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AUTUMN COLLEGE ON PLASMA PHYSICS

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DYNAMIC CURRENT SHEETS - III

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Lecture 1.

CURRENT SHEETS IN SPACE PLASMA

Non adiabatic particle dynamics and
numerical simulations

Lecture 2.

EQUILIBRIUM CONFIGURATIONS OF FORCED SHEETS

Role of trapped and transient populations.

Lecture 3.

FRACTAL STRUCTURES IN CURRENT SHEETS

Multiscale turbulence and current branching

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Lecture 3

FRACTAL STRUCTURES IN CURRENT SHEETS

1. **Introduction:** "Laminar" and "turbulent" sheets.

2. **"Turbulence" in magnetotail**
   1. Universality of power-low spectra.
   2. "Clumps" in plasma sheet.

3. **Parameters of fractal distribution.**
   1. Length (Hausdorff dimension).
   2. Spatial fractal dimension
   3. Index of connectivity
   4. Fractal dimension and Fourier spectra.

4. **Self-consistent fractal model**
   1. Diffusion on fractals.
   2. Effective cross-tail transport.
   3. Scaling of Maxwell equations.
   4. Percolation thresholds and Alexander-Orbach condition.
   5. Characteristics of "turbulence" and indexes of power law spectra.

5. **Numerical model of particle transport in turbulent sheet**
   1. Towards the pressure balance.
   2. Current splitting.
   3. Particle residence time

6. **Structural stability of branched current network.**

7. **Conclusions:** "Turbulence" as an intrinsic property of CS.

Universality of the spectra of low-frequency magnetic “turbulence” in the near-Earth and distant parts of magnetotail

$P(f) \sim f^{-\alpha}$, α $(2 \div 2.5)$

f $(0.01 - 1\ \text{Hz})$

Kinks in the spectrum

$|\mathbf{B}| \sim 10^{-4}\,\mathrm{G}$

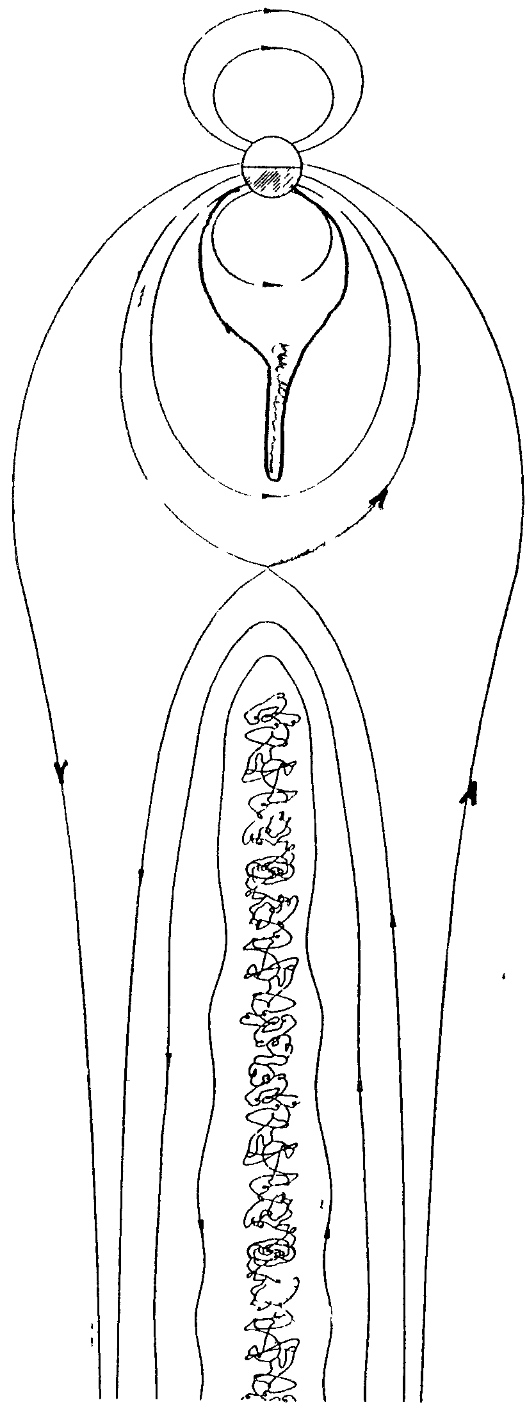
$n \sim 1\ \mathrm{cm}^{-}$

AMPTE – CCE (Ohtani, 1995, 1998)
AMPTE – IRM (Bauer, 1995)
ISEE (Borovsky, 1998)

OQO-5 (Russell, 1972)

INTERBALL-1 (Petrukovich, 1998)

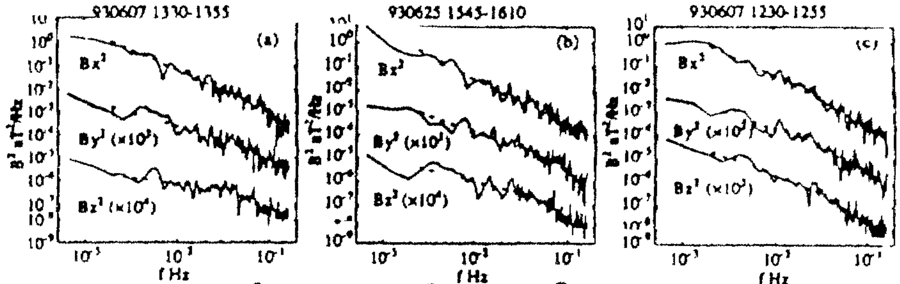
GEOTAIL (Hoshino, 1994)



HOSHINO ET AL TURBULENT PLASMA SHEET

$f > f_* \sim 0.04$ Hz

$|B_z|^2 \sim f^{-S}$



S = 1.78 S = 2.43 S = 2.21

Figure 2 Fourier Power spectral densities of Type C without bipolar signat[ure] and Type B with bipolar signature (c) , observed by *GEOTAIL* The turnover fr[equency of the] continuum appears around 0.04 Hz. The model fitted spectra are plotted as dotte[d]

f_*

AMPTE-CCE

OHTANI ET AL FRACTAL ANALYS[IS]

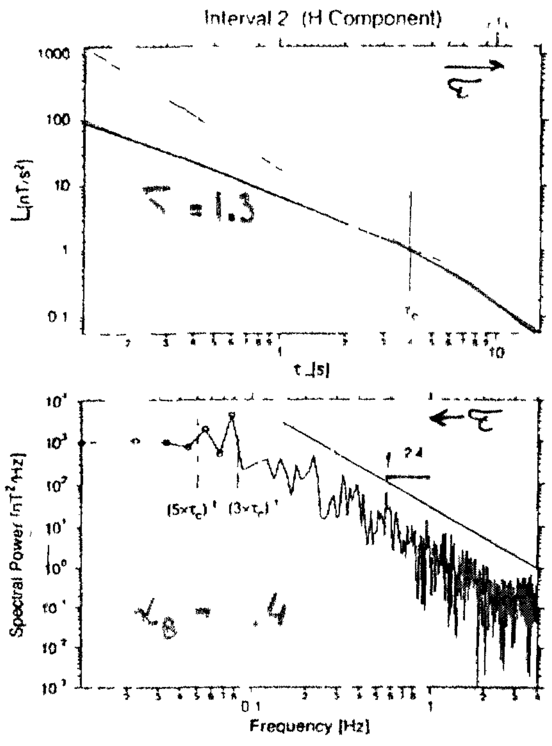
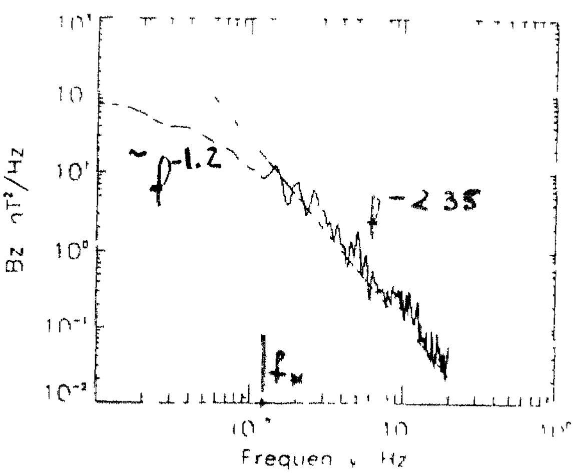


Figure 4 Comparison of the (top) fractal and (bottom) Fourier analyses for the *H* component during interval 2, from 1152 45 to 1154 15 UT The dotted lines in the top panel show the results of the two segment fitting

$\alpha = S - 2\delta$

$\delta = 1.3 \qquad \alpha_B = S - 2(1.3) = 2.4$

Petrukovich et al INTERBALL-1 magnetic ...



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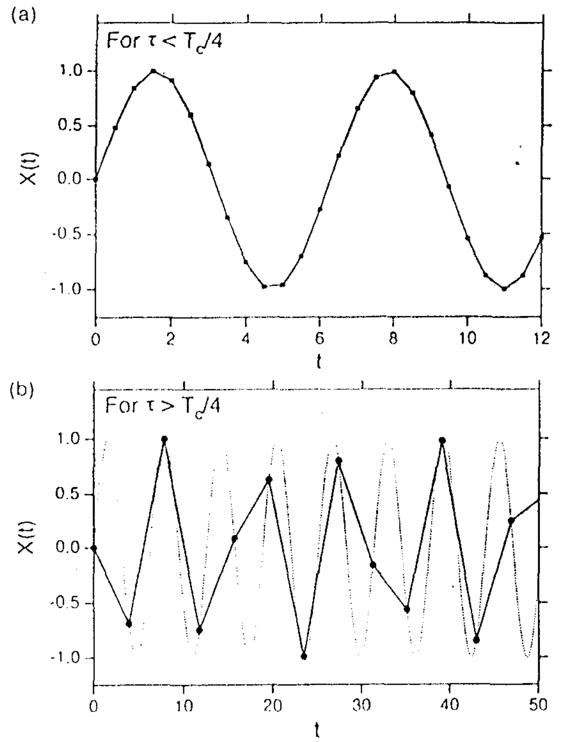


Figure 1. Schematic illustrations of the measurement of the length of fluctuations, $L(\tau)$, in cases in which τ is (a) significantly shorter than and (b) the significant fraction of the characteristic time scale (period) of fluctuations.

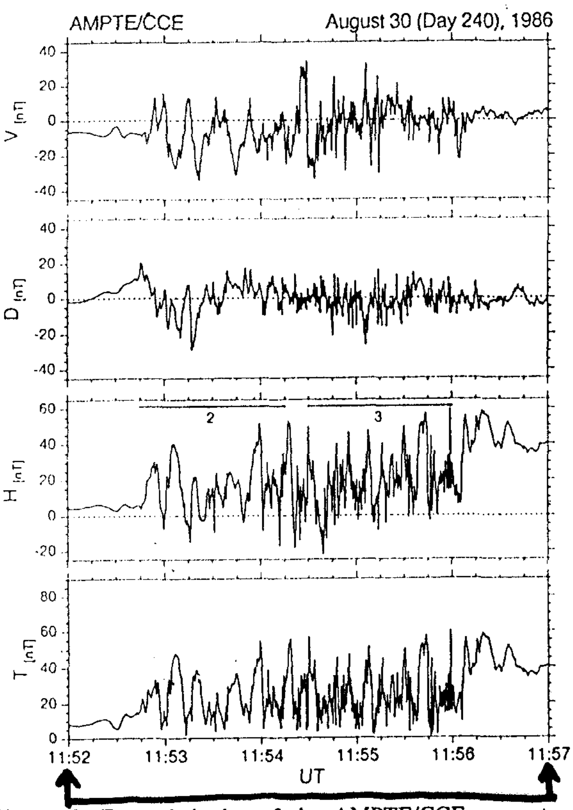


Figure 3. Expanded plot of the AMPTE/CCE magnetometer data from 1152 to 1157 UT.

sets. Since in the Fourier analysis $P(f)$ is calculated for discrete values of f separated equally ($\Delta f = (N\Delta t)^{-1}$, where N is the total number of data points), we do not have many data points in a low-frequency range. On the other hand, the fractal analysis has no methodological restriction in selecting values of t, although the calculation of $L(\tau)$ is less stable for τ closer to the whole duration of the data interval. Since the region of tail current disruption is spatially localized, the spacecraft does not remain in the disruption region for very long, and the dynamics of a current sheet cause rapid motion of the spacecraft relative to the spatial structure of signatures, resulting in abrupt changes in the phase of magnetic fluctuations. Thus these two points are crucial in examining current disruption.

3. Case Study: August 28, 1986, Event

3.1. Outline of the Event

An event we selected for a case study occurred on August 28 (day 240), 1986. This event provides a unique opportunity, because the AMPTE/CCE satellite was in the disruption region at a substorm onset and remained in the region for as long as 3 min. The event was originally reported by *Takahashi et al.* [1987] and was examined later by *Lui et al.* [1992] and *Burkhart et al.* [1993] from different viewpoints.

