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Liquid Metal Experiments on Dynamos: Magnetic Instabilities Driven by Sheared Flow

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These are preliminary lecture notes, intended only for distribution to participants.

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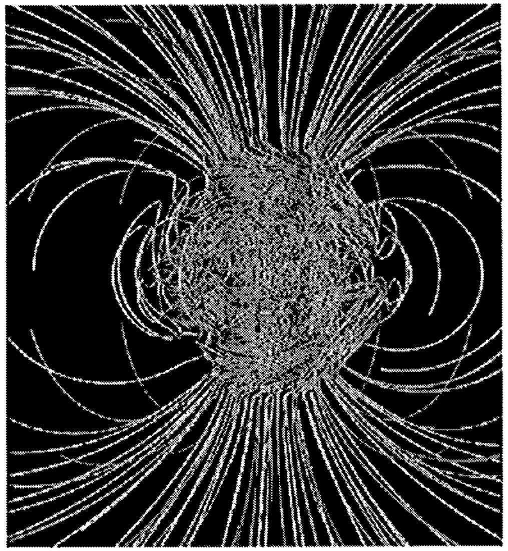
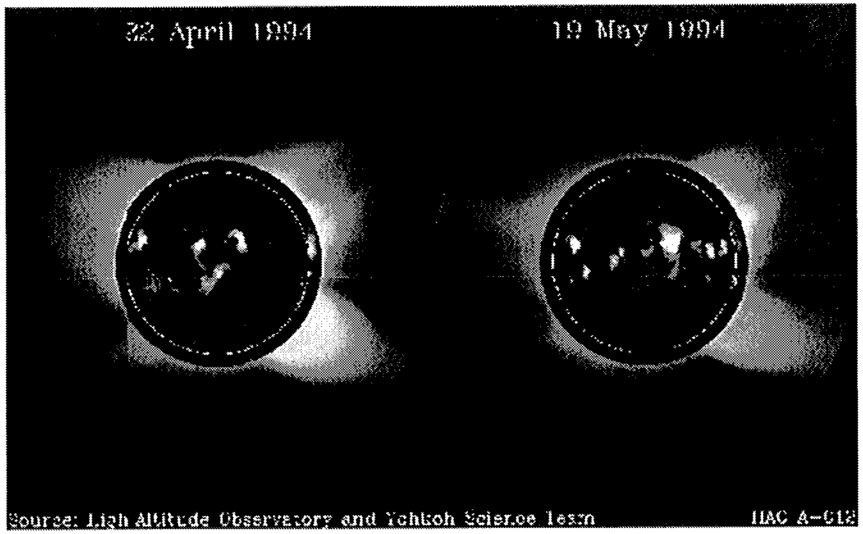
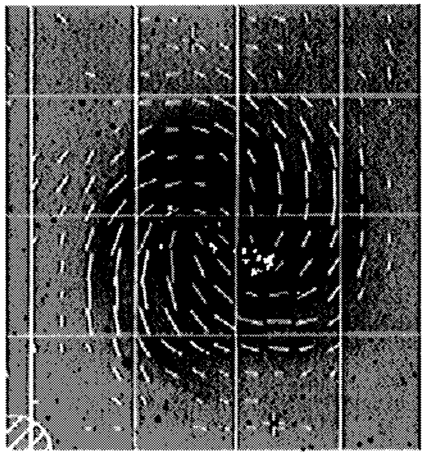
Planets, stars and perhaps the galaxy all have magnetic fields produced by dynamos



Glatzmaier Roberts simulation of geodynamo



Solar prominences and x-rays from sunspots



Magnetic fields observed in M83 spiral galaxy

Outline

- What are dynamos?
- The Madison dynamo experiment
  - A stretch-twist-fold experiment
- Some models of astrophysical and geophysical dynamos
  - An α-Ω experiment
- Questions for experiments to study
  - Fundamental issues in MHD turbulence theory

Physics of magnetic field generation

1. Charged particles moving in a magnetic field experience the Lorentz force

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{V} \times \mathbf{B}\right)$$

2. The motional EMF generates a current

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{V} \times \mathbf{B}\right)$$

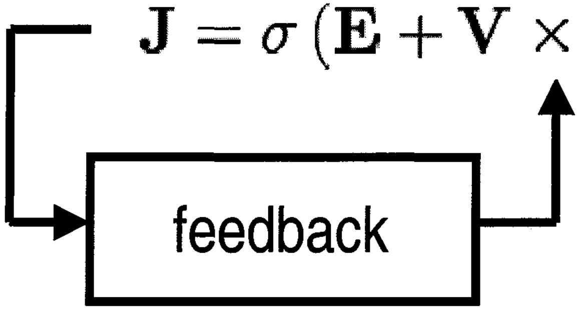
3. Currents produce a magnetic field

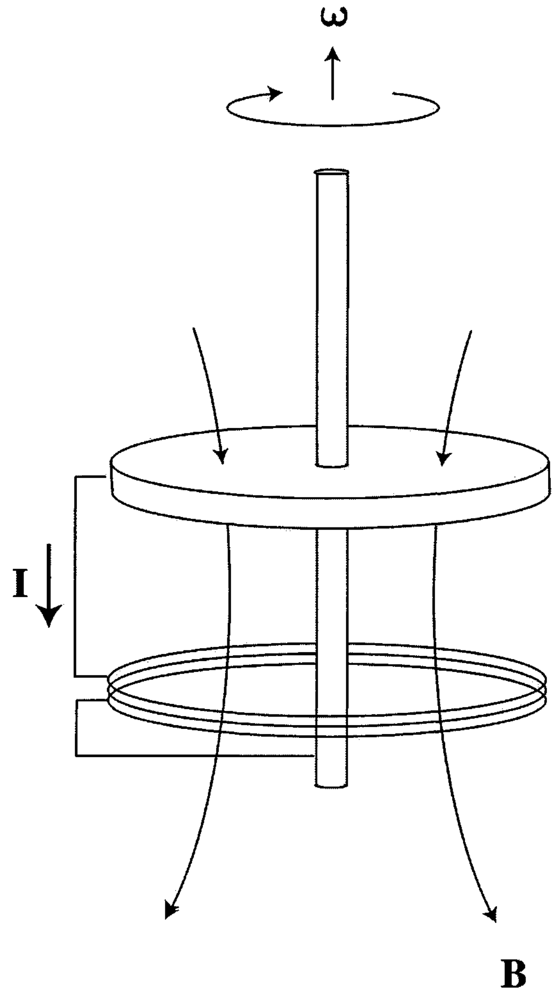
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Dynamos generate magnetic energy from mechanical energy

- this is easy if you allow yourself the luxury of using insulators and solid conductors
- in a conductor, currents are generated by motion across a magnetic field

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{V} \times \mathbf{B}\right)$$





In astrophysical dynamos the conductors are simply connected (no insulators) and can flow

- plasmas or liquid metals
- Magnetohydrodynamics: systems are describe by two vector fields:
  - the magnetic field, is generated by electrical currents in the conducting fluid

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{V} \times \mathbf{B}\right)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma}\nabla^2 \mathbf{B}$$

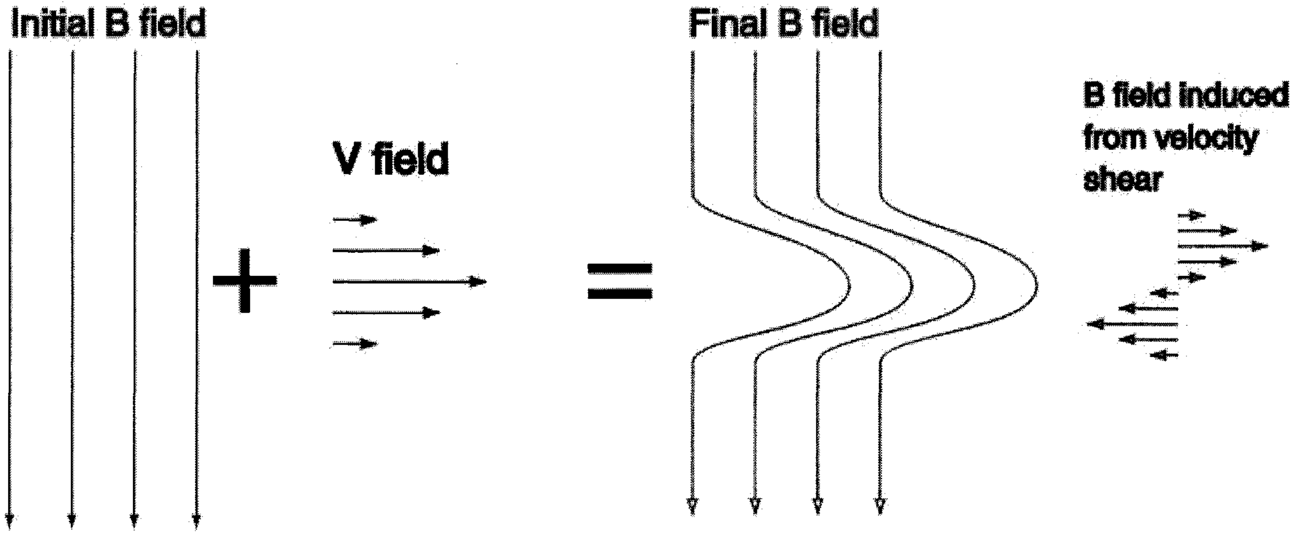
Induction

Resistive diffusion

-The velocity field evolves according to Navier-Stokes + electromagnetic forces

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V}\right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{V}$$

Fluid flow can amplify and distort magnetic fields

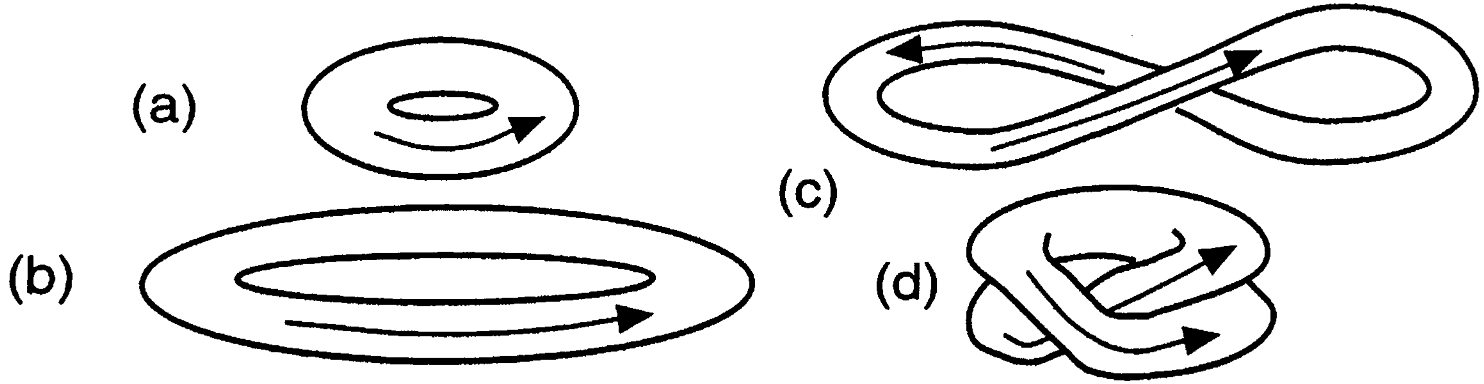


- in a fast moving, or highly conducting fluid, magnetic field lines are frozen into the moving fluid

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \cancelto{0}{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}}$$

- transverse component of field is generated and amplified
- finite resistance leads to diffusion of field lines

Stretch-Twist-Fold Dynamo (Zeldovich and Vainshtein, 1972)



- Final flux tube (d) carries twice the original flux of (a)
- Reconnection is required to change topology
  - Otherwise magnetic tension would tend to pull geometry back to original shape

Laboratory Models of Dynamos

Is it possible to create a MHD dynamo in the laboratory?

What would its parameters have to be to self-excite?

The kinematic dynamo problem

- Start with a sphere filled with a uniformly conducting fluid of conductivity σ, radius a, surrounded by an insulating region
- Find a velocity field $\mathbf{V}(\mathbf{r})$ inside the sphere, which leads to growing $\mathbf{B}(\mathbf{r},t)$
- Ignore the back-reaction of magnetic field on flow

The kinematic dynamo problem (continued)

- Transform to dimensionless variables:

$$\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \\
\frac{\partial \mathbf{B}}{\partial \tau} &= \nabla \times \bar{\mathbf{V}} \times \mathbf{B} + \frac{1}{Rm} \nabla^2 \mathbf{B} \\
Rm &= \mu_0 \sigma a V_{max} \\
\tau &= \mu_0 \sigma a^2
\end{aligned}$$

- Since its linear in B, use separation of variables:

$$\mathbf{B}(\mathbf{x}, t) = \sum_n e^{\lambda_n t} \mathbf{B}_n(\mathbf{x})$$

- Solve eigenvalue equation for given $\mathbf{V}(\mathrm{r})$ profile

$$\lambda_n \mathbf{B}_n = \nabla \times \mathbf{V} \times \mathbf{B}_n + \frac{1}{Rm} \nabla^2 \mathbf{B}_n$$

Solution technique: use spherical harmonic basis function for velocity and magnetic fields (spectral in θ, φ, finite difference in r)

Since fields are incompressible, use vector potential formulation, based on orthogonal spherical harmonic expansion (Bullard and Gellman, 1954)

$$\mathbf{V} = \nabla\times t_n^m(r)\, Y_n^m(\theta,\phi)\,\hat{\mathbf{r}} + \nabla\times\nabla\times s_n^m(r)\, Y_n^m(\theta,\phi)\,\hat{\mathbf{r}}$$
$$\mathbf{B} = \nabla\times T_n^m(r)\, Y_n^m(\theta,\phi)\,\hat{\mathbf{r}} + \nabla\times\nabla\times S_n^m(r)\, Y_n^m(\theta,\phi)\,\hat{\mathbf{r}}$$

