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Active Galactic Nuclei: Accretion Disks and Jets - I

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Active galactic nuclei. Accretion disks and jets. I

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1. – Introduction

Active galactic nuclei and, more generally, the nuclei of normal galaxies are the sites where large luminosities are produced by non-stellar mechanisms connected with strong dynamical activity and all kinds of high-energy events. The energetic requirements are commonly interpreted as gravitational energy release by accretion onto supermassive black holes. In these lectures we review the observational motivations of such model and discuss the plasma physics of accretion disks and of the launch of supersonic and relativistic jets: accretion disks and jets appear to be the key elements in a unifying picture for all galactic nuclei.

2. – Observations of AGNs

The discovery of spiral galaxies with bright nuclei by Seyfert (1943) revealed the importance of galactic nuclei in the global energetics of galaxies. Since then several classes of active galactic nuclei (AGN) have been discovered and also the presence of dormant nuclei in normal galaxies has been inferred.

2·1. *Seyfert galaxies.* – They constitute a class of spiral galaxies with unresolved ($< 100\,\mathrm{pc}$), bright ($M_{\mathrm{B}} > -21.5 + 5\log h_0$, $h_0 = H_0/(100\,\mathrm{km\ s^{-1}\ Mpc^{-1}})$) nuclei with spectra characterized by strong emission lines by highly ionized atoms; these lines require photoionization by non-stellar photon fluxes or collisional ionization by some form of dynamical activity.

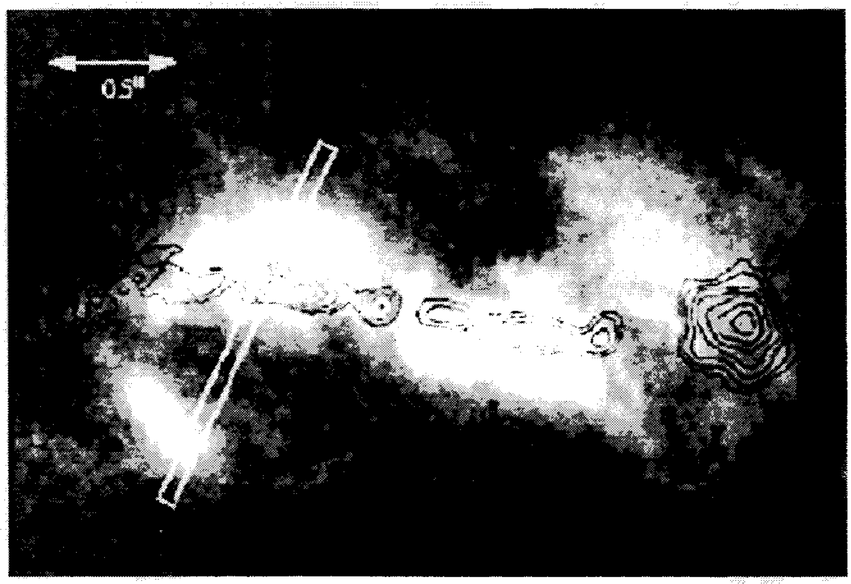

Fig. 1. – HST observations of the nucleus of the galaxy Markarian 3 compared with the structure of its radio jet [1].

Emission lines identify two subclasses:

1. *Seyfert 1* with two sets of emission lines coming from different regions:
   - narrow (permitted and forbidden) lines, corresponding to plasma with velocities $v \sim 10^2\,\mathrm{km\ s^{-1}}$ and densities $n_e \sim 10^3$–$10^6\,\mathrm{cm^{-3}}$;
   - broad (permitted) lines, from plasmas with $v \sim 10^4\,\mathrm{km\,s^{-1}}$ and $n_e \geq 10^9\,\mathrm{cm^{-3}}$.
2. *Seyfert 2* with only narrow emission lines (improved spectroscopic sensitivity allows to detect some weak broad lines in polarized light).

Additional characteristics of both classes are: weak absorption lines, "featureless non-thermal continuum", distinct from stellar continuum, weak in radio, strong in X-rays.

Recent HST imaging (fig. 1) allows to associate the line-emitting regions with elongated structures also observed in radio; if the elongated structures are interpreted as jet-like structures in the nuclear region, the origin of emission lines could be associated with collisional ionization by dynamical sources (shocks, turbulence, etc. [1]).

The velocity dispersion in Seyfert nuclei is typically $\langle v^2 \rangle^{1/2} \sim 10^3\,\mathrm{km\ s^{-1}}$ and correspondingly the equilibrium of the nucleus ($R < 100\,\mathrm{pc}$) requires a total mass

$$M_{\mathrm{Sy}} \sim \frac{\langle v^2 \rangle R}{G} \sim 10^{9\pm1} M_{\odot}.$$

2·2. *Radio galaxies.* – Extended radio galaxies were discovered in the 1950s and later associated with elliptical galaxies. They have a continuum spectrum $S_\nu \propto \nu^{-\alpha}$ with

$\alpha \sim 0.5$–2 extending up to the optical range, and display two opposite well-collimated jets extending in some cases up to few Mpcs and ending in diffuse low-surface brightness radio lobes and halos. Jets are inhomogeneous with bright knots, wiggles and bending by large angles. VLBI observations of powerful sources allow to detect superluminal proper motions in pc scale jets, with knots of size ~ 1 light month.

Fanaroff and Riley [2] proposed a classification of radio galaxies in terms of their total power:

1. FR I with $P_{178\mathrm{MHz}} < 5 \times 10^{25}$ W Hz^{-1}, corresponding to weak nuclei and dominated by highly turbulent jets;

2. FR II with $P_{178\mathrm{MHz}} > 5 \times 10^{25}$ W Hz^{-1}, corresponding to strong nuclei and strong jets, often one-sided.

The optical emission of AGNs associated with radio galaxies also shows narrow and broad emission lines.

2·3. *Quasars and* QSO*s*. – These star-like objects were originally identified in the radio band [3]. They have large redshifts, up to $z > 4$, a time variable continuum flux, $S_\nu \propto \nu^{-\alpha}$, strong UV flux, $U - B < 0$. Radio-quiet quasars (QSOs) were then identified by their UV excess and are 20 times more abundant than radio-loud.