Figure 2 shows a 20-min (1145–1205) plot of the AMPTE/CCE magnetometer data sampled every 0.124 s (see *Potemra*

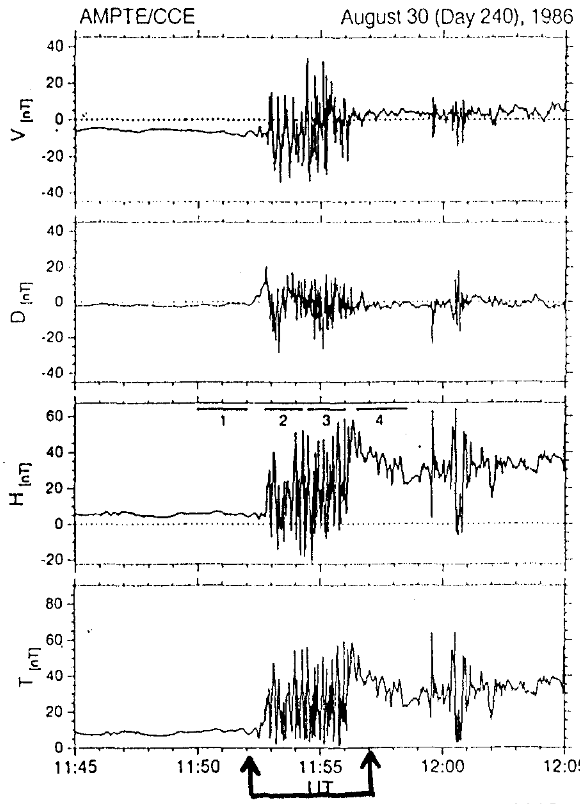
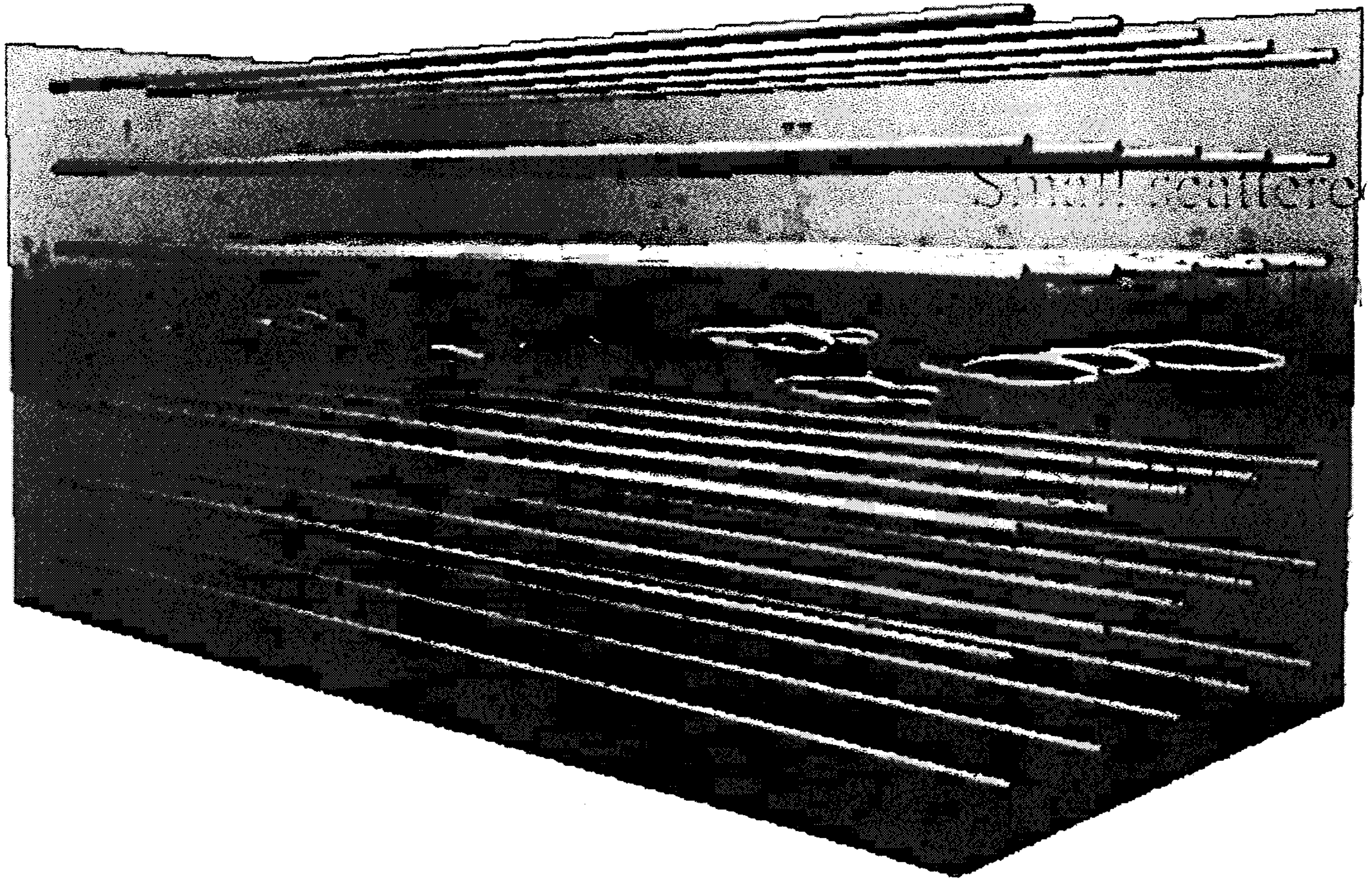


Figure 2. AMPTE/CCE magnetometer data from 1145 to 1205 UT of the August 28, 1986, event. (Top to bottom) The V, D, and H magnetic components and the total field strength. The horizontal bars in the H component panel represent the intervals we selected for the fractal analysis.

Magnetic fields in the equilibrium current sheet:



->Turbulence of small magnetic islands

J. BÜCHNER Kinetic simulations of multiscale turbulence in a Plasma Sheet

Geometrical (FRACTAL) approach to magnetotail high β plasma

- "Turbulence" – ensemble of multiscale hierarchical structures; consisting of magnetic CLUMPS and VOIDS – i.e. is not SPACE Filling
- System reached the stage of **NL** self-consistent

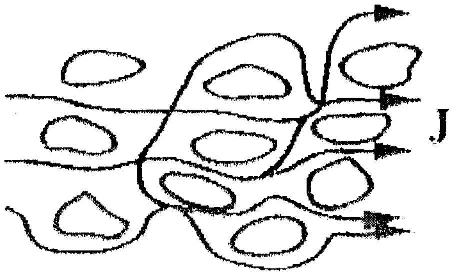
SATURATION

(problem of "Turbulence" generation is BYPASSED)

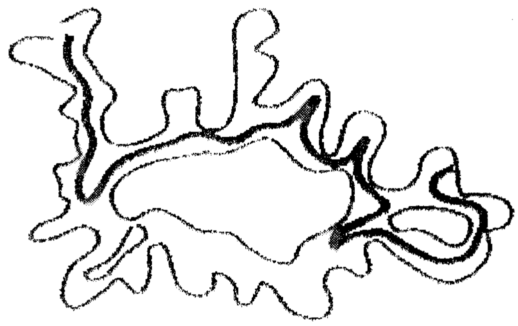
All **NL** effects are accounted in the Geometry of the system

- In certain interval of scales $\alpha<\chi<\xi$ Geometry could be considered as FRACTAL i.e. is Self-Similar.
- Cross tail current $\bar{J}_y = \langle J_y \rangle + \delta J_y$ is organized in Percolating Network

Clumps $\leftrightarrow \delta B_z \leftrightarrow \delta J_y$ | $\langle J_y \rangle \leftrightarrow B_{ox}$ - lobe field



- Coupled patterns of magnetotail current system and structured turbulence



- <u>Geometric analysis</u>:
  Strong NL turbulence is introduced into **TOPOLOGY** of the system.
  Algebraic relations instead of Differential Equations.

- Turbulence is considered as **FRACTAL** object in
  $$\boldsymbol{a} \leq \chi \leq \xi$$

an Example of a Fractal Object

"How long is the coast of Britain ?"

— Mandelbrot (1967).

λ: the spatial resolution $\Rightarrow$

$$L(\lambda) \sim L_0 \lambda^{-s} \to \infty \quad \text{as } \lambda \to 0 \; !$$

$\Rightarrow$ The one-dim. measure is ∞.

$\Rightarrow$ The coast line takes an intermediate position between a smooth rectifiable curve on a plane:

and a finite two-dim. plane area.

$\Rightarrow$ Introduction of the Fractional measure:

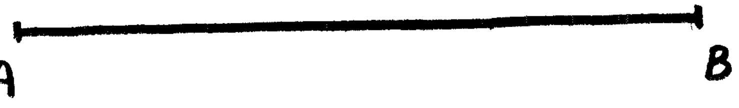
$$1 \leq \delta = 1 + s \leq 2.$$

Why the measure becomes fractional ?

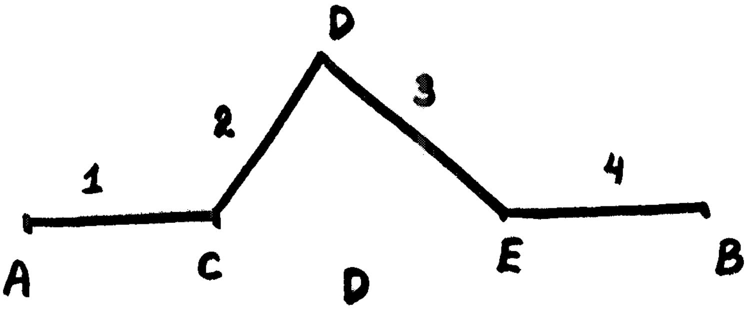
The objects have fine structure on (infinitely) many spatial scales.

a math. example: the von Koch curve

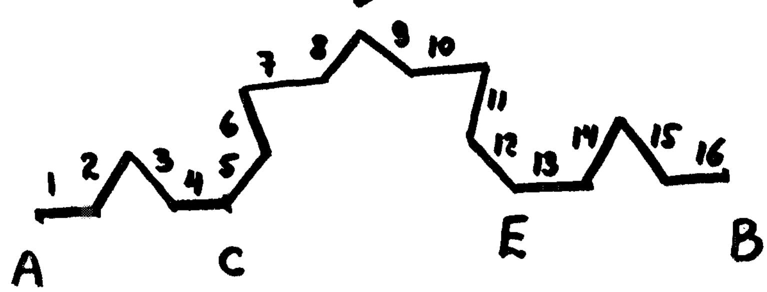
1)



2)



3)



4 - ∞)

$\Rightarrow$ a non-rectifiable continuous curve with the self-similar fine-scale structure

How to take into account the existence of the fine-scale structure on ∞ many spatial scales ?

Kolmogorov (1958):

$$D = -\lim_{\lambda \to +0} \frac{\ln N(\lambda)}{\ln \lambda} \rightarrow \text{HAUSDORFF DIMENSION}$$

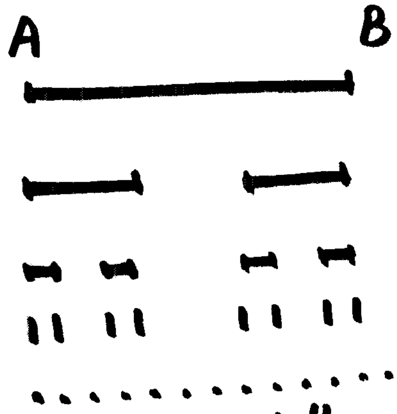
$N(\lambda) \equiv$ the covering number = the min number of d-dim. hypercubes of side λ needed to cover the fractal set.

