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**"Retrieval Approach for the Tropospheric Emission Spectrometer"**

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***Please note: These are preliminary notes intended for internal distribution only.***



# Retrieval Approach for the Tropospheric Emission Spectrometer

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The Tropospheric Emission Spectrometer is a high-resolution Michelson interferometer designed to measure thermal emission from the atmosphere in both limb and nadir modes. It will fly on NASA's Aura satellite in 2003, together with MLS, HiRDLS and OMI. The experimental objectives are to provide a global view of the chemical state of the troposphere and UTLS regions, focussing particularly on the global distribution of tropospheric ozone and the factors that control ozone concentrations.

The spectral sampling of TES is  $0.015\text{ cm}^{-1}$  in the limb mode, and  $0.06\text{ cm}^{-1}$  in the nadir mode. There is an array of 16 detectors, and most of the thermal infrared is measured. The result is a very large quantity of data, about  $10^6$  radiances per profile, so that efficiency of data processing is paramount.

This paper will describe the methods that will be used, including the design of the forward model and its derivatives, the retrieval approach, the selection of spectral windows to be used, the error analysis.

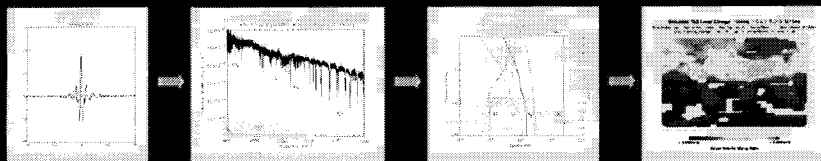
## TES Algorithm Overview

Level 1A: Produces geolocated interferograms.

Level 1B: Produces radiometrically and frequency calibrated spectra with NESR.

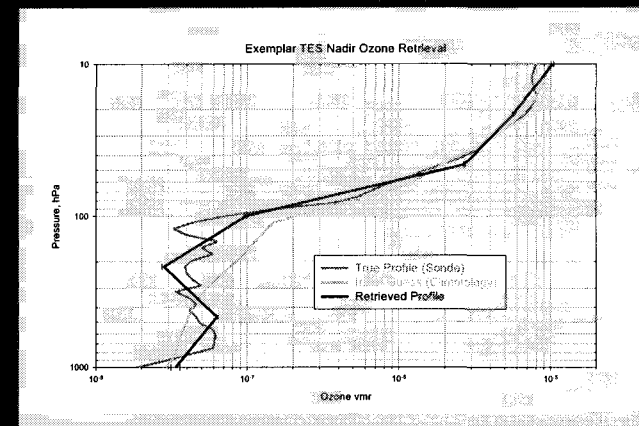
Level 2: Produces VMR and temperature profiles.

Level 3: Produces global maps.

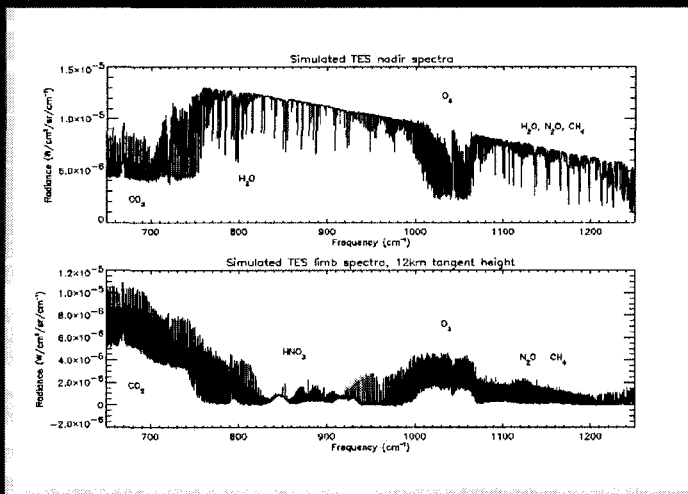


## Level 2 Products

TES Nadir O3 retrievals will typically have 3 layers in the troposphere.

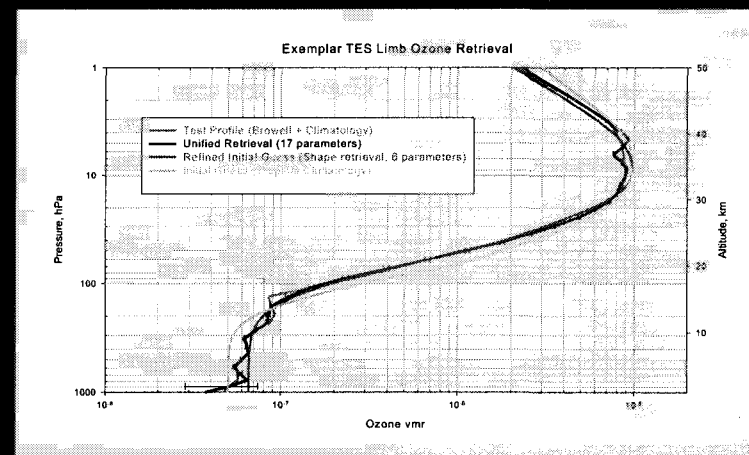


## Level 1B Products

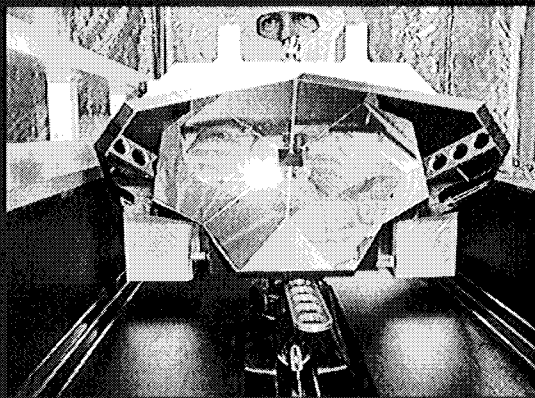


## Level 2 Products

TES Limb O3 retrievals may have as many as 7 layers in the troposphere.



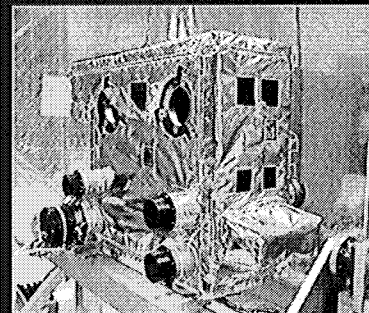
## TES Instrument Specifications



View of the TES engineering model interferometer retroreflector

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## TES Interferometer and Detectors



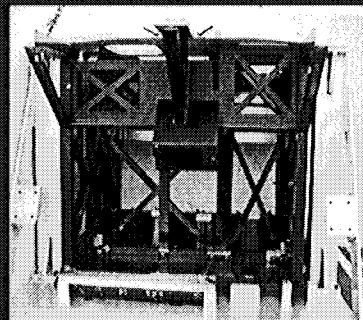
View of the TES engineering model interferometer with thermal blanket

