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### **AUTUMN COLLEGE ON PLASMA PHYSICS**

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## Alfvénic Wave Packets - I & II

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These are preliminary lecture notes, intended only for distribution to participants.

## Alfvénic Wave Packets

- 1. INTRODUCTION
- 2. LINEAR ALFVEN WAVES
- 3. NONLINEAR ALFVÉN WAVES:

OBSERVATIONS

STABILITY

**GOVERNING EQUATIONS** 

- 4. SOLITARY ALFVEN WAVES
- 5. CHAOTIC ALFVEN WAVES
- 6. DISPERSIVE MHD SIMULATIONS
- 7. HYBRID SIMULATIONS
   NONLINEAR LANDAU DAMPING
   ANOMALOUS TRANSPORT

\* SMALL AMPLITUDE (LINEAR) AW ALWAYS STABLE



 $V_A = (B_0^2/4\pi g)^{1/2} = ALFVEN SPEED$ 

DISPERSIVE MHD

FOR R > 0 + RHP

NOTE ;

NO DENSITY FLUCTUATIONS ASSOCIATED WITH LINEAR AW \* LARGE AMPLITUDE (NONLINEAR) AW CAN BE UNSTABLE

LHP

N.L. EVOLUTION :

\* DECAY INSTABILITY



A.W.

\* MODULATIONAL INSTABILITY => \* SOLITARY A.W.

CHARACTERISTIC EVOLUTION EAN. DNLS / MODIFIED DNLS

\* DRIVEN ALFVEN WAVES

-> CHAOS

\* GROWTH RATES FURTHER REDUCED BY KINETIC EFFECT (FINITE ION PRESSURE) FOR SOLAR WIND,  $\frac{\gamma_{max}(B_i \ge \beta_e)}{\gamma_{max}(B_i = 0)} \approx 30\%$ MHD CASE

\* GROWTH RATE OF DECAS INST. MUCH SMALLER FOR LARGER

\* DECAY INST. ONLY QUASI-LINEAR

NOTE

×

DECAJ INST. \_ Jes

RHP ( P<1) > NO MOD. INST.

FURTHER REMARKS

#### TWO MODES OF SOLAR WIND FLOW

PROPERTY (1 AU)
Speed (V)
Density (n)
Flux (nV)
Magnetic Field(B<sub>r</sub>)
Temperatures

Coulomb collisions Anistropies Beams

Structure Composition Waves Minor species

Associated with

Sunspot minimum Sunspot maximum LOW SPEED < 400 km/sec  $\sim 10 / \text{cm}^3$   $\sim 3 \times 10^8 / \text{cm}^2 \text{ sec}$   $\sim 2.8 \text{nT}$   $T_p \sim 4 \times 10^4 \text{ K}$   $T_e \sim 1.3 \times 10^5 \text{K} > T_p$ Important  $T_p$  isotropic None

Filamentary, highly variable  $He/H \sim 1 - 30\%$ Both directions  $n_i/n_p$  variable  $T_i \sim T_p$   $V_i \sim V_p$ Streamers, transiently open field ±15 deg from "equator" Dominant at most latitudes

HIGH SPEED 700 - 900 km/sec $\sim 3 / \text{cm}^3$  $\sim 2 \times 10^8 / \text{cm}^2 \text{ sec}$  $\sim 2.8 \text{ nT}$  $T_p \sim 2 \times 10^5 {\rm K}$  $T_e \sim 10^5 \mathrm{K} < T_p$ Negligible  $T_p(\bot) > T_p(||)$ Fast ion beams + electron "strahl" Uniform, slow changes  $He/H \sim 5\%$ Outwards propagating  $n_i/n_p \sim {\rm constant}$  $T_i \sim AT_p$  $V_i \sim V_p + V_A$ Coronal holes

> 30 deg Less frequent





STOKES << ANTISTOKES

\* KINETIC TREATMENT OF DECAY INST. -> ADDITIONAL BACKWARD PROPAGATING JAW.