Examples:

1) The von Koch curve:

$$\left.\begin{aligned} N(\lambda) &= 4 \\ \lambda &= 1/3 \end{aligned}\right\} \quad D = \frac{\ln 4}{\ln 3} \approx 1.26 > 1$$

2) The Cantor set:



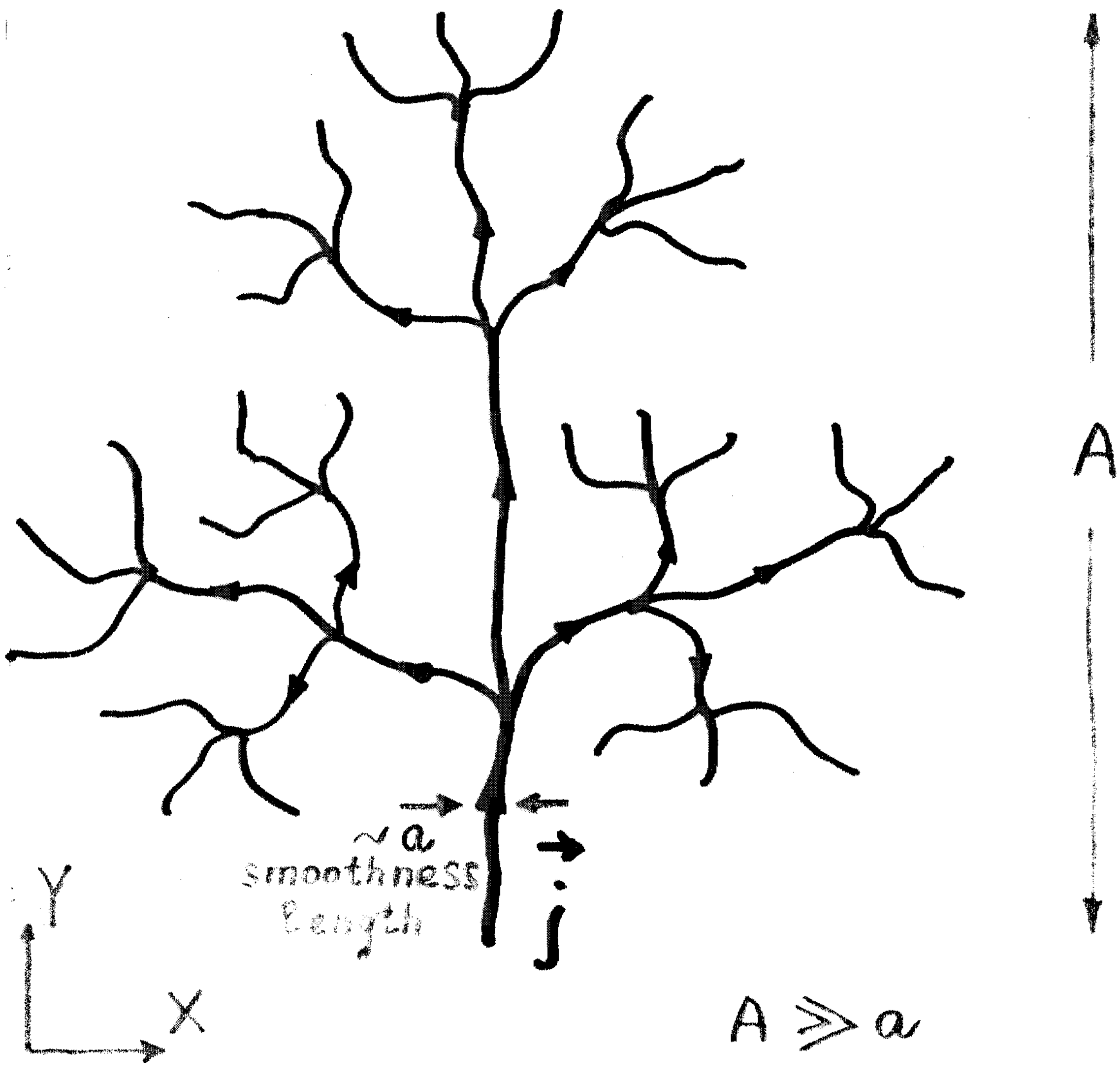
$$\left.\begin{aligned} N(\lambda) &= 2 \\ \lambda &= 1/3 \end{aligned}\right\} \quad D = \frac{\ln 2}{\ln 3}$$

$\approx 0.63 < 1$ "dust"

Fractal distribution of the energy density of the magnetic field fluctuations

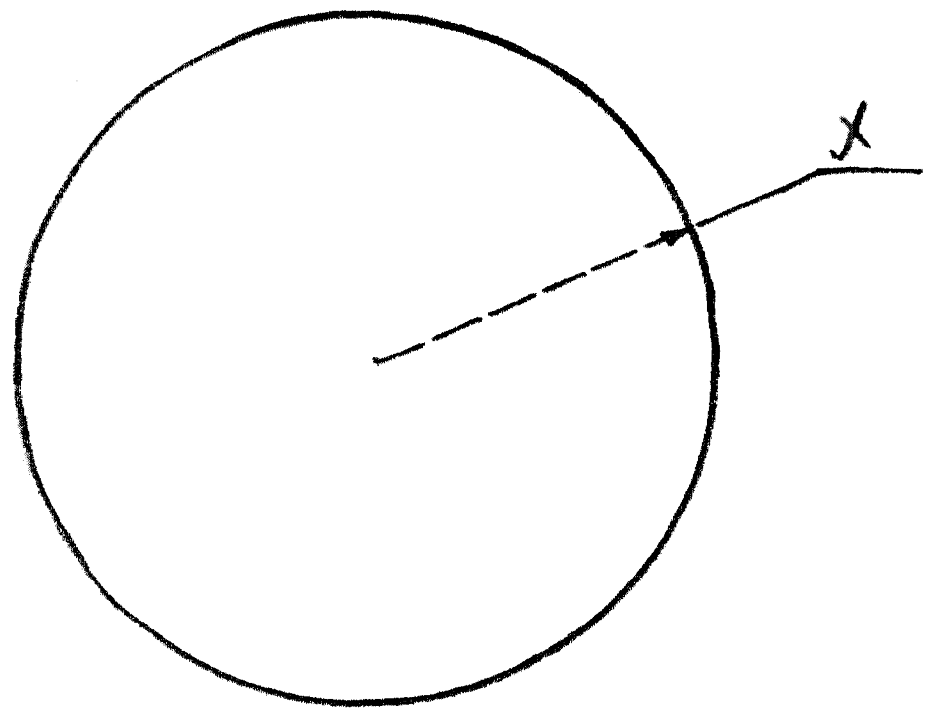
"SAKURA MODEL"

a percolating network of current flows



a "tree-like", branching structure

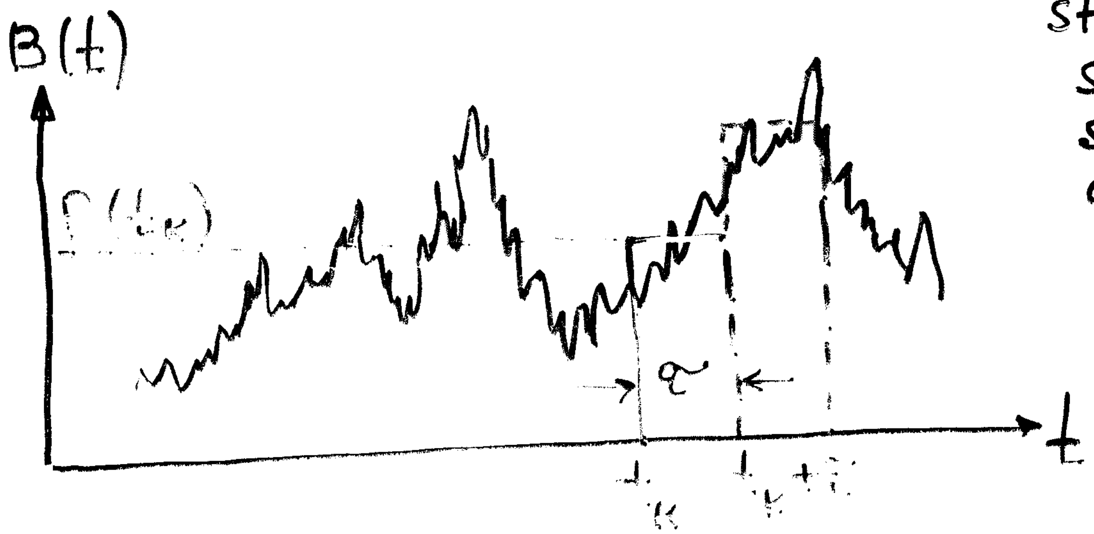
at length scales $a \leq X \leq A$



For FRACTAL Distribution of θ,

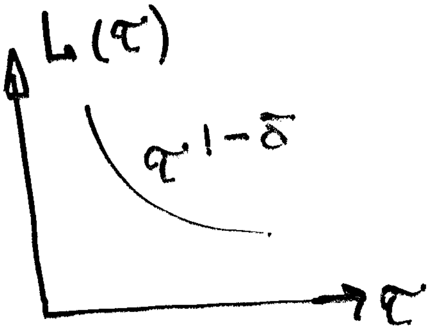
$N(\chi) \propto \chi^{D}$; $n(\chi) \propto \chi^{D-2}$

$\Rightarrow \delta B_z^2 / 8\pi \propto \chi^{D-2}$



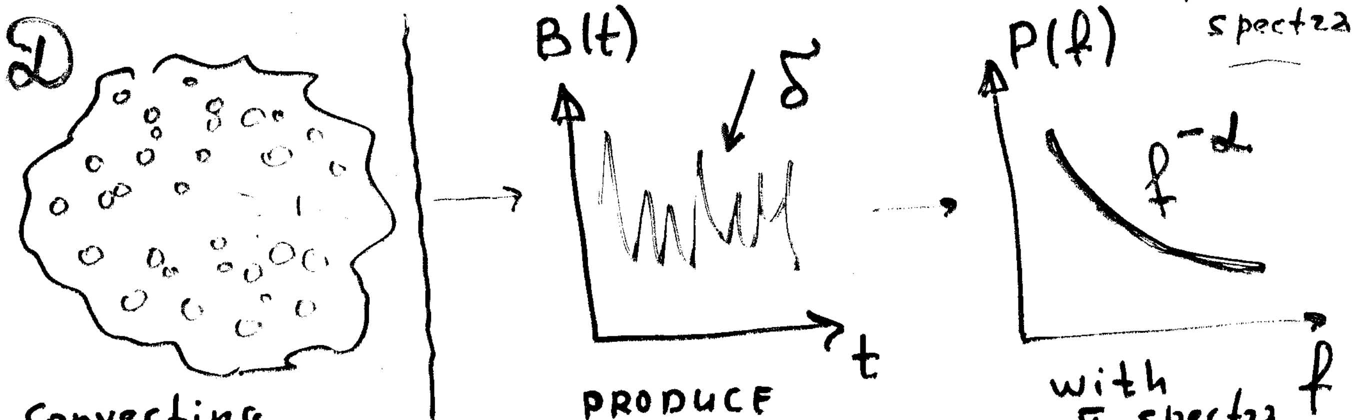
statistically self-similar curve

Length of curve
$$L(\tau) = \sum_{k=1}^{N} \left| B(t_k+\tau) - B(t_k) \right|$$
$$\tau \to 0$$



$$L(\tau) \simeq L_0 \tau^{1-\bar{\delta}}$$

$\bar{\delta}$ – fractal dimension of time series



Convecting FRACTAL OBJECT

PRODUCE TIME SERIES

Fourier spectra

with F. spectra

$$D + \delta = 3$$ (Milov. Zelen. 1994)

$$\alpha = 5 - 2\delta$$ (Berry)
$$= 2D - 1$$

FRACTAL PARAMETERS
IN SPATIAL AND TEMPORAL DOMAINS

Relation of δ - fractal dimension of time series
and D – spatial fractal dimension

$$L(\tau)=\sum_{k=1}^{N}\left|\left(B(t_k+\tau)-B(t_k)\right)\right|=L_0\tau^{1-\delta}$$

We assume that the fractal aggregate convects with the constant velocity V_R

$$X_k \to V_R t_k$$

$$\to L(\chi)=\sum_{k=1}^{N}\left|B(x_k+\chi)-B(\chi_k)\right|$$

$$N(x_k)\sim \chi_k^D \to B(x_k)\sim x_k^{D-2}$$

$$L(\chi)\sim L_0\chi^{D-2} \to L_0\tau^{D-2}$$

$$D-2=1-\delta$$
$$D+\delta=3$$

$\delta = 1$, $D = 2 \to$ Euclidean (nonfractal) case

Power low index of Fourier spectra

$$\alpha = 5 - 2\delta \to 2D - 1$$

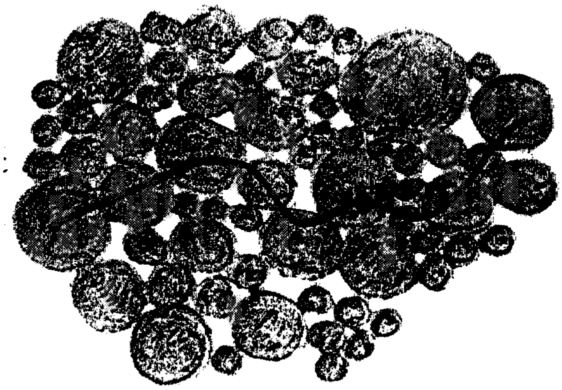
Berry Equation — Steady convection

The Index of Connectivity

Geodesic line on a Fractal object: $d_G = \frac{2+\sigma}{2}$

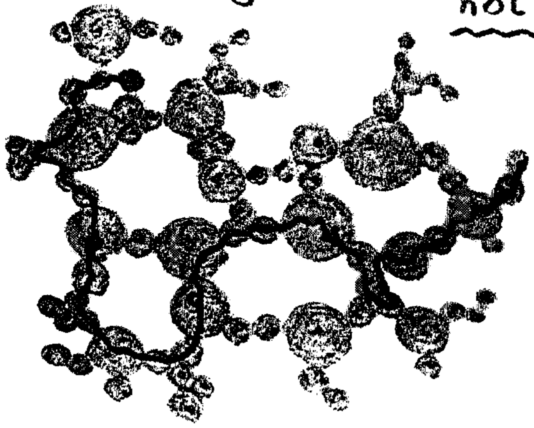
([redacted])