Spectrometer Type	Connes'-type 4-port Fourier Transform Spectrometer
Max. Optical Path Difference	$\pm 8.45$ cm (nadir & calibration) $\pm 33.8$ cm (limb), interchangeable
Scan (integration) Time	4 sec (nadir & calibration) 16 sec (limb)
Interferogram Sampling Metrology	Nd:YAG laser
Spectral Resolution (unapodized)	$0.06$ $\text{cm}^{-1}$ (nadir) $0.015$ $\text{cm}^{-1}$ (limb)
Spectral Coverage	$650$ to $3050$ $\text{cm}^{-1}$ ( $3.2$ to $15.4$ $\mu\text{m}$ )
Detector Arrays	4 (1 x 16) arrays, optically- conjugated, all MCT PV @65K
Signal-to-Noise Ratio (spectral)	Up to 600:1 Minimum requirement is 30:1

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## TES Allocations

Mass	385 kg
Average Power	334 W
Peak Power	361 W
Avg. Data Rate	4.5 Mbps
Peak Data Rate	6.2 Mbps
Actual Size	$1.0 \times 1.3 \times 1.4$ m (with earth shade stowed)
Lifetime	5 years on orbit

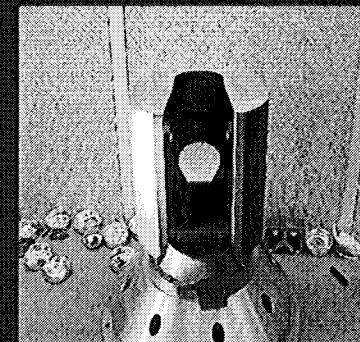


TES Structural Housing

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## TES Pointing and Calibration

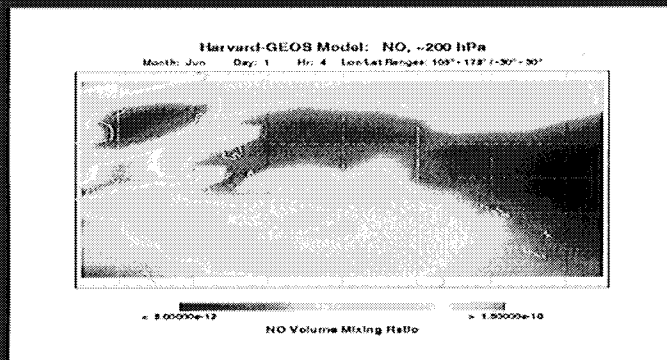
Field of Regard	$45^\circ$ cone about nadir; trailing limb or cold space; internal calibration sources
Pointing Accuracy	$75$ $\mu\text{rad}$ pitch $750$ $\mu\text{rad}$ yaw $1100$ $\mu\text{rad}$ roll
Maximum Stare Time, Nadir	208 sec (40 nadir scans)
Spatial Resolution	$0.5 \times 5$ km (nadir) $2.3 \times 23$ km (limb)
Radiometric Calibration	Internal, adjustable, cavity blackbody (340K) + cold space view
Radiometric Accuracy	$= 1\text{K } 650 - 2500$ $\text{cm}^{-1}$ $= 2\text{K } 2500 - 3050$ $\text{cm}^{-1}$ Internal thin slit source
Detector Array Co- alignment Calibration	



TES gimbal pointing mirror

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**Tropospheric Chemistry Hypothesis 2: Changes in tropospheric  $O_3$  perturb the natural abundance of OH, the main atmospheric oxidant / cleanser, with wide ranging environmental consequences.**



TES will measure most of the important IR-active species in the tropospheric  $O_3$  formation/destruction cycle, from which the OH abundance may be inferred. TES will be able to resolve different sources of  $O_3$  precursors in the upper troposphere, e.g., biomass burning vs. non-anthropogenic sources of NOx such as lightning.

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### TES Standard Products & Required Sensitivity

Product Name	Product Source		Required Sensitivity <sup>1</sup>
	Nadir	Limb	
Level 1A Interferograms	✓	✓	
Level 1B Spectral Radiances	✓	✓	
Atmospheric Temperature Profile	✓	✓	0.5 K
Surface Skin Temperature	✓		0.5 K
Land Surface Emissivity <sup>2</sup>	✓		0.01
Ozone ( $O_3$ ) VMR Profile	✓	✓	1 - 20 ppbv
Water Vapor ( $H_2O$ ) VMR Profile	✓	✓	1 - 200 ppmv
Carbon Monoxide (CO) VMR Profile	✓	✓	3 - 6 ppbv
Methane ( $CH_4$ ) VMR Profile	✓	✓	14 ppbv
Nitric Oxide (NO) VMR Profile		✓	40 - 80 pptv
Nitrogen Dioxide ( $NO_2$ ) VMR Profile		✓	15 - 25 pptv
Nitric Acid ( $HNO_3$ ) VMR Profile		✓	1 - 10 pptv
Nitrous Oxide ( $N_2O$ ) VMR Profile	✓	✓	Control <sup>3</sup>

<sup>1</sup> Sensitivity range maps to expected concentration range.  
NOx measurements may need averaging to meet these requirements.

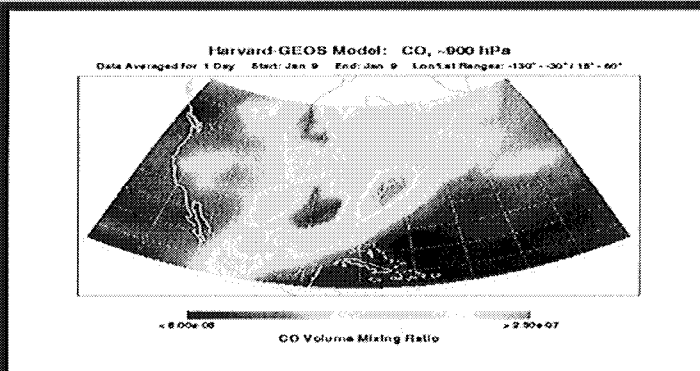
<sup>2</sup> Water (and, probably, snow & ice) emissivities are known and are therefore input, not output parameters.

<sup>3</sup> Tropospheric concentration known.

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**Tropospheric Chemistry Hypothesis 3: Increasing emissions of  $O_3$  and aerosol precursors and CO will impact air quality worldwide.**

- smog reduction by local emission controls could be offset by cumulative increases in global emissions.



TES will measure tropospheric carbon monoxide (CO), a major component of atmospheric pollution that will allow TES to track continental outflows.