6

 $T_{ii}$   $\uparrow$ 

### **OBSERVATIONS:**

Large amplitude Alfvén waves observed in a variety of plasmas:

- \* solar wind
- solar corona
- \* environment of comets
- \* planetary bow shocks
- \* interplanetary shocks etc.

Some Chaotic Features of Solar Wind:

- \* Intermittent Turbulence
- \* Fractal / Multifractal Nature (Dimension)

Solar Wind Spectral Characteristics:

- \* Power Law Spectra
- \* Increase in Spectral Index with heliocentric distance

\* Break in Power Spectra; Break-point moves towards lower frequencies with increasing heliocentric distances

### **COSMIC PLASMAS:**

Implications of the existence of large-amplitude Alfvén waves in many cosmic plasmas have been investigated to model :

- turbulent heating of the solar corona
- \* interstellar scintillations of radio sources
- \* coherent radio emissions
- \* generation of stellar winds and extragalactic jets etc

# DIFFERENT APPROACHES EVOLUTION EQUATION ١. For finite but not very large fluctuations. SB/B # 1 singular perturbation => DNLS/MDNLS Equation 2. MHD (DISPERSIVE) SIMULATIONS DNLS not valid for BNI × For B~1, coupling of Sn and SB significant ¥ Kinetic Effects negligible 3 HJBRID SIMULATIONS to incorporate \* Finite Lannon Radius Effects \* Nonlinear LANDAU Damping

# SOLITARY ALFVEN WAVES

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{J} \times \mathbf{B}$$

 $\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times \left[ (\mathbf{v} \times \mathbf{B}) - \frac{1}{\rho} (\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} \right]$ 

B is normalized to  $B_0$ ,  $\rho$  to  $\rho_0$ , v to  $V_A = B_0/(4\pi\rho_0)^{1/2}$ , t to  $\Omega_i^{-1}$ , the ion cyclotron frequency and l to  $V_A/\Omega_i$ .

Subscript '0' refers to equilibrium quantities.

Nonlinear Evolution Equations integrable systems

- \* KdV (Ion Acoustic Waves)
  - $\frac{\partial t}{\partial \phi} + \phi \frac{\partial \phi}{\partial \phi} + \frac{\partial x}{\partial^3 \phi} = 0$

\*

\*

NLS (Langmuir Waves)  $i \frac{\partial \Phi}{\partial E} + \frac{\partial^2 \Phi}{\partial x^2} + (\Phi)^2 \Phi = 0$ 

DNLS (Polarized Alfvén Waves)  $\frac{\partial \phi}{\partial t} \pm i \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial}{\partial x} \left( |\phi|^2 \phi \right) = 0$ + LH Polarization

RH "

Evolution Eqs. => solitory waves

DNLS :  $\frac{\partial B_{\pm}}{\partial t} + \frac{1}{4(1-\beta)} \frac{\partial}{\partial x} \left( \frac{B_{\pm}}{B_{\pm}} \right)$  $\pm \frac{i\sqrt{3^2B_{\pm}}}{2\pi^{3}X^2} = 0$ 

+ LHP  
- RHP  

$$B_{+} = B_{y} + iB_{z}$$
  
 $B_{-} = B_{y} - iB_{z}$   
 $\vec{R} \parallel \vec{B}_{0} \equiv B_{0}\hat{e}_{x}$ 

$$DNLS \implies SOLITONS$$

$$B \pm = \frac{(\sqrt{2} - 1)^{1/2}}{[\sqrt{2} \operatorname{Bmax} e^{\pm i \oplus (x)}]}$$

