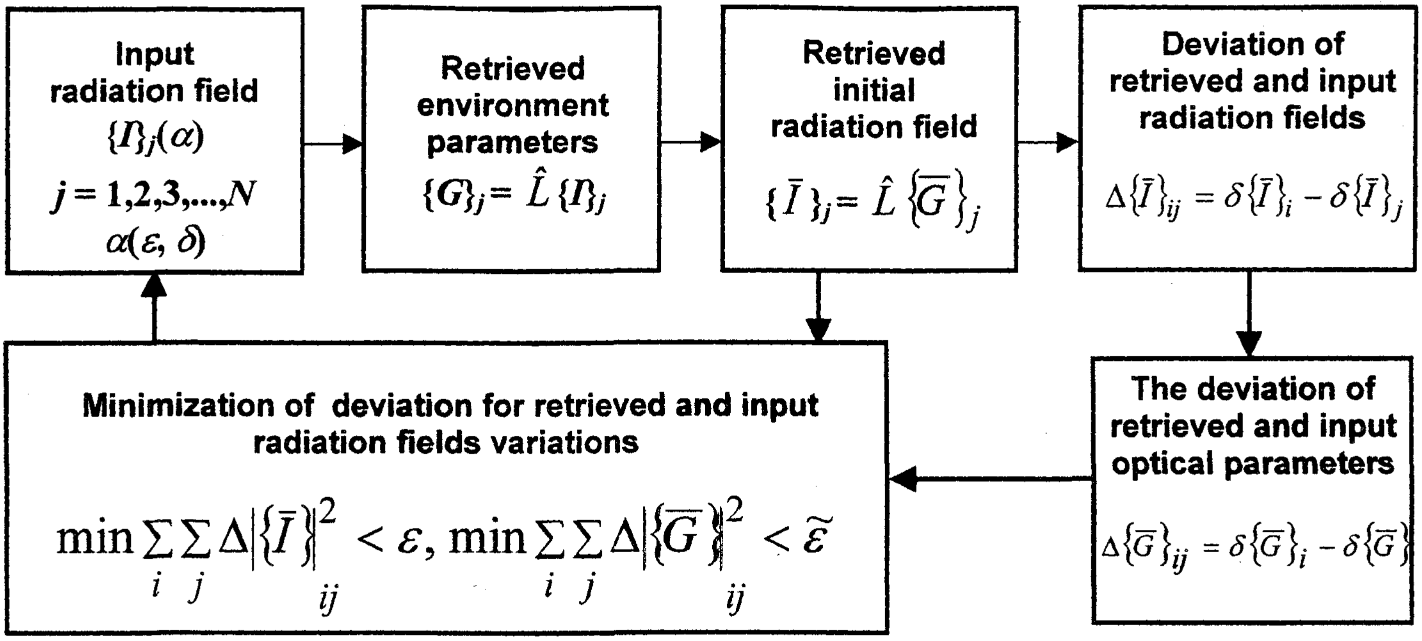
where $$\bar{\rho}^m(\eta,\zeta,\tau_0) = \frac{1}{2\pi}\int_0^{2\pi} \bar{\rho}(\eta,\varsigma,\varphi',\tau_0)\cos m\varphi' d\varphi'.$$

Basic problem

$$\{I\} = \hat{L}\{G\} \Leftrightarrow \{G\} = \hat{L}^{-1}\{I\}$$

retrieving of optical parameters of system "atmosphere-underlying surface"
x_i (i=1,2,...,N) , Λ, τ_0 and A(x,ξ)

General scheme of quality control for inverse problems solutions



The operator's quality at absence a priori information about input optical characteristics is determined by a value of deviation $\Delta\{I\}$. Just

$$If \quad \min\sum_i\sum_j \Delta\left|\{\bar{I}\}\right|^2_{ij} < \varepsilon, \; then \quad \min\sum_i\sum_j \Delta\left|\{\bar{G}\}\right|^2_{ij} < \tilde{\varepsilon} \qquad (1)$$

Thus the solution of inverse problem's will be correct, if to small variations of input radiation field $\{I\}$ will be correspond to small variations of retrieved optical parameters $\Delta\{G\}$.

By selection ε and δ_ε retrievement of $\{G\}$ is achieved with desirable error $\Delta\{G\}$ by means minimization of radiation field deviation $\Delta\{I\}$.

Thus at absence of information a priori about initial calibration model of input optical parameters reasonable criterion of the inverse problems solution quality is following: to small perturbations of input radiation field $\{I\}$ must correspond to small perturbations of retrieved parameters $\{G\}$. Making use of inverse problems solution data we have minimization of following deviation:

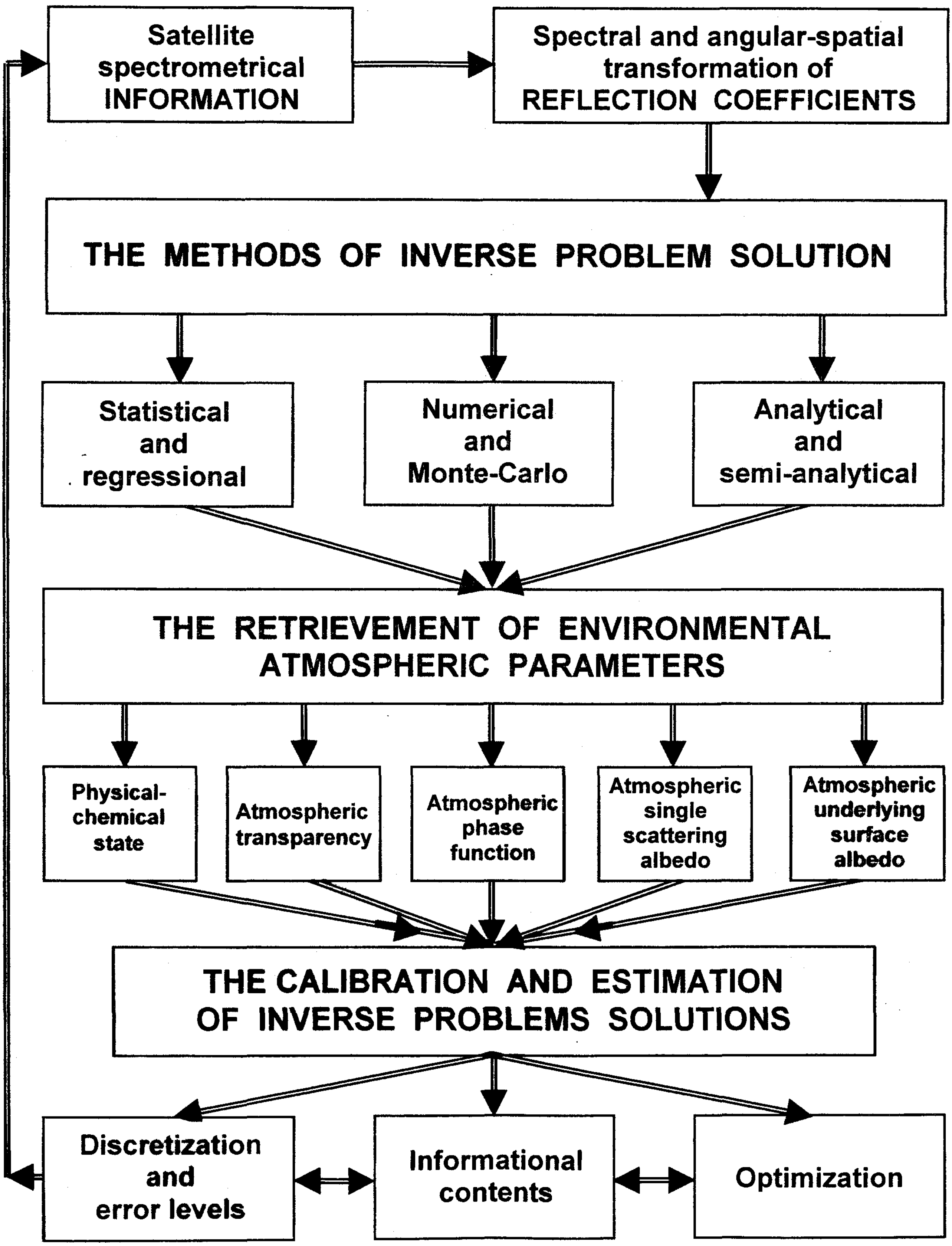
$$\boldsymbol{min} \sum_{j=1}\sum_{i=1}\left[\Delta\{I\}_i - \Delta\{\bar{I}\}_i\right]^2 \le \varepsilon, \qquad (2)$$

From this deviation for radiation field the next optimization for initial and retrieved optical parameters deviation $\Delta\{G\}, \Delta\{\bar{G}\}$ follows

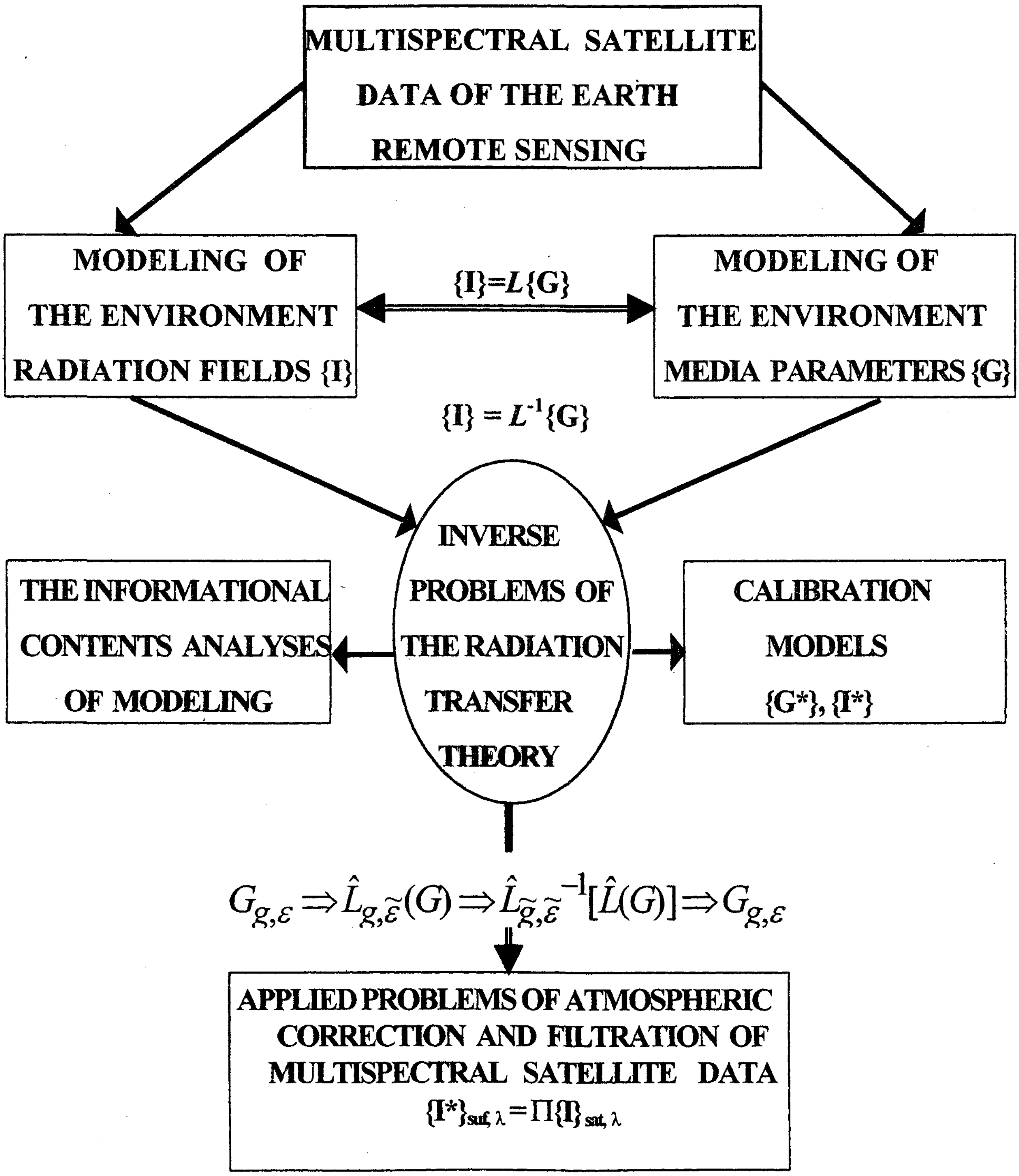
$$\boldsymbol{min} \sum_{i=1}\sum_{j=1}\left[\Delta\{G\}_i - \Delta\{\bar{G}\}_j\right]^2 \le \tilde{\varepsilon}, \qquad (3)$$