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### Potential Special (Research) Products for TES

Chemical Group	Common Name	Formula	Product Source	
			Nadir	Limb
$H_2O_2$	Hydrogen Peroxide	$H_2O_2$		✓
	Monodeuterated Water Vapor	$HDO$	✓	✓
	Ethane	$C_2H_6$		✓
	Acetylene	$C_2H_2$		✓
	Formic Acid	$HC(=O)OH$	✓	✓
	Methyl Alcohol	$CH_3OH$	✓	✓
O-compounds	Peroxyacetyl Nitrate	$CH_3C(=O)OONO_2$		✓
	Acetone	$CH_3C(=O)CH_3$		✓
	Ethylene	$C_2H_4$		✓
	Peroxyacetic Acid	$HO_2C(=O)OH$		✓
	Ammonia	$NH_3$	✓ <sup>1</sup>	✓
N-compounds	Hydrogen Cyanide	$HCH$		✓
	Dinitrogen Pentoxide	$N_2O_5$		✓
	Hydrogen Chloride	$HCl$	✓ <sup>1</sup>	
	Chlorine Nitrate	$ClONO_2$		✓
	Carbon Tetrachloride	$CCl_4$		✓
Halogen compounds	$CF_2$	$CF_2$	✓	✓
	$CF_2$	$CF_2$	✓	✓
	$HOFC-21$	$CHClF_2$		✓
	$HOFC-22$	$CHClF_2$		✓
	Sulfur Dioxide	$SO_2$	✓	✓
S-compounds	Carbonyl Sulfide	$OCS$	✓	✓
	Hydrogen Sulfide	$H_2S$	✓ <sup>1</sup>	✓
8 compounds	Sulfur Hexafluoride	$SF_6$		✓

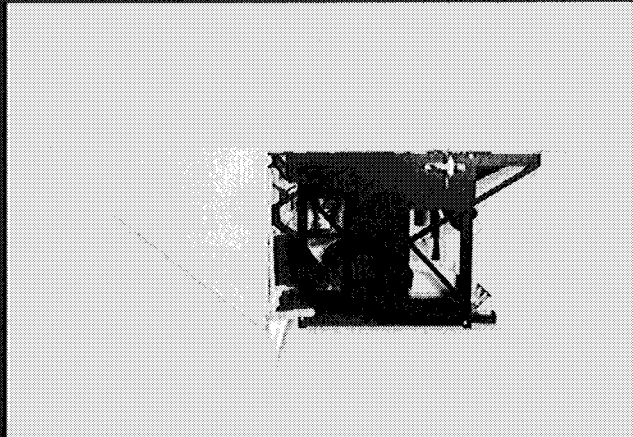
<sup>1</sup> Volcanic/Industrial/biomass burning plume column densities only.

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# The EOS-CHEM Tropospheric Emission Spectrometer (TES)

A summary presentation of tropospheric chemistry issues and how TES measurements will contribute to their understanding



TES model in flight configuration

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## Tropospheric Emission Spectrometer Formal Investigator Team

NAME	INSTITUTION	ROLE
Reinhard Beer	JPL	Principal Investigator
Shepard A. Clough	AER, Inc.	Retrieval Algorithms; Validation
Daniel J. Jacob	Harvard University	Tropospheric Chem. Modeling
Jennifer A. Logan	Harvard University	Tropospheric Chem. Modeling
Frank J. Murcray	University of Denver	Correlative Measurements; Spectroscopy
David M. Rider	JPL	Instrument Scientist; AES Team Leader
Curtis P. Rinsland	NASA Langley	Spectroscopy; Validation
Clive D. Rodgers	Oxford University	Retrieval Algorithms
Stanley P. Sander	JPL	Tropospheric Chemistry
Fredric W. Taylor	Oxford University	Strat-Trop Exchange
Helen M. Worden	JPL	Algorithm Team Leader, AES Analysis

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## The TES Experiment

Global measurements of tropospheric ozone and its precursors from TES combined with *in-situ* data and model predictions will address the following key questions:

*How is the increasing ozone abundance in the troposphere affecting*

- climate change?
- oxidizing reactions that "cleanse" the atmosphere?
- air quality on a global scale?

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Tropospheric Chemistry Hypothesis 1: Changes in tropospheric  $O_3$  are a significant agent for climate change.

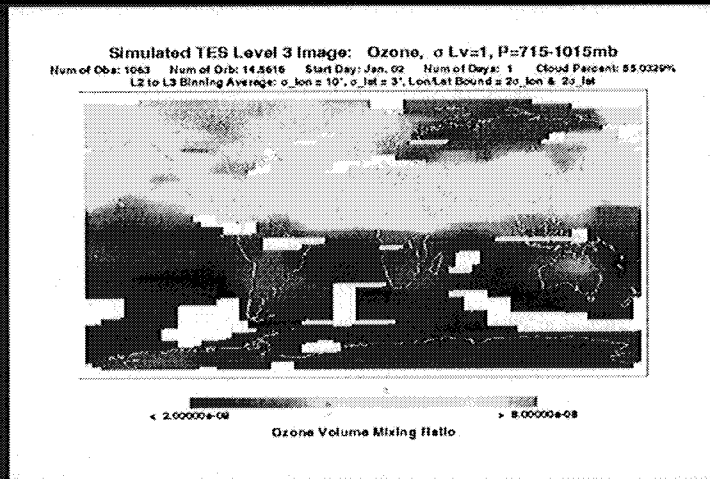
- through direct greenhouse forcing and indirect effects on  $CH_4$ , aerosols, etc.



TES will measure vertical profiles of global tropospheric  $O_3$  and other greenhouse gases. TES will also measure the IR radiative effects of aerosols.

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## Level 3 Products



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## SUMMARY

TES will provide unprecedented information about the state of the Earth's lower atmosphere.

The capability of viewing the troposphere in both limb and nadir has never before existed.

Furthermore, the spectral coverage and resolution available will permit the search for trace gases which may be indicative of unforeseen atmospheric processes.

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## Measurement Overview

- Each orbit comprises 73 sequences of 81.2 seconds
- Each sequence contains calibration spectra, 2 nadir views and 3 limb views.
- Each view uses one of a set of filters, around  $250 \text{ cm}^{-1}$  wide, in a programmable order.
- Detectors are 16 element arrays, so each view comprises 16 spectra, either:
  - a limb array, 2.3km spacing
  - a set of contiguous nadir views,  $0.5 \times 5 \text{ km}$
- Spectra have resolution of  $0.1 \text{ cm}^{-1}$  (nadir) or  $0.025 \text{ cm}^{-1}$  (limb)
- Noise level around 1 K.
- Around  $10^6$  data points per sequence!