These AGNs have very bright (brighter than Seyfert's), spatially unresolved nuclei, $M_\mathrm{B} < -21.5 + 5 \log h_0$, $h_0 = H_0/(100\,\mathrm{km\ s^{-1}\ Mpc^{-1}})$; they are surrounded by a "stellar fuzz": but recent HST data give evidence of association with a galactic structure.

Very often radio-loud nuclei are associated with jets.

Narrow and broad emission lines are always present, with narrow lines always weaker than broad lines. Stellar absorption lines are almost inexistent.

2·4. *Other classes of* AGN*s*. – Depending on the observational criteria used several classifications of peculiar galactic nuclei have been proposed in the literature.

LINERS. Galaxies with strong nuclear low-ionization emission lines, weak nuclei; they are classified in terms of the lines flux ratio $[OIII]/H\beta$: if interpreted by photo-ionization, a flat nuclear continuum is required.

BL Lacs and OVV and blazars. Bright, highly variable compact sources with featureless optical spectrum and with large linear polarization $\gg 1\%$ (in other AGNs $< 1\%$); the absence of strong lines does not allow z measurements; they are typically radio loud, strong emitters also in the X and γ bands.

Narrow emission line X-ray galaxies. Seyfert galaxies with optical spectra reddened and extinguished by dust in the disk.

Starburst galaxies. AGNs with strong B excess, emission lines characteristics of strong HII regions, and possible evidence of the presence of O, B stars and supernova remnants; signatures of recent star formation activity is found in the infrared.

Markarian galaxies. A class of AGNs with strong UV excess in the spectra; it includes quasars, starburst, Seyfert and blue galaxies.

Zwicky galaxies and N galaxies. A class of AGNs with peculiar blue nuclei; it includes Seyfert and quasars.

IRAS galaxies. Ultraluminous nuclei in far infrared, $L_{\rm IR} \sim 10L_{\rm opt}$, due to thermal radiation from dust ($T \sim 100\,\rm K$).

2˙5. *Physical parameters.* – Not every element discussed above is present in all sources, but they all have similar global energetics in some morphological feature.The typical parameters of AGNs derived from observations can be summarized as follows.

1. Bolometric luminosities:
   - both bolometric luminosities and kinetic luminosities reach up to $10^{47}\,\rm erg\ s^{-1}$(blazars have limits 100 times larger, which are interpreted in terms of Doppler boosting).
2. Spectra:
   - non-thermal power law continua extending in many sources from the radio to the γ-ray range;
   - highly excited emission lines corresponding to strong pressure and with large proper velocities of the emitting clouds.
3. Extended features:
   - kinetic luminosities correspond to the emission of supersonic, relativistic jets, collimated over distances $\geq 1\,\rm Mpc$;
   - the activity of these extended features requires a continuous energy input of $\sim 10^{47}\,\rm erg\ s^{-1}$ for time scales $\approx 10^8\,\rm y$.

3. – Unified model of AGNs

The observational results now summarized can be globally interpreted in a unified model based on accretion onto supermassive black holes that liberates large amounts of gravitational energy a large fraction of which is transformed into radiation and dynamical activity. We start examining the motivations of the model.

TABLE I. – *Light-to-mass ratios for normal galaxies and* AGN*s.*

	$M/M_\odot$	R (kpc)	$L/L_\odot$	L/M (erg g^{-1} s^{-1})
Normal galaxy	10^{11}	30	10^{11}	1
Nucleus of normal galaxy	10^{8}	< 1	10^{9}	10
Nuclei of Seyfert	10^{9}	$< 10^{-1}$	$> 10^{11}$	10^{2}
Powerful AGNs	10^{9}	$\leq 10^{-6}$	10^{13}	10^{4}

3·1. *Search for an energy source.* – Thermonuclear burning in solar-type stars is insufficient to support luminosities $\sim 10^{47}\,\mathrm{erg\ s^{-1}}$; in fact the light-to-mass ratio of the Sun (specific power) is

$$\epsilon_\odot = \frac{L_\odot}{M_\odot} \sim \frac{4\times 10^{33}\,\mathrm{erg\ s^{-1}}}{2\times 10^{33}\,\mathrm{g}} \sim 2\,\mathrm{erg\ g^{-1}\ s^{-1}},$$

where one must take into account that only 10% of the total mass of the Sun is actually participating in the nuclear burning at any instant. If we consider total burning instead, the upper limit for thermonuclear reactions gives a maximum specific power

$$\epsilon \sim 30\,\mathrm{erg\ g^{-1}\ s^{-1}}.$$

If one compares this value with the specific powers suggested by observational data on AGNs, as shown in table I, it is clear that thermonuclear burning cannot account for the luminosity of AGNs, and only marginally for the nucleus of normal galaxies.

Alternatives proposed in the literature over many years after the discoveries of AGNs were: enhanced stellar activity and supernova explosions, neutron stars cluster, gravitational energy release around collapsed objects as spinars or supermassive black holes.

3·2. AGN*s and supermassive black holes* (SMBH). – From the above estimates we can define the efficiency factor, *i.e.* the fraction of Mc^2 transformed by thermonuclear burning of a typical star over its whole lifetime τ:

$$\alpha = \frac{L}{Mc^2/\tau} \sim 0.01.$$

This corresponds to the average fraction of rest mass of every particle transformed into radiation during the lifetime of the source. For the case of AGNs:

$$\alpha = \frac{L_{\mathrm{AGN}}}{M_{\mathrm{AGN}}\, c^2/\tau},$$
$$L_{\mathrm{AGN}} \sim 10^{47}\,\mathrm{erg\ s^{-1}}, \qquad \tau \sim 10^8\,\mathrm{y},$$
$$\alpha \sim 1.7 \times 10^7 \left(\frac{M_{\mathrm{AGN}}}{M_\odot}\right)^{-1} L_{47}\tau_8 .$$