which is understood in such sense that under mathematically correct solution of inverse problems any variations of initial optical parameters $\Delta\{G\}$ will be similar to variations of retrieved optical parameters $\{\bar{G}\}$.

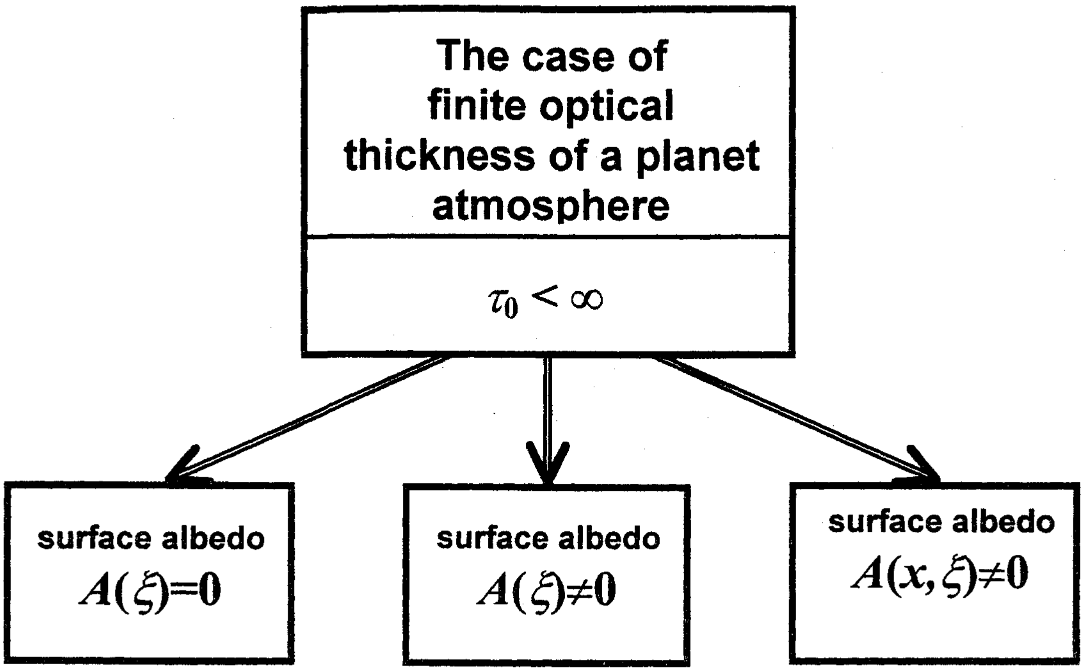
The general structure scheme of the solution of the inverse problems on the base satellite data and radiative modelling



General structural scheme of conciliated description for modeling of direct-inverse problem solutions



Angular structure of reflected radiation fields



a) Absence of underlying surface placed on the level under $\tau = \tau_0$:

$$A \equiv 0, \quad \bar{\rho}(\eta,\zeta,\varphi,\tau_0) \equiv \rho(\eta,\zeta,\varphi,\tau_0)$$

According to V.Ambarzumyan and S.Chandrasekhar :

$$\rho^m(\eta,\zeta,\tau_0) = \frac{\Lambda}{4}\sum_{i=m}^{N}(-1)^{i+m}\frac{(i-m)!}{(i+m)!}x_i \frac{\varphi^m(\eta,\tau_0)\varphi^m(\zeta,\tau_0)-\psi^m(\eta,\tau_0)\psi^m(\zeta,\tau_0)}{\eta+\zeta} \qquad (1)$$

where $$\varphi_i^m(\zeta,\tau_0) = \mathrm{P}_i^m(\zeta) + 2\zeta\int_0^1 P_i^m(-\eta)\,\rho^m(\eta,\zeta,\tau_0)d\eta, \quad m = 0,1,2,\ldots,N, \qquad (2)$$

$$\psi_i^m(\zeta,\tau_0) = \mathrm{P}_i^m(\zeta)\,e^{-\tau_0/\xi} + 2\zeta\int_0^1 P_i^m(\eta)\sigma^m(\eta,\zeta,\tau_0)d\eta, \quad m = 0,1,2,\ldots,N \qquad (3)$$

Basic properties of $\varphi_i^m(\eta,\tau_0)$ and $\psi_i^m(\eta,\tau_0)$ - Ambarzumyan's functions :

$$\psi_i^m(0,\tau_0) = 0, \qquad \varphi_i^m(0,\tau_0) = P_i^m(0), \quad m = 0,1,\ldots,N, \quad i = m, m+1,\ldots,N. \qquad (4)$$

Basic properties of $\varphi_i^m(\eta,\tau_0)$ and $\psi_i^m(\eta,\tau_0)$ - Ambarzumyan's functions for $m=N$:

$$\psi_N^N(\zeta,\tau_0) = \sqrt{\left[\varphi_N^N(\zeta,\tau_0)\right]^2 - \frac{8(2N)!}{\Lambda x_N}\rho^N(\xi,\zeta,\tau_0)\xi}\ , \qquad (5)$$

$$\sigma^N(\eta,\zeta,\tau_0) = \frac{\Lambda x_N}{4(2N)!}\,\frac{\varphi_N^N(\eta,\tau_0)\psi_N^N(\eta,\tau_0)-\varphi_N^N(\zeta,\tau_0)\psi_N^N(\eta,\tau_0)}{\eta-\zeta}. \qquad (6)$$

Basic relations for inverse problem solution:

$$\rho^m(0,\zeta,\tau_0) = \frac{\Lambda}{4}\sum_{i=m}^{N}(-1)^{i+m}\frac{(i-m)!}{(i+m)!}x_i P_i^m(0)\frac{\varphi^m(\zeta,\tau_0)}{\zeta}, \quad m = 0,1,\ldots,N. \qquad (7)$$

b) Orthotropic horizontally uniform underlying surface placed on the level $\tau = \tau_0$

The representation of azimuth's harmonics $\bar{\rho}^m(\eta,\zeta,\tau_0)$ given by V.Sobolev and van de Hulst:

$$\bar{\rho}^m(\eta,\zeta,\tau_0) = \rho^m(\eta,\zeta,\tau_0) + \frac{A(\zeta)}{1-A(\zeta)c(\tau_0)}\mu(\eta,\tau_0)\,\mu(\zeta,\tau_0),$$

$$m = 0,1,\ldots, N, \tag{1}$$

where

$$\mu(\zeta,\tau_0) = e^{-\tau_0/\xi} + 2\int_0^1 \bar{\sigma}^0(\eta,\zeta,\tau_0)\eta d\eta, \tag{2}$$

$$c(\tau_0) = 4\int_0^1 \eta d\eta \int_0^1 \bar{\rho}^0(\eta,\zeta,\tau_0)\zeta d\zeta, \tag{3}$$

It is essential, that in the case of lambertian (orthotropic) underlying surfaces we have

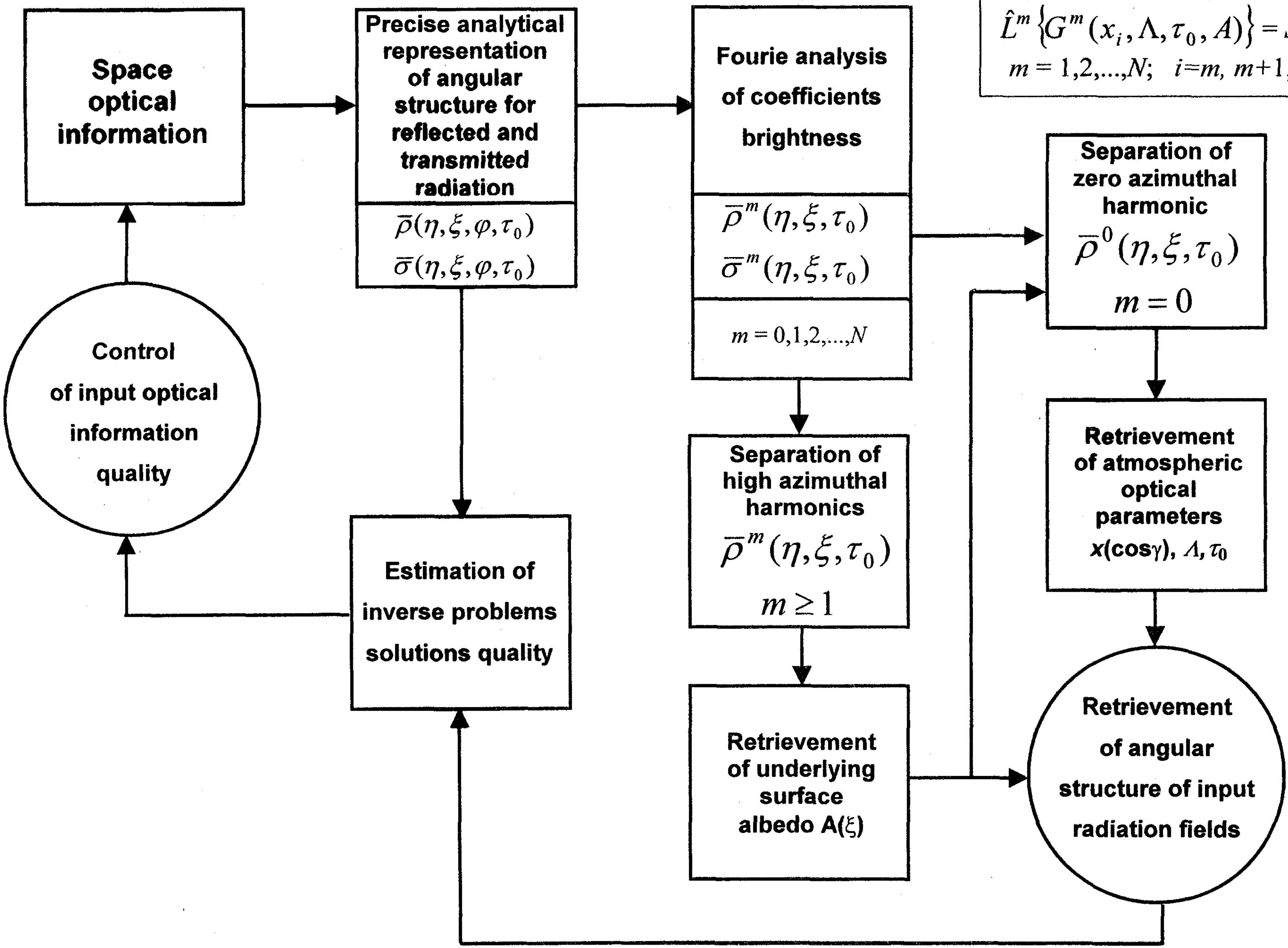
$$\bar{\rho}^m(\eta,\zeta,\tau_0) = \rho^m(\eta,\zeta,\tau_0), \quad m \geq 1. \tag{4}$$