# GLOBAL SURVEY OBSERVATION STRATEGY

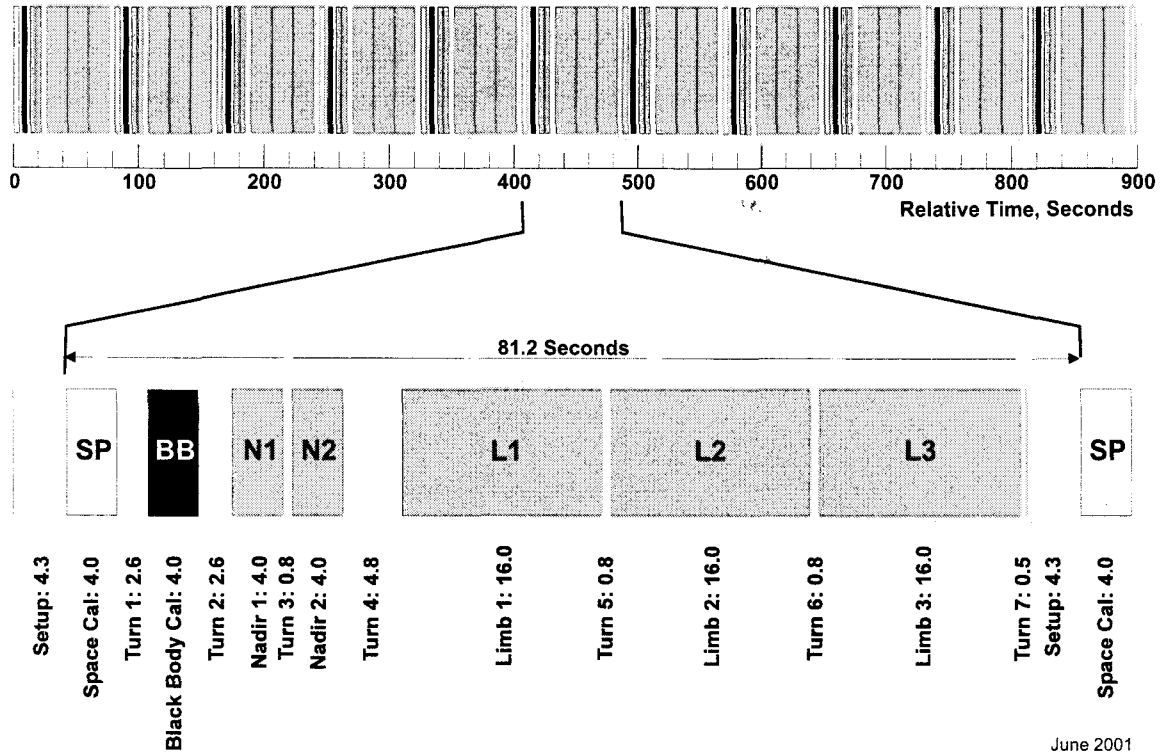
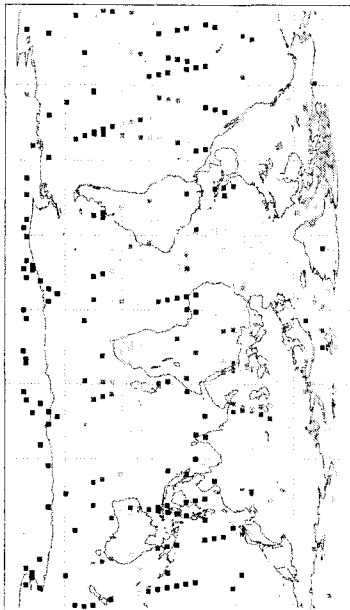
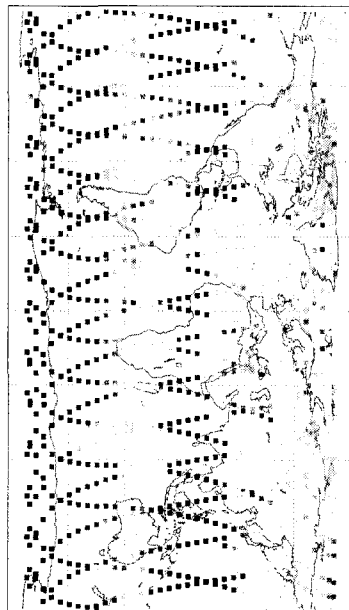


Plate 1.



## Building an Operational Retrieval Method

- Construct a forward model
  - Determine a state vector
  - Determine cost function
  - Carry out characterisation & error analysis
  - Select & build numerical method for minimising cost function
  - Validate the resulting system
- Considering computer resources at all stages:  
Time, storage,...

## Forward Model

- The forward model includes:
  - Monochromatic atmospheric thermal emission model for both nadir and limb modes.
  - Instrument description
- Includes the following physics:
  - Thermal emission radiative transfer
  - Single scattering for aerosol
  - Non-LTE
  - Refraction (for limb)
  - Spectral lines of all relevant molecules
  - Continuum (water vapour)
  - Surface emission, reflectivity and elevation
  - Field of view of the instrument
  - Instrument line shape
  - Geometry of the measurement
- Evaluates Jacobians analytically, mostly

# Radiative Transfer

## Forward Model State Vector

The forward model uses a 'full state vector,' i.e. quantities required to simulate the measured signal. Not all elements of this are retrieved.

Both views:

- Temperature on a pressure grid
- Constituent mixing ratios on a pressure grid
- Aerosol extinction coefficient on a pressure/waveno grid
- Instrument line shape
- Angular field of view function

Nadir only:

- Surface pressure
- Surface radiating temperature
- Surface optical properties: emissivity, albedo, BRDF
- Nadir view angle
- Nadir view location

Limb only:

- Spacecraft position
- Look angle of the boresight from the spacecraft
- Sun angle at nominal tangent point
- Altitude of one pressure level

Constant data:

- Constituent absorption coefficient tables on  $\nu$ ,  $p$ ,  $T$
- Digital elevation model
- Surface characterisation map

- The equation of transfer is:

$$\begin{aligned}
 L(\Omega, \nu) = & \int_0^\infty \Phi(\nu, \nu) \left\{ \int_{z_0}^\infty B(\nu, T(z)) \frac{\partial T(\Omega, z, z_0, \nu')}{\partial z} dz \right. \\
 & \text{(Instrument)} \quad \text{(Upwelling atmospheric emission term)} \\
 & + \left[ \alpha(\nu') \int_{2\pi} \mathcal{R}_{BRDF}(\Omega, -\Omega', \nu') \int_{-\infty}^{z_0} B(\nu', T(z)) \frac{\partial T(-\Omega', z, z_0, \nu')}{\partial z} dz d\Omega' \right. \\
 & \quad \text{(Downwelling, back - reflected, atmospheric emission term)} \\
 & + \varepsilon(\Omega, \nu') \cdot B(\nu', T_{surf}) \\
 & \quad \text{(Surface emission term)} \\
 & \left. + \alpha(\nu') \cdot \mathcal{R}_{BRDF}(\Omega, -\Omega_0, \nu') \cdot E_s(\nu') \cdot \Omega_s \cdot T(-\Omega_0, \infty, z_0, \nu') \right] T(\Omega, z_0, \infty, \nu') \} d\nu' \\
 & \quad \text{(Reflected sunlight term)}
 \end{aligned} \tag{3.5}$$

where

- $L(\Omega, \nu)$  = radiance at frequency  $\nu$  into upward, directed, solid angle  $\Omega$
- $\Phi(\nu, \nu')$  = ILS, i.e. spectral response of  $\nu$  due to incident radiance at  $\nu'$
- $B(\nu, T)$  = Planck function for temperature  $T$
- $T_{surf}$  = surface temperature
- $T(\Omega, z, z', \nu')$  = atmospheric transmittance at frequency  $\nu'$  in a direction  $\Omega$  between altitudes  $z$  and  $z'$
- $\alpha(\nu')$  = surface albedo
- $\mathcal{R}_{BRDF}(\Omega, -\Omega', \nu')$  = surface biconical reflectance function for incident (downward) solid angle  $-\Omega'$  and emergent (upward) solid angle  $\Omega$
- $\varepsilon(\Omega, \nu')$  = surface emittance at frequency  $\nu'$  in the upward direction  $\Omega$
- $E_s(\nu')$  = disk-average solar radiance
- $\Omega_s$  = solar solid angle at Earth

The same equation holds for limb emission sounding if the last three (surface-related) terms are omitted. While the equation, therefore, seems much simpler, the geometry of the light path (especially in the lower atmosphere) becomes much more complicated because of strong refraction effects.

but I won't go over this in detail...