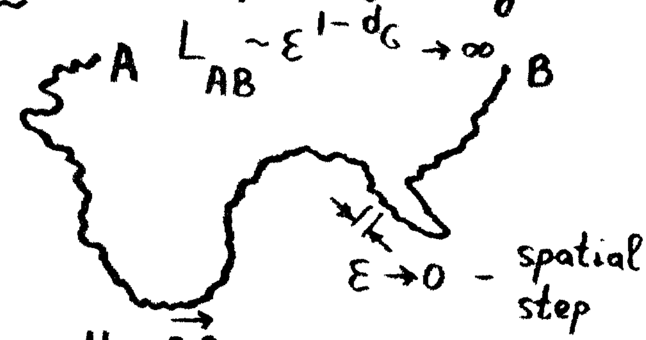
1) $\sigma = 0$, $d_G = 1$ path-connected homogeneous cluster "space-filling"



path $\vec{AB}$:
smooth rectifiable curve

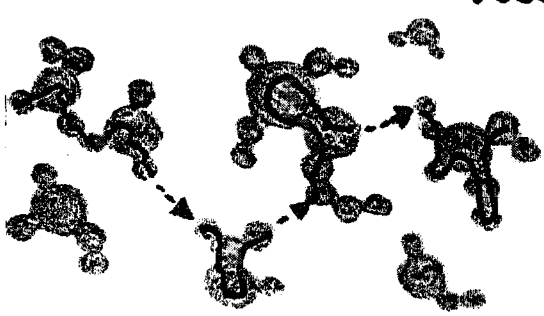
2) $\sigma > 0$, $d_G > 1$ path-connected cluster with "holes" - not "space-filling"

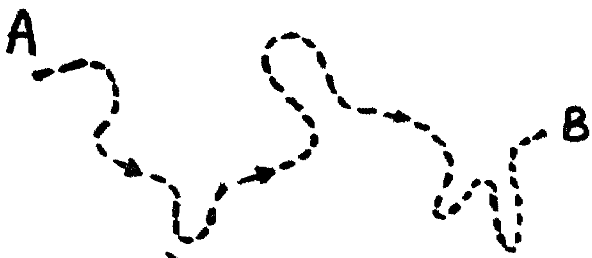




path $\vec{AB}$
self-affine fractal curve

3) $\sigma < 0$, $d_G < 1$ disconnected cluster





path $\vec{AB}$:
everywhere disconnected set

GLOSSARY of TERMS:

- [redacted]

- "CLUMPY" turbulence (not space filling)

- Hausdorff Fractal Dimension – **D**
(generalization of the topological dimension **E = 1. 2. 3)**

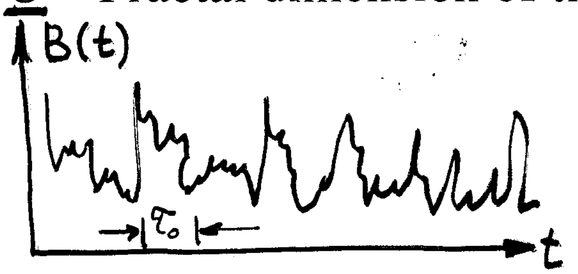
- Anomalous ion diffusivity: $<\delta\chi^2> = (_\Delta t)^\alpha$
  $\alpha = 1$ (Einstein diff.) $\rightarrow \alpha = 2/(2+\underline{\sigma})$

  - subdiffusion ! $<\sigma\chi^2> \sim (_\Delta t)^{0.8}$

- σ - connectivity: "normal" diffusion
  superdiffusion

- Percolation Threshold (Alexander – Orbach Conjecture) $\leftrightarrow$SOC

- $\underline{\delta}$ - Fractal dimension of time series

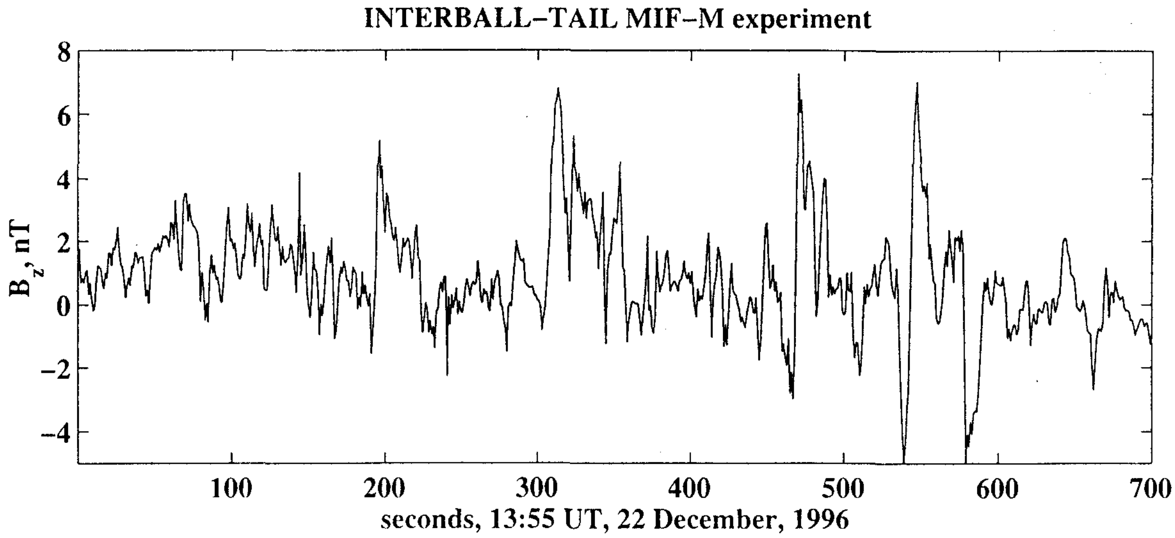


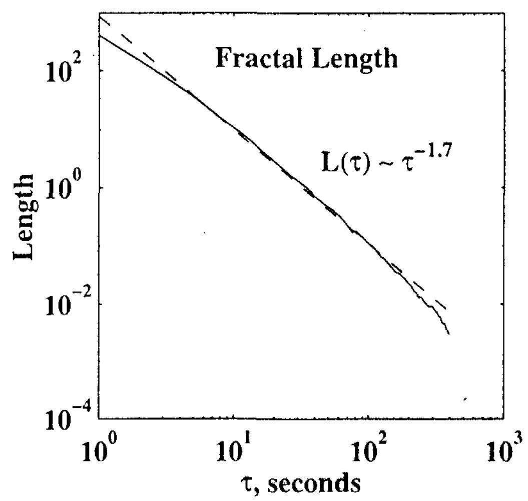
$L(\tau_0) = L_0\tau_0^{1-\underline{\delta}}$
length of curve

- D + δ = 3 for convection of turbulence with flow
(Milovanov,, Zelenyi, 1993)

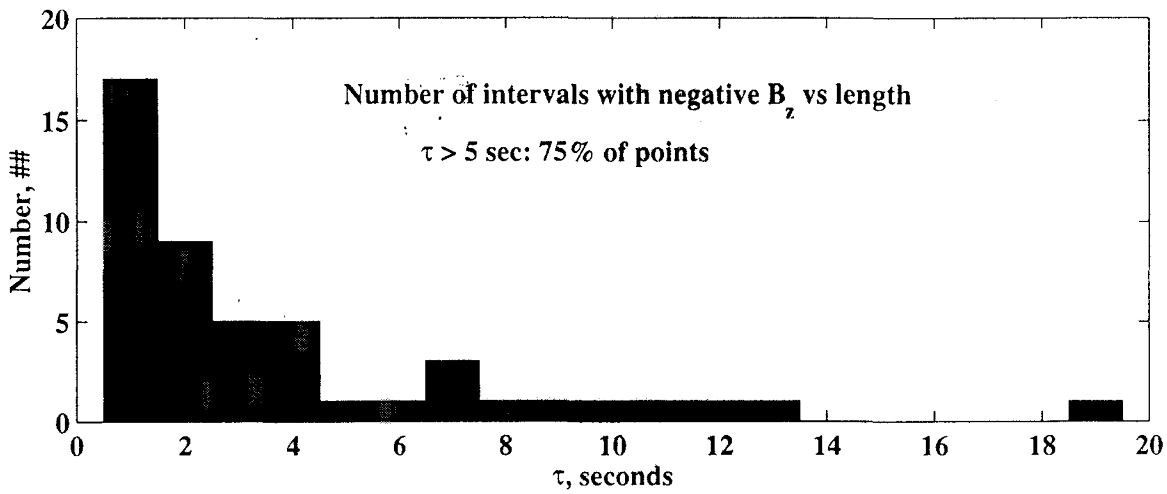
- $S(f) \sim f^{-\alpha}$ power low index $\boxed{\alpha = 5 - 2\underline{\delta} \quad \text{(Berry, 1979)}}$

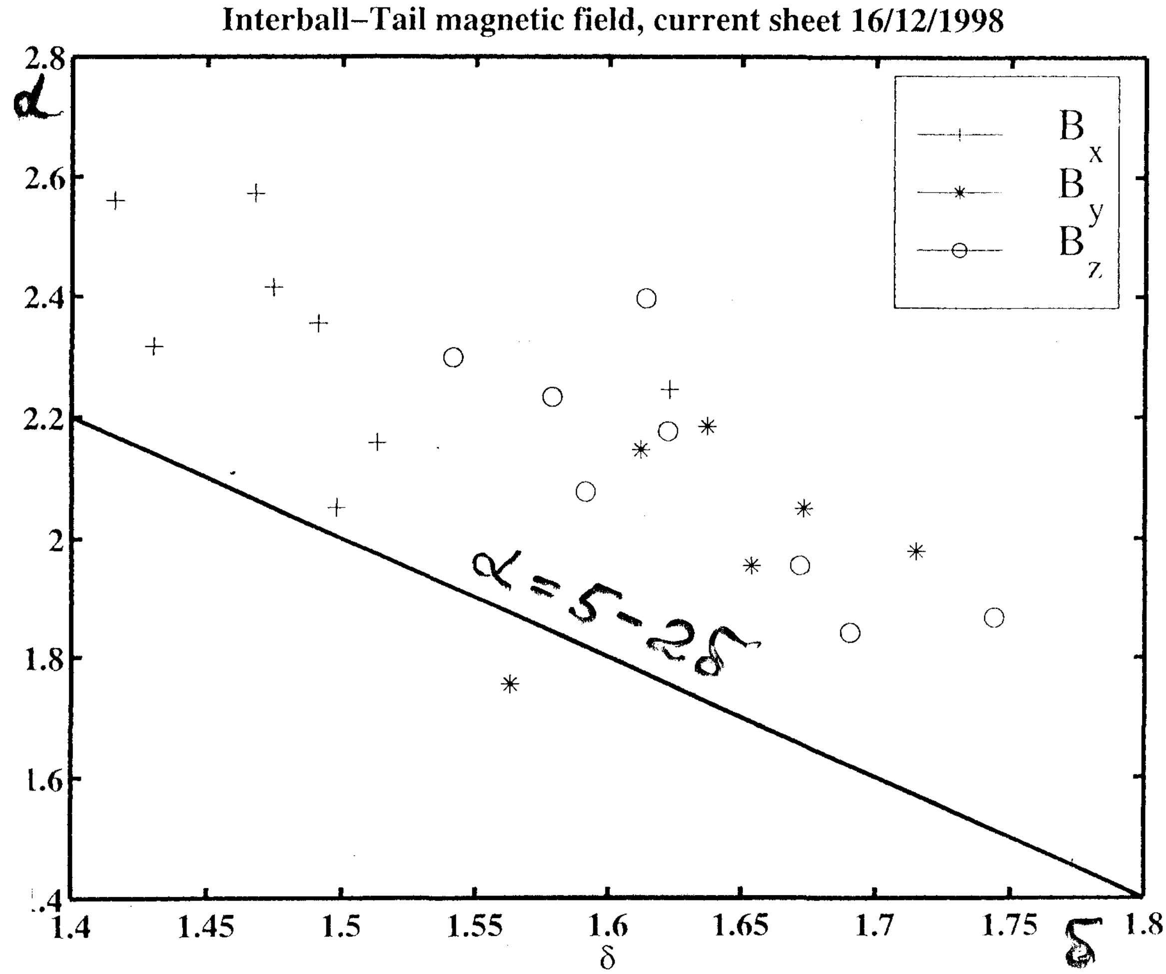
A. Petrukovich





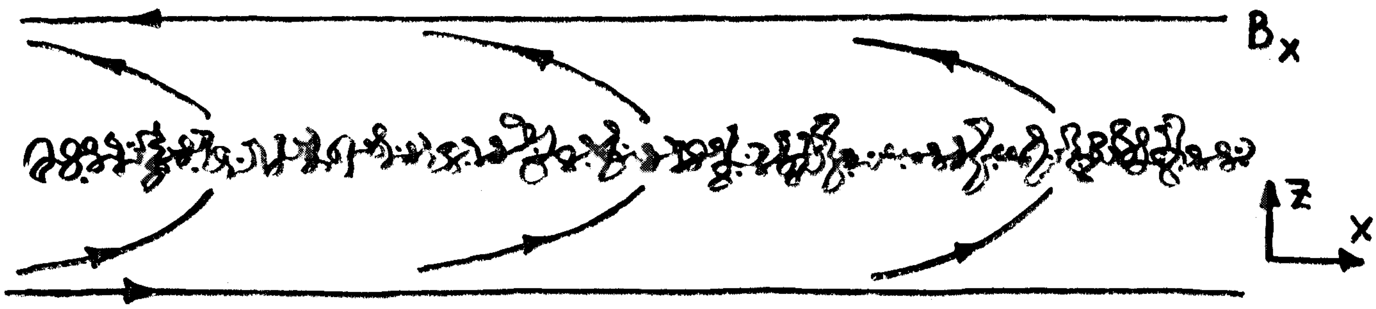
$\delta_{TH} \approx \frac{5}{3}$





A. Petrukovich, 2001

Self-organized steady state of cross-tail current and developed magnetic turbulence (3