Thus only zero harmonics $\bar{\rho}^0(\eta,\zeta,\tau_0)$ contains the information about albedo of underlying surface $A(\xi)$. Thus, angular structure of azimuth's harmonics $\rho^m(\eta,\zeta,\tau_0)$ in the case $A \neq 0$ same as without underlying surface ($A = 0$).

This property determines identical informational contents of azimuthal harmonics ρ^m, for $m \geq 1$, in both cases $A(\xi) \neq 0$ and $A(\xi) = 0$ from the point of view the atmospheric optical characteristics x_i, Λ, τ_0 retrieving.

The angular structure method of inverse problems solutions ($\tau_0 < \infty$)

$$\hat{L}\{G(x_i,\Lambda,\tau_0,A)\} = S\xi\rho(\eta,\zeta,\varphi,\tau_0) \Leftrightarrow$$
$$\hat{L}^m\{G^m(x_i,\Lambda,\tau_0,A)\} = S\xi\rho^m(\eta,\zeta,\tau_0),$$
$$m = 1,2,\ldots,N; \quad i = m, m+1,\ldots,N$$



The case of three-term atmospheric phase function

$x(\cos\gamma) = 1 + x_1\cos\gamma + x_2P_2\cos\gamma$

(molecular light scattering)

$$\Lambda x_1 = \frac{\widetilde{b}_1}{a_{11}} = 8\zeta\frac{\rho^1(\zeta,0,\tau_0)}{\varphi_1^1(\zeta,\tau_0)} = \frac{8\zeta\rho^1(\zeta,0,\tau_0)}{\left[P_1^1(\zeta) + 2\zeta\int_0^1 P_1^1(-\eta)\rho^1(\zeta,\eta,\tau_0)d\eta\right]} \tag{1}$$

$$\Lambda x_2 = \frac{\widetilde{b}_2}{a_{22}} = \frac{32\zeta\rho^2(\zeta,0,\tau_0)}{\varphi_2^2(\zeta,\tau_0)} = \frac{32\zeta\rho^2(\zeta,0,\tau_0)}{\left[P_2^2(\zeta) + 2\zeta\int_0^1 P_2^2(-\eta)\rho^2(\zeta,\eta,\tau_0)d\eta\right]}, \tag{2}$$

$$\Lambda = \frac{\widetilde{b}_0}{a_{00}} = \frac{4\zeta\rho^0(\zeta,0,\tau_0)}{\varphi_0^0(\zeta,\tau_0)} = 16\zeta\rho^2(\zeta,0,\tau_0)\frac{\varphi_2^0(\zeta,\tau_0)}{\varphi_0^0(\zeta,\tau_0)\varphi_2^2(\zeta,\tau_0)}. \tag{3}$$

$$\tau_0 = \zeta\ln\left|\frac{P_N^N(\zeta)}{\sqrt{\left[\varphi_N^N(\zeta,\tau_0)\right]^2 - 2P_N^N(0)\frac{\rho^N(\xi,\zeta,\tau_0)}{\rho^N(\xi,0,\tau_0)}}\varphi_N^N(\zeta,\tau_0) - 2\zeta\int_0^1 P_N^N(\eta)\sigma^N(\eta,\zeta,\tau_0)d\eta}\right| \tag{4}$$

Primary optical parameters retrievement of system "atmosphere - underlying surface" in the case of finite optical atmospheric thickness ($\tau_0 < \infty$)

1. <u>The absent of underlying surface (A = 0):</u>

The system of linear algebraic equations of (N+1) rank for (N+1) unknown atmospheric optical parameters Λx_i(i=0,1,...,N):

$$\rho^m(\eta,\zeta,\tau_0) \text{ for } \eta = 0$$

$$\rho^m(0,\zeta,\tau_0) = \frac{\Lambda}{4}\sum_{i=m}^{N}(-1)^{i+m}\frac{(i-m)!}{(i+m)!}x_i P_i^m(0)\frac{\varphi^m(\xi,\tau_0)}{\xi}, \quad (1)$$

$$m = 0,1,...,N.$$

<u>*Formal algebraic representation of basic algebraic system*</u>

$$y_m = \Lambda x_m, \qquad b_m = 4\rho(\zeta, 0, \tau_0)\zeta \qquad (1)$$

$$\mathrm{AY} = \mathrm{B}, \qquad (2)$$

$$\mathrm{A} = \begin{pmatrix} a_{00} & a_{01} & a_{02} & \dots & a_{0N} \\ 0 & a_{11} & a_{12} & \dots & a_{1N} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_{NN} \end{pmatrix}, \quad \mathrm{Y} = \begin{pmatrix} y_0 \\ y_1 \\ \dots \\ y_N \end{pmatrix}, \quad \mathrm{B} = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{pmatrix} \qquad (3)$$

$$a_{mi} = \begin{cases} 0, & i = m + 2k - 1 \\ (-1)^k \dfrac{(2k-1)!!}{2(m+k)!!}\varphi_i^m(\zeta) \end{cases} \qquad (4)$$

$$\det \mathrm{A} = \prod_{m=0}^{N} a_{mn} \Rightarrow \frac{\varphi(\eta,\tau_0)\varphi(\xi,\tau_0)}{(2m)!} \qquad (5)$$

<u>*Unique solution of algebraic system:*</u>

$$y_m = \frac{\widetilde{b}_m}{a_{mm}}, \quad m=0,1,2,...,N, \qquad (6)$$

where

$$\widetilde{b}_{N-k}^{m} = b_{N-k} - \left[\frac{a_{N-k,N}}{a_{NN}}\widetilde{b}_N + \frac{a_{N-k,N-1}}{a_{N-1,N-1}}\widetilde{b}_{N-1} + \frac{a_{N-k,N-k+1}}{a_{N-k+1,N-k+1}}\widetilde{b}_{N-k+1}\right],$$

$$k=1,2,...,N. \qquad (7)$$

The case of isotropic light scattering

x(cosγ) = 1

$$\Lambda = \frac{4\eta\rho^0(\eta,0,\tau_0)}{1+2\eta\int_0^1 \rho^0(\eta,\eta',\tau_0)d\eta'} \tag{1}$$

$$\tau_0 = \zeta \ln\left|\frac{P_N^N(\zeta)}{\sqrt{\left[\varphi_N^N(\zeta,\tau_0)\right]^2 - 2P_N^N(0)\dfrac{\rho^N(\xi,\zeta,\tau_0)}{\rho^N(\xi,0,\tau_0)}\varphi_N^N(\zeta,\tau_0) - 2\zeta\int_0^1 P_N^N(\eta)\sigma^N(\eta,\zeta,\tau_0)d\eta}}\right| \tag{2}$$

Λ = 1

In the case of arbitrary atmospheric phase function *x(cosγ)* for $\Lambda = 1$:

$$\int_0^1 \varphi_{2i}^0(\eta)(\tau_0+\eta)d\eta - \int_0^1 \psi_{2i}^0(\eta)\,\eta d\eta = 2\tau_0\delta_{2i,0} \tag{3}$$

$$\int_0^1 \varphi_{2i+1}^0(\eta)(\tau_0+\eta)d\eta + \int_0^1 \psi_{2i+1}^0(\eta)\,\eta d\eta = \frac{2}{3}\delta_{2i+1,1}$$

$$\int_0^1 \left[\varphi_i^0(\eta) + (-1)^i\psi_i^0(\eta)\right]d\eta = 2\delta_{i,0} \tag{4}$$

$$\tau_0 = \frac{\int_0^1 \left\{4\eta\rho^0(\eta,0,\tau_0) - \sqrt{8\eta\left\{2\eta\left[\rho(\eta,0,\tau_0)\right]^2 - \rho(\eta,\eta,\tau_0)\right\}}\right\}\eta d\eta}{1 - \frac{1}{2}\int_0^1 \left\{4\eta\rho(\eta,0,\tau_0) - \sqrt{8\eta\left\{2\eta\left[\rho(\eta,0,\tau_0)\right]^2 - \rho(\eta,\eta,\tau_0)\right\}}\right\}d\eta} \tag{5}$$

b) The case of a planet atmosphere bounded from below an arbitrary horizontally uniform orthotropic underlying surface with albedo A(ξ)

$$\bar{\rho}^m(\eta,\zeta,\tau_0) = \rho^m\ (\eta,\zeta,\tau_0) + \frac{A(\zeta)}{1-A(\zeta)c(\tau_0)}\mu(\eta,\tau_0)\ \mu(\zeta,\tau_0)\,,$$

$$m = 0,1,..., N, \qquad (1)$$

Surface albedo $A(\xi)$ is determined by zero azimuthal harmonics $\bar{\rho}_0(\eta,\zeta,\tau_0)$ only.

Optical parameter Λx_i (i=1,2,…,N) are retrieved from azimuthal harmonics $\bar{\rho}^m(\eta,\zeta,\tau_0)$ for $m \geq 1$ only.