Therefore thermonuclear burning (with $\alpha \le 0.1$) could match the AGN luminosity only for galactic nuclei with mass

$$\left(\frac{M_{\mathrm{AGN}}}{M_\odot}\right) \ge 1.7 \times 10^8 \alpha^{-1} L_{47}\tau_8 ,$$

typically larger than $10^9 M_\odot$. However, such a large mass is not consistent with observations as it would correspond to objects inside the gravitational horizon that cannot emit radiation. In fact the gravitational radius corresponding to this mass is

$$R_{\mathrm{g}} = \frac{2GM_{\mathrm{AGN}}}{c^2} \sim 3 \times 10^5 \left(\frac{M_{\mathrm{AGN}}}{M_\odot}\right) \mathrm{cm} \sim 5.2 \times 10^{13} \alpha^{-1} L_{47}\tau_8\,\mathrm{cm}$$

and for any value of α (even for efficiencies approaching unity) R_{g} is very close to (or higher than) $R_{\mathrm{AGN}} \le 0.001\,\mathrm{pc}$. There seems to be no way out from the conclusion that AGNs are close to their gravitational radius and cannot escape the collapse to black-hole state.

Observational evidence in favor of the presence of SMBH in galactic nuclei has been reviewed by Kormendy and Richstone [4]. In particular the strongest support comes from the study of the kinematics of water masers in some spatially resolved galactic nuclei (Miyoshi *et al.* [5]).

3·3. *Fuelling black holes: accretion power.* – These are the motivation that led to propose that AGNs are SMBH fuelled by accretion. General Relativity allows to calculate the energy liberated by particles falling from infinity to the gravitational horizon R_{g}; in particular we have: $E \sim 0.05mc^2$ for Schwarzschild black holes, and $E \sim 0.42mc^2$ for rotating Kerr black holes. It is clear that the efficiency of energy release through accretion onto Kerr black holes is much higher than that of thermonuclear reactions. However the luminosity produced by accretion depends on the accretion rate $\dot{M}$. We can estimate the value of $\dot{M}$ required to produce the AGN luminosities, in terms of η, the fraction of the particle rest mass energy mc^2 that is actually transformed into radiation (η is in fact the instantaneous value of the efficiency that was called α previously):

$$L = \eta \dot{M} c^2,$$
$$\dot{M} \sim \frac{1.8}{\eta} L_{47}\ M_\odot\, \mathrm{y}^{-1},$$

a rather modest requirement if $\eta > 0.1$. In order to understand how η is related to the accretion rate onto an object of mass M and radius R, we can simply write

$$L = \epsilon \frac{\mathrm{d}\phi}{\mathrm{d}t} = \epsilon \frac{GM\dot{M}}{R},$$

where ϵ = accretion/radiation coupling < 1 and the coupling factor is obviously related to the efficiency η:

$$\eta = \epsilon \frac{GM}{Rc^2} = \frac{\epsilon R_\mathrm{g}}{2R}.$$

3·4. *Eddington limit.* – The luminosity of an accreting object cannot produce a pressure larger than the gravitational pressure, otherwise the inflow would be stopped. This is a limit derived by Eddington, applying the balance condition between gravitational and radiation pressure forces on particles (hydrogen gas):

$$F_\mathrm{grav} = -\frac{GM(m_\mathrm{p} + m_e)}{r^2} \geq F_\mathrm{rad} = \sigma_\mathrm{T} \left(\frac{L}{4\pi r^2 h\nu} \right) \frac{h\nu}{c} = \sigma_\mathrm{T} \frac{L}{4\pi r^2 c}.$$

For a given mass this yields an upper limit on luminosity:

$$L \leq L_\mathrm{E} = \frac{4\pi G c m_\mathrm{p} M}{\sigma_\mathrm{T}} \sim 1.3 \times 10^{46} \left(\frac{M}{10^8 M_\odot} \right) \mathrm{erg\ s}^{-1}$$

or for a given luminosity the minimum mass allowed to maintain equilibrium:

$$M \geq M_\mathrm{E} \sim 8 \times 10^8 L_{47} M_\odot$$

corresponding to a gravitational radius

$$R_\mathrm{g} = \frac{2GM_\mathrm{E}}{c^2} \sim 2.2 \times 10^{14} L_{47}\,\mathrm{cm}.$$

Comparison with table I shows that AGNs must contain a supermassive black hole SMBH [6].

The Eddington luminosity corresponds to an Eddington accretion rate:

$$\dot{M}_\mathrm{E} = \frac{L_\mathrm{E}}{\eta c^2} = 0.2 \eta^{-1} \left(\frac{M}{10^8 M_\odot} \right) M_\odot\, \mathrm{y}^{-1}.$$

It must be noticed that the Eddington limit has been derived in conditions of spherical symmetry; a non-symmetric configuration, where accretion and radiation emission insist on different regions, allows super-Eddington luminosities.

4. – Accretion flows

The presence of rotation in galactic nuclei, recently confirmed also by data on the kinematics of water masers, suggests that accretion flows take the form of disks inside which "viscous" dissipation of the inflow kinetic energy can be highly efficient, $\epsilon \to 1$ [6-8]. For accretion onto black holes with particles following quasi-Keplerian circular orbits the innermost stable orbit is $R = 3R_{\rm g}$, corresponding to an efficiency $\eta = \epsilon/6$: the modelling of accretion inflows requires a detailed analysis of the angular momentum removal by "viscous" and/or magnetic stresses, as discussed in the following sections.

Accreting gas may come from interstellar uncondensed matter or from the disruption of stars spiraling into the gravitational well of the galactic nucleus. In order to disrupt stars into gas (otherwise their infall into the central black hole would only produce bursts of gravitational waves), the Roche limit for disruption puts an upper limit to the central black-hole mass:

$$r_{\rm R} = 2.4\left(\frac{\rho_{\rm BH}}{\rho_{\rm star}}\right)^{1/3} R_{\rm g},$$

$$\frac{r_{\rm R}}{R_{\rm g}} = 2.4\left(\frac{3M_{\rm BH}}{4\pi R_{\rm g}^3 \rho_{\rm star}}\right)^{1/3} > 1,$$

$$M_{\rm BH} < 6 \times 10^8 \rho_{\rm star}^{-1/2} M_\odot.$$

This puts an upper limit on the SMBH mass that is not very stringent for normal to compact stars.