$$B \pm = \frac{(\sqrt{2} - 1)^{1/2}}{[\sqrt{2} \cosh(2\sqrt{3}X) - 1]^{1/2}}$$

$$\Theta(X) = -\sqrt{3}X + 3\tan^{-1}[(\sqrt{2} + 1) \tanh(2\sqrt{3}X)]$$

$$V_{S} = \frac{(\sqrt{2} - 1)^{1/2}}{8(1 - 7^{2})} = SOLITON \text{ SPEED}$$

$$X = (X - t)$$

BMON = AMPLITUDE OF SOLITON

SOLITARY AW IN INHOMOGENEOUS STREAMING PLASMAS

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v_r) = 0$$
$$\rho \frac{dv_r}{dt} = -\frac{\partial p}{\partial r} - \frac{\partial}{\partial r} \frac{B_\perp^2}{2} - \frac{B_\perp^2}{2}$$
$$\rho \frac{dv_\perp}{dt} = \frac{B_r}{r} \frac{\partial}{\partial r} (r\mathbf{B}_\perp)$$

and

$$\frac{\partial \mathbf{B}_{\perp}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (B_r v_{\perp} - v_r B_{\perp}) + \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{B \hat{\mathbf{e}}_r}{r \rho} \times \frac{\partial (\mathbf{B}_{\perp} r)}{\partial r} \right]$$

$$\mathbf{B}_{\perp} = (B_{\theta}, B_{\phi}), \mathbf{v}_{\perp} = (v_{\theta}, v_{\phi}), \text{ and } B_{\perp}^2 = (B_{\bullet}^2 + B_{\phi}^2)$$

$$p\rho^{-\gamma} = const.$$

$$B_0(r) r^2 = const$$

 $\rho_0(r) U(r) r^2 = const.$ 

$$\frac{\partial B}{\partial \eta} + \frac{3 U}{2 V \eta} B + \frac{B}{4 V (V - U)} \frac{\partial}{\partial \eta} \left( V^2 - U^2 \right) + \frac{(V - U)}{4 B_0^2(\eta) V^2 (1 - \beta(\eta))} \frac{\partial}{\partial \xi} \left( B \mid B \mid^2 \right) + \frac{i (V - U)^2}{2 V^3 B_0(\eta)} \frac{\partial^2 B}{\partial \xi^2} = 0$$

MDNLS

$$\eta = \epsilon^2 r; \quad \xi = \epsilon \left[ \int \frac{dr}{V(r)} - t \right]$$

 $\epsilon$  is the stretching parameter and V(r) is the phase velocity of the Alfvén

$$V(r) = U(r) + \frac{B_0(r)}{\rho_0^{1/2}(r)}$$

U is the equilibrium streaming plasma velocity.

For 
$$S_0(k) = 1$$
,  $B_0(k) = 1$ ,  $U = 0$ 

$$B(\xi, r_0) = \frac{B_{max} e^{i\theta(\xi)}}{\cosh^{1/2} \psi},$$



Fig. 20



OBSERVATIONS TURBULENCE POWER SPECTRA etc.



Figure 6. The turbulence in the solar wind proton parameters over the period from 0500 SET to 1900 SET on March 13th 1986 when the spacecraft was travelling from 4.5 million km to 1.2 million km from the nucleus. The FLD coordinate system has the y axis magnetic field aligned, the z axis in the  $-v \ge B$  direction and the x axis forming a right handed set. The mean value of the parameters has been subtracted. The frequency range indicated is set at the low frequency end by the digital filter and at the high end by the sample rate of the instrument.



**Figure 10.** An example of an Alfvén wave with a phase-steepened edge (rotational discontinuity [RD]). From left to right, dashed vertical lines give the start and stop times of intervals of analyses.



Fig. 2. Magnetic field (vector components and magnitude) power spectral densities at different heliographic distances. The 'mean field' coordinate system (Z axis parallel to the average direction of the vector magnetic field) was used throughout the paper. The spectra were computed from 40-s averages of the magnetic field over 11 1/2 hours on January 24, 1900 UT to January 25, 0623 UT-, 1976 (0.97 AU) and on April 14, 2300 UT to April 15, 1023 UT, 1976 (0.29 AU). Also given are the 95% confidence limits for the spectral density computed from an equivalent of 32 degrees of freedom.

From

JGR, 87, 1982.