Afterword the initial azimuthal harmonics $\bar{\rho}^m_{retr}(\eta,\zeta,\tau_0)$ for m=0,1,…,N are retrieded accoding to direct scheme of radiative transfer problem solution. Finally surface albedo A(ξ) is retrieved according to following relation

$$A(\zeta)=\frac{\left|\bar{\rho}^0(\eta,\zeta,\tau_0)-\rho^0(\eta,\zeta,\tau_0)\right|}{\left|\bar{\rho}^0(\eta,\zeta,\tau_0)-\rho^0(\eta,\zeta,\tau_0)\right|c(\tau_0)+\mu(\eta,\tau_0)\mu(\zeta,\tau_0)}\geq 0. \qquad (2)$$

c) Orthotropic horizontally non-uniform underlying surface placed on the level $\tau = \tau_0$

The generalization of Sobolev's- Van de Hulst's representation following conditions are:

- primary light reflection from underlying surface is calculated exactly for real values of surface albedo A(x,ξ)
- multiple light reflection from underlying surface is calculated approximately for average values of surface albedo $\bar{A}$ (x,ξ)
- surface reflected radiation is transformed by system "atmosphere-underlying surface" uniformly for all spatial frequencies ν

Basic radiation field approximation, given by O.Smokty:

$$\bar{\rho}^m(x-\Delta x,\eta,\varphi,\tau_0) = \rho^m(\eta,\xi,\varphi,\tau_0) + \frac{A(x-\Delta x,\xi)\mu(\xi,\tau_0)}{\bar{A}(x-\Delta x,\xi)c(\tau_0)},$$

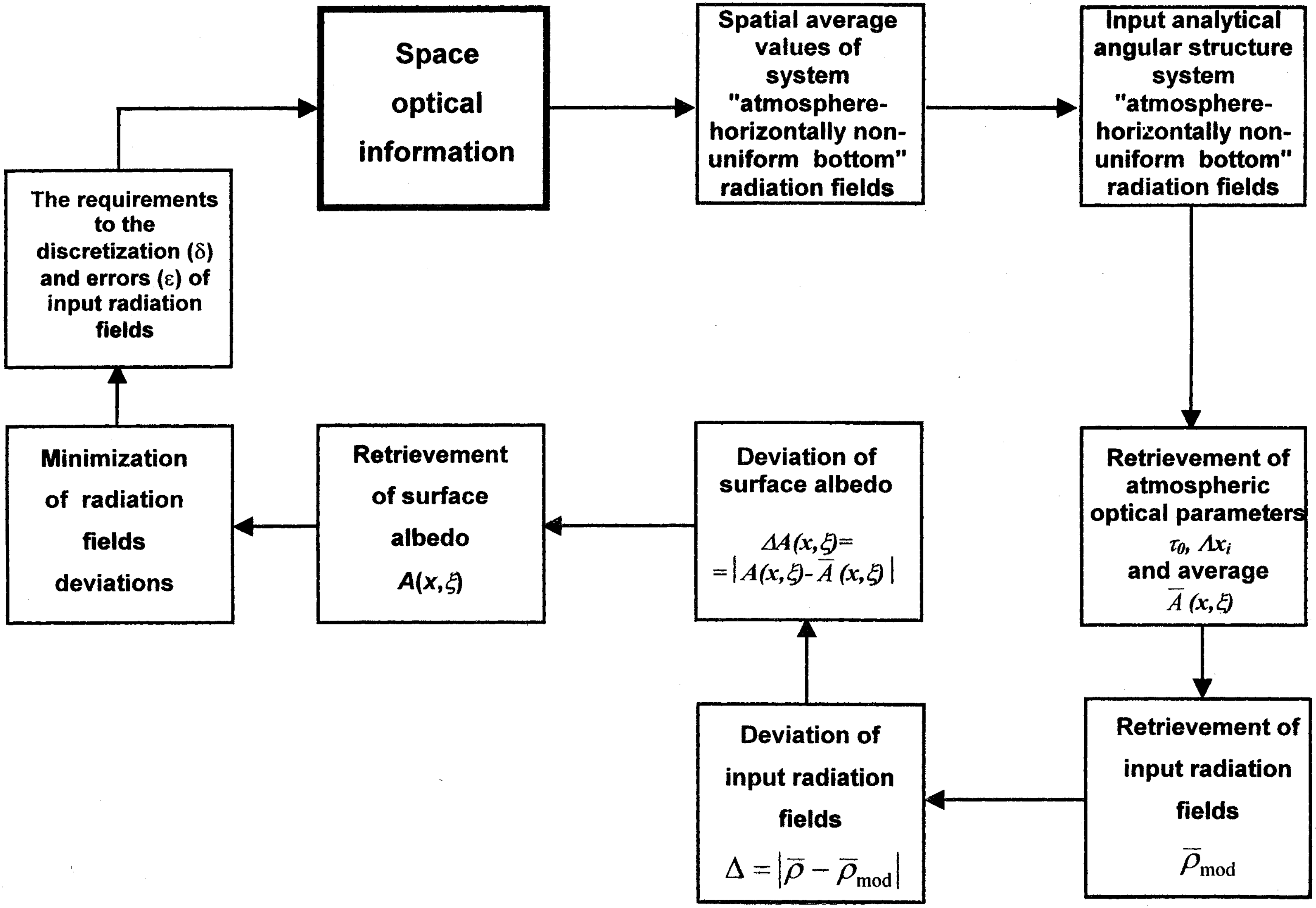
$$m = 0,1,\ldots,N, \qquad (1)$$

where horizontal shift Δx of coordinate x due to inclined vision is equal

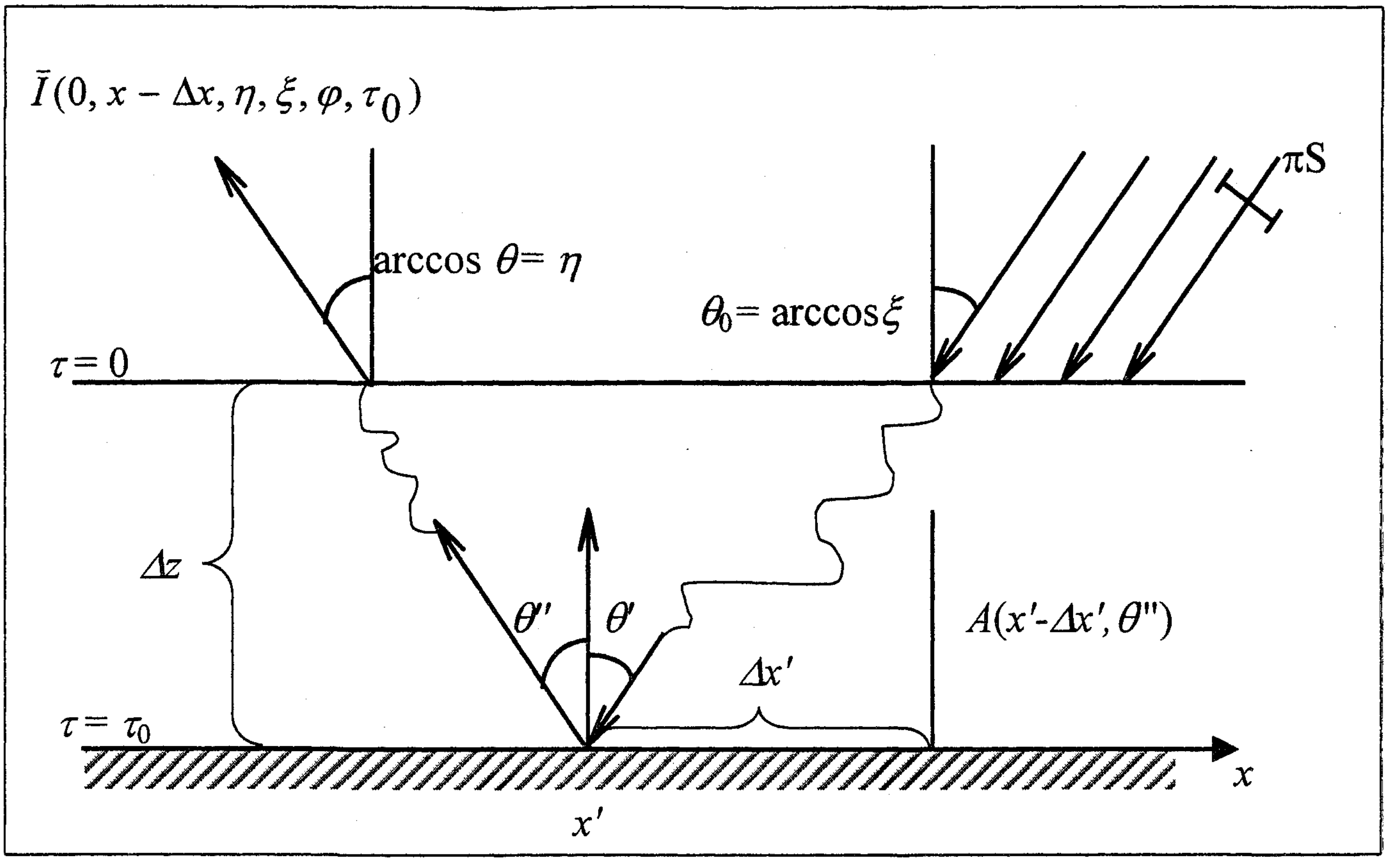
$$\Delta x = \sqrt{1-\eta^2}\,\frac{\Delta z}{\eta}\cos\varphi. \qquad (2)$$

As well as in the case $A(\xi) \neq 0$ angular structure of reflected radiation field determines atmospheric optical parameters from azimuth's harmonics ρ^m ($m \geq 1$). The zero harmonics ρ^0 ($m = 0$) determines real albedo of underlying surface $A(x, \xi)$.

General scheme of optical parameters retrievement for system "atmosphere-reflecting bottom" in the case of horizontally non-uniform orthotropic underlying surface



Retrievement primary of optical parameters for system "atmospheric-reflecting bottom" in the case of arbitrary horizontally non-uniform orthotropic underlying surface



Azimuthal harmonics $\bar{\rho}^m(x-\Delta x, \eta, \varphi, \tau_0)$ according to approximations given by O.Smokty:

$$\bar{\rho}^m(x-\Delta x, \eta, \varphi, \tau_0) = \rho^m(\eta, \xi, \varphi, \tau_0) + \frac{A(x-\Delta x, \xi)\mu(\xi, \tau_0)}{\bar{A}(x-\Delta x, \xi)c(\tau_0)}, \; m = 0,1,\ldots, N, \tag{1}$$

1) Horizontally average value of $\bar{\rho}^m(x-\Delta x, \eta, \varphi, \tau_0)$:

$$\bar{\bar{\rho}}(\eta, \xi, \varphi, \tau_0) = \frac{1}{\mathcal{D}} \int_{\mathcal{D}} \bar{\rho}\ (x'-\Delta x', \eta, \xi, \varphi, \tau_0) dx' \tag{2}$$

2) The retrievement of atmospheric optical parameter τ_0, Δx_i and average surface albedo $\bar{A}(x, \xi)$ according to referred above scheme $A(x, \xi)$.