Each proton infalling onto the SMBH can emit its energy at most into a single photon of maximum frequency

$$\nu = \eta \frac{m_{\rm p}c^2}{h} = \frac{\epsilon R_{\rm g}}{2R}\frac{m_{\rm p}c^2}{h}.$$

TABLE II. – *Maximum frequencies produced by protons accreting onto stellar objects.*

	η/ϵ	$h\nu$	
Sun	2×10^{-6}	$< 1\,\mathrm{keV}$	UV/soft X-rays
White dwarf	5×10^{-4}	$5 \times 10^2\,\mathrm{keV}$	hard X-rays
Neutron star	2×10^{-2}	$50\,\mathrm{MeV}$	γ-rays
Black hole	0.18	$0.2\,\mathrm{GeV}$	γ-rays

Table II lists the maximum frequency produced by accretion onto different stellar configurations.

4.1. *Generalities on accretion disks.* – The inflow of matter can settle to a configuration of a stationary disk with particles following quasi-Keplerian orbits provided angular momentum is transported outwards by viscous and/or magnetic stresses:

$$G_{\mathrm{stress}} = (GMr_0)^{1/2} - (GMr)^{1/2}.$$

Rotational energy must be dissipated locally into heat. If one assumes that the disk is optically thick and in thermal equilibrium, the virial theorem applies (half of the gravitational energy released goes into radiation and half into heating) and local emission is a blackbody:

$$L = \frac{GM\dot{M}}{2r} = 2\pi r^2 \sigma T^4(r).$$

In these conditions the local disk temperature is (the numerical factors are taken from an accurate solution)

$$T(r) = \left[\frac{GM\dot{M}}{8\pi\sigma R_{\mathrm{g}}^3}\right]^{1/4} \left(\frac{r}{R_{\mathrm{g}}}\right)^{-3/4} \approx 6.3 \times 10^5 \left(\frac{\dot{M}}{\dot{M}_{\mathrm{E}}}\right)^{1/4} \left(\frac{M}{10^8 M_\odot}\right)^{-1/4} \left(\frac{r}{R_{\mathrm{g}}}\right)^{-3/4} \mathrm{K}.$$

For $M = 10^8 M_\odot$ and $\dot{M} = \dot{M}_{\mathrm{E}}$ the peak of the blackbody spectrum is at $\nu_{\max} = 2.8kT/h \approx 3.6 \times 10^{16}$ Hz, *i.e.* typically in UV for the AGNs (for a $1M_\odot$ the peak is in hard X-rays).

We can imagine the following scenarios.

Low accretion rates $\dot{M} < \dot{M}_{\mathrm{E}}$ and high opacities. In this case one expects the formation of geometrically thin disks, with strong radiation emission $\eta \to 1$ and energy eventually advected into the BH much less than the radiated energy. The spectrum is superposition of blackbodies; for stellar mass BH the spectrum extends from X-rays (inner regions) to optical (outer regions of disk).

Large accretion rates $\dot{M} \gg \dot{M}_{\mathrm{E}}$. Due to the inadequate radiation efficiency (electron scattering opacity), the energy remains trapped in the disk with the formation of geometrically thick disks and *tori.* The emission comes out as star-like thermal blackbody $T \approx 10^4$ K.

Very low accretion rates $\dot{M} \leq 0.1\dot{M}_{\mathrm{E}}$ with low opacities. The disks are optically thin, and therefore electrons cool rapidly by radiation, while ions do not. Given the low densities in the flow, electrons and ions do not interact efficiently and a two-temperature plasma is formed, as thermalization between ions and electrons is slow, with $T_{\mathrm{el}} < T_{\mathrm{ion}}$: ions reach the virial temperature

$$kT_{\mathrm{vir}} \approx \frac{GMm_{\mathrm{p}}}{3r} = \frac{m_{\mathrm{p}}c^2}{6}\frac{R_{\mathrm{g}}}{r} \approx 160\,\frac{R_{\mathrm{g}}}{r}\,\mathrm{MeV},$$

$$T_{\mathrm{ion}} \approx 2\times 10^{12}\,\frac{R_{\mathrm{g}}}{r}\,\mathrm{K},$$

while electrons cool down rapidly. These disks are called *ion tori* as they are supported by the ion pressure.

5. – The physics of accretion

The study of accretion flows is based on hydrodynamic and magnetohydrodynamic models. The global problem is based on the solution of time-dependent partial differential equations, including special and general relativistic effects, magnetic fields and multidimensionality. Idealized models can be solved analytically, but numerical simulations allow to explore less trivial regions of the parameter space.

5·1. *Spherical accretion.* – The stationary Newtonian hydrodynamics of a gas of density ρ accreting in spherical symmetry onto the potential well generated by a mass M was studied originally by Bondi [9] with the following equations:

$$\nabla\cdot(\rho\mathbf{u}) = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2\rho u) = 0,$$

$$u\frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} - \frac{GM}{r^2},$$

$$P = K\rho^{\Gamma}, \qquad a_{\mathrm{s}} = \left(\frac{\mathrm{d}P}{\mathrm{d}\rho}\right)^{1/2}.$$

The solution produces topologies (fig. 2) with u monotonically increasing from $r=\infty$ towards $r=0$ passing through a critical sonic point at $r=r_{\mathrm{s}}$, where the flow goes from subsonic to supersonic:

$$u_{\mathrm{s}} = a_{\mathrm{s}} = \frac{1}{2}\frac{GM}{r_{\mathrm{s}}},$$

$$a_{\mathrm{s}}^2 = \frac{2}{5-3\Gamma}a_{\infty}^2, \qquad r_{\mathrm{s}} = \frac{5-3\Gamma}{4}\frac{GM}{a_{\infty}^2}.$$