Integration over angles and selection rules produce a coupled set of differential equations, which are solved numerically, using finite differences in the radial direction

$$\begin{aligned}\frac{\partial^2 S_\gamma}{\partial r^2} - \lambda S_\gamma \quad &- \quad \frac{\ell_\gamma(\ell_\gamma+1)}{r^2} S_\gamma = \frac{Rm}{r^2}\sum_{\alpha,\beta} t_\alpha S_\beta \rightarrow S_\gamma \\ &+ \quad t_\alpha T_\beta \rightarrow S_\gamma + s_\alpha T_\beta \rightarrow S_\gamma + s_\alpha S_\beta \rightarrow S_\gamma \\ \frac{\partial^2 T_\gamma}{\partial r^2} - \lambda T_\gamma \quad &- \quad \frac{\ell_\gamma(\ell_\gamma+1)}{r^2} T_\gamma = \frac{Rm}{r^2}\sum_{\alpha,\beta} t_\alpha T_\beta \rightarrow T_\gamma \\ &+ \quad t_\alpha S_\beta \rightarrow T_\gamma + s_\alpha T_\beta \rightarrow T_\gamma + s_\alpha S_\beta \rightarrow T_\gamma\end{aligned}$$

Solutions for complex eigenvalues are found using inverse iteration

Three wave interaction terms describe induction terms

$$
\begin{aligned}
s_\alpha S_\beta \to S_\gamma &= -\frac{K_{\alpha\beta\gamma}}{N_\gamma}\left\{\nu_\alpha\mu_\alpha s_\alpha\frac{\partial S_\beta}{\partial r} - \nu_\beta\mu_\beta\frac{\partial s_\alpha}{\partial r}S_\beta\right\} \\
t_\alpha S_\beta \to S_\gamma &= -\frac{L_{\alpha\beta\gamma}}{N_\gamma}\nu_\beta t_\alpha S_\beta \\
s_\alpha T_\beta \to S_\gamma &= -\frac{L_{\alpha\beta\gamma}}{N_\gamma}\nu_\alpha s_\alpha T_\beta \\
s_\alpha S_\beta \to T_\gamma &= \frac{L_{\alpha\beta\gamma}}{N_\gamma}\left\{\nu_\alpha s_\alpha\frac{\partial^2 S_\beta}{\partial r^2} + \left(-\mu_\gamma\frac{\partial s_\alpha}{\partial r} - \nu_\alpha\frac{s_\alpha}{r}\right)2\frac{\partial S_\beta}{\partial r}\right. \\
&\left. +\nu_\beta\left(\frac{\partial^2 s_\alpha}{\partial r^2} - \frac{2}{r}\frac{\partial s_\alpha}{\partial r}\right)S_\beta\right\} \\
t_\alpha S_\beta \to T_\gamma &= \frac{K_{\alpha\beta\gamma}}{N_\gamma}\left\{\left[\nu_\beta\mu_\beta + \nu_\gamma\mu_\gamma\right]t_\alpha\frac{\partial S_\beta}{\partial r} + \nu_\beta\mu_\beta\left(\frac{\partial t_\alpha}{\partial r} - \frac{2t_\alpha}{r}\right)S_\beta\right\} \\
s_\alpha T_\beta \to T_\gamma &= -\frac{K_{\alpha\beta\gamma}}{N_\gamma}\left\{\nu_\alpha\mu_\alpha s_\alpha\frac{\partial T_\beta}{\partial r} + \left[\nu_\alpha\mu_\alpha\left(\frac{\partial s_\alpha}{\partial r} - \frac{2s_\alpha}{r}\right) + \nu_\gamma\mu_\gamma\frac{\partial s_\alpha}{\partial r}\right]T_\beta\right\} \\
t_\alpha T_\beta \to T_\gamma &= -\frac{L_{\alpha\beta\gamma}}{N_\gamma}\nu_\gamma t_\alpha T_\beta \\
t_\alpha T_\beta \to S_\gamma &= 0. \qquad (1)
\end{aligned}
$$

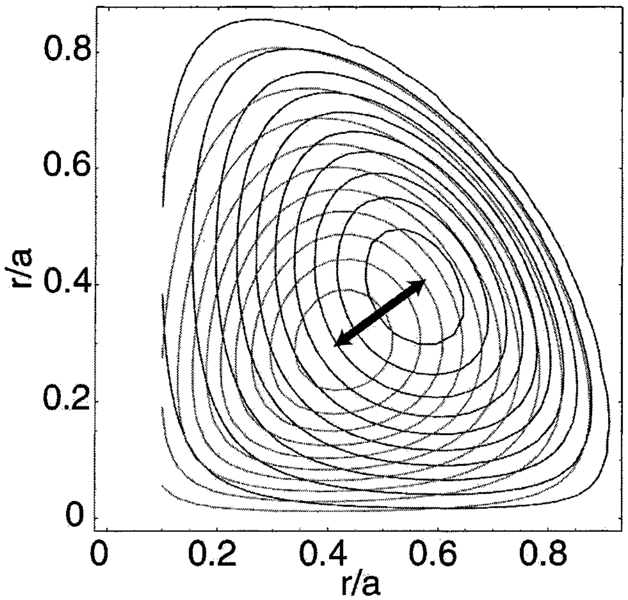
where $\nu_\alpha = \ell_\alpha(\ell_\alpha + 1)$, $\mu_\alpha = \frac{1}{2}(\nu_\alpha - \nu_\beta - \nu_\gamma)$, and $N_\gamma = 4\pi\nu_\gamma/2\ell_\gamma + 1$. $K_{\alpha\beta\gamma}$ and $L_{\alpha\beta\gamma}$ are the Gaunt and Elssaser Integrals which obey particular selection rules.

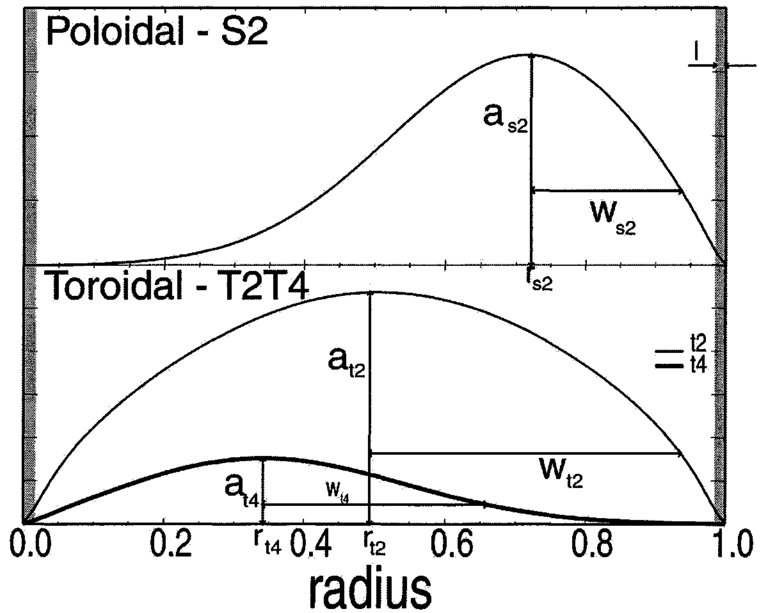
Numerical search is used to find optimized velocity profiles for given flow topology

- Radial function is parameterized and search algorithm is used to find low Rm_{crit}

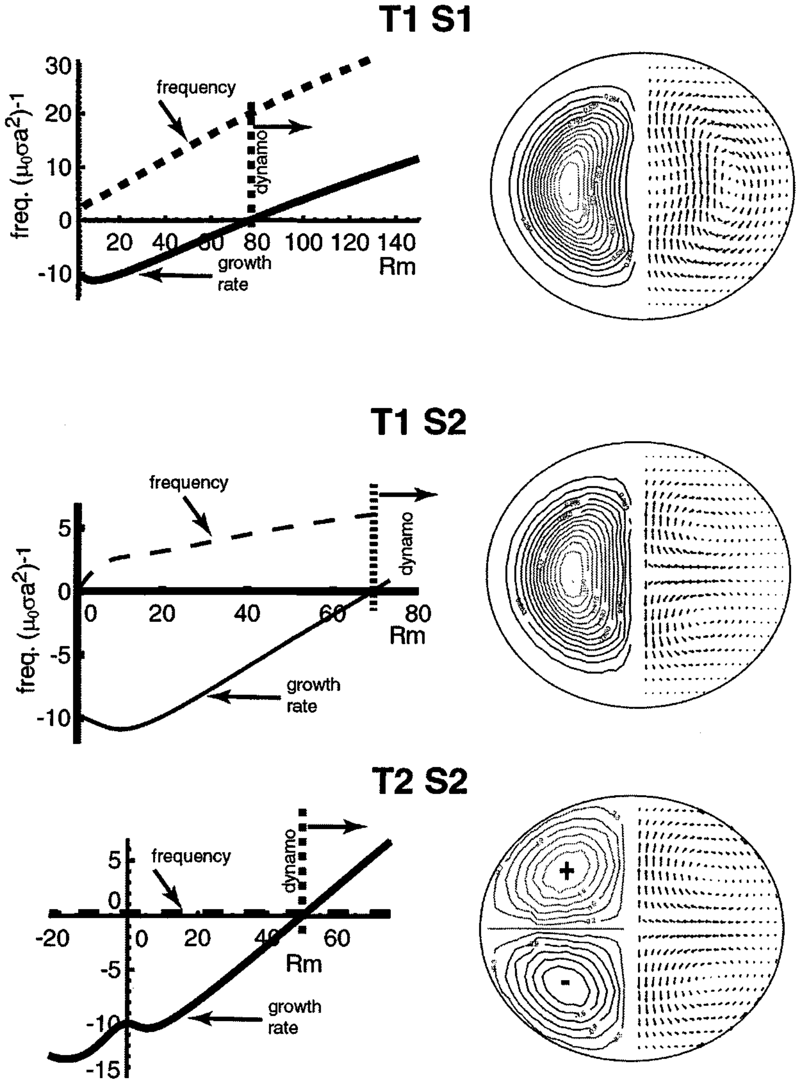
$$t_2^0(r) = \exp\left(-\frac{\delta}{r} + \frac{\delta}{1-r} - \frac{(r-r_t)^2}{w_t^2}\right)$$

$$s_2^0(r) = \exp\left(-\frac{\delta}{r} + \frac{\delta}{1-r} - \frac{(r-r_s)^2}{w_s^2}\right)$$

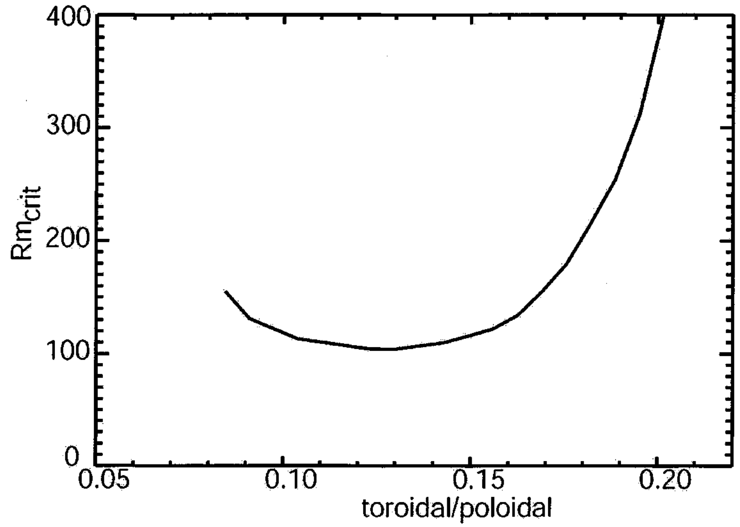




Growing magnetic fields are predicted for simple flow topologies in a sphere

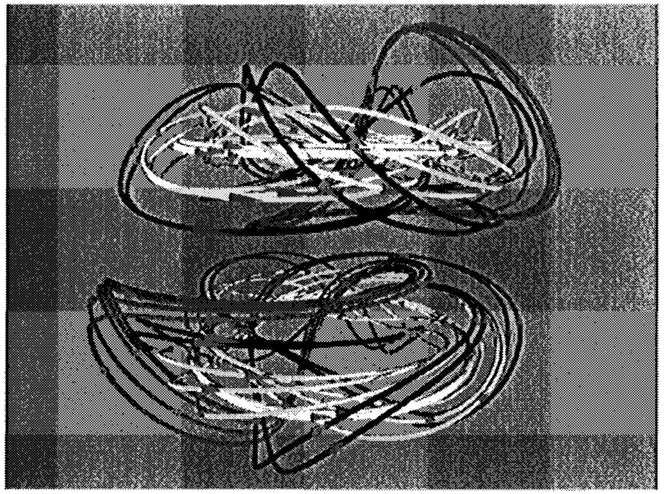


- induction equation is solved numerically
- flow fields are axisymmetric
- growth rate depends upon Rm
- $Rm = \mu_0\, \sigma\, a\, V_{max}$
  $\propto$ conductivity $\times$ size $\times$ velocity
- sensitive to ratio of poloidal and toroidal velocity

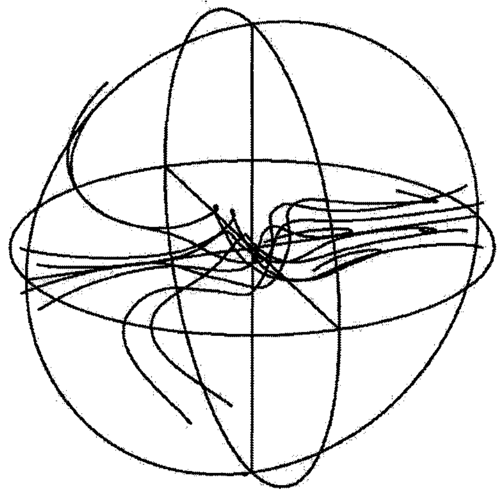


An axisymmetric, double vortex flow produces a growing magnetic field which is an equatorial dipole