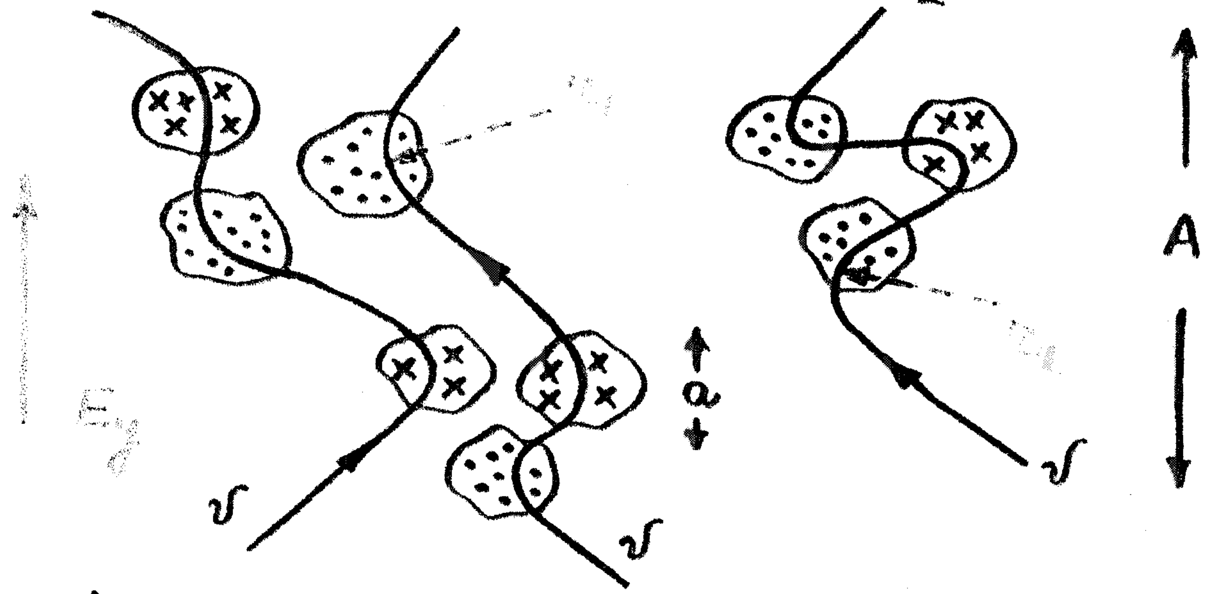
- multiscale branching of the current system
- self-organized generation of the magnetic field fluctuations in the current sheet: $\delta \vec{j} \leftrightarrow \delta \vec{B}_n$
- lobe component of the field: $\langle j_y \rangle \leftrightarrow B_x$

- stability of the current system:

highly "knotted" web-like percolating network self-consistently "pulled" on the multiscale magnetic field "clumps".

The role of the finite ION Larmor radius:

Effective ion Larmor radius (i.e., the curvature radius of the ion trajectories) self-consistently controls the characteristic spatial scale of the magnetic fluctuations in the current sheet: $a \lesssim r_L \ll A$.



$$\frac{\delta j}{\langle j_y \rangle} \sim \frac{a}{r_L} \qquad a \sim \sqrt{\frac{v\, m c L}{e B_x}}$$

The cross-tail ion current $\leftrightarrow$ the diffusion of particles on the self-consistently generated magnetic field turbulence

Anomalous ion diffusivity:

$$\langle \delta x^2 \rangle \propto \Delta t^1 \Rightarrow \langle \delta x^2 \rangle \propto \Delta t^{\varkappa \neq 1}$$

Einstein   Generalized

Anomalous particle diffusion on a FRACTAL object.

$$\left\langle \delta\chi^2 \right\rangle = 2D\theta(\Delta t/\theta)^{\frac{2}{2+\sigma}}$$

(O'Shagnessey Procaccia, 1995. Diffusion on Fractal Objects.)

Diffusion depends on microscopic time scale

of particle random motion $\theta \sim \dfrac{a}{V}$

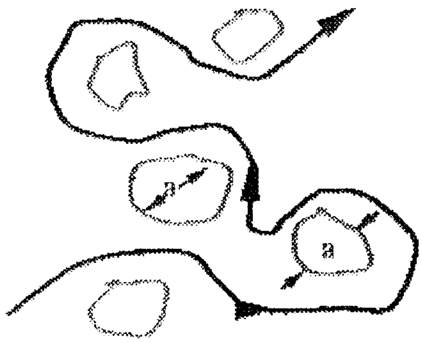
$\theta \sim \dfrac{a}{V} < \Delta t \sim \dfrac{r_L}{V}$ -characteristic time

$r_L^{eff} \rightarrow r_L$ - effective radius of curvature of particle trajectories.

$\sigma = 0$ Dependence on microscopic time disappears

$\left\langle \delta\chi^2 \right\rangle = 2Dt$ Classical Einstein Diffusion

$\sigma > 0$



Antipersistent Diffusion

$-1 < \sigma < 0$

Persistent Diffusion

<u>Introduction to the problem:</u>

D. ben-Avraham, S. Havlin "Diffusion and Reactions in Fractals..."

Estimates of Anomalous Diffusion and Effective Conductivity.

$$|\delta\chi| \sim r_L^{eff}, \quad r_L^{eff} = \frac{e|\widetilde{B}_z|}{m_i c}, \quad \theta \sim \frac{a}{V}, \quad \Delta t \sim \frac{r_L}{V}$$

$$\boxed{D \sim r_L^2 \frac{V}{a}\left(\frac{a}{r_L}\right)^{\frac{2}{2+\sigma}}}$$

($\sigma = 0$, $D \sim Vr_L$ - classical strong (BOHM) diffusion limit)

Effective conductivity
(scattering on magnetic clumps)

$$\sum = \frac{ne^2\tau}{m_i}, \qquad \tau = \frac{a^2}{D} \Rightarrow$$

$$\sum \sim \frac{1}{D} \sim \delta B_Z^{\frac{2(1+\sigma)}{2+\sigma}}$$

Scaling Properties of Maxwell Equations.

- $\nabla \times \overline{\boldsymbol{B}}_x = \dfrac{4\pi}{c} \langle \bar{j}_y \rangle$

- $\nabla \times \overline{\delta \boldsymbol{B}_Z} = \dfrac{4\pi}{c} \delta \bar{j}$

$$\langle j_y \rangle = \sum E_y \,, \quad \sum \sim \frac{ne^2 \tau_{eff}}{m_i}$$

- **Current fluctuations** $\leftrightarrow$

the finite ion Larmor radius effects $\dfrac{\delta j}{\langle j_y \rangle} \sim \dfrac{a}{r_L} \propto \delta B_Z$

- **Estimate of cross-tail current.**

Effective OHM's law due to scattering on fractal clumps

$$\langle j_y \rangle \sim \sum E_y \propto \delta B_Z^{2(1+\sigma)/(2+\sigma)}$$

Differentiation on Fractal field: $\nabla \times \rightarrow \chi^{-\frac{\sigma}{2+\sigma}}$

Self - Consistency Relation.

From the scaling of Maxwell Equations:	From the Definition of Fractal Dimension D
$\delta B_Z^2 \sim \chi^{-\frac{\sigma}{1+\sigma}}$	$\delta B_Z^2 \sim \chi^{D-2}$

$$\Downarrow$$

- $D = 2 - \sigma/(1+\sigma)$ **Self-Consistency Condition**

$\sigma = 0$, $D = 2$ - homogeneous "classical" turbulence

$\sigma > 0$, $D < 2$ - Turbulence is ***not space – filling***

$\sigma \uparrow$ **Trajectory between magnetic obstacles becomes more "wiggled" <u>(current is getting more branched).</u>**

The Alexander-Orbach conjecture

Current Network is at THRESHOLD of PERCOLATION

$$\frac{2D}{2+\sigma} \approx \frac{4}{3}$$ (AO, 1982)

Condition that network is "minimal", but already sufficient to maintain the global cross-tail current structure.

• CRITICALITY CONDITION ↔ SOC

- **Originally checked numerically for many studies** (AO, 1982)
  - **Recently proved analytically** (Milovanov, 1997)

Fractal parameters and Fourier spectra in the interval of kinetic self-consistency

$$\begin{cases} \bullet\ D = 2 - \dfrac{\sigma}{1+\sigma} < 2 & \text{(Scaling EM EQs)} \\ \bullet\ \dfrac{2D}{2+\sigma} \approx \dfrac{4}{3} & \text{(AO conjecture)} \end{cases}$$

$$D \cong \frac{5}{3} \qquad \sigma \cong \frac{1}{2}$$

u – velocity of plasma convection
(decelerated SW speed in magnetotail)

$$[\chi] \Rightarrow [\mathbf{f}] \sim \mathbf{D} + \delta = \mathbf{3}$$

$$\mathbf{P(f)} \sim \mathbf{f}^{-\alpha}$$

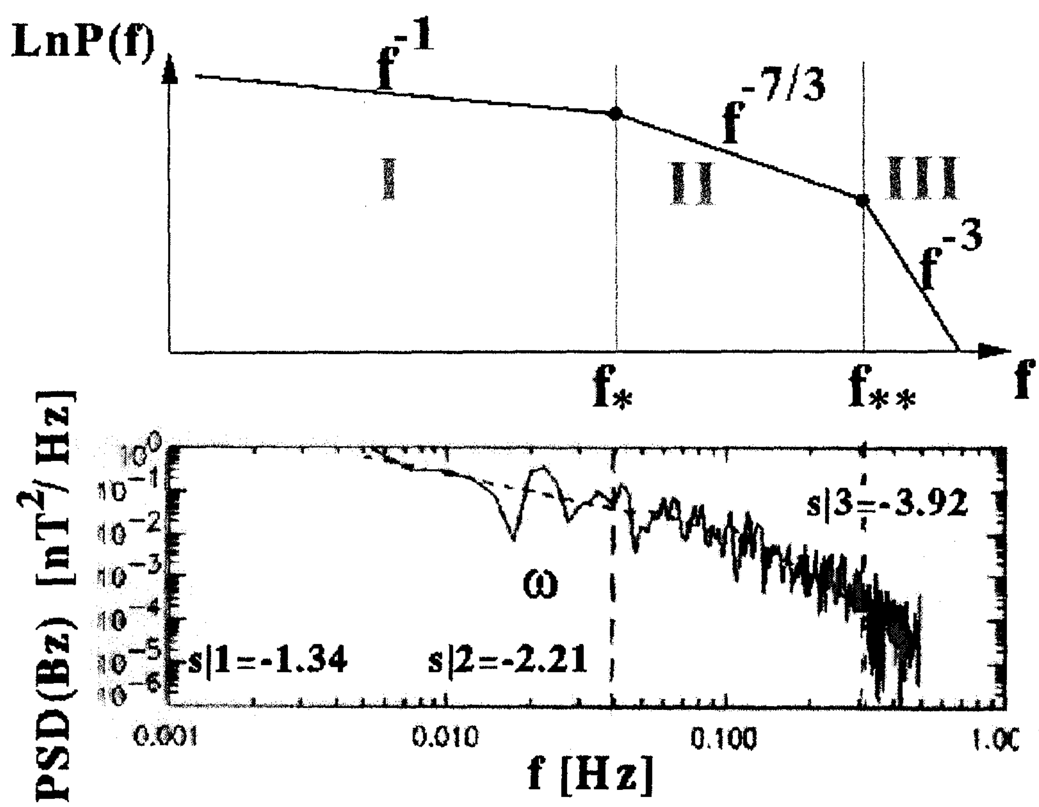
$$\underset{\text{Berry Equation}}{\alpha = 5 - 2\delta} \Rightarrow \underset{\text{plasma convection}}{2\mathbf{D} - 1} \Rightarrow 7/3$$

$$\frac{u}{\xi} \le f \le \frac{u}{a} \qquad \alpha = \frac{7}{3}$$

Universal power law spectrum !