## Radiative Transfer 2

- This must be evaluated monochromatically and then integrated over angle and wavenumber:

$$L = L_0 \exp(-\tau_0) + \int B(\tau) \exp(-\tau) d\tau$$

- Numerically this is evaluated in a way which is based on the differential form of the equation of transfer,  $dL/d\tau = B - L$ :

$$L_{i+1} = L_i T_i + B_i^e (1 - T_i)$$

where:

$T_i$  is the transmittance of the  $i$ th layer, i.e.  $\exp(-\tau_i)$

$\tau_i$  is the optical depth of the  $i$ th layer,

$B^e$  is the effective Planck function of the layer (an approximation to  $B(\tau)$  weighted with  $\exp(-\tau)$  across the layer)

- For speed, both  $B(\nu, T)$  and  $\exp$  are pretabulated rather than being

## Optical Depth Calculation

- Optical depth is  $\tau = \int k(\nu, p, T) du$ , where  $u$  is absorber amount.
- The calculation of the absorption cross-section  $k(\nu, p, T)$  can be time or storage consuming. Options are:

– Calculate it line-by-line

Takes a lot of time - HITRAN has a lot of lines!

– Precompute it, and store it as cross-section tables

Takes a lot of space:

Point spacing  $\sim 10^{-4} \text{ cm}^{-1}$ ,  $2000 \text{ cm}^{-1}$  gives  $2 \times 10^7$  points  
50 pressures, 10 temperatures, 40 gases gives  $4 \times 10^{11}$ ...

- TES has chosen to store cross-section tables for microwindows in the operational code.

# Path Integrals and Ray Tracing

## Path Integrals

The optical depth calculation requires an integral along the path of the ray from/to the satellite:

$$\int k(v,p,T) (du/dx) dx$$

This requires knowledge of the ray, as well as an efficient method of doing the integrals.

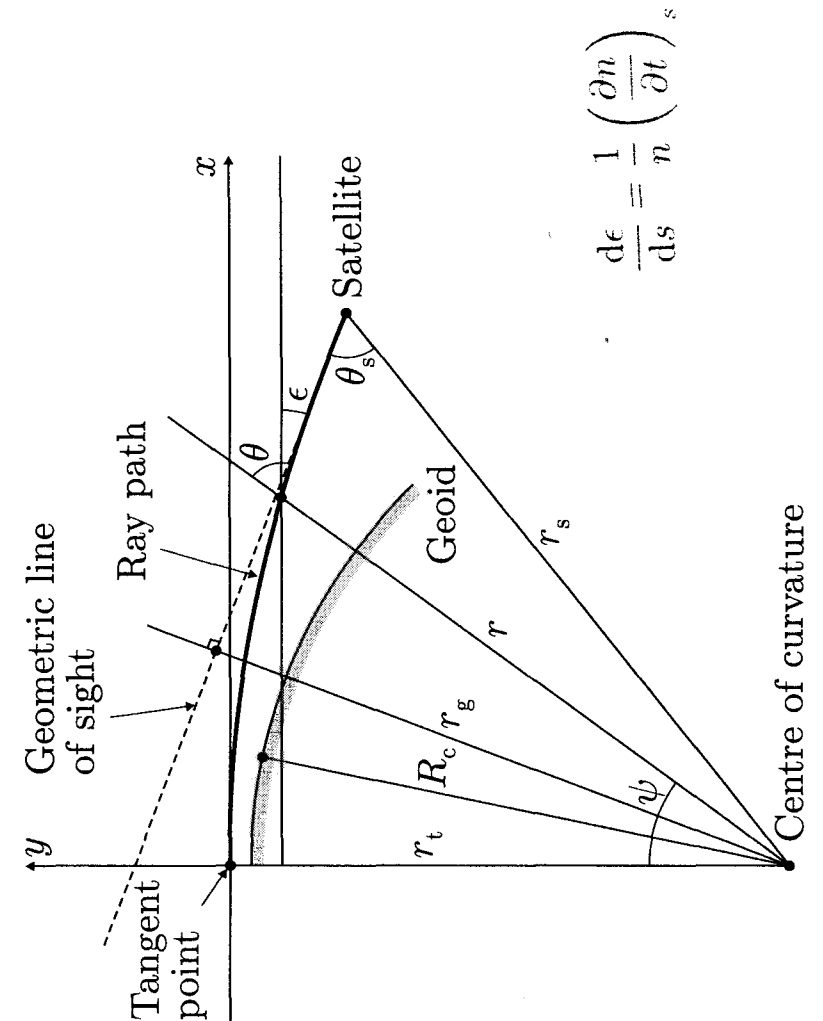
## Ray Tracing

Nadir view: Straightforward spherical trigonometry.  
Refraction is negligible.

Limb view: Refraction is important.

## Jacobians

- It is possible, but complicated, to evaluate analytic derivatives of the ray trace and the path integrals
- This is such a small part of the forward model that the perturbation methods is not computationally expensive.



## Field of View Integration

- Nadir: This is unimportant, because the state vector doesn't vary within the instrument field of view.
- Limb: The atmospheric limb radiance profile varies significantly over the field of view, so the integration has to be carried out.
- For the horizontally homogeneous case
  - consider rays from each tangent point pressure level to the satellite
  - refraction equation provides an angle  $\theta_s$  of the ray at the satellite

$$\sin \theta_s = n(r_t)r_t/r_s$$

where tangent point has refractive index  $n$  at radius and the satellite is at  $r_s$

- ray trace from each tangent point pressure level (hydrostatic eqn first!) to satellite altitude, and compute radiance spectrum
  - integrate over angle on an unequally spaced grid using a four point Lagrange interpolation for the radiance to the grid that the FOV is stored on.
- The horizontally inhomogeneous case is seriously messy.

## Retrieval Method Outline

- The usual nonlinear least squares to minimise the cost function:

$$J = (\mathbf{y} - \mathbf{F}(\mathbf{x}, \mathbf{b}))^T \mathbf{S}_e^{-1} (\mathbf{y} - \mathbf{F}(\mathbf{x}, \mathbf{b})) + (\mathbf{x} - \mathbf{x}_0)^T \mathbf{H} (\mathbf{x} - \mathbf{x}_0)$$

- with respect to  $\mathbf{x}$
  - $\mathbf{y}$  is a subset of the spectral data
  - $\mathbf{S}_e$  is the measurement noise: diagonal
  - $\mathbf{H}$  and  $\mathbf{x}_0$  may be *ad hoc* constraints or statistical *a priori*
- Minimisation uses Levenberg-Marquardt method
  - An *ad hoc* 'shape retrieval' using a simplified state vector provides a first guess.
  - The subset of the spectrum used comprises 'microwindows' – limited regions in wavenumber and tangent altitude (limb case) containing



## Retrieval Strategy

- Retrieval process is table-driven
- Each stage of retrieval is defined by such things as:
  - Shape retrieval or full retrieval
  - Subset of products to be jointly retrieved  
(e.g. T, ozone, etc)
  - Constraints  
(e.g. Tikhonov, climatology, forecast,...)
  - Microwindows
  - First guess  
(e.g. climatology, forecast, previous stage, previous retrieval...)
  - Products to be stored or archived
- A typical table might provide for retrieval stages as follows:
  1. Pointing and temperature around the tropopause (limb)
  2. Pointing and temperature (limb)
  3. Temperature and water (nadir)
  4. N<sub>2</sub>O limb & column (cloud check)
  5. Ozone (limb)
  6. Other constituents in some order TBD
  - n.* Ozone (nadir)

## Retrieved State Vector

The retrieval state vector is a subset of the full state vector:

*Both views:*

- Temperature on a subset pressure grid
- Constituent log mixing ratios on a subset pressure grid
- Aerosol extinction coefficient on a subset pressure/waveno grid

*Nadir only:*

- Surface emitting temperature per detector
- Surface emissivity

*Limb only:*

- Look angle of the boresight from the spacecraft

Temperature and log mixing ratio are linear in  $\ln(p)$

Quantities not retrieved are treated as forward model parameters for the purpose of error analysis.