Velocity Streamlines, T2S2

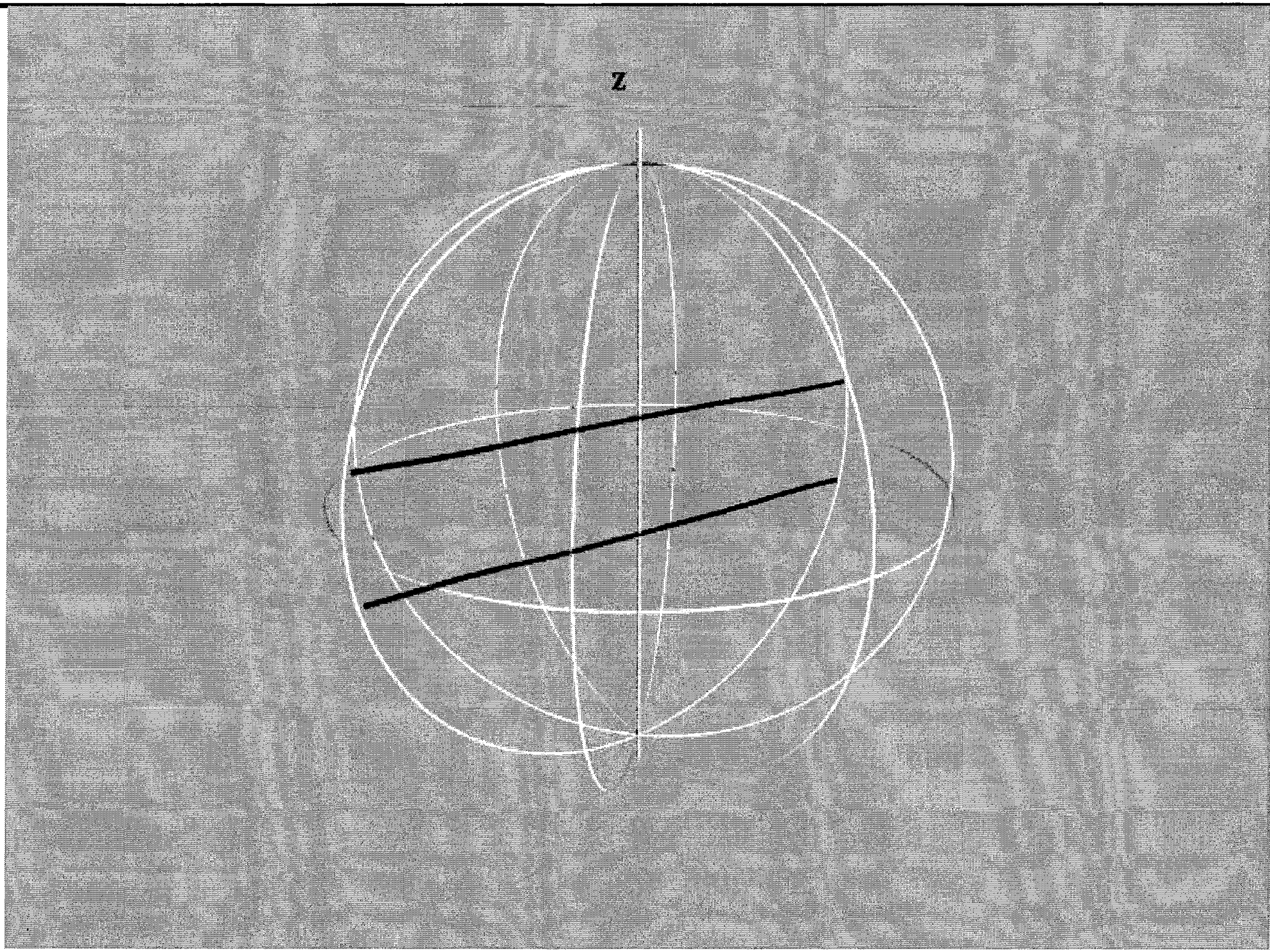


magnetic eigenmode

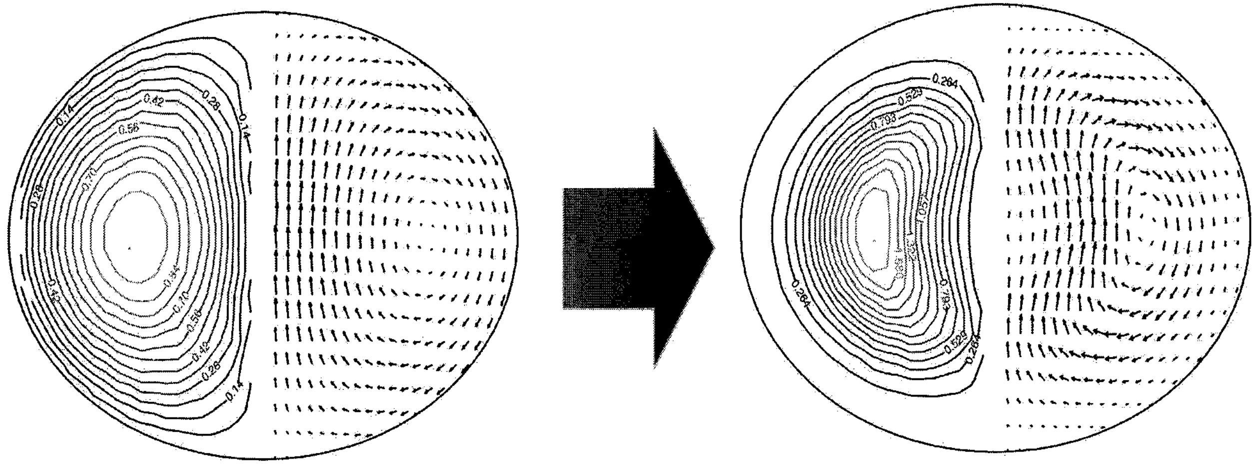


- Magnetic field is not axisymmetric: dynamo satisfies Cowling's theorem

Dynamo is of the stretch-twist-fold type: field line stretching and reinforcement leads to dynamo



Optimized solutions have low critical Rm



Dudley & James t1s1 flow
$Rm_{crit} = 160$

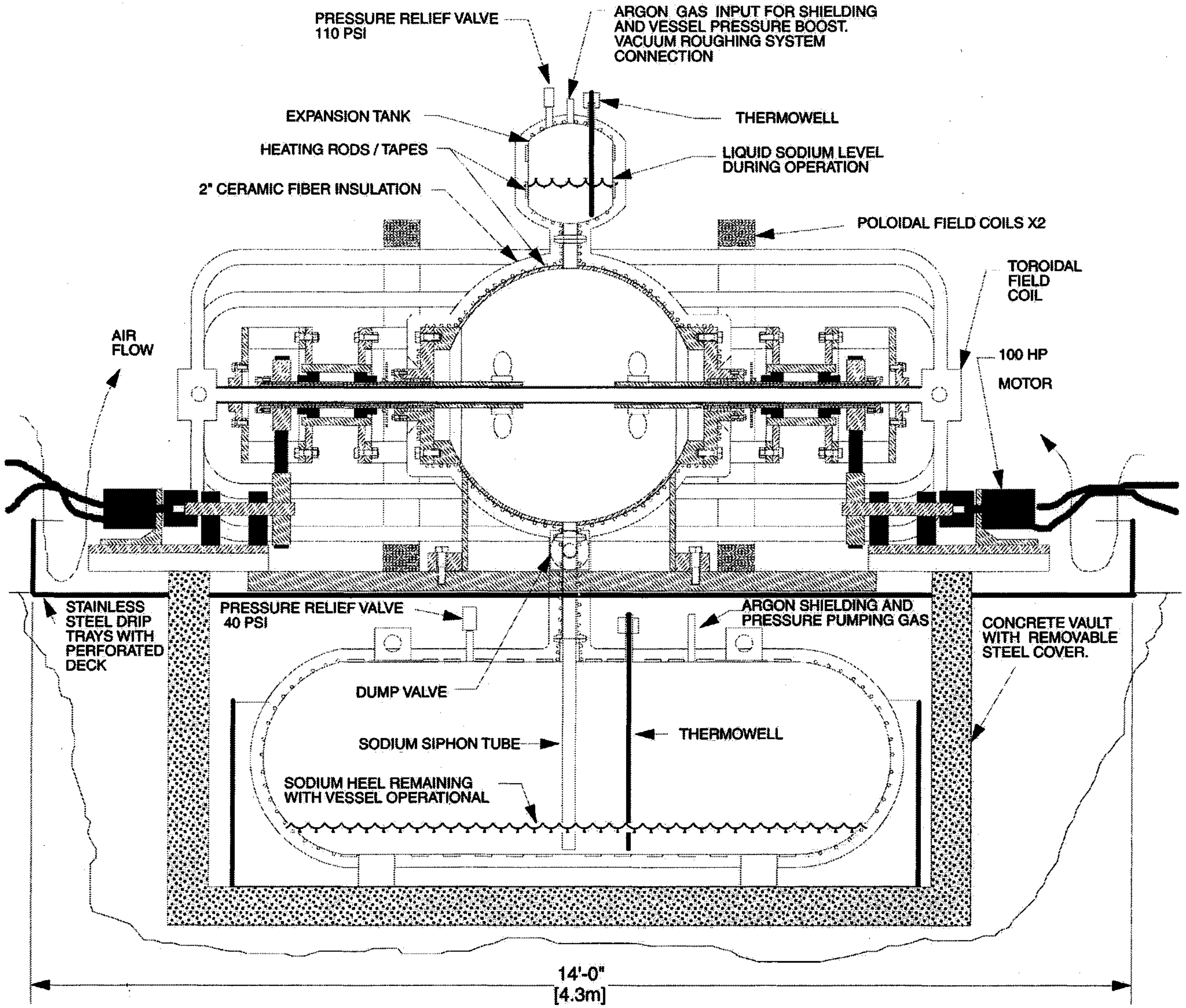
Optimized t1s1 flow
$Rm_{crit}=79$

Flow Configuration	D&J	Optimized
t1s1	150	79
t1s2	95	73
t2s2	55	42

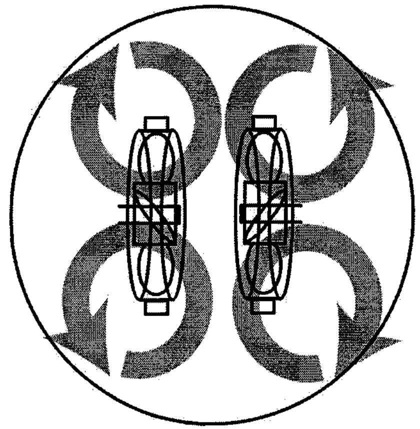
Can these flows be produced in a laboratory?

- How big and how much power is required?
- Is this simply an linear eigenmode problem in a turbulent/mechanically homogeneous system?
- Estimates suggest 100 kW, and a=0.5 m with sodium should provide Rm=100

The Dynamo Experiment



- 1 m diameter
- 200 Hp
- sodium
- Rm=200

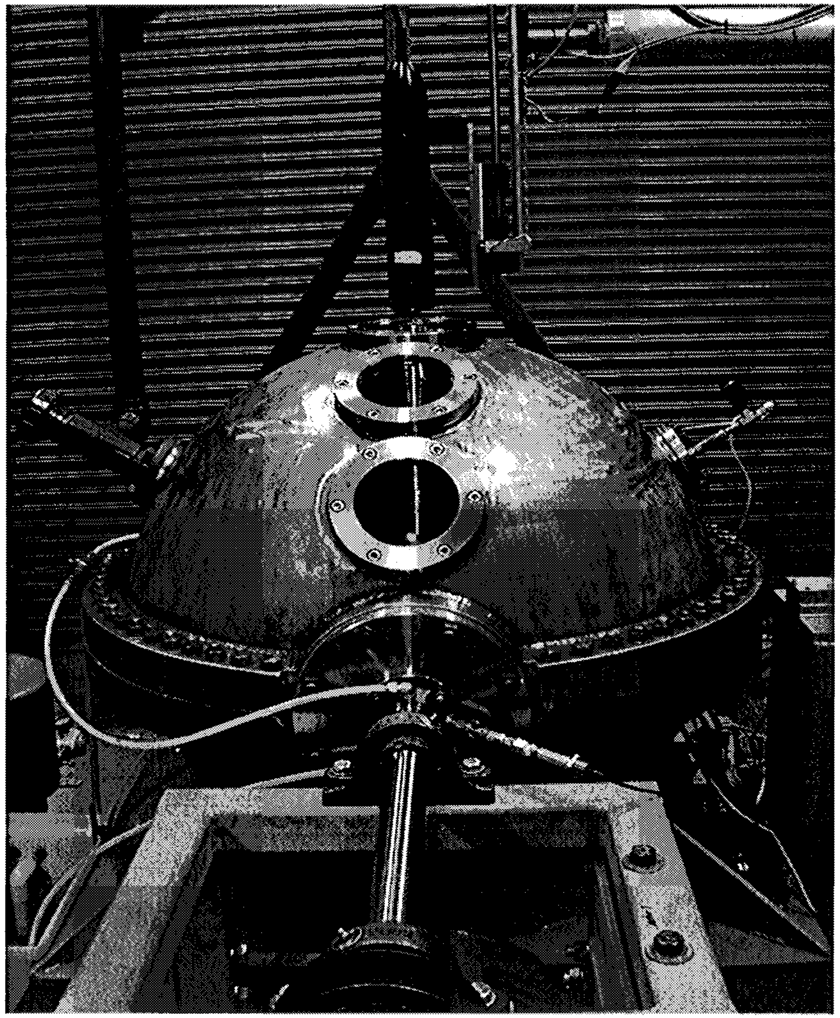


Why sodium?