TWO KINKS in spectra: $f_* = \dfrac{u}{\xi}$ $\qquad f_{**} = \dfrac{u}{a}$

Fourier spectrum of multiscale structured "turbulence"



Interball-Tail observations (18.11.96)
The $[B_Z(Z=0)]^2_\omega$ spectrum

I. $\frac{1}{f}$ - Flicker noise. Explicit time dependence of structures.

II. Interval of self-consistency:
$\omega' = \omega - \boldsymbol{kV}$, $\omega < \boldsymbol{kV}$, $\omega' \approx \boldsymbol{kV}$

III. $\lambda < \boldsymbol{a}$ ~ smoothness interval: $\boldsymbol{D} \approx \mathbf{2}$, $\alpha = \mathbf{2}\boldsymbol{D} - \mathbf{1} \to \mathbf{3}$
At small scales turbulence becomes space filling

PROPERTIES of “TURBULENCE” at “Kinetic” scales ($\geq \rho_i$)

Why MHD – approach breaks at certain scale lengths?

MHD interpretation of turbulence:

$\alpha_V \approx \alpha_B \approx 5/3$

fluid-like turbulence with frozen in $\overline{B}$

non – MHD kinetic model (fractal field)

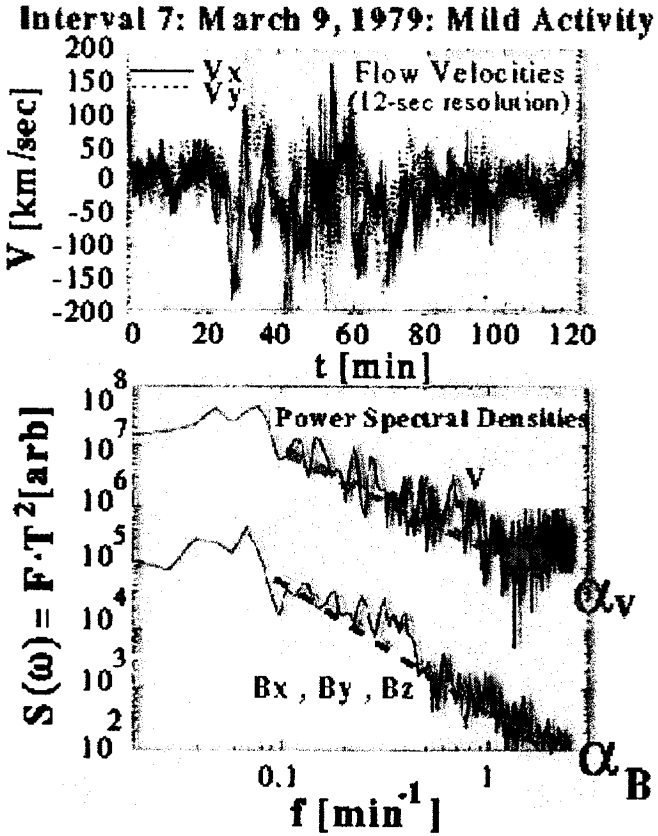
$$D_B \rightarrow (5/3) \rightarrow \alpha_B = 2D_B - 1 = \frac{7}{3}$$

$$D_V = 3 - D_B = 4/3 \qquad \neq$$

$$\alpha_V = 2D_V - 1 = \frac{5}{3}$$

Field fluctuations are decoupled from flow fluctuations

BOROVSKY, 1997 Magnetotail as Laboratory for turbulence



$S_V(f) = f^{-\alpha_V}$ $\qquad \alpha_V \cong 1.5$

$S_B(f) = f^{-\alpha_B}$ $\qquad \alpha_B \cong 2.2$

- Interval of Kinetic selfconsistency for low-frequency "turbulence" in the high β plasma.

particle dynamics **Maxwell law**

$$\xi \geq \lambda \geq \alpha \sim \rho_i \qquad \delta \bar{B} \rightarrow \delta U \rightarrow \delta J_\perp$$

S.C.

- "CLUMPY" properties of $\delta \bar{B}$ in Self-consist regime

$$D \sim 5/3 < 2$$

. . . Universal Fourier spectra $S(f) \sim f^{-\alpha}$ of fluctuations in the ($\delta B \rightarrow \delta U \rightarrow \delta j_\perp$) interval $\alpha_B = 7/3, \alpha_V = 5/3$

- non-MHD characteristics

. . . "Universal" interval

$$10^{-2} \div 10^{-1}Hz < f < 10^{-1}Hz - 10^{1}Hz$$

Weakness of theory $\rightarrow$
Rescaling of Spatial scales to Temporal

. . . . **KINKS** of power-law spectra at both ends of

the "Universal" interval.

Substorm **ONSET** as Structural Instability of percolating current network

Scenario:

- Accumulation of MAGN. ENERGY in tail

$$\delta W_F \uparrow, \quad \langle J_Y \rangle \uparrow$$

- 2**D** highly-branched current system at the THRESHOLD of CRITICALITY could not support the growing $\langle J_Y \rangle$
- OVERCRITICAL CONDITIONS:

  Simplification of current network:
- Reduced branching, straightening of current filaments:

$$D\uparrow, \ \sigma\uparrow, D^{+}\downarrow, \sigma^{+}\downarrow$$

Cascade of energy to smaller f $\longrightarrow$ larger SCALES

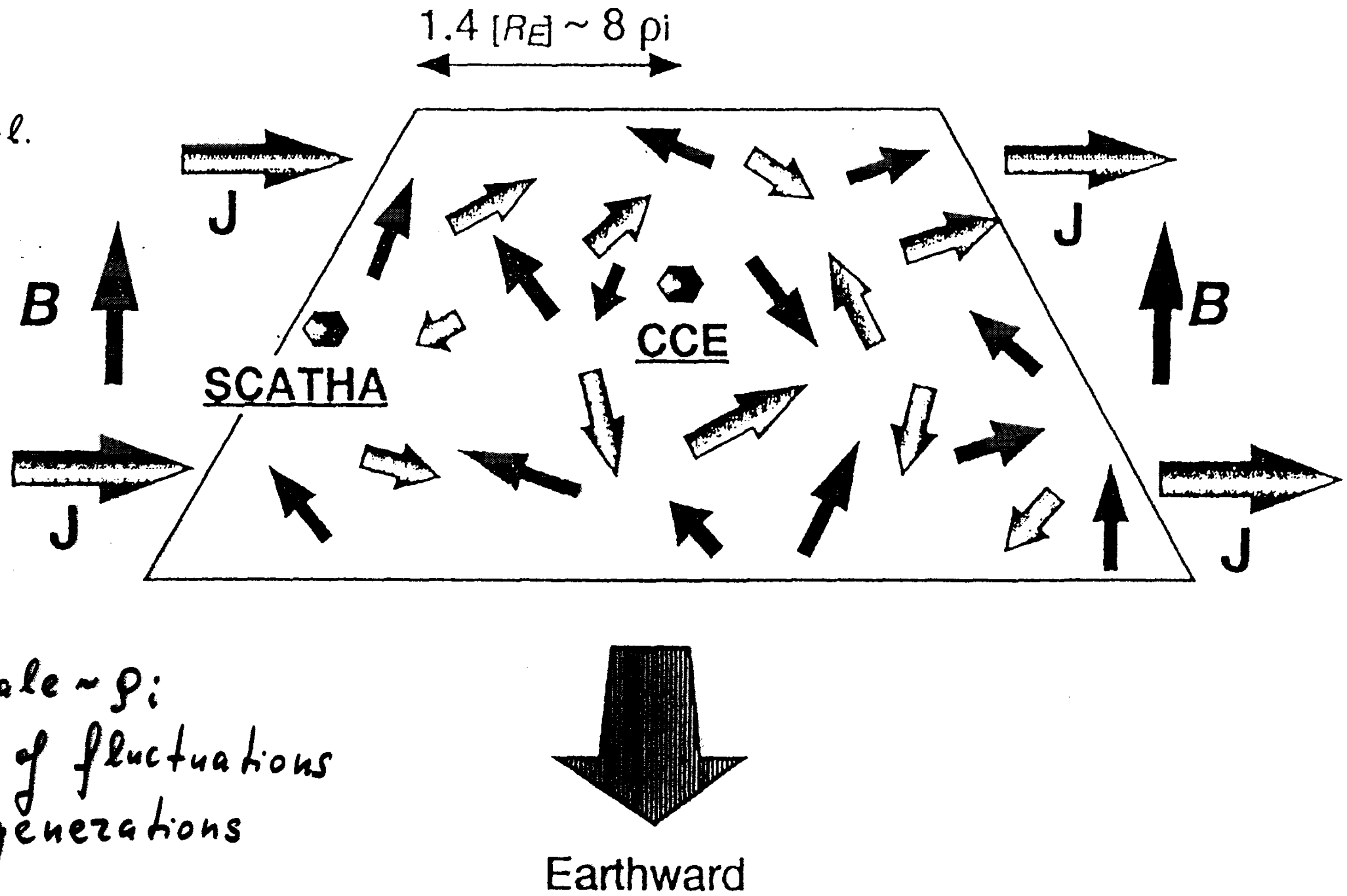
- Reduction of ELASTICITY of current network

  $\sim \sigma^{+}\downarrow$, due to decrease of BRANCHING
- Structural instability of floating current LOOPS. ~ ??

  Current WEDGE?

  Rapid 3-dimensionalization of cross-tail current

OHTANI et.al.
1998



- Spatial scale ~ ρ_i
- Convection of fluctuations
- local generations

Figure 10. A schematic diagram of the tail current disruption illustrating the chaotic features of magnetic field and electric current perturbations in the onset region. The basic concept was adopted from *Lui et al.* [1988, Figure 4].

Simultaneous observations of substorm-associated magnetic fluctuations

Self-consistent fractal description:

"Physical" properties of the turbulence are analyzed by means of "**unconventional**" geometric parameters.

GLOSSARY of TERMS

- **D** = Hausdorff fractall dimension describing spatial distribution of magnetic turbulence "clumps" accross the current sheet plane $1 \le D \le 2$;
  ($D < 2$ – not "space-filling" turbulence).

- σ = connectivity of the fractal set composed of the magnetic "clumps" $0 \le \sigma < 1$.

- $\mathbf{D}^+$ = Hausdorff fractal dimension of the current density percolating network $1 \le D^+ \le 2$.

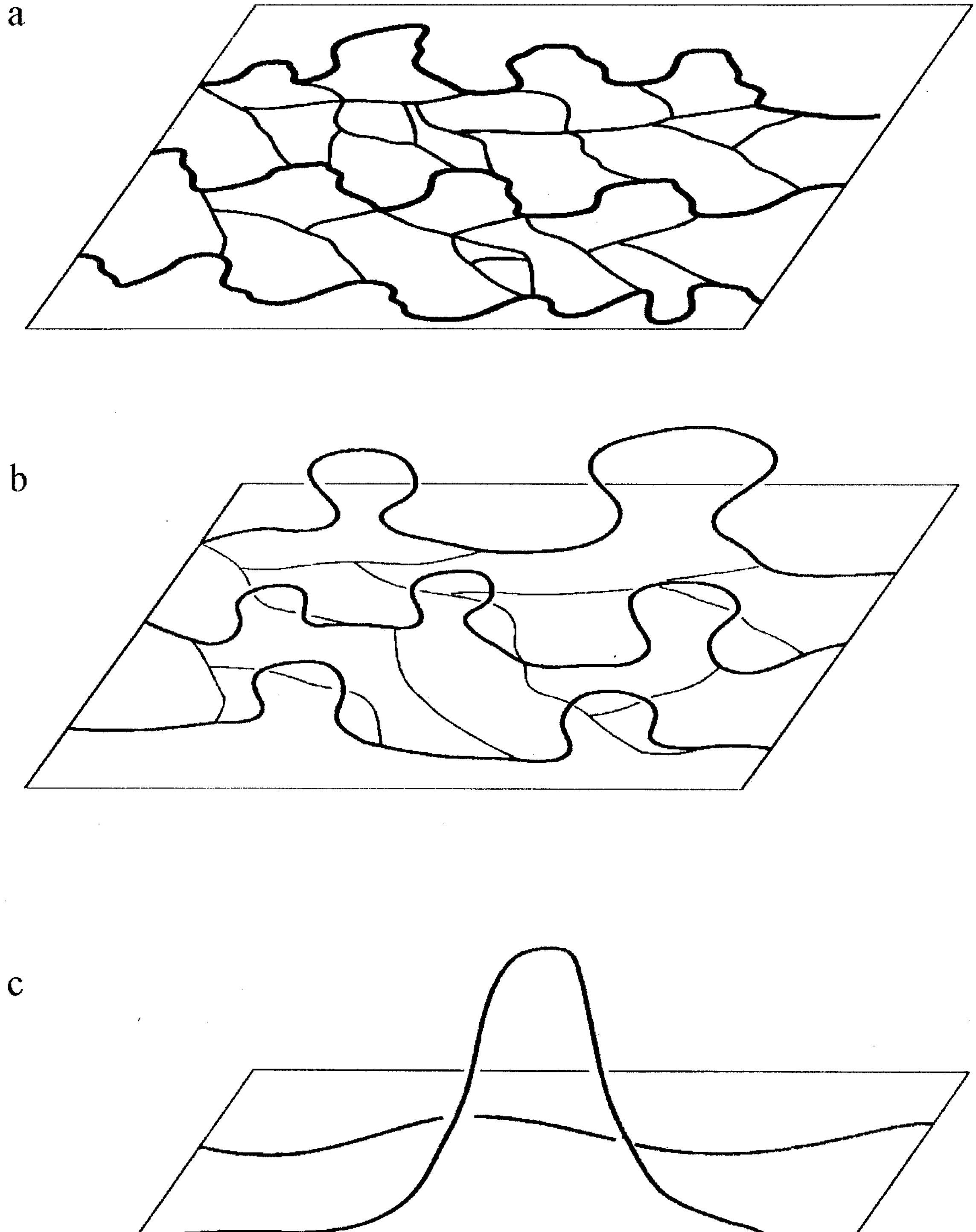
- σ^+ = connectivity of the network $0 \le \sigma^+ < 1$