Obviously no stable solutions exist for $\Gamma > 5/3$. There is no guarantee that a hydrodynamic behavior is possible in these conditions in astrophysics. The accretion rate is easily estimated:

$$\dot{M} = 4\pi\rho_{\infty}u_{\mathrm{s}}r_{\mathrm{s}}^2\left(\frac{a_{\mathrm{s}}}{a_{\infty}}\right)^{2/(\Gamma-1)} = 4\pi\lambda_{\mathrm{s}}\left(\frac{GM}{a_{\infty}^2}\right)^2\rho_{\infty}a_{\infty}$$

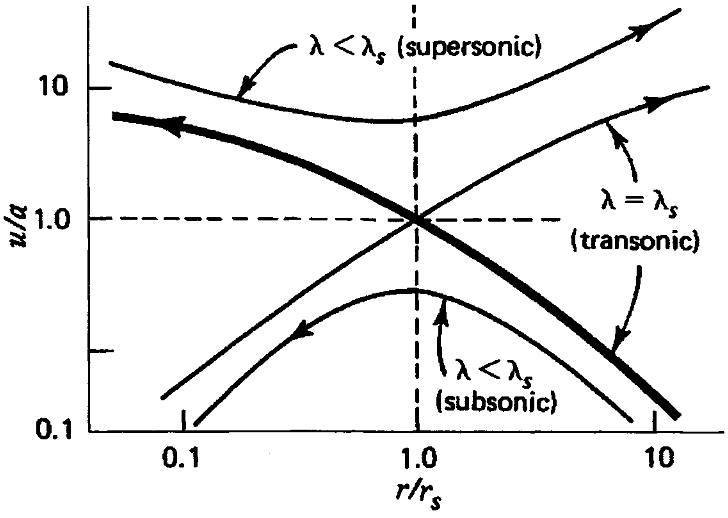

Fig. 2. – Topologies of the spherical wind/accretion solutions (see text). The heavy line corresponds to steady accretion.

in terms of the conditions at infinity and the equation of state through

$$\lambda_s = \left(\frac{1}{2}\right)^{(\Gamma+1)/2(\Gamma-1)} \left[\frac{5-3\Gamma}{4}\right]^{-(5-3\Gamma)/2(\Gamma-1)}.$$

The ensuing luminosity comes from thermal bremsstrahlung by the compressed gas at the horizon and has a very low efficiency:

$$\varepsilon = \frac{L_{\rm br}}{\dot{M}c^2} \approx 6 \times 10^{-11} \left(\frac{n_\infty}{1\,{\rm cm}^{-3}}\right) \left(\frac{T_\infty}{10^4\,{\rm K}}\right)^{-3/2} \left(\frac{M}{M_\odot}\right),$$

which makes this type of accretion unsuitable for AGNs. The spectrum is flat up to $h\nu \approx kT_{\rm BH} \approx$ 10–100 MeV followed by a rapid exponential decay.

It is not clear how a collection of independent particles falling radially can actually reach a hydrodynamic behavior. However recent papers have shown that the expressions for the accretion rate and luminosity of the Bondi solution are confirmed by numerical simulations (see lectures by Lamb in these *Proceedings*, this volume pp. 325, 349).

5·2. *Thin accretion disks.* – We consider the inflow of matter with angular momentum onto a gravitational potential well and calculate the formation of a flat, thin disk structure with particles on quasi-Keplerian orbits. In order to reach a stationary situation angular momentum must be transported outwards and away from disk rings at a rate sufficient to liberate enough gravitational energy for supporting radiation emission (fig. 3). Understanding the physics of this process has been the crucial issue since the original ideas of using disks for modeling binary X-ray stars and AGNs (see the review by Balbus and Hawley [10]). This aspect of the problem is discussed in more detail below.

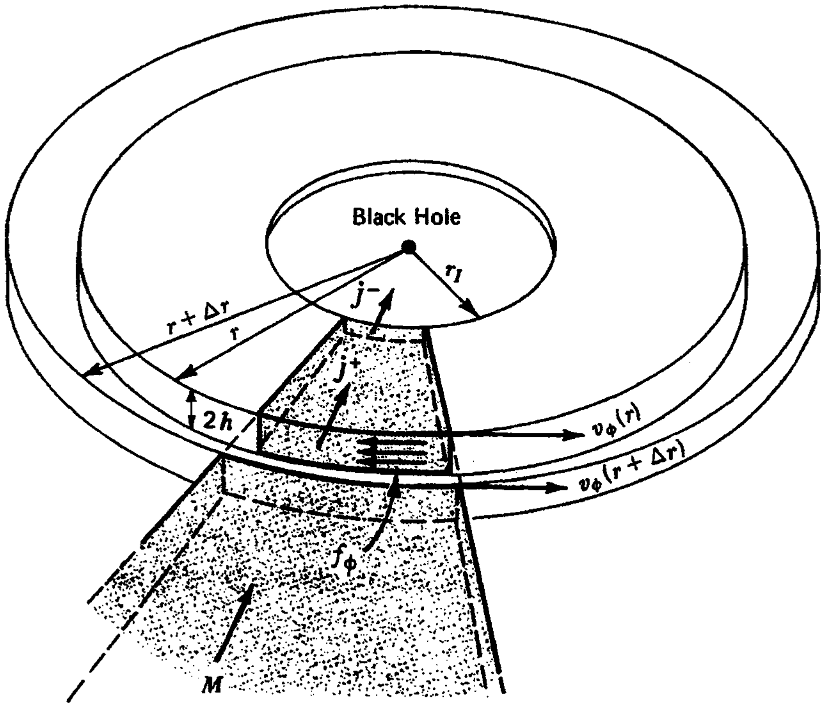

Fig. 3. – Thin accretion disk.