- the control parameter is the magnetic Reynolds number
  - $Rm = \mu_0 \sigma a V_{max} \propto$ conductivity $\times$ size $\times$ velocity
  - quantifies relative importance of generation of B field by velocity and diffusion (decay) of B due to resistive decay of electrical currents
  - must exceed critical value for system to self-excite
- Sodium is more conducting than any other liquid metal (melts at 100 C)
  - $Rm = 120$ for $a=0.5$ m, $V_{max}=15$ m/s

Dimensionally identical water experiment is used to create flows and test technology

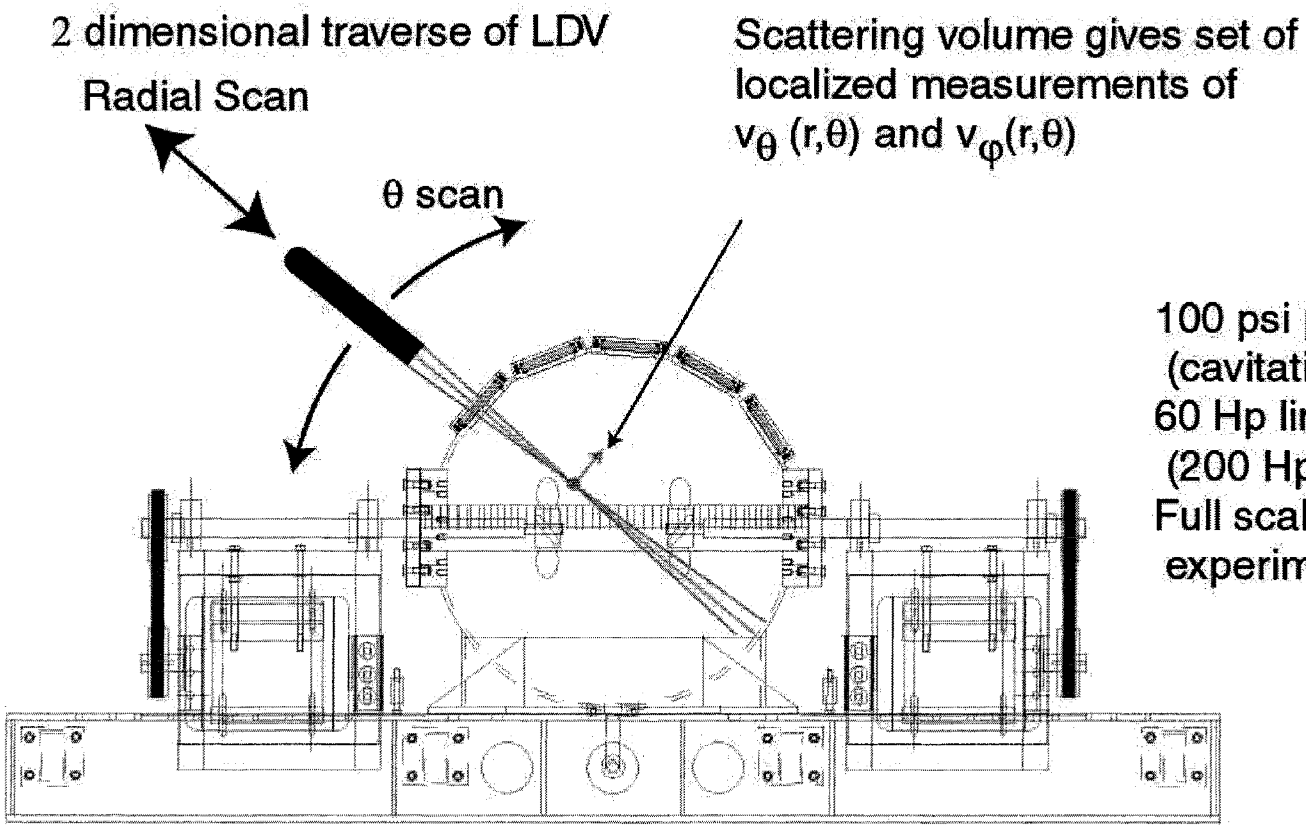
- Laser Doppler velocimetry is used to measure vector velocity field
- Measured flows are used as input to MHD calculation
- Full scale, half power



	Sodium	Water
Temperature	$110^\circ C$	$50^\circ C$
viscosity	$0.65 \times 10^{-6}\ \mathrm{m^2\,sec^{-1}}$	$0.65 \times 10^{-6}\ \mathrm{m^2\,sec^{-1}}$
mass density	$0.925\ \mathrm{gm\,cm^{-3}}$	$0.988\ \mathrm{gm\,cm^{-3}}$
resistivity	$10^{-7}\ \Omega\mathrm{m}$	

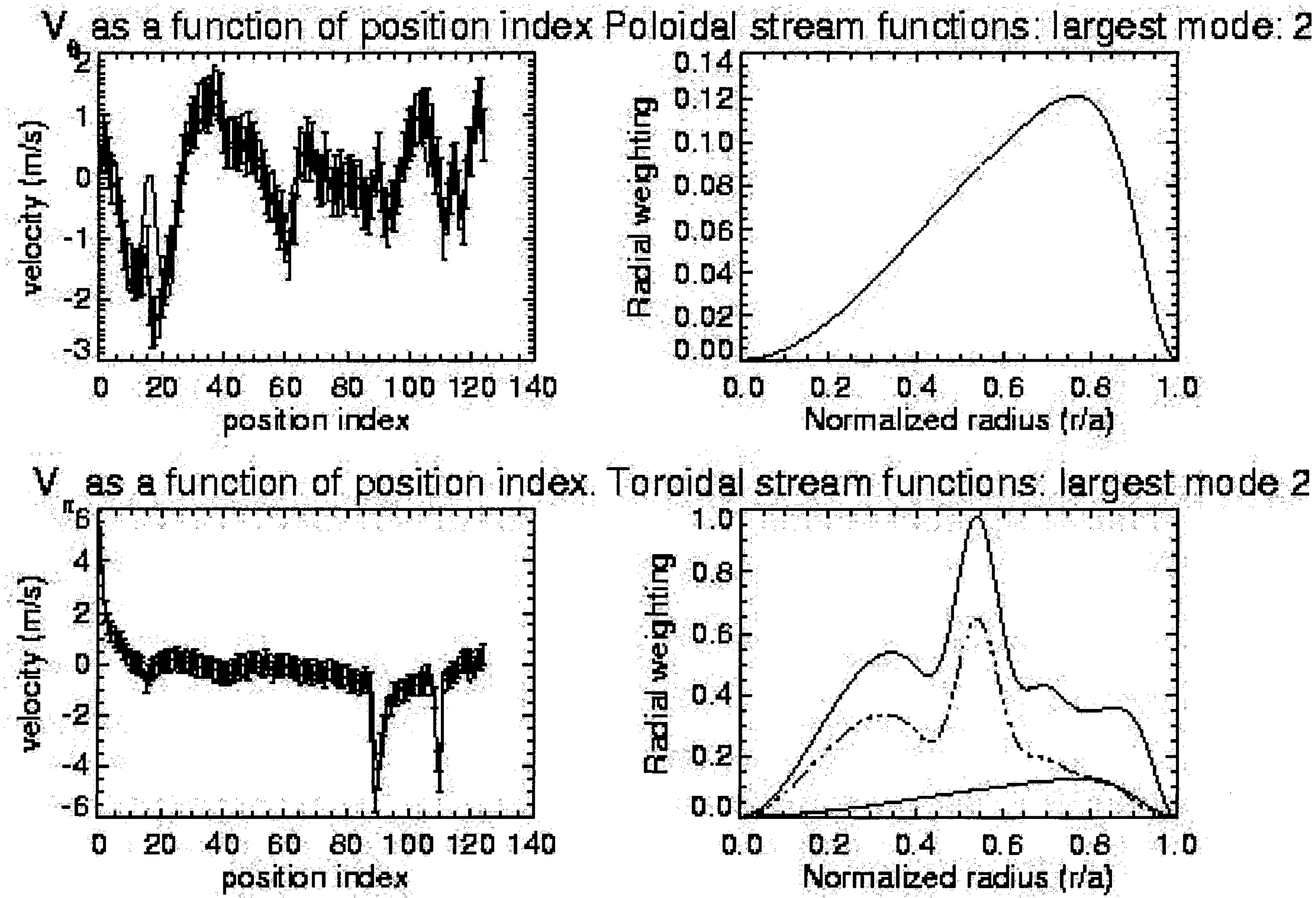
$$\longrightarrow \quad Rm = \frac{\mu_0 a V}{\eta} = 4\pi\, a(m) V(m/s)$$

Measurements at discrete positions are used to find radial profile functions for input to eigenvalue code

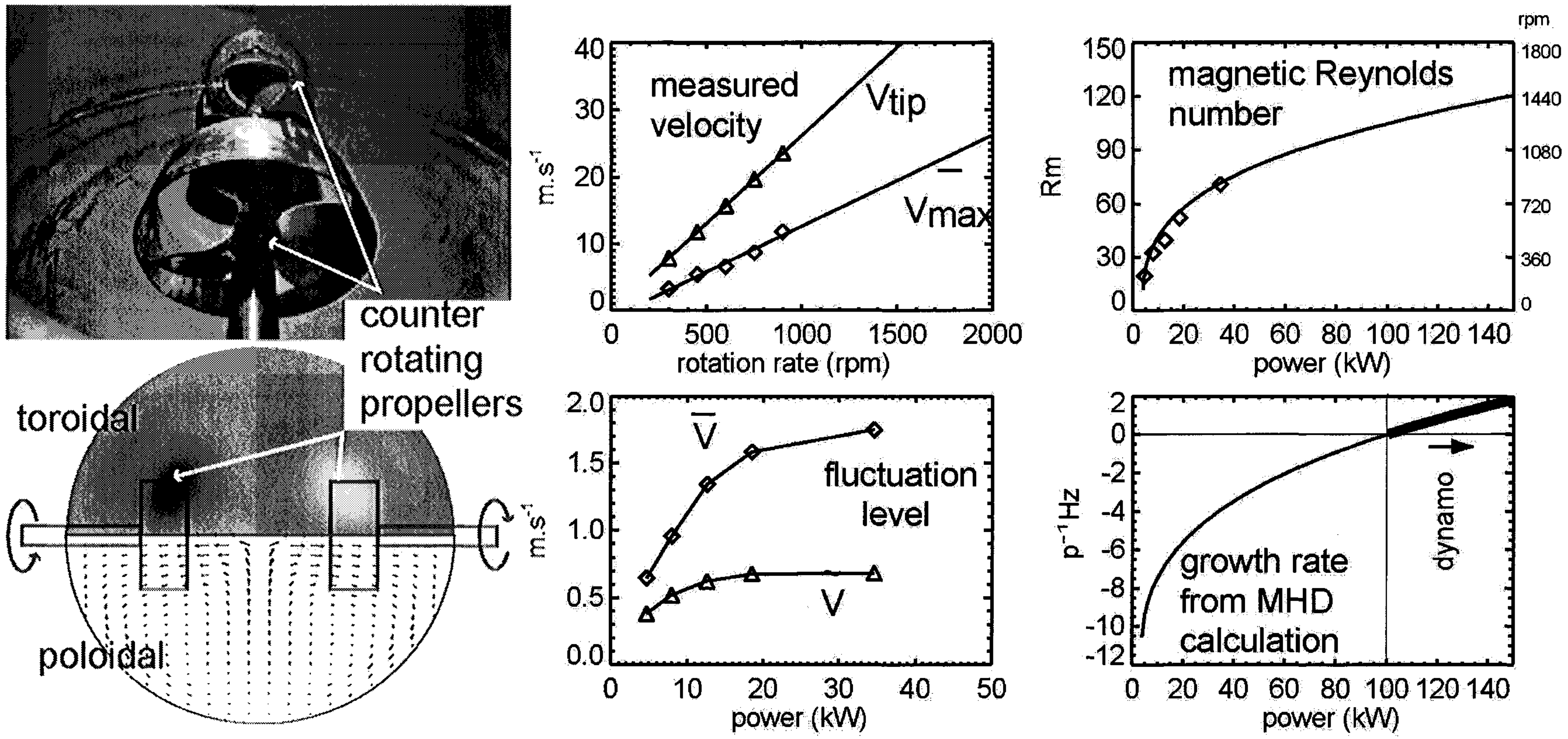


100 psi pressure vessel
(cavitation issue)
60 Hp limit on motors
(200 Hp on sodium device)
Full scale model of sodium experiment

Data are fit by profile functions

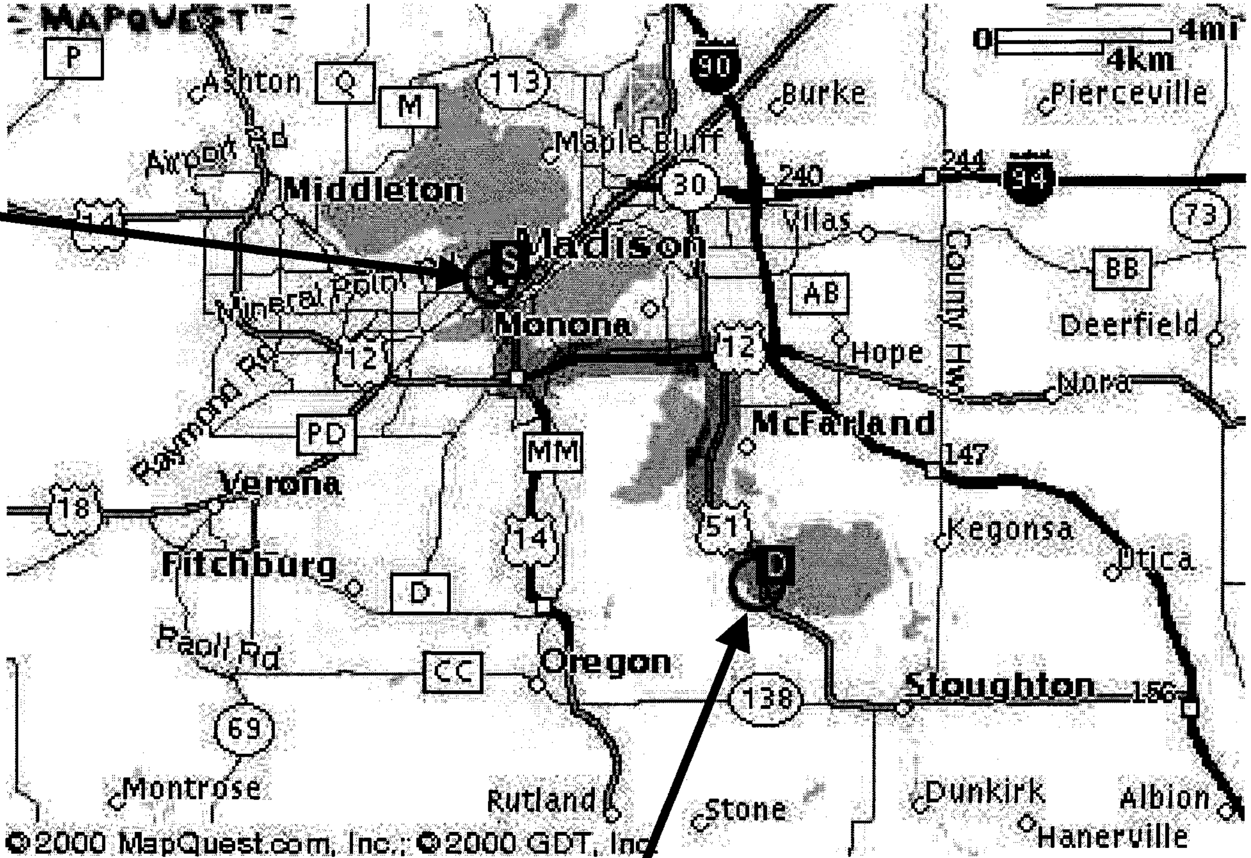


Velocity fields can be generated in water which lead to dynamo action (a=0.5 m,σ=10^7 mhos for sodium)