• Тр/Тө



beta

ISEE-3 Hourly averages



# CHAOTIC ALFVEN WAVES

**DRIVEN DNLS EQUATION:** 

$$rac{\partial B}{\partial t} + rac{1}{4 \; (1-eta)} rac{\partial}{\partial x} \left( B \mid B \mid^2 
ight) + rac{i}{2} rac{\partial^2 \; B}{\partial x^2} = A \; e^{i(\kappa x - \Omega t)}$$

### NUMERICAL SOLUTION

Spectral-collocation method with periodic boundary conditions used

Initial condition:

$$B \ (x, \ t=0) \ = \ rac{(2^{1/2}-1)^{1/2} \ B_{max} \ e^{i heta \ (x)}}{[2^{1/2} \ \cosh \ (2 \ V_s \ x) \ - \ 1]^{1/2}}$$

 $B_{max}$  is the amplitude of the soliton

 $\theta (x, t = 0) = -V_s x + 3 \tan^{-1} \left[ (2^{1/2} + 1) \tanh(2V_s x) \right]$ 

 $V_s$  is the soliton speed defined by,

$$V_s = rac{(2^{1/2}-1) \ B_{max}^2}{8 \ (1 \ - \ eta)}$$



A = 0

 $\mathcal{B} = 0.1$ 

CHAOTIC AW

RHP





Fig

POWER SPECTRA



**FIGURE** Shows Spectra for magnetic field turbulence generated through chaos for a) RHP with A = 0.3, b) LHP with A = 0.1, c) LHP with A = .03 and d) LHP with A = .003.

SLOW SOLAR WIND (~ IAU) *B* large; *B*~1 *T*i < *T*e *FAST* SOLAR WIND: *B* large *T*i > *T*e SIMILAR FEATURES IN VICINITY OF SHOCKS

LARGE AMPLITUDE AW SB/B, 21

\* ION HOLES

\* COLLAPSE > TRANSPORT OF ENERGY

 $\Rightarrow$ 

HIGH RESOLUTION SIMULATIONS

ADEQUATE

HJBRID SIMULATIONS

\* FOR L-F WAVES (w2< s2)

\* KINETIC EFFECTS CRUCIAL

\* SN - SB COUPLING SIGNIFICANT

NOT ADEQUATE

DNLS / MDNLS / KNLS

\* MODELS BASED ON

\* SIMULATIONS RESOLUTION NOT VERS HIGH

WIND9, 1999) \* DISCUSSED DISRUPTION OF WAVE PACKETS (SB, B not very large)

(BUT! et al., GRL, 1998 VELLI, BUTI, GOLDSTEIN, SOLAR

HALL MHD SIMULATIONS OF ALFVENIC WAVE PACKETS

B FINITE BUT % 1 RESOLUTION NOT VERSHIGH × \* DISCUSSED DECAY INST. and MODULATIONAL INST. FOR LKP and RHP AW

SB FINITE BUT NOT LARGE \*

SIMULATIONS of AMPLITUDE -MODULATED O POLARIZED AN (MACHIDA et al., JGR, 1987)

## MHD SIMULATIONS

$$rac{\partial 
ho}{\partial t} + rac{\partial}{\partial x}(
ho v_x) = 0$$

$$\frac{\partial}{\partial t}(\rho v_x) + \frac{\partial}{\partial x}(\rho v_x^2) + \frac{\partial\beta}{\partial x} + \frac{\partial}{\partial x}\left(\frac{B^2}{2}\right) = 0$$

$$rac{\partial}{\partial t}(
ho ilde{v})+rac{\partial}{\partial x}(
ho v_x ilde{v})-rac{\partial B}{\partial x}=0$$

$$rac{\partial B}{\partial t}+rac{\partial}{\partial x}(v_xB- ilde v)=-irac{\partial}{\partial x}\left(rac{1}{
ho}rac{\partial B}{\partial x}
ight)$$

$$rac{\partialeta}{\partial t}+rac{\partial}{\partial x}(eta v_x)+(\gamma-1)etarac{\partial v_x}{\partial x}=0$$