## Capturing Tropospheric Ozone Time Variability

### A Study Using TES Nadir Retrievals.

K. Bowman<sup>1</sup>, J. Worden<sup>2</sup>, T. Steck<sup>1</sup>, H. Worden<sup>1</sup>, S. Clough<sup>2</sup>, C. Rodgers<sup>3</sup>

<sup>1</sup>Jet Propulsion Laboratory, Pasadena, CA

<sup>2</sup>Atmospheric and Environmental Research, Cambridge, MA

<sup>3</sup>Oxford University, Oxford, England

9/25/01

## Goal and assumptions of study

- Determine, with simulated retrievals, whether retrieved TES nadir retrievals are sensitive enough to capture the time variability in a set of “real” ozone measurements.
- Assumptions for study:
  - Surface temperature, atmospheric temperature, and water amounts have been accurately retrieved prior to the ozone retrievals. Temperature errors, however, are quantified for ozone retrievals.
  - The signal-to-noise (SNR) ratio is 300, which is based on a conservative radiometric model
  - There are no spectral systematic biases.

9/25/01

## Retrieval problem

- Given:  $y = F(x) + n$  where  $y$  is the measured spectrum,  $x$  is the true ozone profile,  $F$  is the forward model, and  $n$  is additive white Gaussian noise. The retrieval problem is to find  $x$  given  $y$ .
- The fine vertical structure and variability of ozone profiles lead to nonlinearities in the retrieval that result in numerical instabilities or unacceptably expensive computations. We address these problems by separating the retrieval into two steps.

9/25/01

## 2-step retrieval

- First, estimate the broad features of the ozone using the “shape retrieval”
- $$\hat{x}_s = M_s \cdot \min_z \|y - F(M_s z_s)\|_{S_s^{-1}}^2$$
- Second, estimate the fine features using a “level” retrieval with smoothing constraint.

$$\hat{x} = M_z \cdot \min_z \left( \|y - F(M_z z)\|_{S_z^{-1}}^2 + \|z - M_z^* \hat{x}_s\|_{\Lambda}^2 \right)$$

9/25/01

## 2-step retrieval (con't)



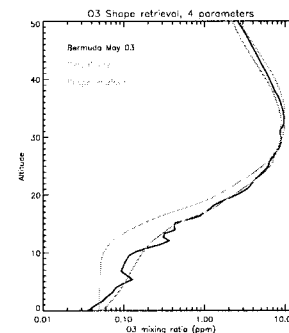
- The retrieval minimizes the cost function with respect to the retrieval vector  $\mathbf{z}_s$  for the shape retrieval, and  $\mathbf{z}$  for the level retrieval.
- The retrieval vectors are mapped to the ozone profile,  $\mathbf{x}$ , through  $\mathbf{M}_s$  for the shape retrieval and  $\mathbf{M}_z$  for the level retrieval.
- The level retrieval uses a smoothness constraint based on the Tikhonov-type constraint matrix  $\Lambda$  and constraint vector  $\mathbf{M}_z^* \hat{\mathbf{x}}$

9/25/01

## Shape Retrieval



- We retrieve four parameters:
  1. A scaling parameter that scales a pre-configured stratospheric profile
  2. A “shift” parameter that vertically adjusts the pre-configured stratospheric profile.
  3. A scaling parameter for the troposphere
  4. Tropospheric lapse rate.



9/25/01

## Regularization of level retrieval



- A Tikhonov-type constraint is used to insure a smooth estimate.
- Constraint scaled by the atmospheric profile for smoothness in the troposphere
- Strength calculated based on estimated error.

$$\Lambda = \alpha \mathbf{B}^{-1} \mathbf{L}^T \mathbf{L} \mathbf{B}^{-1}$$

$$\mathbf{L}^T \mathbf{L} = \begin{pmatrix} 1 & -1 & 0 & \dots & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -1 & 2 & -1 \\ 0 & \dots & \dots & 0 & -1 & 2 \end{pmatrix}$$

$$[\mathbf{B}]_{ii} = [\mathbf{M}_i^* \hat{\mathbf{x}}]_i$$

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## Error analysis



- Estimate from shape retrieval is used as initial guess and *a priori* to level retrieval.
- Hence, error analysis must be extended to handle a variable *a priori*.

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## Linear retrieval



The linear estimate of the 2-step retrieval is

$$1. \mathbf{x}_s = \mathbf{x}_0 + \mathbf{A}_{xx}^s (\mathbf{x}^* - \mathbf{x}_0) + \mathbf{M}_s \mathbf{G}_s \mathbf{n}$$

$$2. \hat{\mathbf{x}} = \mathbf{x}_s + \mathbf{A}_{xx}^z (\mathbf{x}^* - \mathbf{x}_s) + \mathbf{M}_z \mathbf{G}_z \mathbf{n}$$

where  $\mathbf{A}_{xx}^z$  and  $\mathbf{A}_{xx}^s$  are the averaging kernels,  $\mathbf{G}_z$  and  $\mathbf{G}_s$  are the gain matrices,  $\mathbf{M}_z$  and  $\mathbf{M}_s$  are the maps for the level and shape retrieval, respectively.  $\mathbf{x}_0$  is the initial guess,  $\mathbf{x}^*$  is the true profile,  $\mathbf{x}_s$  is the estimate from the shape retrieval, and  $\hat{\mathbf{x}}$  is the estimate of the ozone.