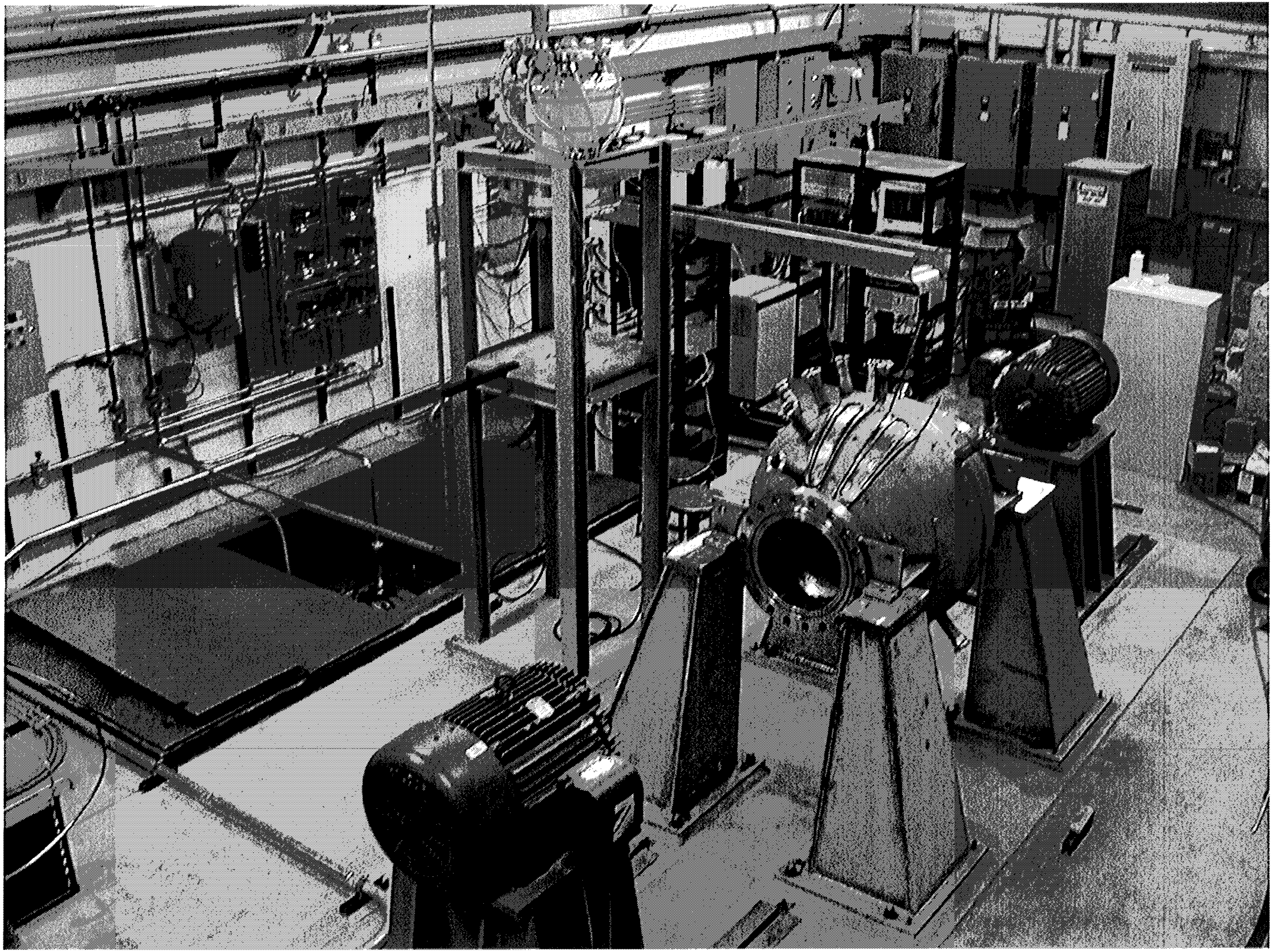


Sodium laboratory is 20 miles from Water laboratory (remember high school chemistry)

Physics Department



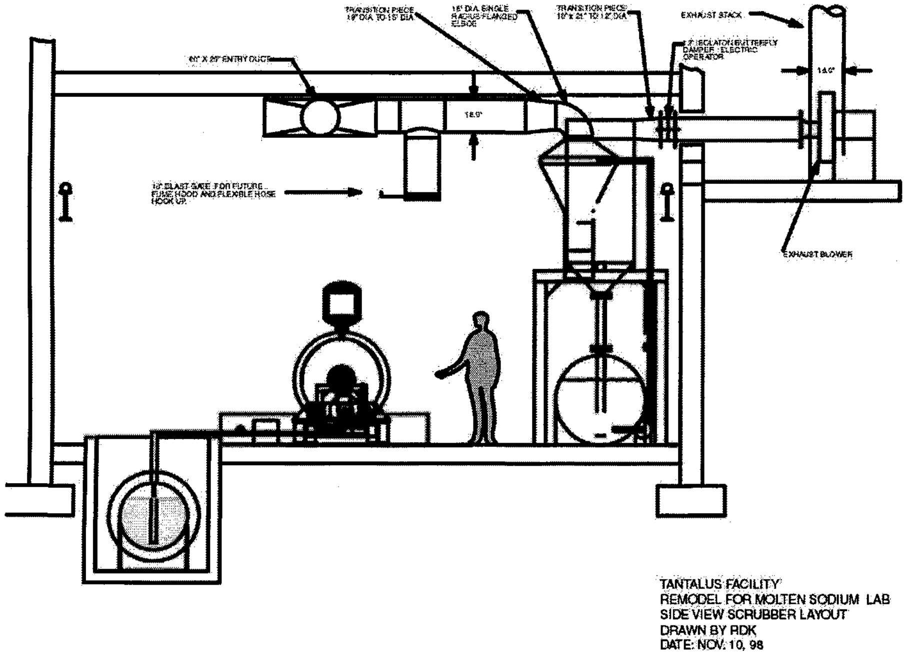
Liquid sodium laboratory

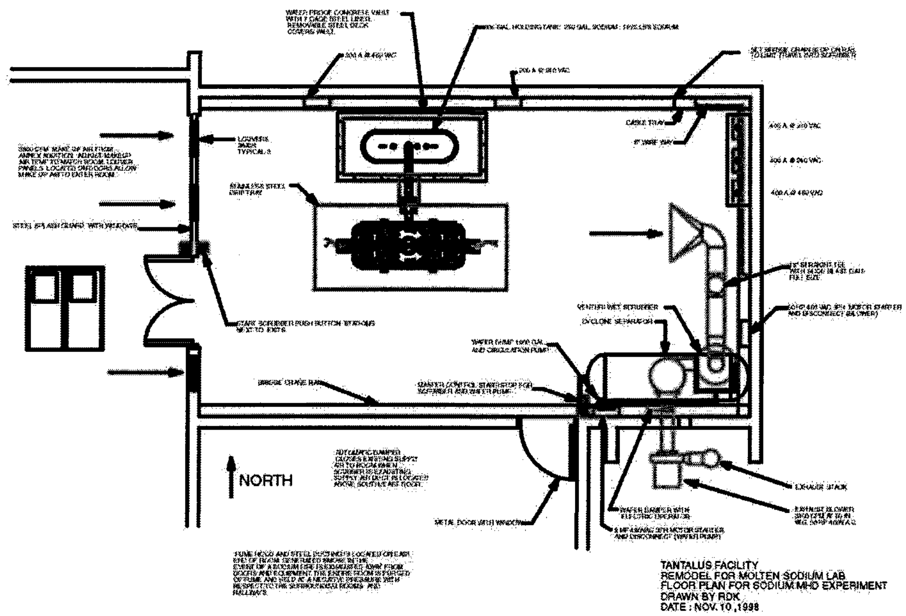


Molten sodium laboratory is operational

- A new laboratory has been constructed for housing the dynamo experiment

LABORATORY LAYOUT





Major technical challenges for engineering of sodium experiment

- Sodium is flamable, corrosive, and melts at 100 C
  - All stainless steel construction
  - All components are heated (expansion)
  - Remote operation
- Wetted seals
  - Double mechanical seals
- 200 Hp mechanical energy
  - Vibrations
  - Cooling required
- Cavitation on propellers
  - Entire system pressurized to increase peak velocity
- Diagnostics
  - Hall probes
  - ultrasound

The dynamo experiment will address several dynamo issues experimentally

- Do dynamically consistent flows exist for kinematic dynamos?
- How does a dynamo saturate?
  - role of Lorentz force on fluid velocity
  - How big is B?
- What role does turbulence play in a real dynamo?
  - energy equipartition of velocity fields and magnetic fields
  - enhanced electrical resistivity
  - current generation

Back-reaction and saturation is due to non-linearities in MHD Equations

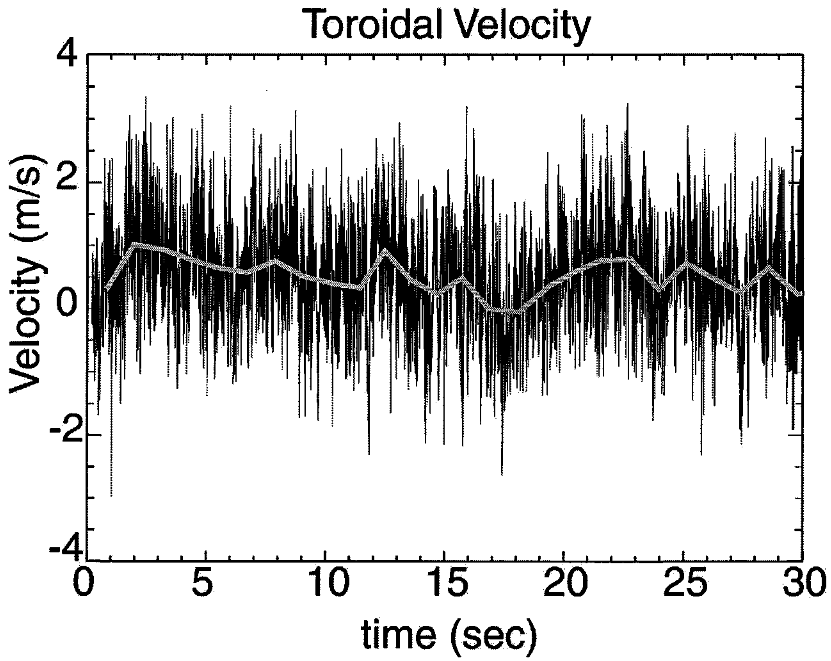
- Induction equation by itself is linear in B
  - **If V is given, linear solutions can be found (kinematic dynamo problem)**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

- Induction equation is non-linear if V is affected by B
- Navier-stokes is nonlinear in V and JxB
  - **V is naturally turbulent**
  - **J × B force modifying V we can call the back-reaction**

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{V} + F_{prop}$$

Flows are turbulent and large scale velocity field varies on resistive time scales



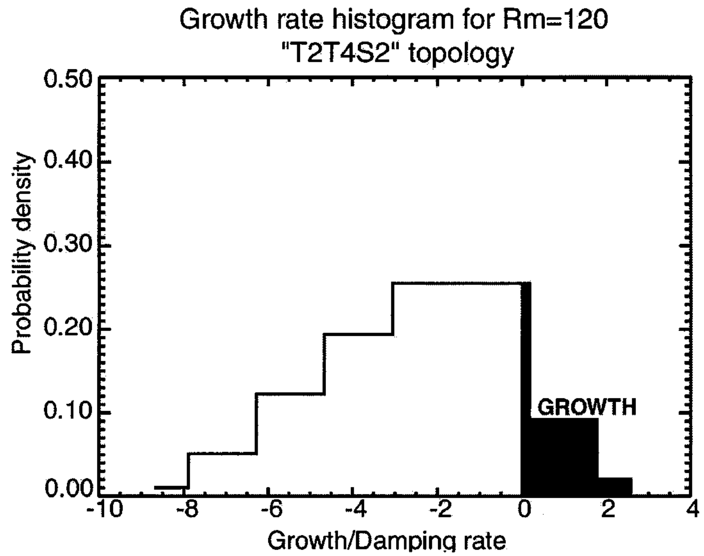
- Flows are turbulent

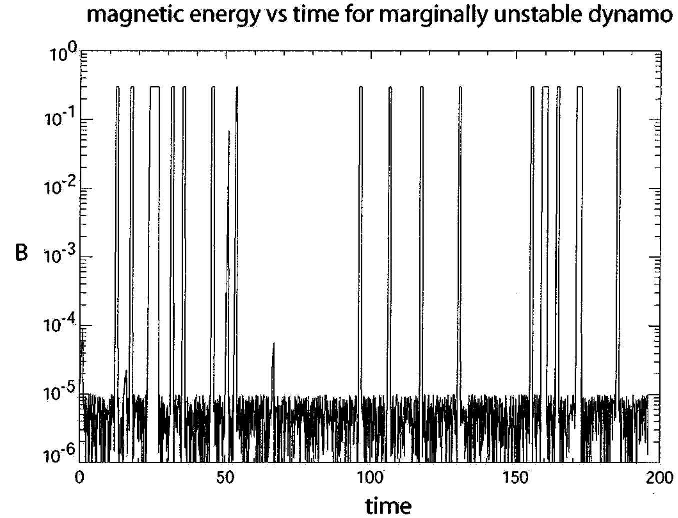
$$Re = \frac{1}{\nu \mu_0 \sigma} Rm = 10^5 Rm$$

- Large scale velocity field varies with time
  - Mechanically homogeneous
- Magnetic eigenmode analysis is only valid on correlation time of velocity fluctuations
- Turbulence from small scales may play important role

Flow should be linearly unstable some fraction of the time

- Assume mean flow is bounded by rms fluctuation levels
- Specific geometry corresponds to a family of flow profiles
- Some fraction of flow profiles self-excite
- Monte-Carlo analysis shows estimates fraction of time in growing phase
- Temporal intermittency for dynamo?