 $v_x$  is flow velocity along direction of propagation,  $B = (B_y + i B_z)$  and  $\tilde{v} = (v_y + i v_z)$ . Adiabatic equation of state:  $p\rho^{-\gamma} = \text{const.}$ .  $\gamma$  is ratio of specific heats. Initial condition:

$$B(x, t=0) = \frac{(2^{1/2}-1)^{1/2} B_{max} e^{i\theta(x)}}{[2^{1/2} \cosh(2 V_s x) - 1]^{1/2}}$$

 $B_{max}$  is the amplitude of the soliton

 $\theta(x,t=0) = -V_s x + 3 \tan^{-1} \left[ (2^{1/2}+1) \tanh(2V_s x) \right]$ 

 $V_{\bullet}$  is the soliton speed defined by,

$$V_{s} = \frac{(2^{1/2} - 1) B_{max}^{2}}{8(1 - \beta)}$$
  
SN (x,t=0) =  $\frac{1}{2} \frac{1}{(1-\beta)} |B(x,t=0)|^{2}$   
Bo = Boex



Figure 1. (a) The evolution of the magnetic and density fluctuations, B and  $\delta\rho$ , respectively, for a RHP soliton for  $\beta = 0.3$  at  $t \approx 40\Omega_p^{-1}$  (dashed line) and at 5000  $\Omega_p^{-1}$  (solid line). (b) Same as (a) but for  $\beta = 1.5$ .

$$SS = \frac{1}{2(1-73)} |B|^2$$



Figure 2. (a) Same as Fig. 1a but for LHP soliton. (b) Same as Fig. 1b but for LHP soliton.



73=0.2



B= 0.2



B= 0.2

# HYBRID SIMULATIONS COLLAPSE OF ALFVENIC WAVE PACKETS

## DYNAMICAL TURNING POINT → CHANGE OF POLARIZATION

RADIATIONS

## Simulation Model

*	One - Dimensional Hybrid Code
*	Electrons as Isothermal Fluid
*	Protons as Particles Maxwellian , BIMAXWELLIAN etc.
*	HIGH RESOLUTION:
	Simulation Box Length 860 Ion Inertial Lengths (VA/SLi)
	Number of Cells 2048
	number of Particles / Cell 200
	Simulation Time 1000 Ion Gyro-Periods (Sci)
	Time resolution 0.01

Siz = 104 B

Initial condition:

$$B(x, t=0) = \frac{(2^{1/2}-1)^{1/2} B_{max} e^{i\theta(x)}}{[2^{1/2} \cosh(2 V_s x) - 1]^{1/2}}$$

 $B_{max}$  is the amplitude of the soliton

$$\theta (x, t = 0) = -V_s x + 3 \tan^{-1} \left[ (2^{1/2} + 1) \tanh(2V_s x) \right]$$

 $V_s$  is the soliton speed defined by,

$$V_{s} = \frac{(2^{1/2} - 1) B_{max}^{2}}{8 (1 - \beta)}$$
  
SN (x,t = 0) =  $\frac{1}{2} \frac{1}{(1-\beta)} (B(x,t=0))^{2}$   
(DRIVEN by PONDER MOTIVE FORCE)

COLLAPSE



P10b

LHP

 $B = \cdot 2$  $T_e = T_i$ 

> ENERGY TRANSPORT



DUNAMICAL TURNING POINT

NONLINEAR DISPERSION RELATION IN WAVE FRAME OF REFERENCE

$$\omega = \kappa |B|^2 / (-B) \pm \mu \kappa^2$$

- LHP



RHP



R10b





# ION HOLES NONLINEAR LANDAU DAMPING







# LOW FREQ. WAVES ION DUNAMICS CRUCIAL ION ACOUSTIC WAVES (IAW) ALFVÉN WAVES (AW)

EVOLUTION OF IAW :

 $\omega^{2} = \frac{2 \cdot 2}{k \cdot \sqrt{2}} \left[ \frac{B_{i} + \frac{B_{e}}{(1 + k \cdot 2 \cdot \sqrt{2})}}{(1 + k \cdot 2 \cdot \sqrt{2})} \right]$   $\approx k^{2} C_{s}^{2}$ 

SMALL AMPLITUDE LINEAR WAVES

LARGE AMPLITADE >

GOVERNING EAN. \_ NLS >> COMPRESSIVE SOLITARY WAVES SN > 0

### OBSERVATIONS

- \* DENSITY CAVITIES in AURORAL PLASMA
- \* SCALE LENGTH for THE CAVITIES VERY SMALL (~ a few ELECTON INERTIAL LENGTH)

\* PLASMA B << 1

\* DENSITY CAVITLES in INTER-PLANETARY MEDIUM ?