The structure equations are simply the conservations of mass, radial momentum, angular momentum and energy:

$$\frac{\mathrm{d}}{\mathrm{d}R}(\rho R H v_R) = 0,$$

$$v_R \frac{\mathrm{d}v_R}{\mathrm{d}R} - \Omega^2 R = -\Omega_K^2 R - \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}R},$$

$$v_R \frac{\mathrm{d}(\Omega R^2)}{\mathrm{d}R} = \frac{1}{\rho R H}\frac{\mathrm{d}}{\mathrm{d}R}(\nu \rho R^3 H \Omega'),$$

$$\rho v_R T \frac{\mathrm{d}s}{\mathrm{d}R} = \nu\rho(R\Omega')^2 - Q^- \equiv f\nu\rho(R\Omega')^2,$$

where v_R is the infall velocity, Ω the angular velocity ($\Omega \approx \Omega_K \approx \sqrt{GM/R^3}$), $\Omega' = \mathrm{d}\Omega/\mathrm{d}R$, Q^- the rate of (radiative) losses and ν the kinematic viscosity; typically $\nu = v_{\mathrm{turb}}\lambda$.

The right-hand-side term of the third equation is the local torque on the angular momentum that is derived in terms of the torque across a surface $2\pi R$ from an outer to an inner adjacent ring as

$$G(R) = 2\pi R \nu \Sigma R^2 \left(\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right),$$

where Ω is the angular velocity and $\Sigma = \rho H$ the surface density with H the disk thickness; typically $\lambda < H$ and $v_{\rm turb} < a_{\rm s}$ and therefore $\nu \ll a_{\rm s} H$. The total torque flowing across a ring of thickness $\mathrm{d}R$ is

$$G(R + \mathrm{d}R) - G(R) = \frac{\partial G}{\partial R}\mathrm{d}R.$$

The first term on the right-hand side of the fourth equation is the rotational energy dissipation rate by viscous torques. The work done on rotation is

$$\Omega\frac{\partial G}{\partial R}\mathrm{d}R = \left[\frac{\partial}{\partial R}(G\Omega) - G\Omega'\right]\mathrm{d}R,$$

the first term corresponding to advection of rotational energy by the infalling matter and the second term to the energy dissipated in the gas and radiated:

$$\dot{D}(R) = \frac{G\Omega'}{4\pi R} = \frac{1}{2}\nu\Sigma(R\Omega')^2.$$

The equations for the stationary solutions for the radial structure can be integrated:

$$R\Sigma v_R = \text{const},$$
$$R\Sigma v_R\left(R^2\Omega\right) = \frac{G}{2\pi} + \frac{C}{2\pi}, \quad C = \text{const}.$$

They yield the disk structure equation

$$-\nu\Sigma\Omega' = \Sigma(-v_R)\Omega + \frac{C}{2\pi R^3};$$

assuming Keplerian orbits and expressing C as the rate of angular momentum inflow absorbed by a central torque at the inner boundary R_*,

$$C = -\dot{M}R_*^2\Omega = -\dot{M}\left(GMR_*\right)^{1/2},$$

we obtain

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right],$$

that show clearly the dependence on viscosity. Then we derive the dissipation rate at the surface of the disk:

$$D(R) = \frac{1}{2}\nu\Sigma\left(R\Omega'\right)^2 = \frac{3GM\dot{M}}{8\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

that results independent of ν. Correspondingly the luminosity produced by the whole disk from $R = R_*$ to $R = \infty$ if all the energy dissipated goes into radiation is calculated to be

$$L_{\text{disk}} = 2 \int_{R_*}^{\infty} D(R) 2\pi R \, \mathrm{d}R = \frac{GM\dot{M}}{2R_*} = \frac{1}{2} L_{\text{accr}}.$$

The other $L_{\text{accr}}/2$ is convected by the matter down to R_*, in principle still available if not swallowed by the potential well.

The emission spectrum can be evaluated from the vertical structure of the disk. For an optically thick gas with viscous torques releasing the energy locally we can apply a blackbody approximation and write the temperature T_{c} at the disk midplane:

$$\frac{4\sigma}{3\tau} T_{\text{c}}^4 = D(R),$$

where τ is the optical depth. Correspondingly, the temperature distribution at the disk photosphere is

$$T_{\text{phot}}(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4},$$

again independent of ν.

In conclusion the local structure of thin disks is governed by the following equations:

$$\rho = \frac{\Sigma}{H}, \qquad P = \frac{\rho k T_{\text{c}}}{\mu m_{\text{p}}} + \frac{4\sigma}{3c} T_{\text{c}}^4,$$

$$H = a_{\text{s}} \left(\frac{R^3}{GM} \right)^{1/2}, \qquad a_{\text{s}}^2 = \frac{P}{\rho},$$

$$\nu \Sigma = \frac{\dot{M}}{3\pi} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right],$$

$$\nu = \nu(\rho_{\text{c}}, T_{\text{c}}, \Sigma, \alpha, \ldots),$$

$$\frac{4\sigma}{3\tau} T_{\text{c}}^4 = \frac{3GM\dot{M}}{8\pi R^3} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right],$$

$$\tau = \Sigma \kappa_R(\rho_{\text{c}}, T_{\text{c}}) = \tau(\Sigma, \rho_{\text{c}}, T_{\text{c}}).$$

In 1973 Shakura and Sunyaev [11] gave a complete analysis of the problem scaling the solutions for the case of stellar mass BH in the framework of the study of binary X-ray stars. They assumed the formal α-prescription for the viscosity:

$$\nu = \alpha a_{\text{s}} H,$$

leaving the value for α as a free parameter, and used the approximations for cool gas and Kramer opacity:

$$P_{\rm rad} \ll P_{\rm gas}, \quad \kappa_R = 6.6 \times 10^{22} \rho T_{\rm c}^{-7/2}\, {\rm cm}^2\ {\rm g}^{-1}.$$