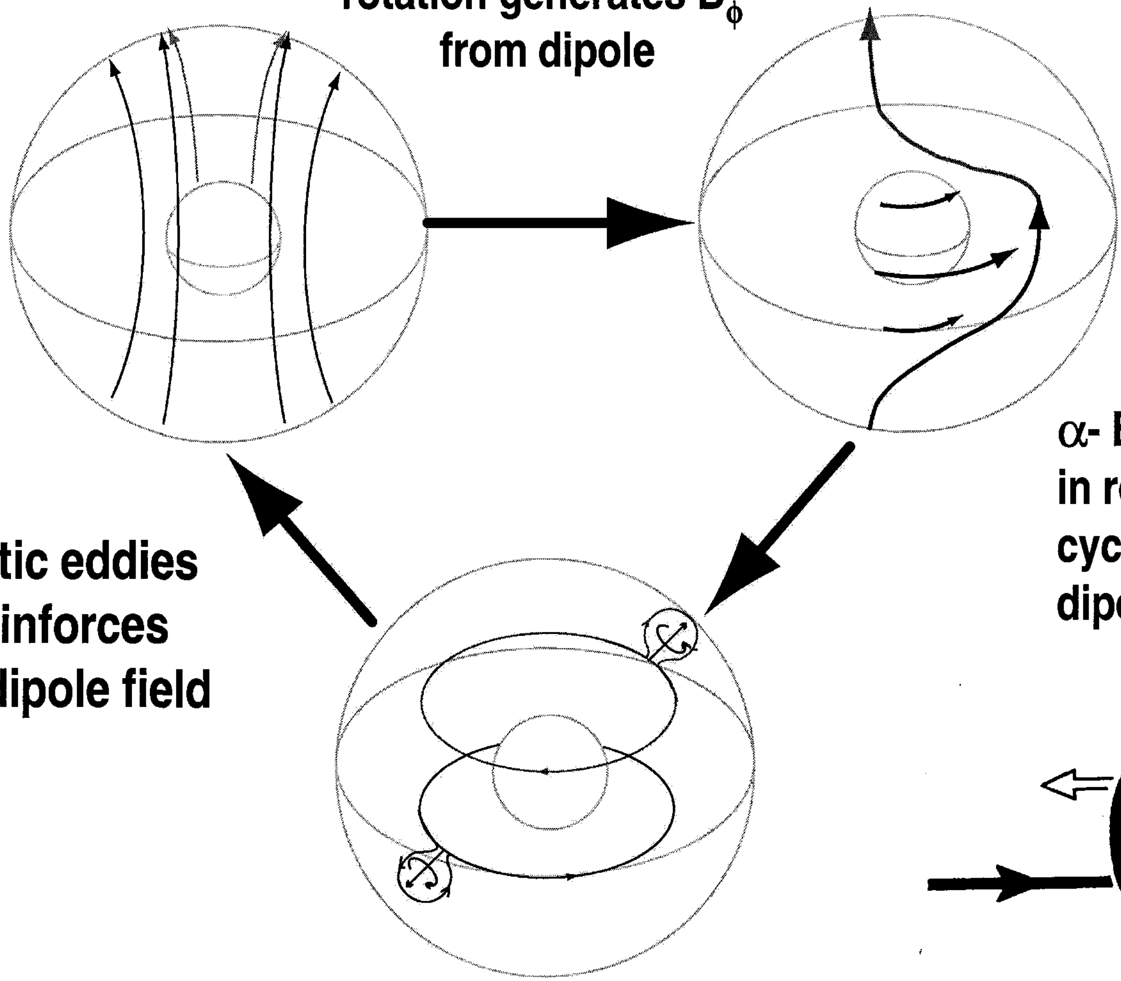
MHD Turbulence experiments

1. Cascades of magnetic and mechanical energy
2. An α-Ω dynamo (Is there an α effect?)
3. Anomalous resistivity

α–ω model for the solar dynamo; small scale helical eddies can contribute to a large scale magnetic field [Parker, 1955]

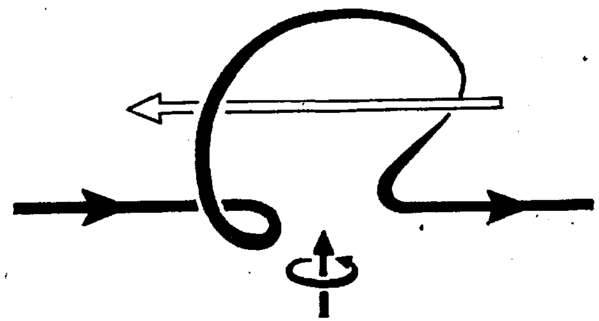
Ω-Effect: differential rotation generates B_ϕ from dipole

$$B_\phi \propto Rm\, B_p$$



α- Effect: magnetic buoyancy in rotating sphere generates cyclonic eddies which produce dipole field from B_ϕ

magnetic eddies field reinforces initial dipole field



Fluctuations can generate currents: mean-field electrodynamics

Separate fields into mean fields and fluctuations:

$$\mathbf{V} = \bar{\mathbf{V}} + \tilde{\mathbf{v}},\ \mathbf{B} = \bar{\mathbf{B}} + \tilde{\mathbf{b}},\ \mathbf{B} = \langle \mathbf{B} \rangle \tag{1}$$

Quasilinear EMF arises from fluctuations

$$\left\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \right\rangle = \int^{t} \left\langle \tilde{\mathbf{v}} \times \nabla \times \left(\tilde{\mathbf{v}} \left(t' \right) \times \bar{\mathbf{B}} \right) \right\rangle dt' \tag{2}$$

Assuming isotropic and homogeneous turbulence

$$\begin{aligned} \mathcal{E}_{turb} &= \left\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \right\rangle \\ &= \alpha \mathbf{B} - \beta \nabla \times \mathbf{B}, \end{aligned}$$

where

$$\alpha = \int_0^{\infty} \left\langle \tilde{\mathbf{v}} \left(0 \right) \cdot \nabla \times \tilde{\mathbf{v}} \left(t' \right) \right\rangle dt' \ \beta = \int_0^{\infty} \left\langle \tilde{\mathbf{v}} \left(0 \right) \cdot \tilde{\mathbf{v}} \left(t' \right) \right\rangle dt' \tag{3}$$

Mean-Field Electrodynamics: the α and β effects, and turbulent conductivity

Mean-field Ohm's law is

$$\begin{aligned}\mathbf{J} &= \sigma\left(\mathbf{E}+\mathbf{V}\times\mathbf{B}+\alpha\mathbf{B}-\beta\nabla\times\mathbf{B}\right)\\ &= \frac{\sigma}{1+\mu_0\sigma\beta}\left(\mathbf{E}+\mathbf{V}\times\mathbf{B}+\alpha\mathbf{B}\right)\end{aligned}$$

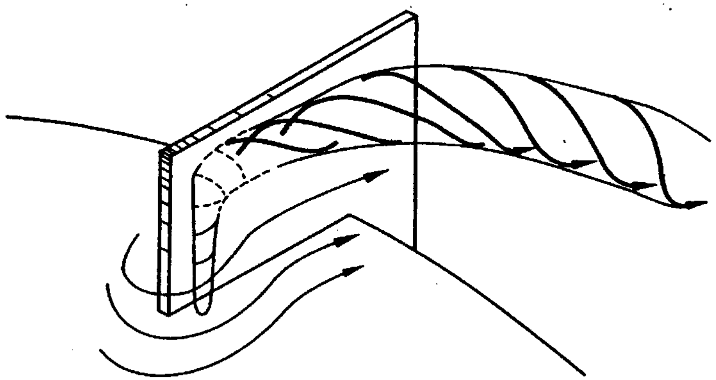
giving the turbulent conductivity

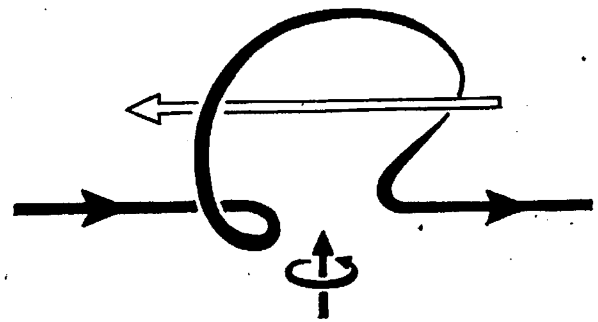
$$\sigma_T = \frac{\sigma}{1+\mu_0\sigma\beta} \tag{1}$$

Simplified scaling gives

$$\beta = \frac{\tau_{corr}}{3}\left\langle\tilde{\mathbf{v}}^2\right\rangle \tag{2}$$

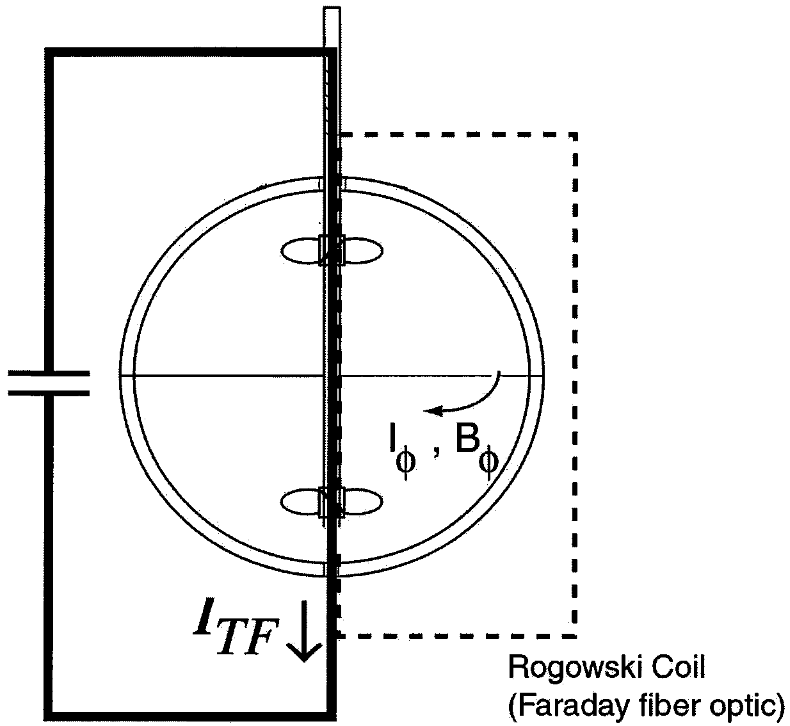
Mixing propellers are great at producing turbulent helicity (its what they were designed to do); differential rotation is easy



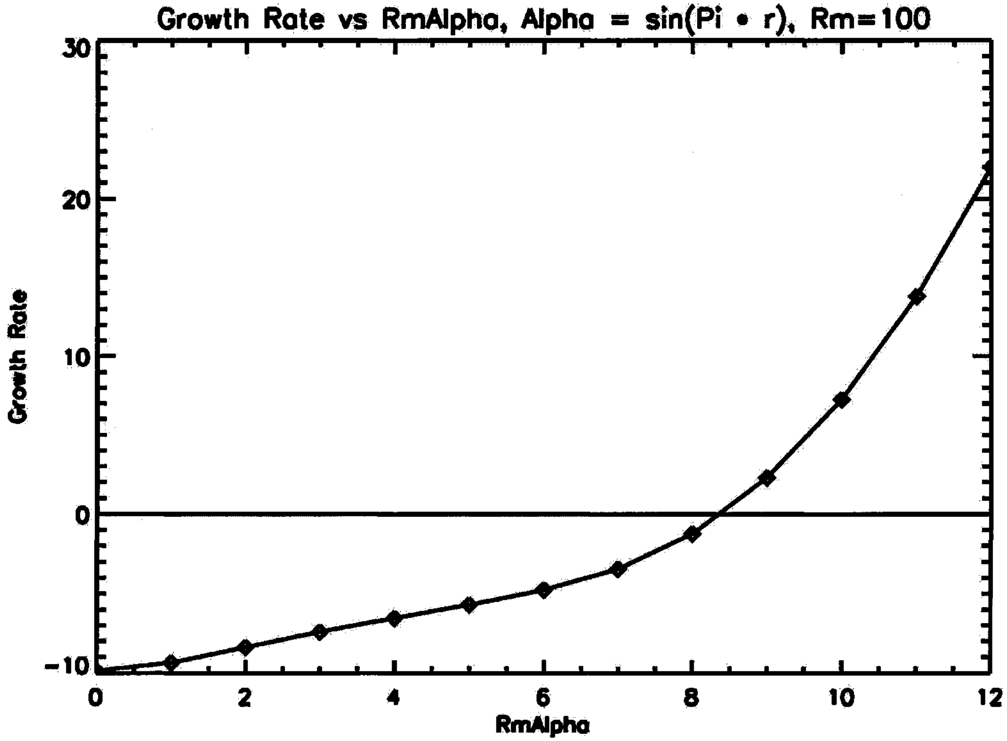


Experiment to measure turbulent α-effect

- Apply toroidal field. If toroidal current is observed, symmetry breaking fluctuations must be responsible
- Increase B and look for modifications to α-effect
- Study role of large scale B on small scale fluctuations (saturation through Alfven effect)