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## Error for 2-step retrieval



The error in the linear estimate is

$$\tilde{\mathbf{x}} = \hat{\mathbf{x}} - \mathbf{x}^* = \underbrace{(\mathbf{A}_{xx}^z - \mathbf{I}) (\mathbf{x}^* - \mathbf{x}_s)}_{\text{Smoothing error}} + \underbrace{\mathbf{M}_z \mathbf{G}_z}_{\text{Noise error}} \mathbf{\epsilon}.$$

The total error covariance is then

$$\mathbf{S}_{\tilde{\mathbf{x}}} = \underbrace{(\mathbf{I} - \mathbf{A}_{xx}^z)(\mathbf{I} - \mathbf{A}_{xx}^s) \mathbf{S}_{\mathbf{x}^*} (\mathbf{I} - \mathbf{A}_{xx}^s)^T (\mathbf{I} - \mathbf{A}_{xx}^z)^T}_{\text{Smoothing error}} + \underbrace{\left[ \mathbf{M}_z \mathbf{G}_z - (\mathbf{I} - \mathbf{A}_{xx}^z) \mathbf{M}_s \mathbf{G}_s \right] \mathbf{S}_{\mathbf{\epsilon}} \left[ \mathbf{M}_z \mathbf{G}_z - (\mathbf{I} - \mathbf{A}_{xx}^z) \mathbf{M}_s \mathbf{G}_s \right]^T}_{\text{Noise error}}.$$

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## Column error



- The column amount is simply  $\hat{\mathbf{c}} = \mathbf{H} \hat{\mathbf{x}}$ . The error  $\tilde{\mathbf{c}} = \hat{\mathbf{c}} - \mathbf{c}$  is then

$$\mathbf{S}_{\tilde{\mathbf{c}}} = \mathbf{H} \mathbf{S}_{\tilde{\mathbf{x}}} \mathbf{H}^T$$

- The column operator,  $\mathbf{H}$ , can be calculated for any region of the atmosphere.

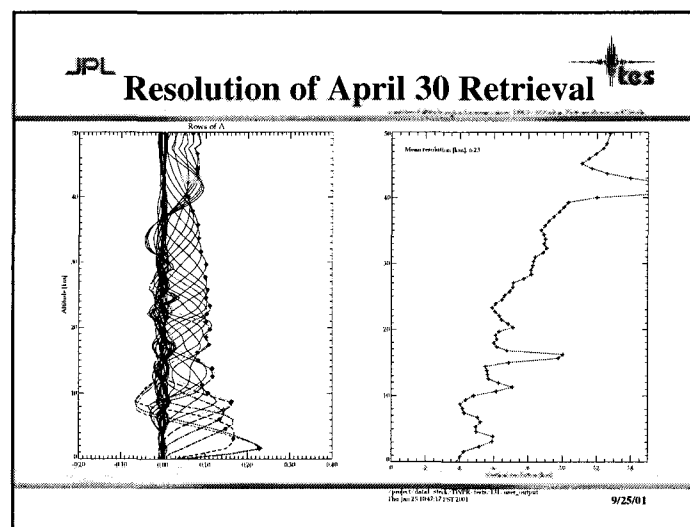
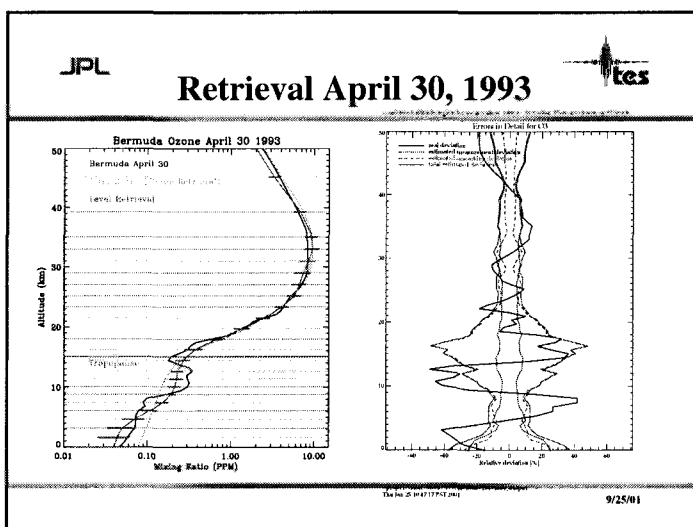
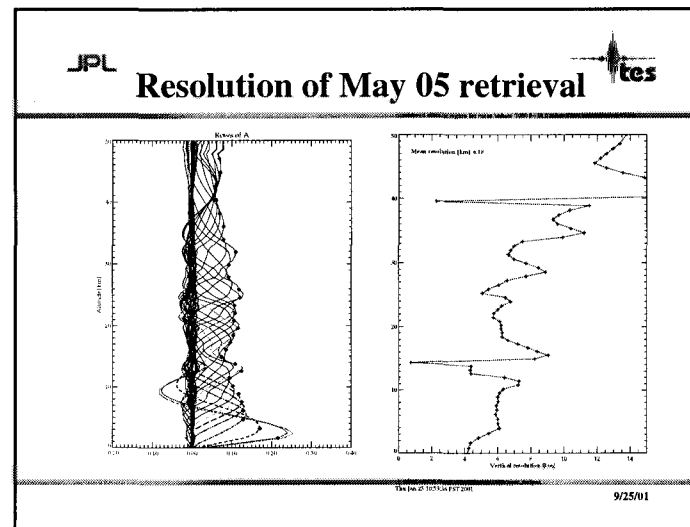
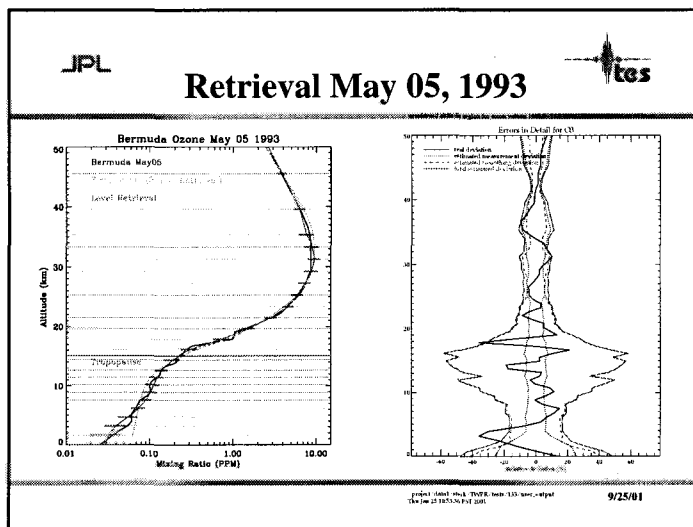
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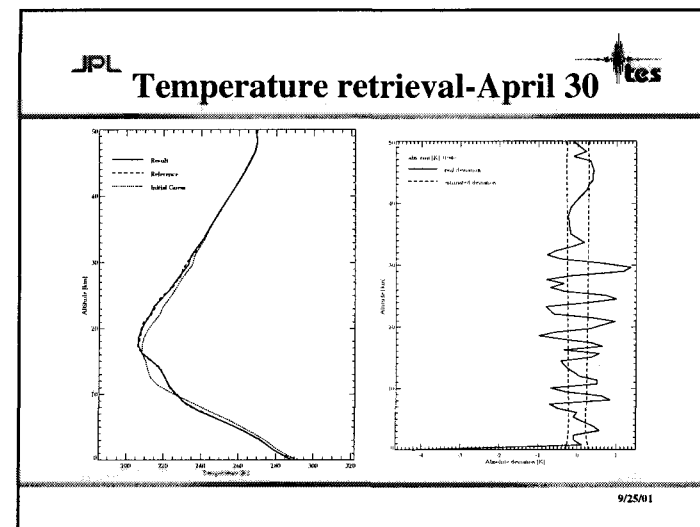
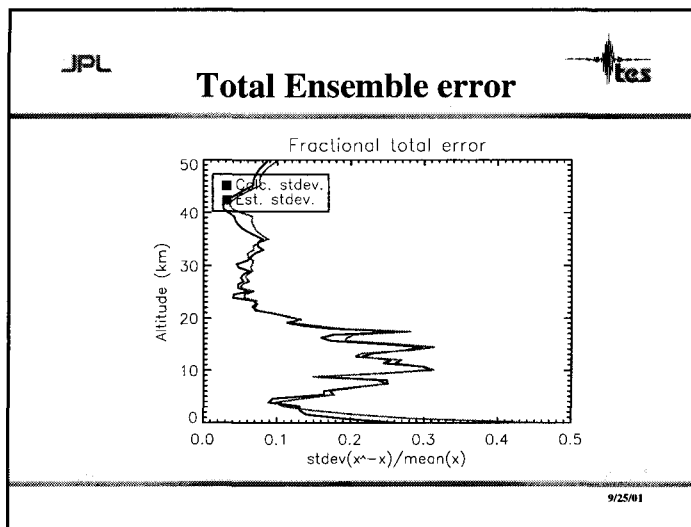
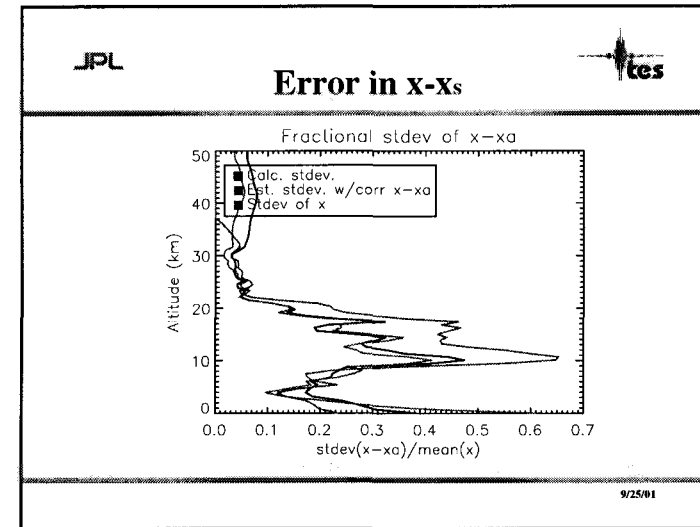
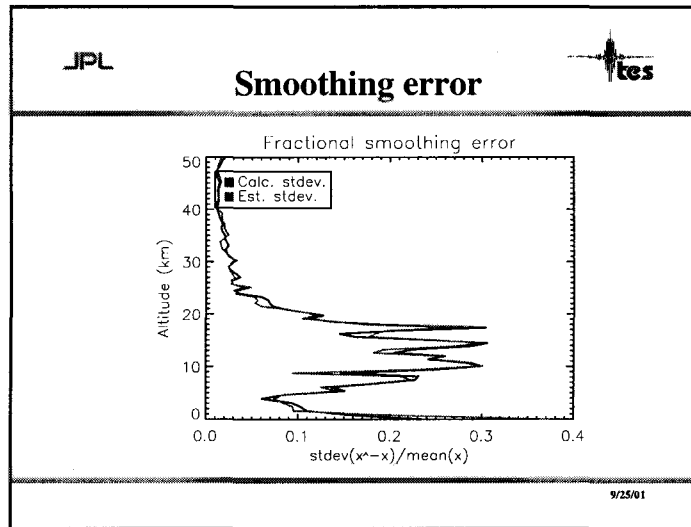
## Simulation of Radiances