The scaling laws for SMBHs, with $f \equiv 1 - (R_*/R)^{1/2} = 1 - (6GM/Rc^2)^{1/2}$, are

$$\begin{aligned}
\Sigma &= 5.2 \times 10^6 \alpha^{-4/5} \dot{M}_{26}^{7/10} M_8^{1/4} R_{14}^{-3/4} f^{14/5}\, {\rm g\ cm}^{-2},\\
H &= 1.7 \times 10^{11} \alpha^{-1/10} \dot{M}_{26}^{3/20} M_8^{-3/8} R_{14}^{9/8} f^{3/5}\, {\rm cm},\\
\rho &= 3.1 \times 10^{-5} \alpha^{-7/10} \dot{M}_{26}^{11/20} M_8^{5/8} R_{14}^{-15/8} f^{11/5}\, {\rm g\ cm}^{-3},\\
T_{\rm c} &= 1.4 \times 10^6 \alpha^{-1/5} \dot{M}_{26}^{3/10} M_8^{1/4} R_{14}^{-3/4} f^{6/5}\, {\rm K},\\
\tau &= 3.3 \times 10^3 \alpha^{-4/5} \dot{M}_{26}^{1/5} f^{4/5},\\
\nu &= 1.8 \times 10^{18} \alpha^{4/5} \dot{M}_{26}^{3/10} M_8^{-1/4} R_{14}^{3/4} f^{6/5}\, {\rm cm}^2\ {\rm s}^{-1},\\
v_R &= 2.7 \times 10^4 \alpha^{4/5} \dot{M}_{26}^{3/10} M_8^{-1/4} R_{14}^{-1/4} f^{-14/5}\, {\rm cm\ s}^{-1},\\
T_{\rm phot} &= 2.2 \times 10^5 \dot{M}_{26}^{1/4} M_8^{1/4} R_{14}^{-3/4}\, {\rm K}
\end{aligned}$$

and show that the temperature is very low for the high-frequency emission from AGNs. A complete calculation shows that the spectrum is almost a blackbody: $F_\nu \propto \nu^2$ for $h\nu/kT \ll 1$, $\propto \nu^{1/3}$ for $h\nu/kT \sim 1$ and $\propto e^{-\nu}$ for $h\nu/kT \gg 1$.

5·3. *Optically thin accretion disks.* – Shapiro, Lightman and Eardley [12] considered in more detail the radiation transport in the disk pointing out that its inner part can become optically thin due to the high temperature. However the gas pressure may still dominate as ions are heated by viscous dissipation and reach a local energy equilibrium with electrons through Coulomb collisions close to the virial temperature (at $R_* = 6GM/c^2$), while electrons cool down efficiently, mostly through Compton scattering against a high-density soft-photon distribution:

$$\begin{gathered}
T_{\rm i} \le \frac{GMm_{\rm p}}{R_* k} \sim \frac{1}{6} m_{\rm p} c^2 \sim 2 \times 10^{12}\, {\rm K},\\
T_{\rm e} \sim 10^{-2} T_{\rm i} \sim 10^9\, {\rm K}.
\end{gathered}$$

The hot disk portion is inflated by the ion pressure with respect to the cold disk solution and corresponds to a geometrically thick disk. The Comptonized emission from electrons in the hot inner part of the disk yields a hard X-ray spectral component that is of interest for AGNs. This solution appears however subject to thermal instabilities.

Ichimaru [13] proposed a scenario in the limit of low accretion rate without local energy balance between ions and electrons. The gas temperature again increases towards the ion virial limit and supports a geometrically thick structure: the effective opacity decreases and the plasma becomes optically thin. Radiative losses cool down electrons rapidly and the inner hot parts of the disk emit very poorly: instead most of the energy dissipated by viscosity is advected into the black hole. Such a solution is not unstable to thermal effects.

A similar solution was discussed in the context of lobe-dominated radio galaxies by Rees, Begelman, Blandford and Phinney [14].

5·4. *Advection-Dominated Accretion Flows:* ADAF *solutions.* – The continuum spectrum of AGNs requires that a large fraction of the energy is radiated at high frequencies, in the X- and γ-ray band, while the standard cold thin disk, scaled to AGNs, can at most provide the blue-bump component and, in the optically thin disk limit, some of the X-ray emission.

Further analysis of Ichimaru's solution has led to the models called ADAF (Advection-Dominated Accretion Flow) proposed by Narayan and Yi [15]. Let us examine again the full system of equations for stationary disks. The equation for the conservation of energy (see above) can be written as

$$\rho v_R T \frac{\mathrm{d}s}{\mathrm{d}R} = Q^+ - Q^- = \nu\rho(R\Omega')^2 - Q^- \equiv f\nu\rho(R\Omega')^2,$$

where Q^+ is the rate of energy generated by viscous dissipation and Q^- is the rate of radiative cooling. Calling briefly $Q^{\mathrm{adv}} = \rho v_R T(\mathrm{d}s/\mathrm{d}R)$ the energy advected by the inflowing gas, we can discuss the various types of inflows in terms of this equation:

$$Q^{\mathrm{adv}} = Q^+ - Q^-.$$

Three regimes of accretion are possible:

1. $Q^+ \sim Q^- \gg Q^{\mathrm{adv}}$, corresponding to cooling-dominated inflow; all energy released by viscous dissipation is radiated; geometrically and optically thin disks pertain to this branch;

2. $Q^{\mathrm{adv}} \sim Q^+ \gg Q^-$, corresponding to an advection-dominated inflow; all energy released by viscous dissipation is deposited into the BH; this is the typical ADAF;

3. $Q^{\mathrm{adv}} \sim Q^- \gg Q^+$, corresponding to negligible energy generation; entropy is lost into radiation; Bondi accretion and Kelvin contraction of stars are in this branch.