Growing eigenmode is possible if helical turbulence is strong enough together with Ω-Effect (for dipole along symmetry axis)



Dipole eigenmode generated by T1 flow and α-effect depends upon $Rm \times R_\alpha$, where $R_\alpha = \mu_0 \sigma a \alpha_{max}$

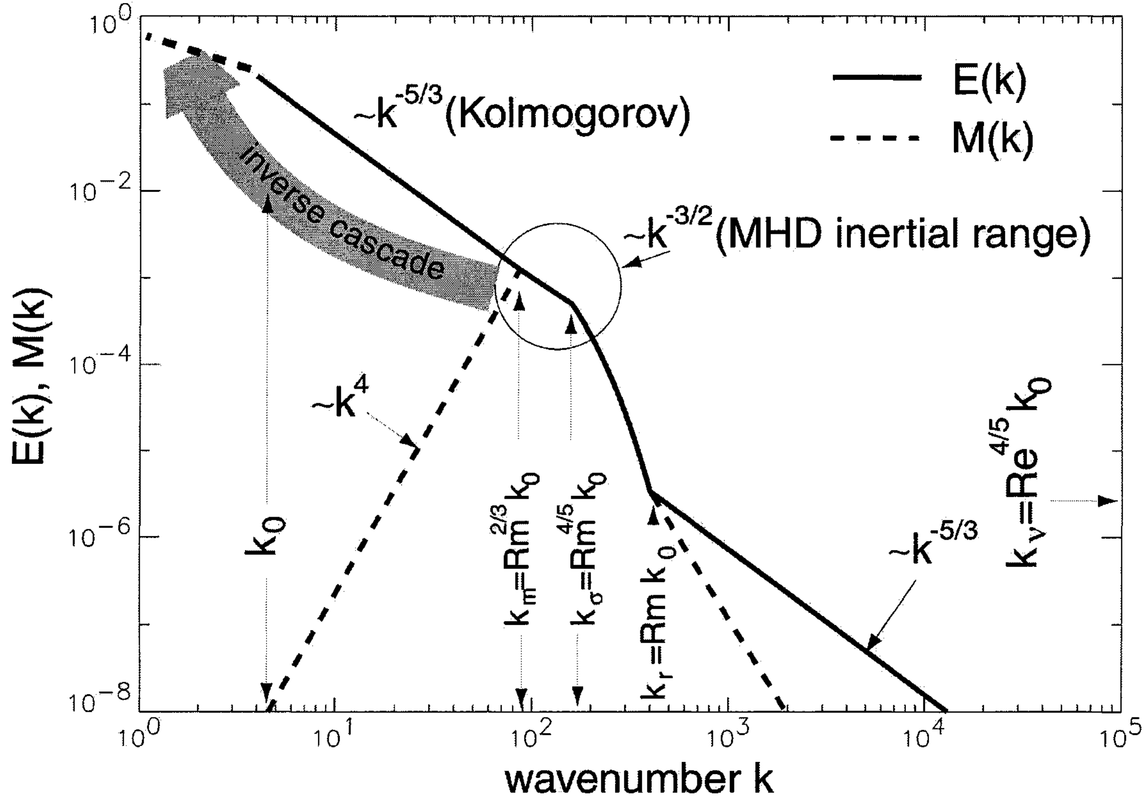
Question: How does the α-effect saturate? What are the non-linear effects in mean-field electrodynamics?

- α-effect as derived is a kinematic treatment
- Current is generated by turbulent flow helicity

$$
\begin{aligned}
\mathbf{E} &= \left\langle \delta\tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \right\rangle + \left\langle \tilde{\mathbf{v}} \times \delta\tilde{\mathbf{b}} \right\rangle \\
\delta\tilde{\mathbf{b}} &= \int_{-\infty}^{t} \nabla \times \left(\tilde{\mathbf{v}}\left(t'\right) \times \mathbf{B}_0 \right) dt' \\
\delta\tilde{\mathbf{v}} &= \int_{-\infty}^{t} \left(\nabla \times \tilde{\mathbf{b}}\left(t'\right) \right) \times \mathbf{B}_0 dt' \\
\alpha &= \frac{\tau_{corr}}{3} \left(\langle \tilde{\mathbf{v}} \cdot \nabla \times \tilde{\mathbf{v}} \rangle - \left\langle \tilde{\mathbf{b}} \cdot \nabla \times \tilde{\mathbf{b}} \right\rangle \right) \\
&\propto \frac{\langle \tilde{\mathbf{v}} \cdot \nabla \times \tilde{\mathbf{v}} \rangle}{1 + \frac{\sigma \tau_{corr}}{\rho} B^2}
\end{aligned}
$$

See Gruzinov and Diamond's Quasilinear Treatment of backreaction term

Alternative viewpoint: turbulent cascades and inverse cascades of magnetic energy (Kraichnan, 1965)



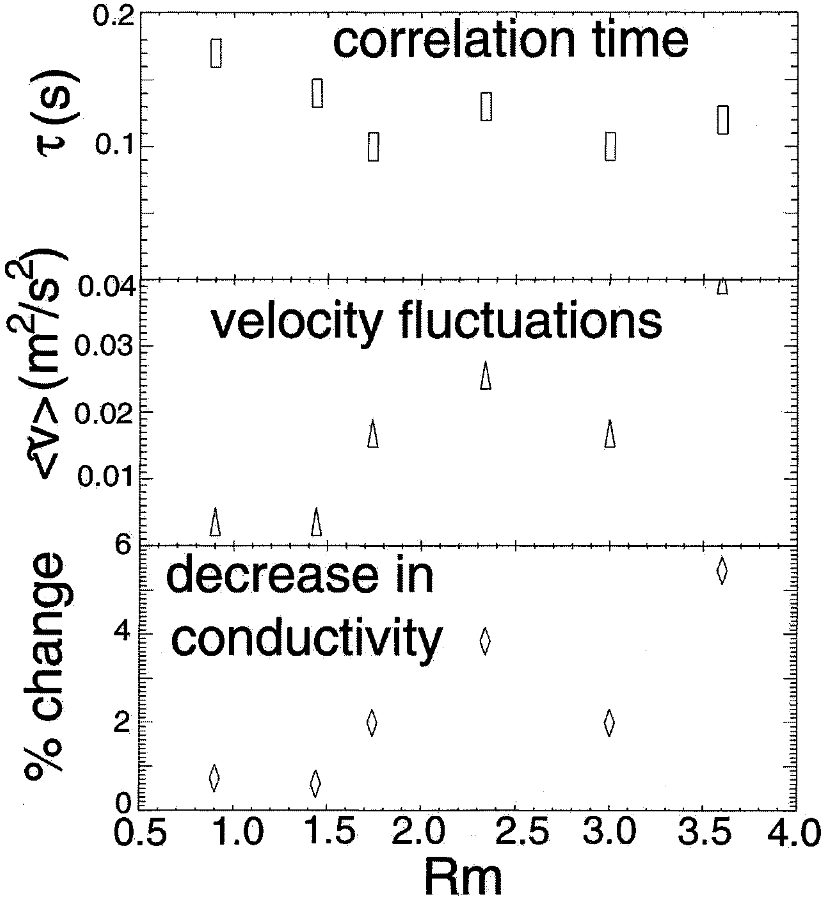
- **equipartition between magnetic and kinetic energy is predicted for small scales**
- **resistivity enhancement due to mixing of magnetic fields on small scales is predicted**
- **Currents can be generated by helical velocity fluctuations on small scales**

Measured turbulence levels already indicate resistivity enhancement may be important

- Turbulence levels are measured on small experiment

$$\sigma_T = \frac{\sigma}{1+\mu_0\sigma\beta}$$

$$\beta = \frac{\tau_{corr}}{3}\left\langle \tilde{\mathbf{v}}^2 \right\rangle$$

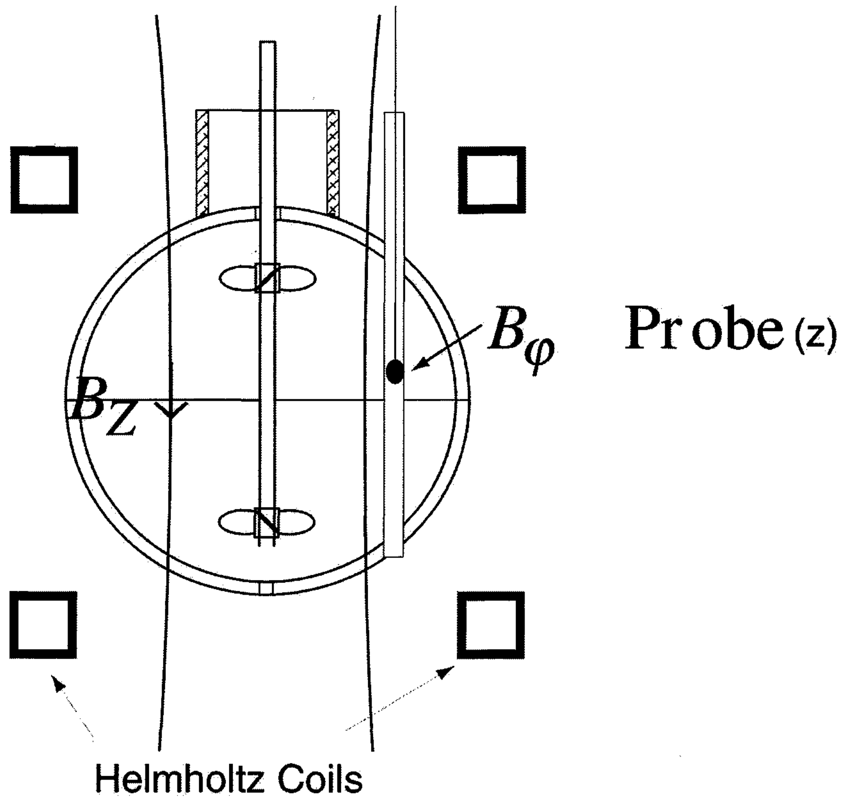


Experiment to measure β-effect

- Use transient techniques to estimate resistivity (measure electrical skin depth of system)

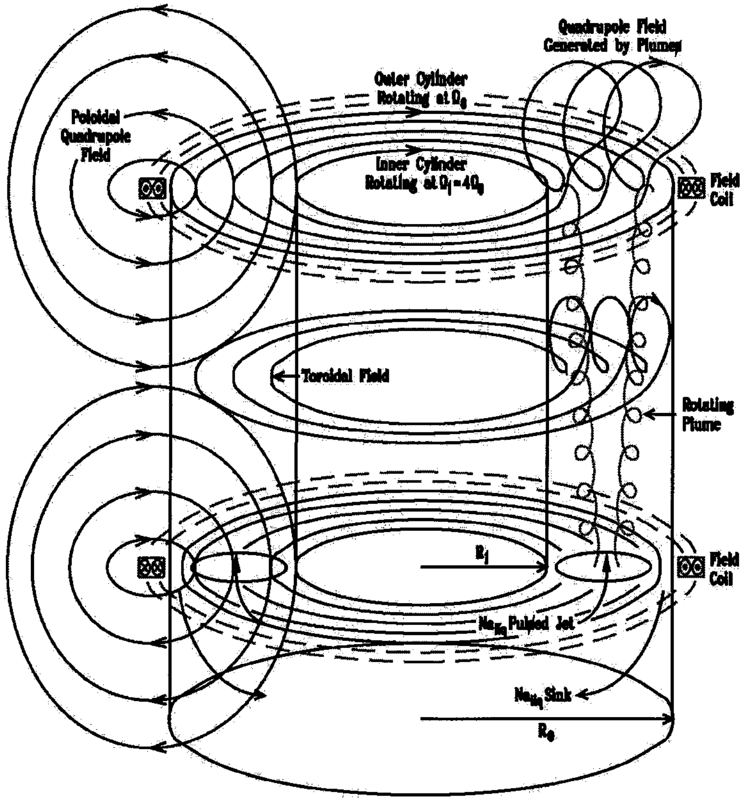
- Use ω-effect to measure Rm with applied poloidal field

$$B_\phi \propto \sigma_{turb} \omega B_z$$

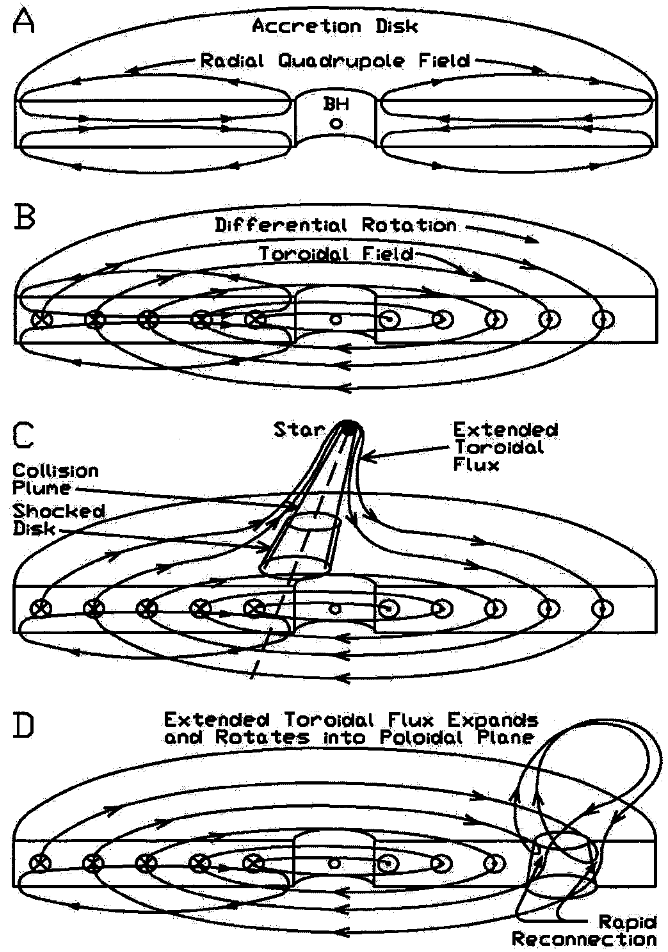


α–ω model for the galactic magnetic field [Colgate]