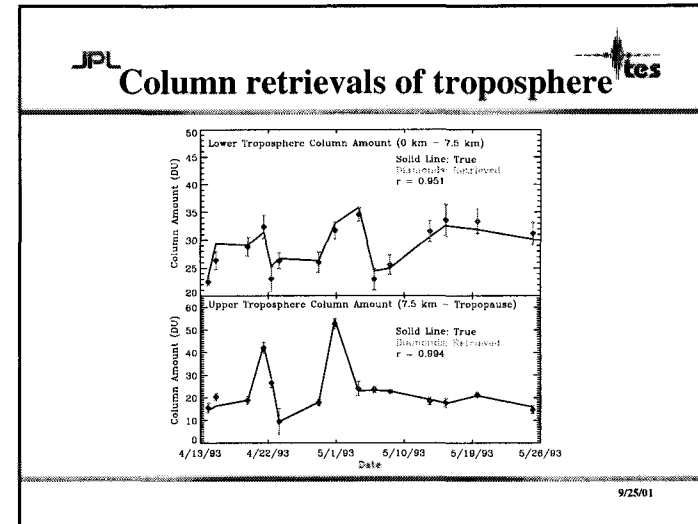
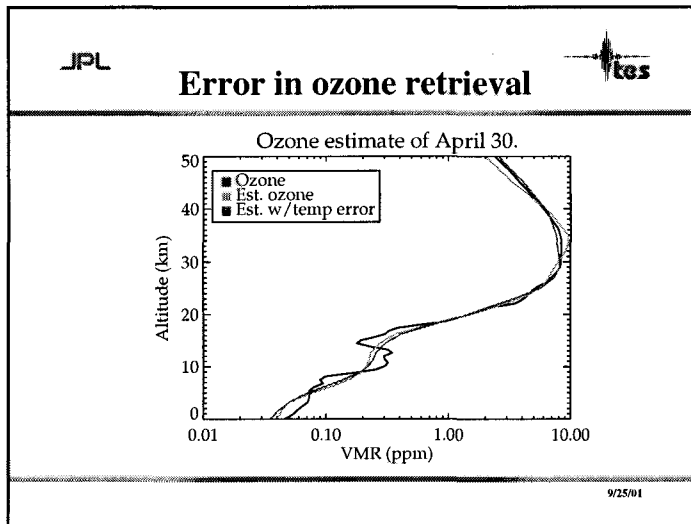
- Radiances calculated from ozone-sondes released over Bermuda from April 14, 1993 to May 25, 1993<sup>1,2</sup>. These sondes included profiles of temperature, water, and ozone.
- Profiles mapped to UARS pressure grid:  
 $\log p = 3 - i/N$ ,  $N=24$ ,  $i=0, \dots, 85$ .
- URAP climatology added to ozone-sonde data from 10 mb to .1 mb<sup>3</sup>.


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- ### Conclusions
- 15 ozone-profiles were estimated using nadir simulated radiances.
  - We employed a novel 2-step retrieval algorithm that is robust to significant deviations between initial guess and the true ozone profile.
  - The average resolution was about 6 km leading to about 2 pieces of information in the troposphere.
  - The retrieved columns in the lower and upper troposphere were correlated at .95 and .99, respectively.
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- ### References
- <sup>1</sup> Dataset obtained from private communication with Jennifer Logan, Harvard University.
  - <sup>2</sup> S.J. Oltmans, *et al.*, "Summer and spring ozone profiles over the North Atlantic from ozonesonde measurements", *J. Geophys. Res.*, 101: (D22) 29179-29200 Dec 20 1996
  - <sup>3</sup> <http://hyperion.gsfc.nasa.gov/Analysis/UARS/urap/home.html>
- 9/25/01