SELF-CONSISTENT REGIME:

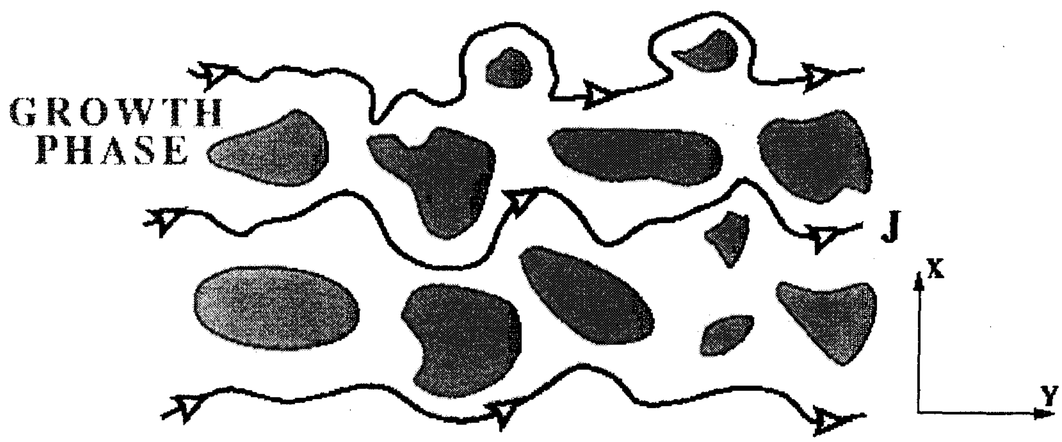
magnetic field and current density fractal structures are mutually "conjugated"

$$(D, \sigma) \leftrightarrow (D^+, \sigma^+)$$

"duality" property


a
b
c

PRE-ONSET EVOLUTION
Simplification of the current system: $\delta W_F \uparrow$



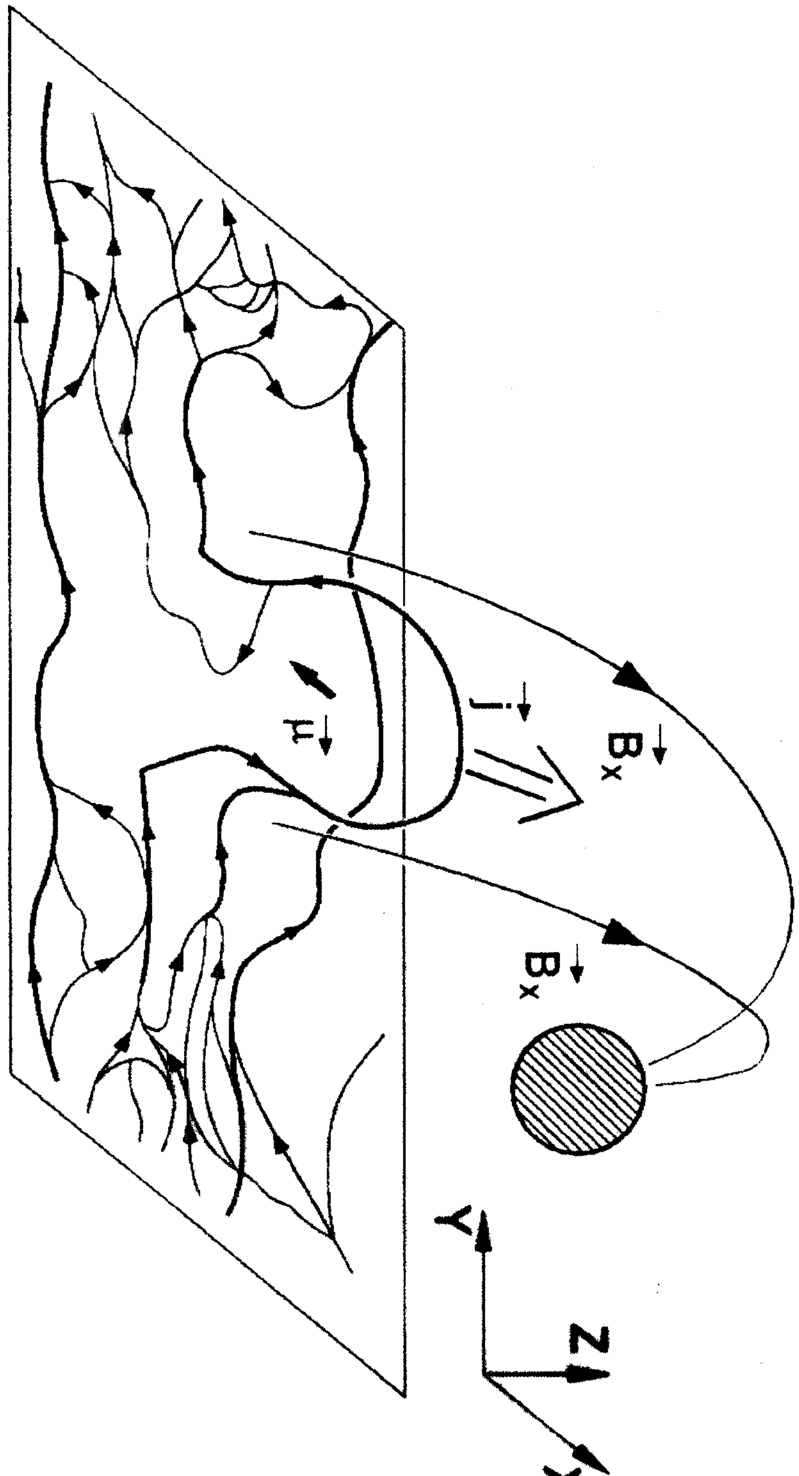
Reduced branching of current: $\langle j_Y \rangle \uparrow$
(straightening of current filaments)

$\underline{\sigma^+ \rightarrow 0}$

Graining of magnetic structure becomes more coarse:
(Cascade of energy to large scales from small scales)

$\mathbf{D \rightarrow 0}$

Weakly knotted current system
becomes TOPOLOGICALLY UNSTABLE


$\vec{B}_x$
$\vec{B}_x$
$\vec{j}$
$\vec{\mu}$
Y
Z
X

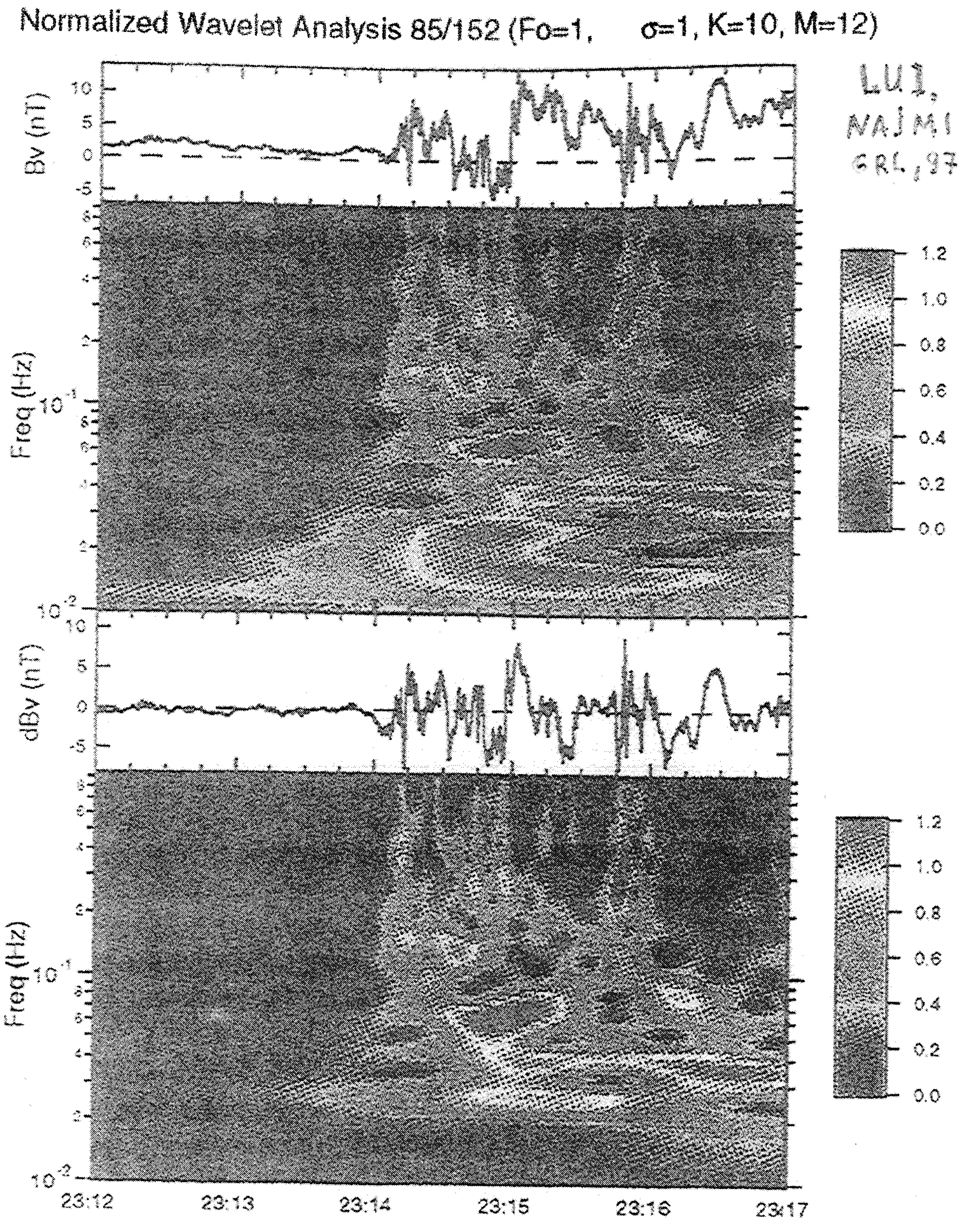


Plate 2. Wavelet transform for magnetic field fluctuations observed in a current disruption event. Several components of the signal can be seen, including a signal suggesting waves at high frequency cascading to lower frequency as time progresses.

CONCLUSIONS - I

1. **Structural stability of the stretched magnetotail could be provided by self-organized turbulence preventing the coalescence of current filaments.**

2. **Turbulence is not "space-filling" and self-consistently supports multiscale current network "pulled" on the magnetic field "clumps".**

3. **Turbulence structures could be described within "geometric" approach involving such topological parameters as Hausdorff fractal dimension & index of connectivity.**

4. **Self-consistent currents and fields topologies are mutually "conjugated" (dual to each other).**

5. **At "basic" steady state with minimum of free energy the system saturates at the threshold of percolation.**

6. **In the course of free energy accumulation in the tail (late growth phase) current system overcomes the marginal percolation threshold, and 3D current filaments start to "emerge" from the current sheet due to simplification of the current network.**

7. **Evolution of the current network above the percolation threshold has catastrophic character and resembles the second-order phase transition. Current system becomes essentially 3D and has features often associated with the Substorm Current Wedge.**

MAGNETIC TURBULENCE AND ION DYNAMICS IN THE DISTANT MAGNETOTAIL

Numerical model

1. Modelling of the Ion Dynamics 1D sheet 3D-turbulence
   J. G. R. 1998, 103, N A7, pp 14987-14910
   J. G. R. 2001 (in press)
2. Multiscale structure.