Self-similar solutions for ADAF, assuming Newtonian gravity and f independent of

R, have been derived in various limits by Narayan and collaborators:

$$v_R = -\frac{5+2\varepsilon'}{3\alpha^2} g(\alpha,\varepsilon')\alpha v_{ff},$$

$$\Omega(R) = \left[\frac{2\varepsilon'(5+2\varepsilon')}{9\alpha^2} g(\alpha,\varepsilon')\right]^{1/2} \frac{v_{ff}}{R},$$

$$a_s^2 = \frac{2(5+2\varepsilon')}{9\alpha^2} g(\alpha,\varepsilon') v_{ff},$$

with

$$v_{ff} = \left(\frac{GM}{R}\right)^{1/2}, \qquad \varepsilon' = \frac{\varepsilon}{f} = \frac{1}{f}\frac{5/3-\gamma}{\gamma-1} \quad (\gamma = \text{adiabatic index}),$$

$$g(\alpha,\varepsilon') = \left[1 + \frac{18\alpha^2}{(5+2\varepsilon')^2}\right]^{1/2} - 1.$$

The main features of results are:

1. ADAF require typically large viscosity parameters $\alpha \sim 0.2$–0.3 and rapid accretion velocities $v_R \geq 0.1 v_{ff}$;
2. gas is partially supported by the centrifugal force and partially by the radial pressure gradient;
3. gas pressure is high because of ion's low radiative losses: $H \sim R$, as in spherical accretion (but the dynamics is different from the Bondi solution);
4. the flow has positive Bernoulli constant: if reverted could reach infinity (see later the discussion about jets);
5. entropy increases with decreasing radius.

Boundary conditions must be studied through global solutions matching with cool disk solutions on large scales and to advection onto the SMBH at the inner ring. Some numerical studies have been preliminarily attacked using pseudo-Newtonian gravity. For low $\alpha \leq 0.01$ the flow has difficulty to transfer angular momentum and is super-Keplerian, marginally stable in the inner regions; a funnel is formed along the rotation axis and the global shape is toroidal. For larger $\alpha \geq 0.01$ the dynamics is dominated by viscosity and radial velocity becomes close to free fall; no funnel is formed and the structure is quasi-spherical down to the sonic point. However several aspects have still to be worked out in detail, and many uncertainties connected with the outer boundaries may still change important elements of the general scenario. For instance we should mention that ADAF solutions require relatively large viscosity parameters, while at the same time want to avoid dissipation to advect most energy to the BH.

ADAFs can be applied to astrophysical conditions in two typical scenarios:

- high accretion rate, high-$\dot{M}$ optically thick solution (still poorly studied, especially the emitted spectrum);

- low accretion rate, low-$\dot{M}$ optically thin, two-temperature solution as in the case of thin disks (applied to low-luminosity AGNs).

In this last case one assumes that coupling between ions and electrons is only due to Coulomb collisions; other more efficient forms of couplings, as for instance magnetized plasma instabilities, have not been fully analyzed yet. Again electrons radiate more efficiently than ions and cool down rapidly. Assuming equipartition magnetic fields, the following scaling laws can be derived:

$$
\begin{aligned}
v_R &= -1.1 \times 10^{10} \alpha r^{-1/2}\,\mathrm{cm\ s^{-1}},\\
\Omega &= 2.9 \times 10^{4} m^{-1} r^{-3/2}\,\mathrm{s},\\
n_{\mathrm{e}} &= 6.3 \times 10^{19} \alpha^{-1} m^{-1} \dot{m} r^{-3/2}\,\mathrm{cm^{-3}},\\
B &= 7.8 \times 10^{8} \alpha^{-1/2} m^{-1/2} \dot{m}^{1/2} r^{-5/4}\,\mathrm{G},\\
P &= 1.7 \times 10^{16} \alpha^{-1} m^{-1} \dot{m} r^{-5/2}\,\mathrm{g\ cm^{-1}\ s^{-2}},\\
Q^{+} &= 5 \times 10^{21} m^{-2} \dot{m} r^{-4}\,\mathrm{erg\ cm^{-3}\ s^{-1}},\\
\tau_{\mathrm{es}} &= 24 \alpha^{-1} \dot{m} r^{-1/2}, \qquad a_{\mathrm{s}}^{2} = 1.4 \times 10^{20} r^{-1}\,\mathrm{cm^{2}\ s^{-2}}
\end{aligned}
$$

with $m = M/M_{\odot}$, $r = R/R_{\mathrm{g}}$, $\dot{m} = \dot{M}/\dot{M}_{\mathrm{Edd}}$. This optically thin, two-temperature solution exists only for $\dot{m} \leq \dot{m}_{\mathrm{crit}}$ that is obtained by verifying the condition $Q^{+} \geq Q^{-}$, $\dot{m}_{\mathrm{crit}} \sim \alpha^{2} r^{-1/2}$ saturating at $0.3\alpha^{2}$ for large $\dot{m}$.

Concerning the emitted spectrum, radiation by electrons comes from synchrotron, bremsstrahlung and inverse Compton in the range of radio, optical, X-rays; radiation by protons contributes γ-rays through π^{0} decay of high energy ions. The model has been fitted to the data of Sag A* and NGC 4258. The high accretion rate solution should be the one applicable to AGNs. However, observational data on radio spectra do not appear to confirm these models.

5·5. *The viscosity problem.* – As mentioned before, all disk solutions depend on the adopted values for the viscosity. In a first instance hydrodynamic shear viscosity in differentially rotating fluids was considered, but it was clear that standard microscopic viscosities produced inadequate fuelling rates for accretion disks [16]. However a strong *effective viscosity* could arise if the disk were turbulent (Boussinesq and Prandtl proposed it in their early works). But the origin of turbulence is not clear especially in supersonic flows without external boundaries.

The most convincing study of the origin of turbulence by shearing instability in differentially rotating disks has been proposed recently by Balbus, Hawley and collaborators [10]. They showed by analytic and numerical works that weak magnetic fields pervading the disk give rise to shearing instability under very general conditions. Actually this process had been discovered by Velikhov [17] in connection with Couette flows and by Chandrasekhar [18] for more general rotation laws, in all cases in the presence of vertical magnetic fields.

The physical interpretation of this weak-field shearing instability is the following. Two elements of fluids connected by magnetic field are displaced from their equilibrium position, one towards the center of the accretion disk (where angular velocity is larger) and one away from the center (where angular velocity is smaller). The connecting magnetic field determines a pull on the two elements, slowing down the inner one and accelerating the outer one. This corresponds to extraction of angular momentum from the inner element and its transfer to the outer element. The inner element is forced to fall further down, the outer element to move further outwards. This effect increases the stretching of the magnetic lines and the process runs away.

More details on these instabilties can be found in the lecture by Coppi in these *Proceedings* (this volume, p. 25).

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