3) The retrievement of average azimuthal harmonics: $\bar{\rho}_{\text{mod}}(\eta, \xi, \tau_0)$

4) Values of radiation fields deviation: $\Delta\bar{\rho}^0 = \left| \bar{\rho}^0 - \bar{\rho}_{\text{mod}} \right|$ (3)

5) Values of surface albedo deviation: $\Delta A(x, \xi) = \left| A(x, \xi) - \bar{A}(x, \xi) \right|$ (4)

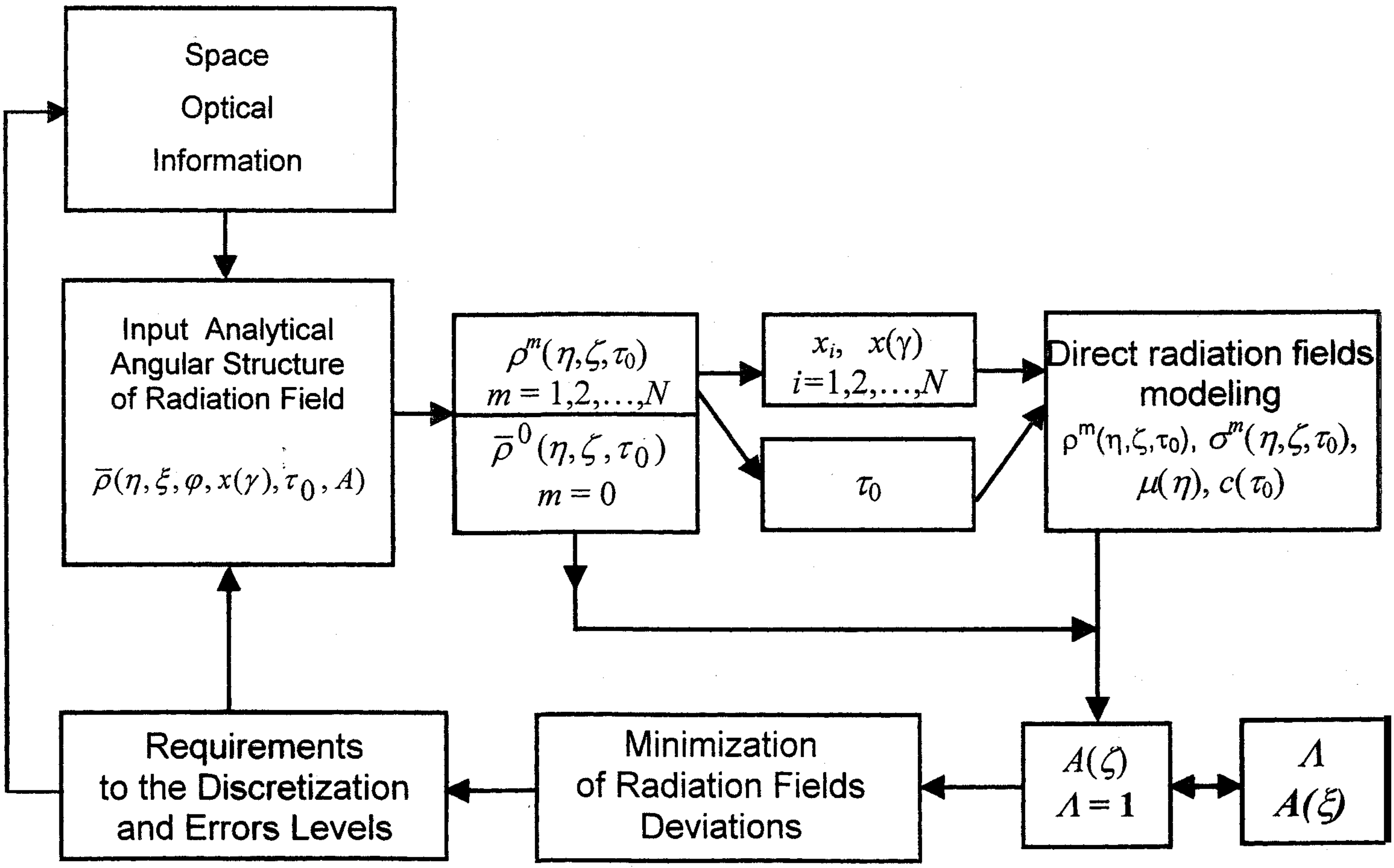
6) Retrievement values of surface albedo $A(x, \xi)$:

$$A(x, \xi) = \frac{\Delta\bar{\rho}^0(x-\Delta x, \xi, \eta, \tau_0)\left[1 - \bar{A}(\xi)c(\tau_0)\right]}{\mu(\xi, \tau_0)\mu(\eta, \tau_0)} \tag{5}$$

General scheme of optical parameters retrievement for the atmosphere bounded from below an orthotropic underlying surface

Unknown optical parameters

- phase function $x(\gamma)$
- optical thickness τ_0
- albedo of underlying surface $A(\zeta)$
- single scattering albedo Λ for $A(\zeta)$ = **0**



Numerical Results:

- *solution's stability,*
- *informational contents compression,*
- *influence of errors and discretization levels*

Atmospheric phase function $x(\cos\gamma)$:

$$x_g(\cos\gamma) = \frac{1-g^2}{(1+g^2-2g\cos\gamma)}, \qquad g = 0.2 - 0.9$$

Atmospheric optical thickness τ_0: $\tau_0 = 0.1 - 0.6$

Surface albedo $A(\xi)$: $A(\xi) = 0.0 - 0.9$

Angular variables (η,ξ): $\xi \in [0,1], \quad \eta \in [1,1]$

Informational contents compression:

- current frequency of spatial-angular discretization (**ε**) for input (retrieved) optical models $\{G\}$
- highest frequency of spatial-angular discretization (***N***) for azimuthal harmonics of input (retrieved) radiation fields

$$I^m(\tau,\eta,\xi,\tau_0) = \hat{L}\{G^m\}$$

According Nyquist-Kotelnikov's theorem: $\varepsilon \geq N$.

For primary light atmospheric scattering and surface reflection the empirical estimation in the frame of accuracy $\sim 10^{-4}$ is given by O.Smokty:

$$\varepsilon \approx 2N + 30$$

Finally empirical correlation for multiple light atmosphere scattering and surface reflection is following:

$$\varepsilon \geq 2qN, \qquad q \sim 1/6$$

Real calculations: total number of harmonics $N \sim 200 - 300$, angular compressions from $\varepsilon \approx 400 - 600$ points to $\delta \approx 130 - 200$ points for given $\delta \sim 10^{-4}$.

Optical models:

RADCOM aerosol model:

$$\left.\begin{aligned}\tau_0 &= 0.226 \div 0.680 \\ \Lambda &= 0.880 \div 0.942\end{aligned}\right\} \quad \lambda = 0.4 \div 0.8 \quad \text{mkm}$$

Henny-Greenstein phase function:

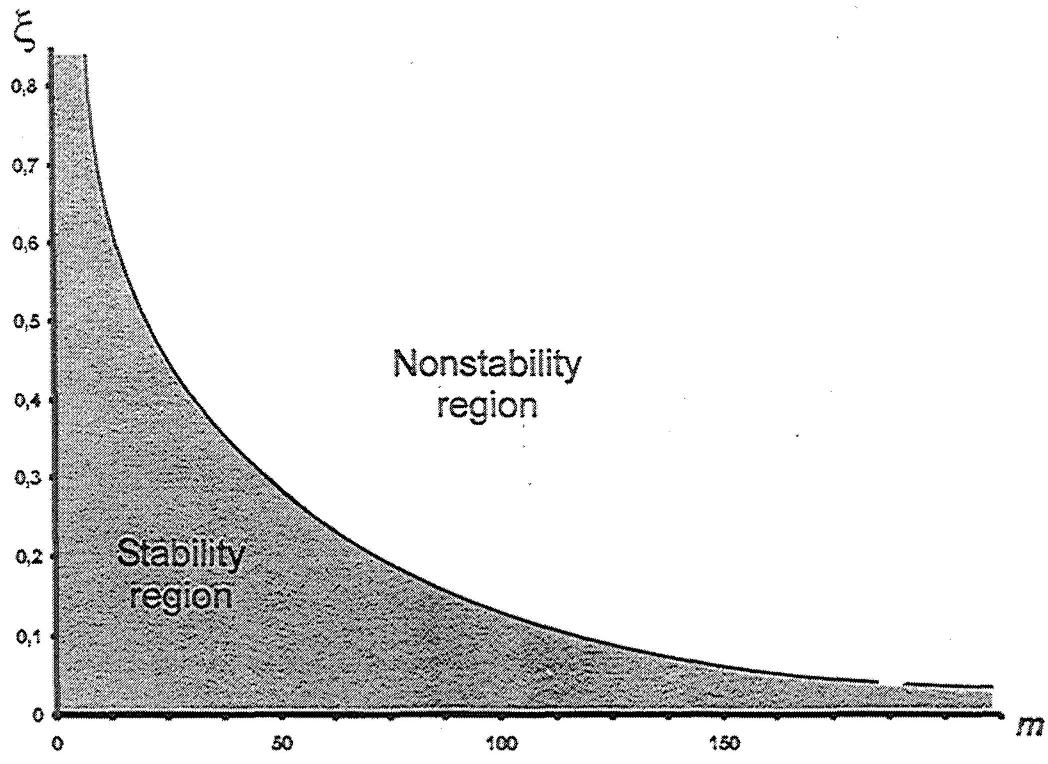
$$x(\gamma) = \frac{1 - g^2}{\left(1 + g^2 - 2g\cos\gamma\right)^{3/2}}, \quad |g| < 1$$