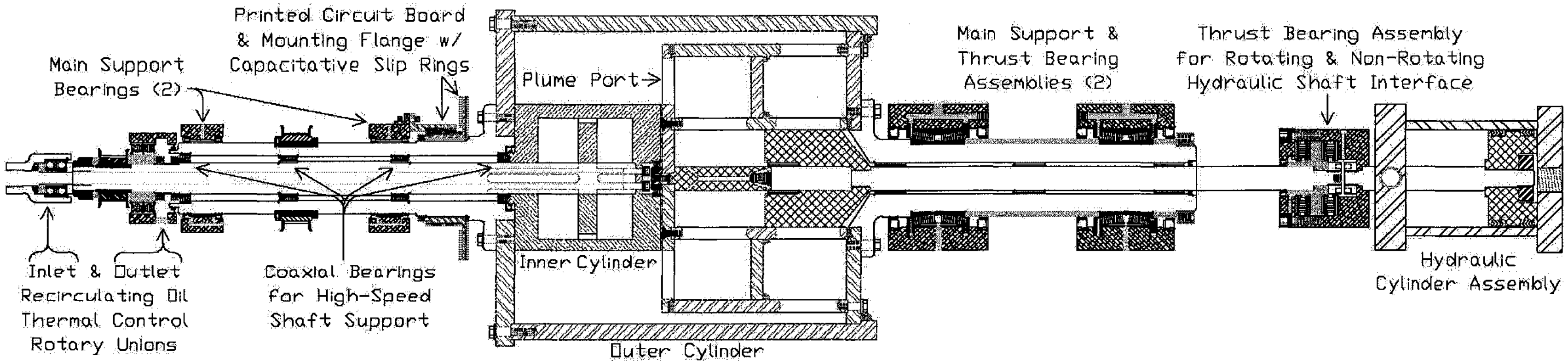
New Mexico Institute of Mining and Technology Experiment

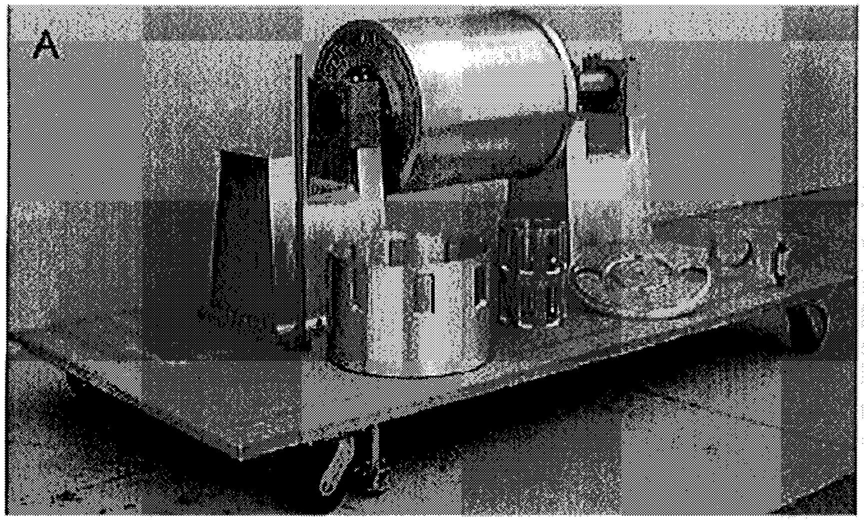


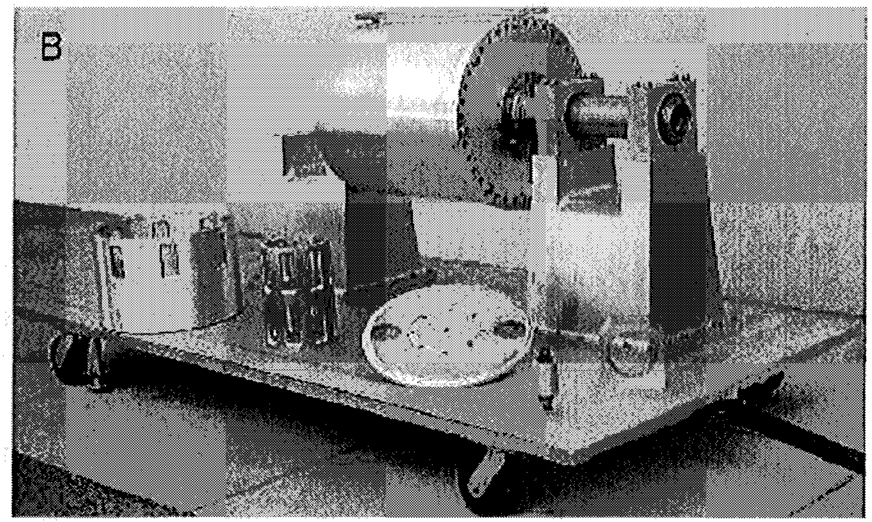
Galaxy



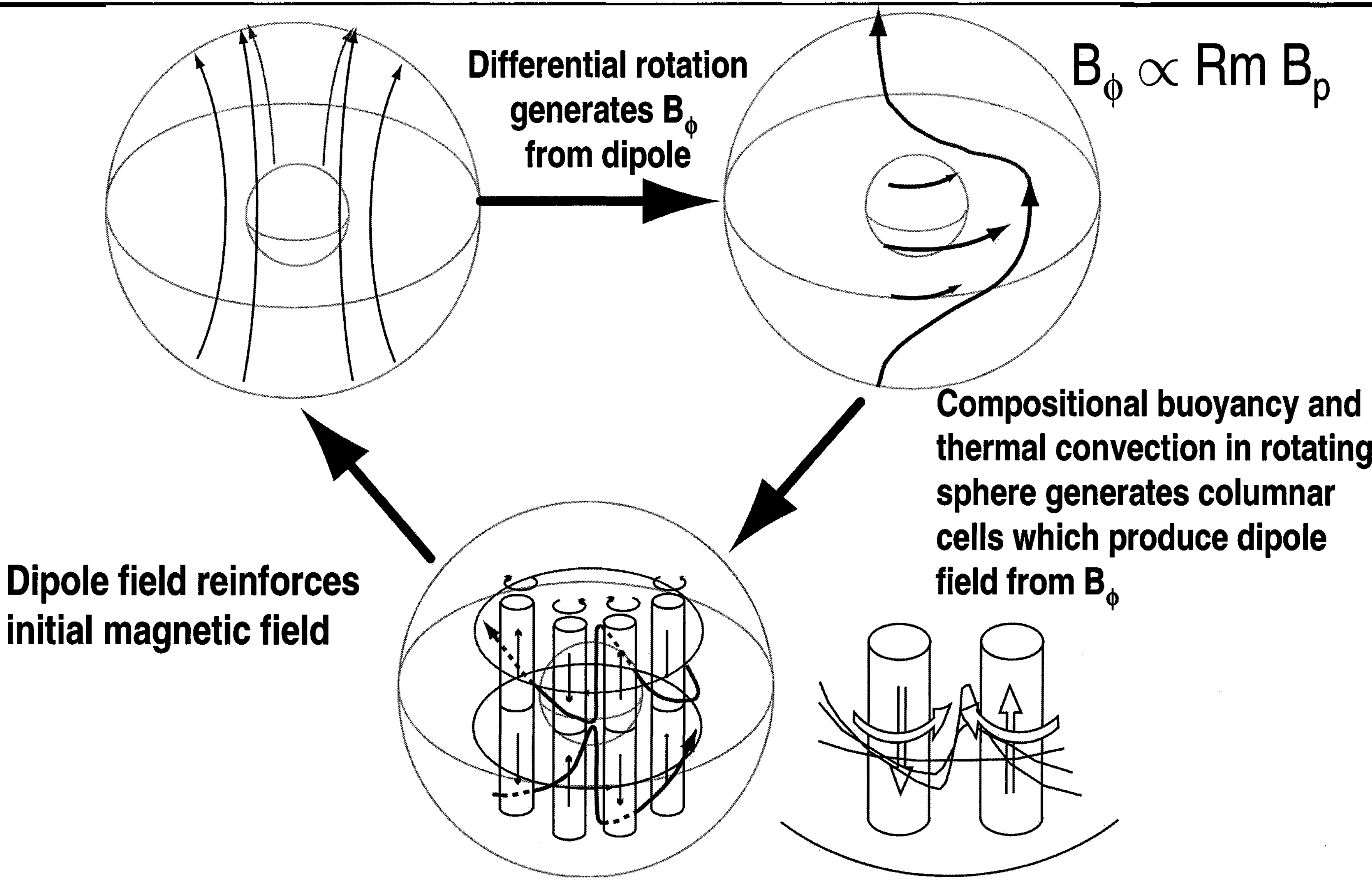
New Mexico Institute of Mining and Technology α-Ω experiment Magnetized Couette Flow plus plums (led by Stirling Colgate) Magnetorotational instability and α-effect



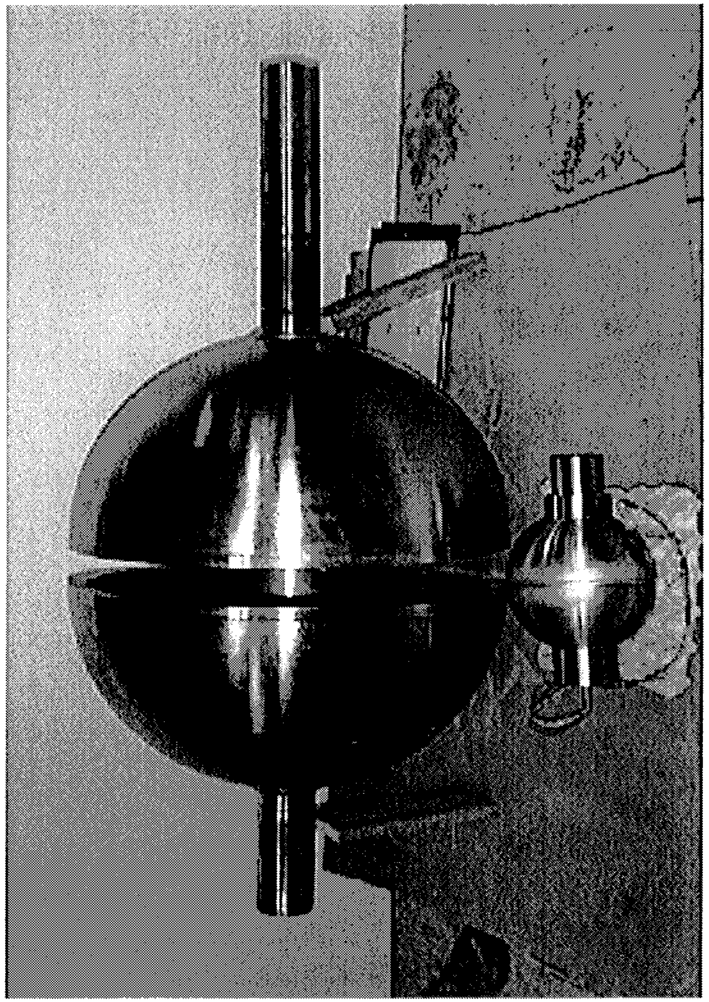


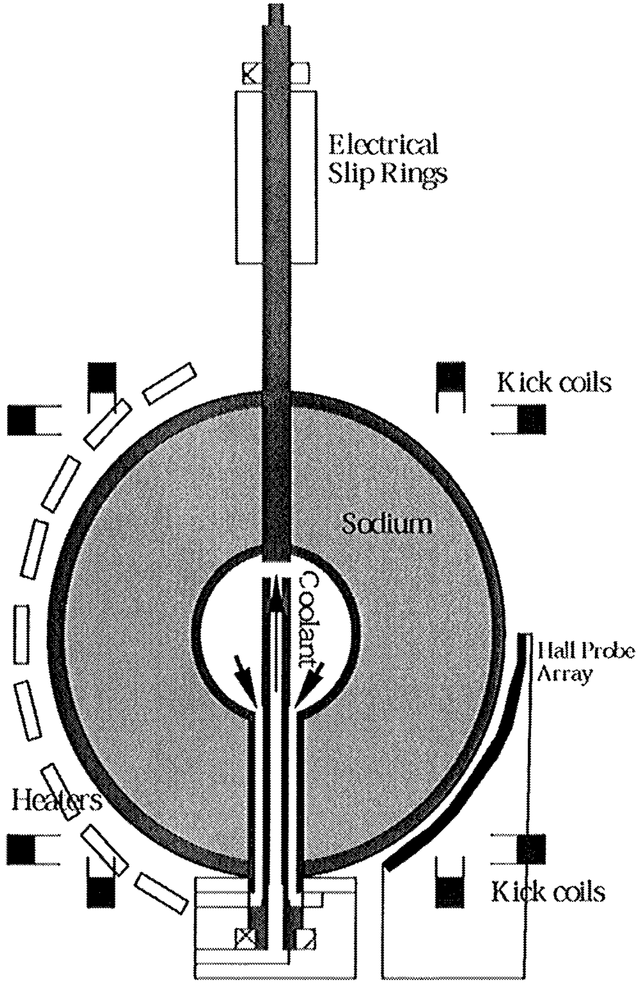


Cartoon of a laminar geodynamo driven by convection and differential rotation [Busse, Kageyama and Sato]



University of Maryland experiments (Dan Lathrop) study rotating convection and mechanically driven systems





Summary

- water experiments have demonstrated flows which may lead to a stretch-twist-fold dynamo experiment
- sodium laboratory facility is completed
  - sodium operation is imminent
- initial experiments will search for growing eigenmodes and begin studies of MHD turbulence

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