ION DENSITY HOLES WITH
 SCALE LENGTH ~ ~ few
 ION INERTIAL LENGTH ||
 ION INERTIAL LENGTH = VA /SIL
 B LARGE

ARE ION HOLES (SN<0) FEASIBLE UNDER WHAT CONDITIONS ? YES 1. IN MULTISPECIES PLASMAS MODEL : (BUTI, PHUS, LETT, 76A 251, 1980) a) COLD IONS b) HOT and COLD ELECTRONS 2) VERY LARGE SN  $\left[\frac{N_{H}T_{c}^{2}+N_{c}T_{H}^{2}}{(N_{H}T_{c}+N_{c}T_{H})^{2}}\right] > 3$ 4) 11 1 for TH = Te FBCc30 HK, c = MH, c/no

MH+MC = 1



B= 0.8







Bs	Ti / Te	<b>BP IA HOLES</b>	FP IA HOLES
		Speed	Speed
0.5	1/7	0.61 Va	0.65 Va
0.5	1/3	0.6 Va	0.63 Va
0.5	1	Α	0.46 Va
0.5	3	Α	Α
0.5	9	Α	Α
Bs	Ti / Te	В	Ν
		Speed (Va)	Speed (Va)
0.5	1/7	0.83	0.81
0.5	1/3	0.75	Α
0.5	1	0.66	Α
0.5	3	0.56	0.54
0.5	9	0.32	0.34
	•		
0.7	1/3	0.3	A
0.7	9	Α	Α

### CONCLUSIONS

\* Governing Evolution Equation for Nonlinear ALFVEN Waves is DNLS / MDNLS

\* DNLS  $\Rightarrow$  Stable Solitary Waves

- \* Inhomogeneities destroy the coherent structures
- \* Driven DNLS can  $\Rightarrow$  Chaos
- \* Chaotic Route to Turbulence with k<sup>-1</sup> spectra
- \* For  $\beta \approx 1$  and  $\delta B / B \approx 1$  Simulations needed

### MHD SIMULATIONS :

- \*  $\delta N$  and  $\delta B$  coupling very significant
- \* Evolution of RHP Alfvenic Wave Packet very different than LHP Wave Packet
- \* LHP  $\Rightarrow$  Blow up
  - $RHP \Rightarrow$  Steepening and High Frequency Radiations

#### **HYBRID SIMULATIONS :**

- \* For Large  $\beta$ ,  $\delta N \delta B$  coupling as well as Wave -Particle interactions very crucial
- 1. LHP EVOLUTION:
- Blow up observed in LHP case in MHD Simulations arrested by Kinetic Effects
- \* COLLAPSE  $\Rightarrow$  Transport of Energy to larger k
- \* Appearance of Turning Point due to Structural Instability in the evolution of LHP
- \* Turning Point function of  $\beta$  and the initial amplitude of the Wave Packet
- \* At the Turning Point LHP  $\Rightarrow$  RHP
- \* Ion Acoustic Waves (Coherent Ion Holes) generated for  $T_e > T_i$
- \* For  $T_e >> T_i$ , forward and backward (in inertial frame) propagating Ion Holes
- \* Nonlinear Landau damping for  $T_e \leq T_i$
- \* Forward propagating Magnetosonic waves in some cases

- 2. RHP EVOLUTION :
- \* High Frequency Radiations for relatively lower  $\beta$
- \* Only Backward Propagating Ion Holes
- \* No Magnetosonic waves generated
- \* More robust compared to LHP

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