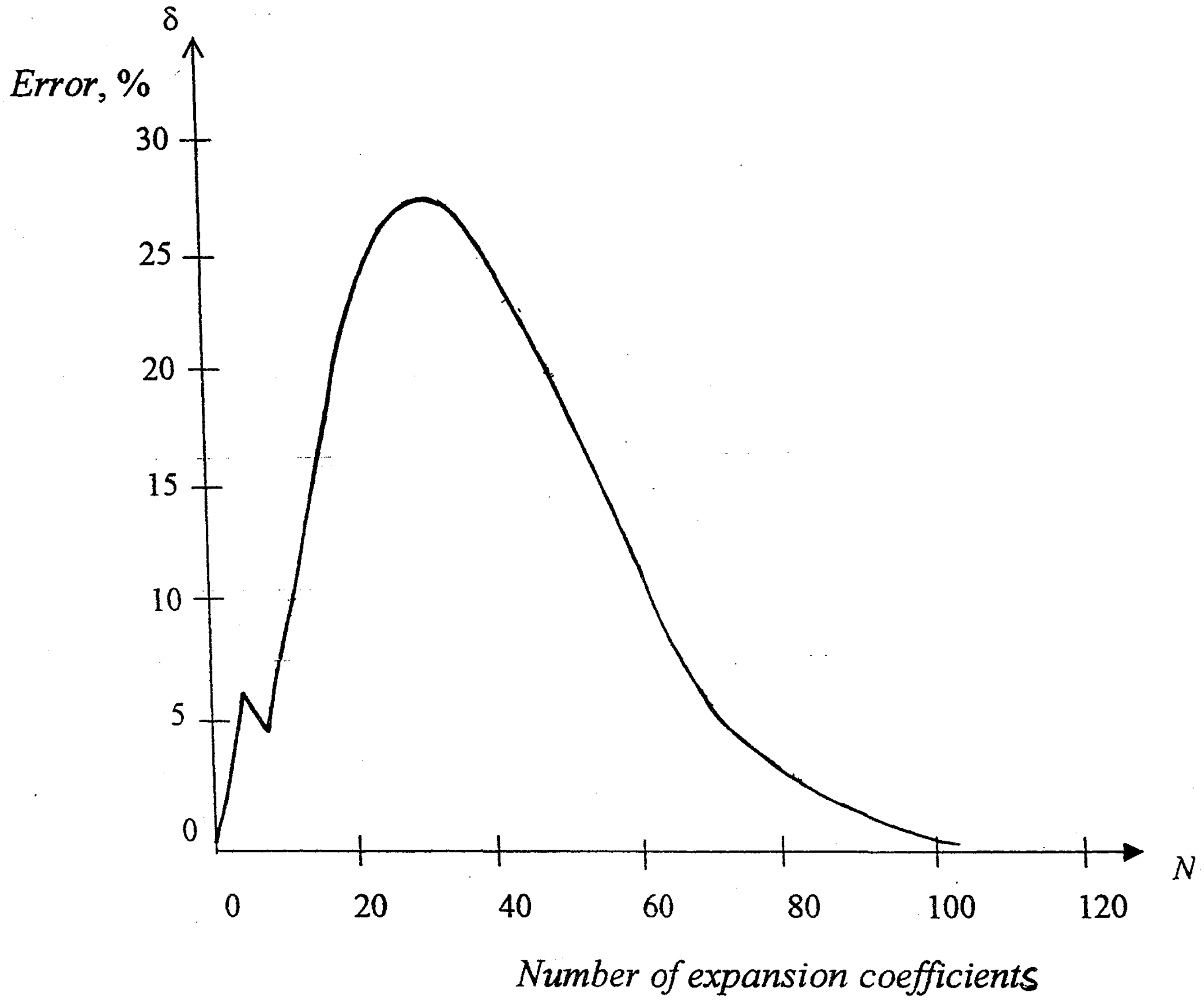
Surface albedo:

$$A = 0.15 \div 0.8$$

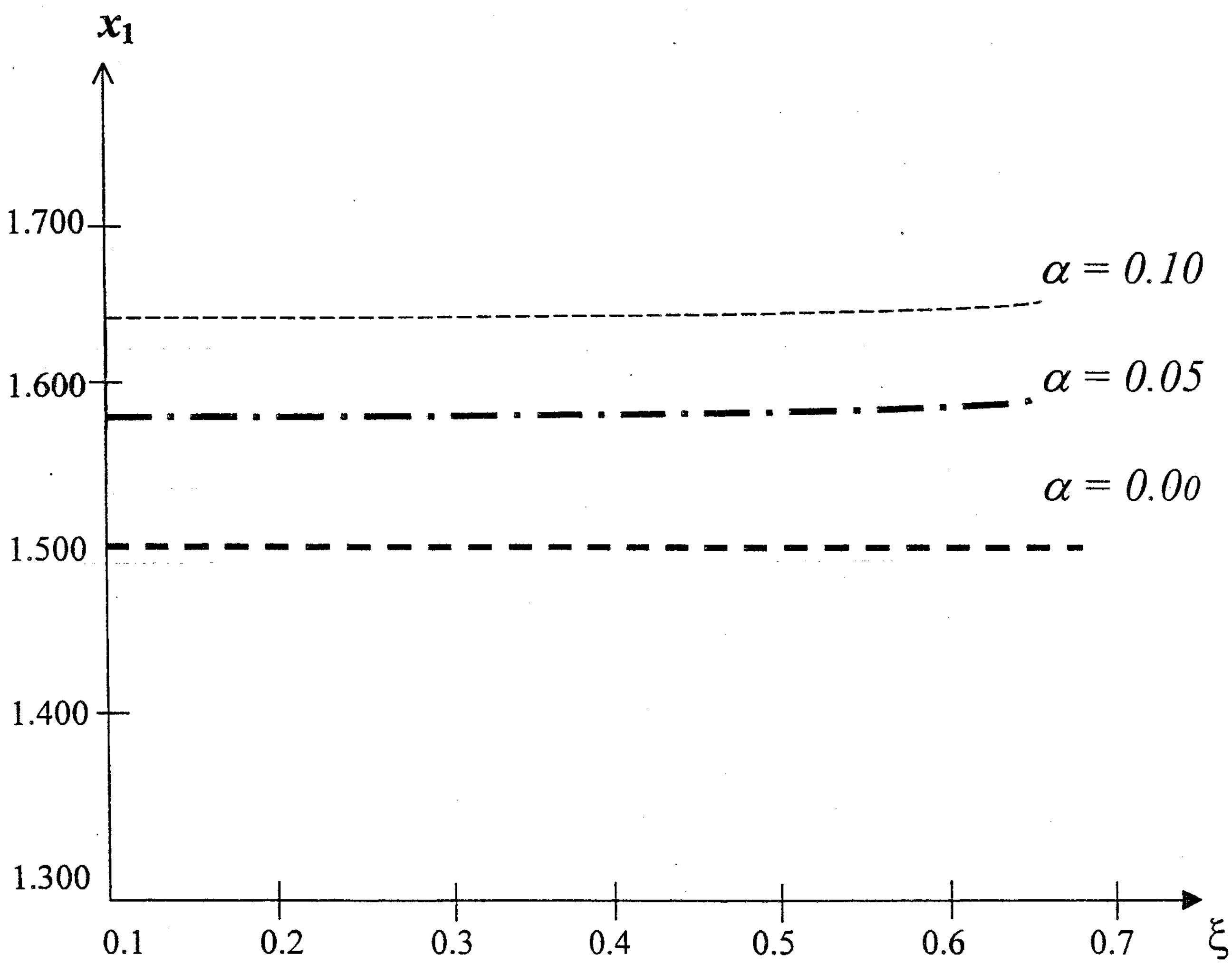
The criteria of inverse problem solutions stability:

$$\xi < \left[\frac{6m + 5}{(2m+3)(2m+1)}\right]^{1/2}$$

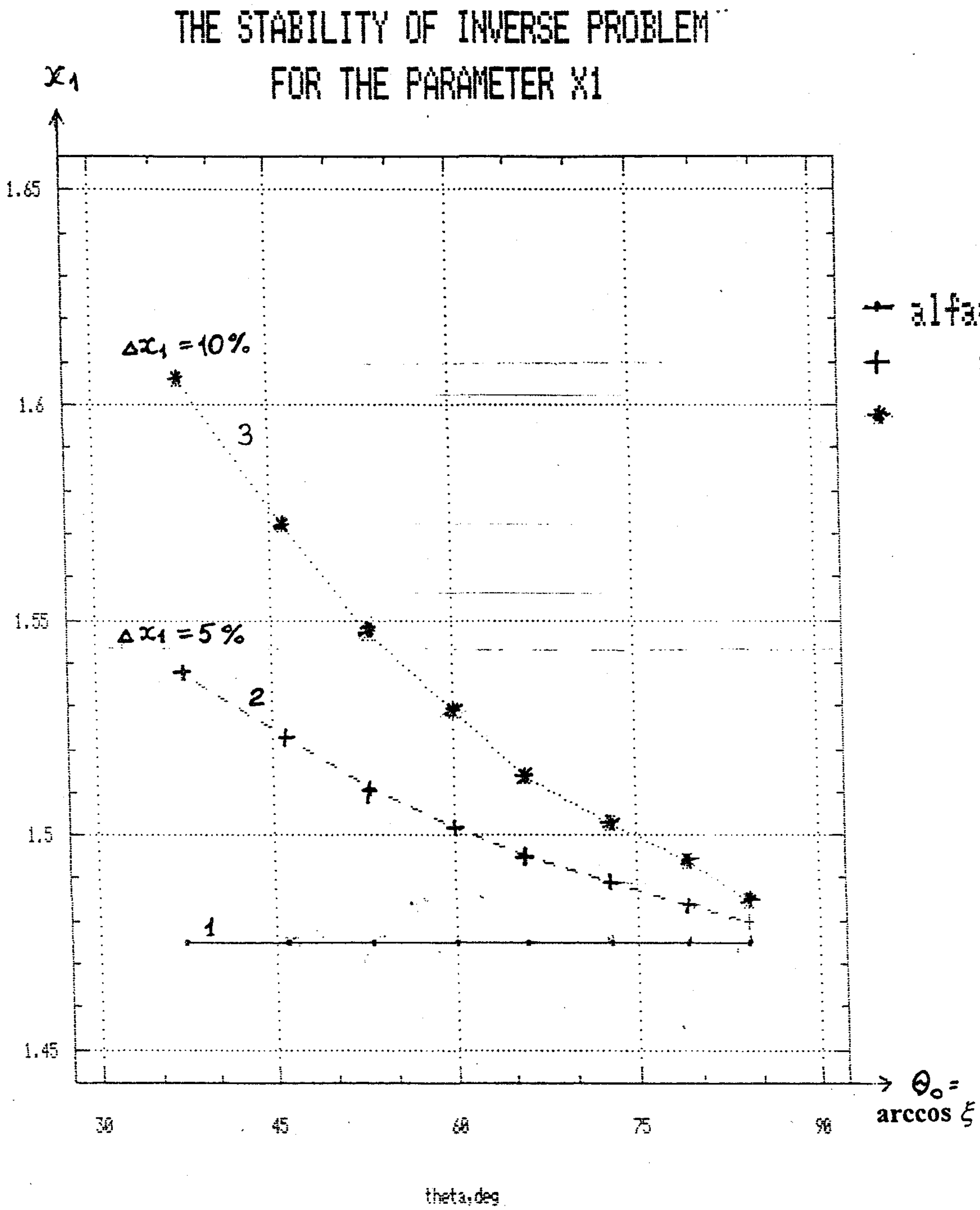




Error in expansion coefficient, ζ_m=0,2 ; g=0,9; N≈170

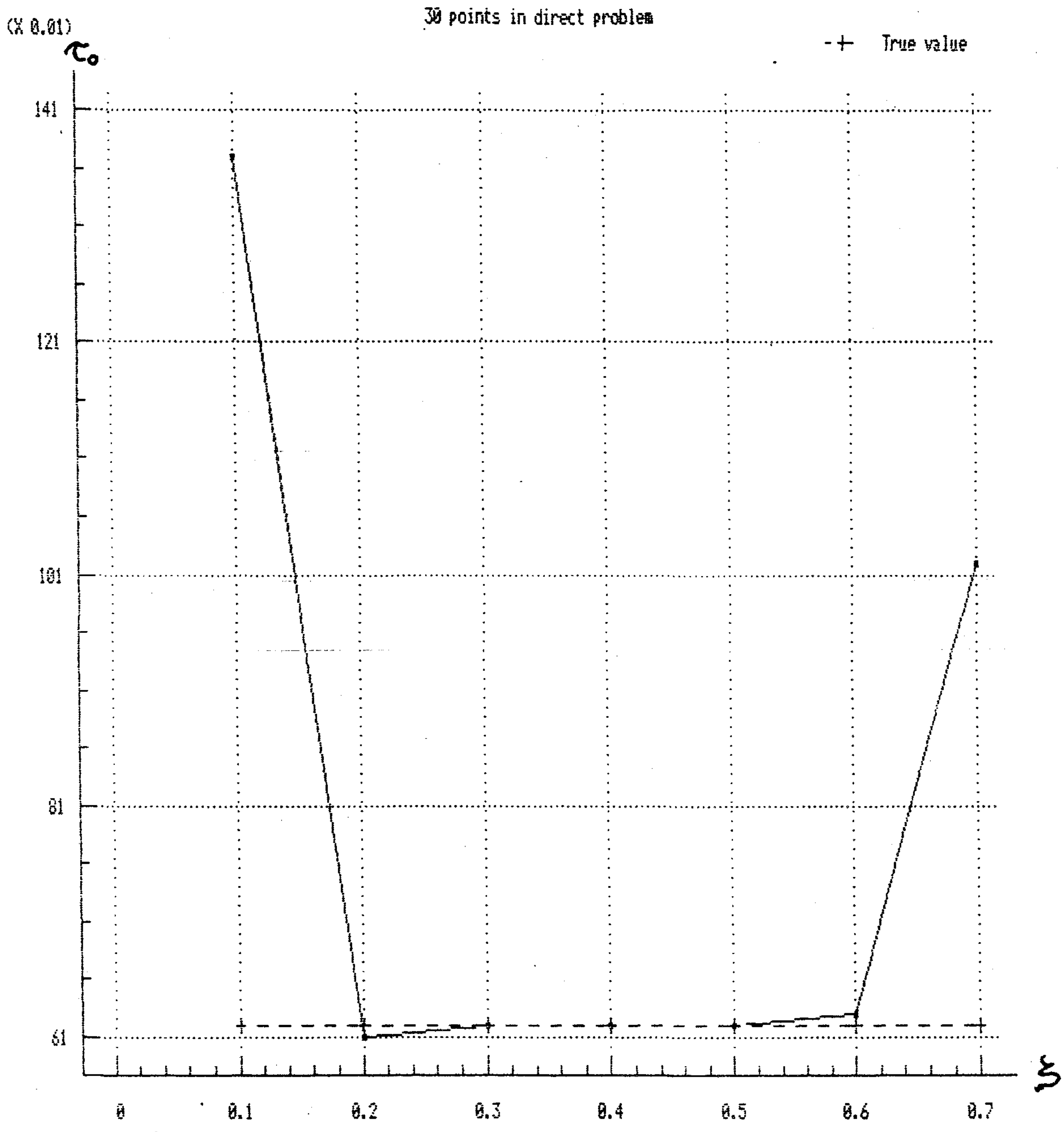


Stability of expansion coefficient x_1 retrieving in dependence on perturbation parameter α and angular value ξ.

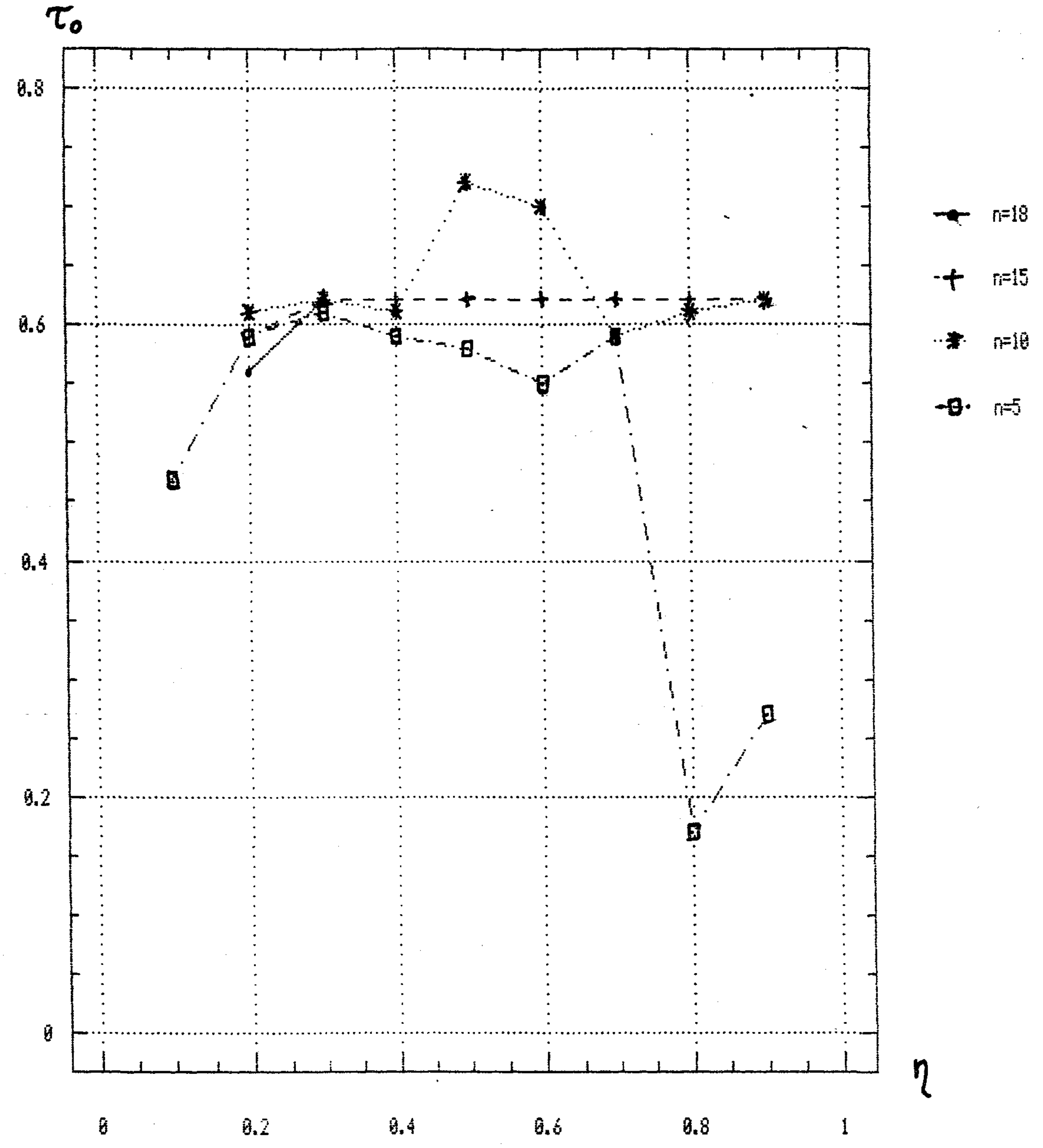


The stability of inverse problem for the parameter x_1:

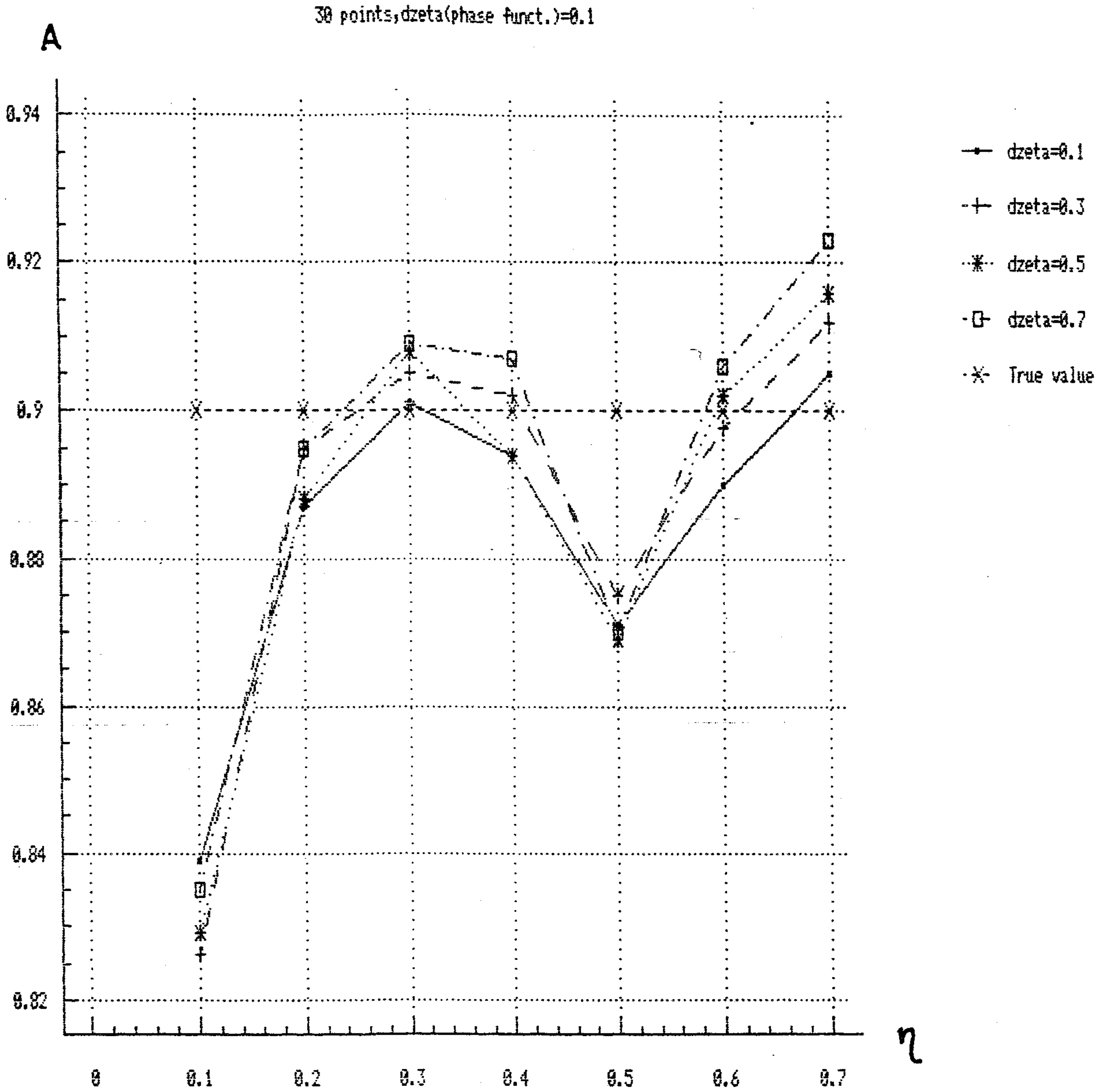
1 - $\alpha = 0{,}00$
2 - $\alpha = 0{,}05$
3 - $\alpha = 0{,}10$



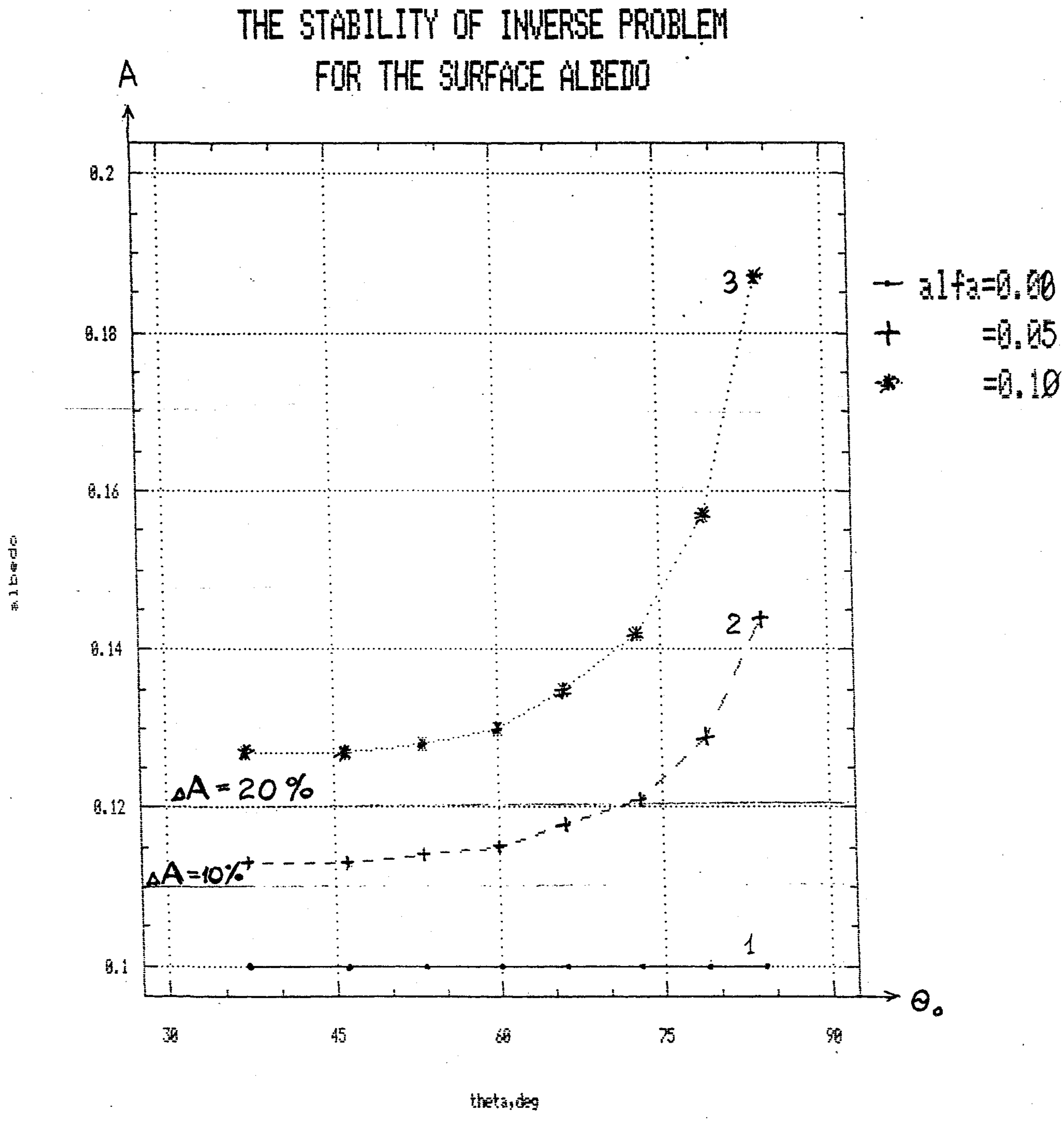
Optical depth retrieving in dependence on angular value ξ and discretization level n ($g = 0.5$).



Optical depth retrieving in dependence on angular value ξ
(discretization level n =30 and g = 0.5).

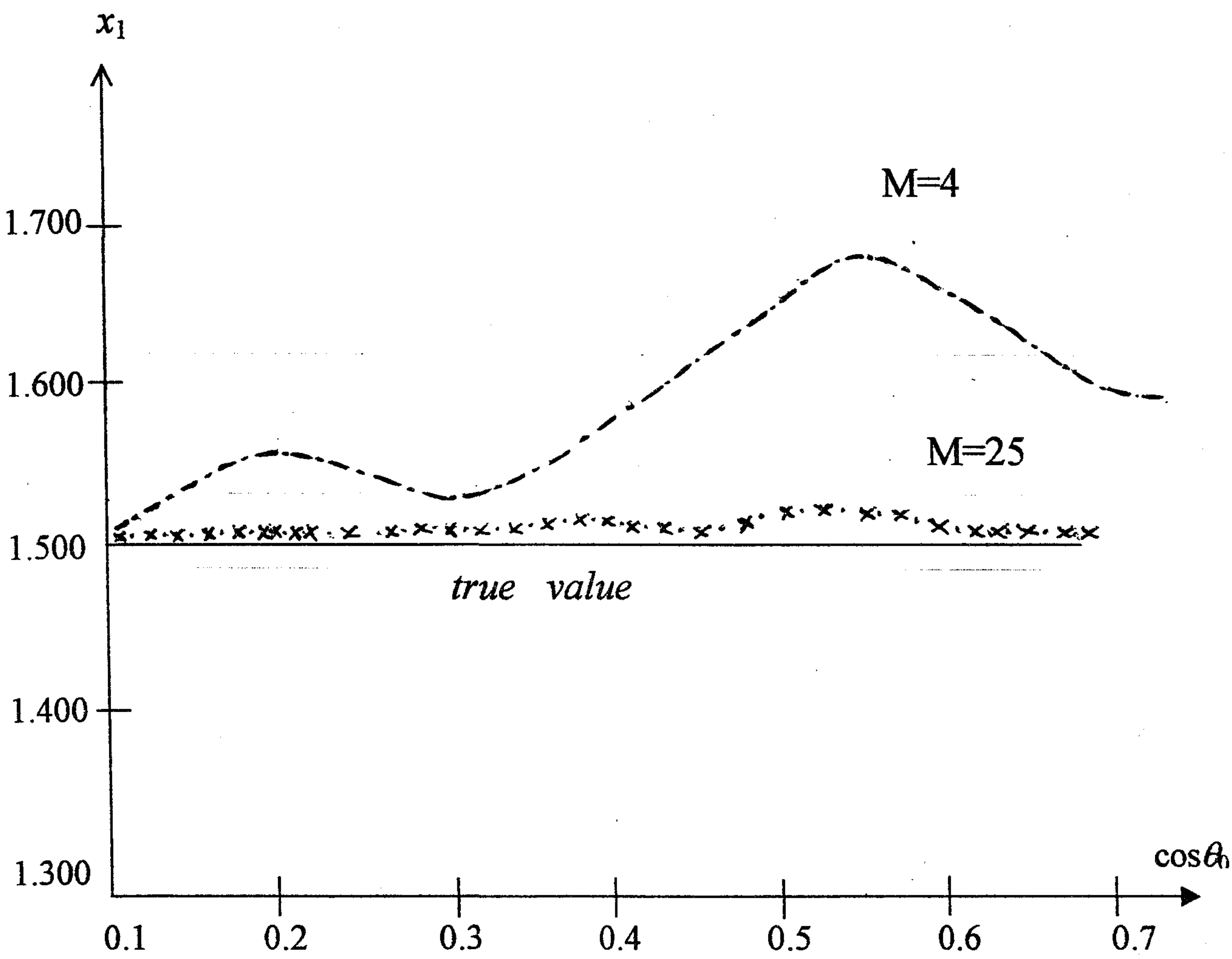


Surface albedo retrievement in dependence on angular value ξ and η (discretization level n =30 and g = 0.5).



The stability of inverse problem for the parameter for the surface albedo:

1 - $\alpha = 0{,}00$
2 - $\alpha = 0{,}05$
3 - $\alpha = 0{,}10$



The atmospheric phase function retrieving in dependence on angular grid points (M).

CONCLUSION

1. Analytical approach to conciliated construction of precise direct ($\hat{L}$) and inverse ($\hat{L}^{-1}$) operators for radiative transfer theory has been developed making use of Ambarzumyan's angular structure of input radiation fields.

2. In the case of the atmosphere bounded from below an arbitrary orthotropic underlying surface the retrievement of primary optical parameters has been carried out ($\tau_0 < \infty$).

3. The problem of stability of "atmosphere-underlying surface" system and errors-discretisation levels for retrieval optical parameters has been considered

Recommended References

1. **V. Ambarzumyan**, Naychuye Trudy, v.1. ,. Erevan, 1960.

2. **S. Chandrasekhar**, Radiative Transfer, Oxford, 1950.

3. **V. Sobolev** Light Scattering in the Planetary Atmosphere, Pergamon Press, 1975.

4. **Van de Huest** Multiple Light Scattering: Tables, Formula and Calculations, Academic Press, 1992.

5. **O. Smokty** Modelling of Radiation Fields in the Problems of Space Spectrophotometry, Pub. co. Nauka, St. Petersburg, 1986.

6. **O. Smokty** Solving of the Inverse Problems of Atmospheric Optics on the Basis of Environment Space Remote Sensing Information, In Proc. IRS '88, Lille, France, 1998, pp. 221-223

7. **O. Smokty**, K. Kondratyev, V. Kozoderov. Remote Sensing of the Earth from Space: Atmospheric, Correction, Springer Verlag, 1992, 592 p.

8. **O. Smokty** The Angular Structure Method of Inverse Problems Solution of Radiative Transfer Theory. In Proc. IGARSS '93, Tokyo, 1993, pp. 2155-2157.

9. **O. Smokty** The Basic Informational Contents Levels for Satellite Ecological Investigation. In Proc. ICIECC ' 97, v.3, St. Petersburg, pp 1337-1344.

10. **O. Smokty**, A. Anikonov. Inverse Problems Solutions in Radiative Transfer Theory. In Proc. SPIIRAS, 1998, p.57.

11. **O. Smokty** Applied Mathematical Problems of Satellite Data Filtration and Atmospheric Correction. In Proc. International Conf. Atmospheric Correction of Satellite Data, Chiba Univ., Tokyo, Japan, 1998, pp. 352-360.

12. **O. Smokty** Calibrated Method of Direct-Inverse Problems Solutions for Multiple Light Scattering in a Vertically Uniform Slab, Proc. IRS '2000, St. Petersburg, Publ. C. Deepak, 2000.