   Attempts of Self-Consistent Description
   J. G. R. 1996, 101, N 9, p. 19903
   J. G. R 2001, 106, N A4, p. 6921

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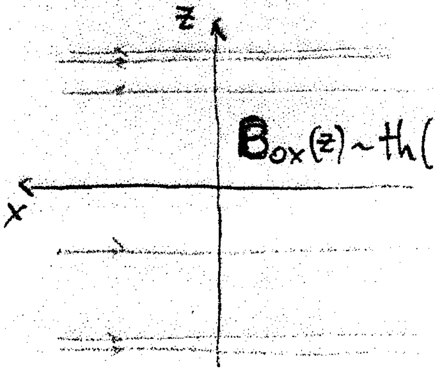
MAGNETIC Field Geometry

$$\vec{B} = B_{0x}(z)\cdot\hat{e}_x + \delta\vec{B}(\vec{r}) + B_z\cdot\hat{e}_z$$

$$\vec{E} = E_y\cdot\hat{e}_y = \text{const}$$



$B_{0x}(z) \sim th\left(\frac{z}{\lambda}\right)$

$$\delta\vec{B} = \sum_{\vec{k}} \delta B(\vec{k})\, e^{i(\vec{k}\vec{r}+\phi_k)}$$

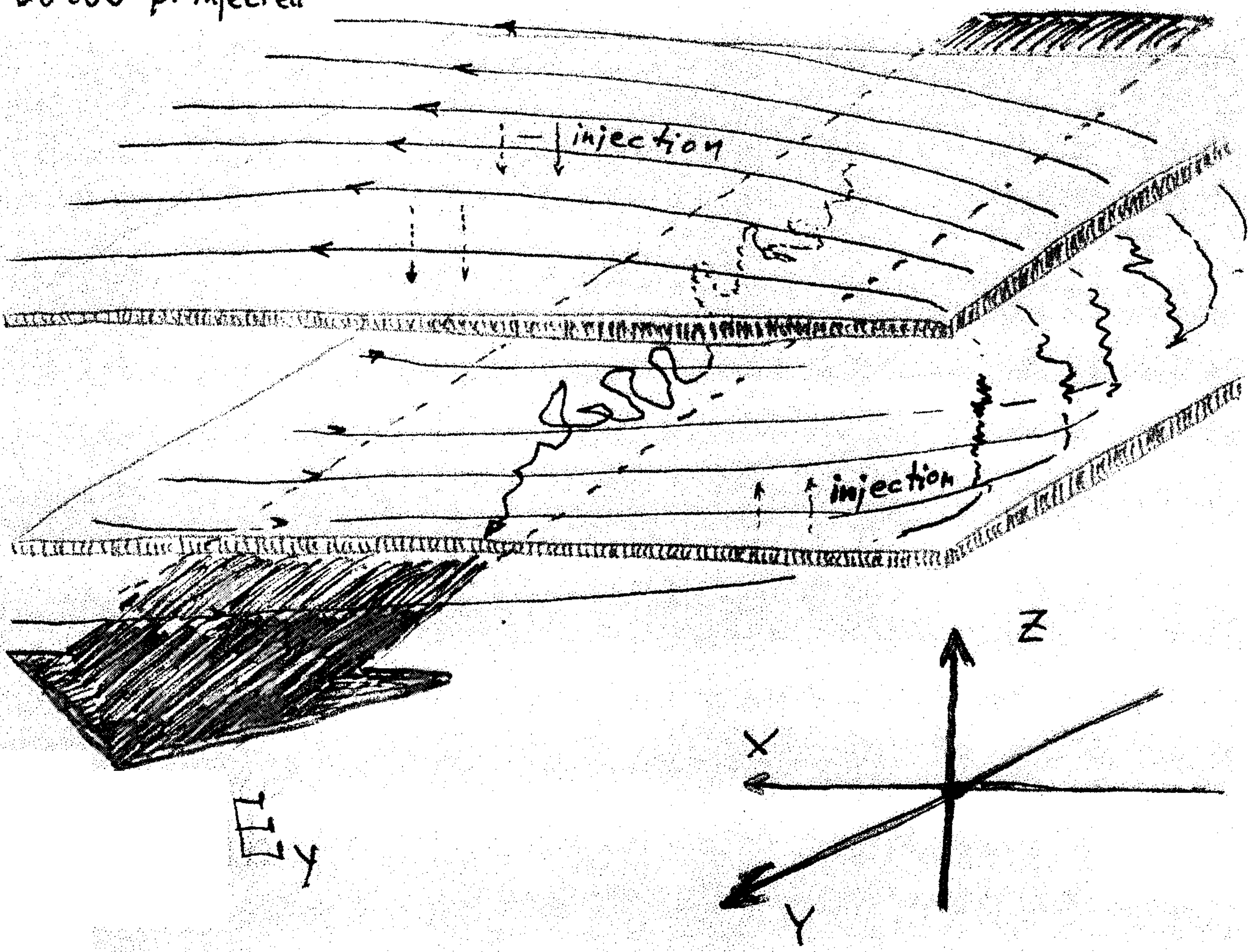
$$\delta B(\vec{k}) \sim \left(\sum_i^{2} k_i l_i^2\right)^{-\alpha}$$

3550 modes

$B_z = \text{const}$

TEST PARTICLES (IONS) - SIMULATION

80 000 p. injected



Magnetic Field Model

Modified Harris profile with fluctuations:

$$\vec{B} = B_{ox}(z) \cdot \hat{e}_x + \sum_{\vec{k}\sigma} \delta B_\sigma(\vec{k}) e_\sigma(\vec{k}) e^{i(\vec{k}\vec{r} + \varphi_{\vec{k}}^{\sigma})}$$

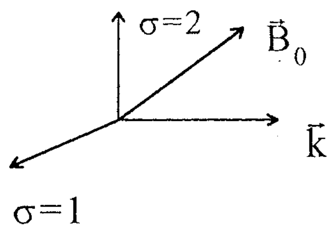
$$B_{ox}(z) = \mathbf{B}_0 \frac{\tanh(z/\lambda) - (z/\lambda)\cosh^{-2}(L/2\lambda)}{\tanh(L/2\lambda) - (L/2\lambda)\cosh^{-2}(L/2\lambda)}$$

for $|z| = L/2$ $\quad \partial B_{ox}(z)/\partial z = 0$

λ - sheet half thickness, L- scale of the simulation box

$$\delta B_\sigma(\vec{k}) \sim \frac{1}{(k_x^2 l_x^2 + k_y^2 l_y^2 + k_z^2 l_z^2 + 1)^{\alpha/4 + 1/2}}$$

1). 2-polarizations for each mode:



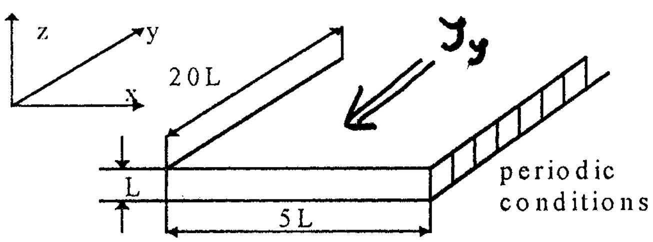
1) Parity conditions

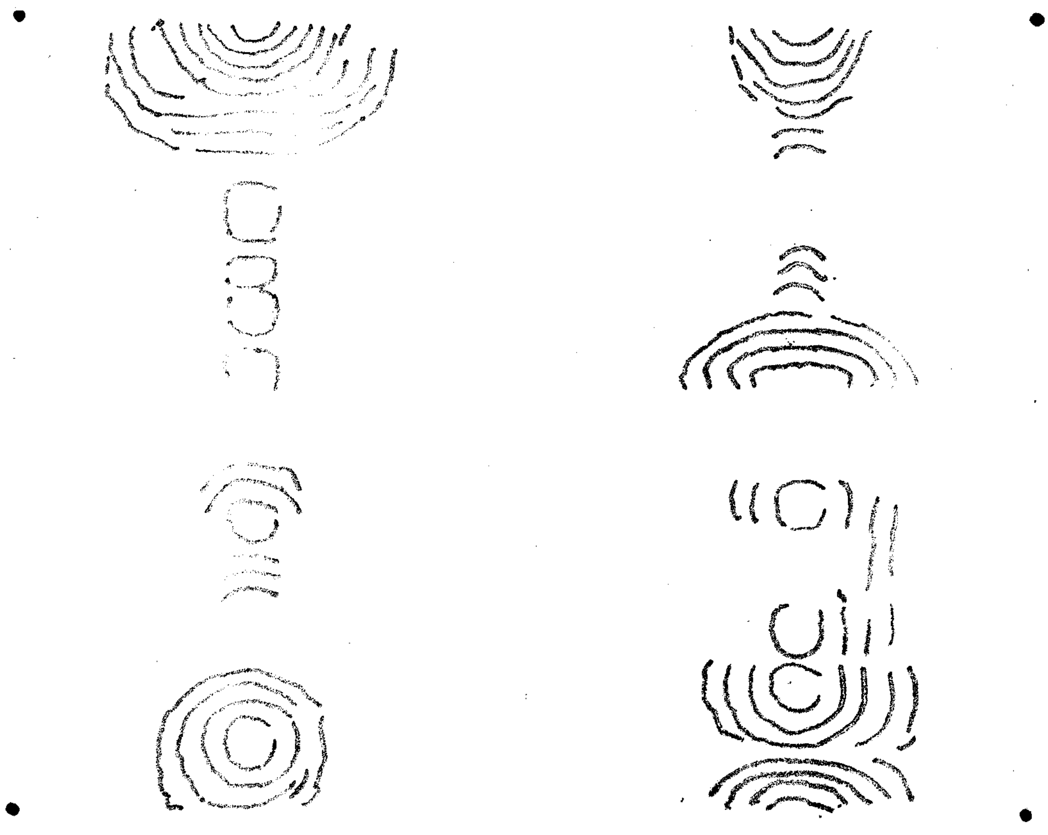
$$\delta B_{x,y}(x, y, z) = -\delta B_{x,y}(x, y, -z)$$
$$\delta B_z(x, y, z) = \delta B_z(x, y, -z)$$
$$\delta B_\sigma(\vec{k}) = \delta B_\sigma(-\vec{k})$$

3). Correlation lengths: $\quad l_x = l_y = 0.25L \qquad l_z = 0.05L$

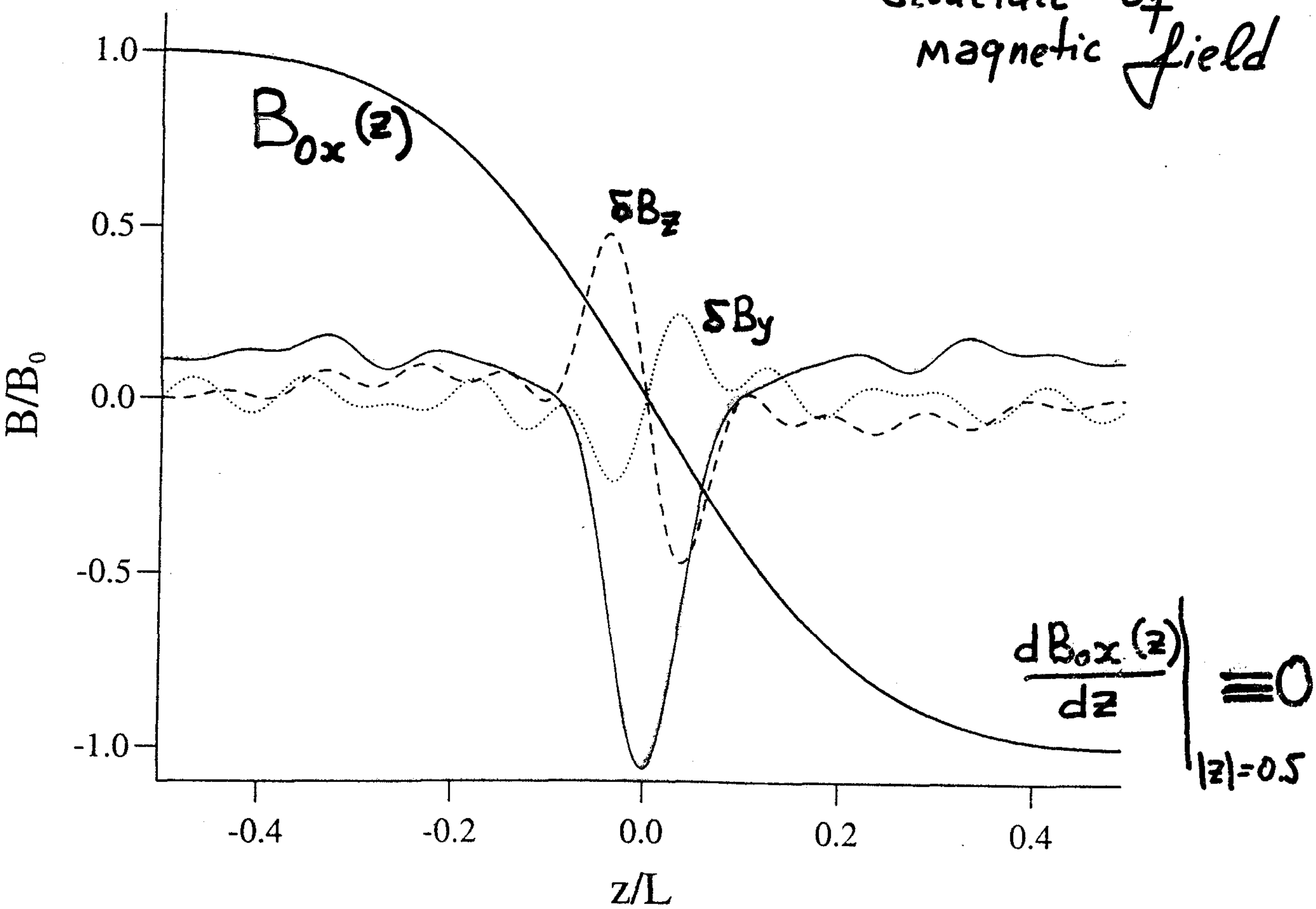
4). Harris sheet half thickness $\quad \lambda = 0.25L$

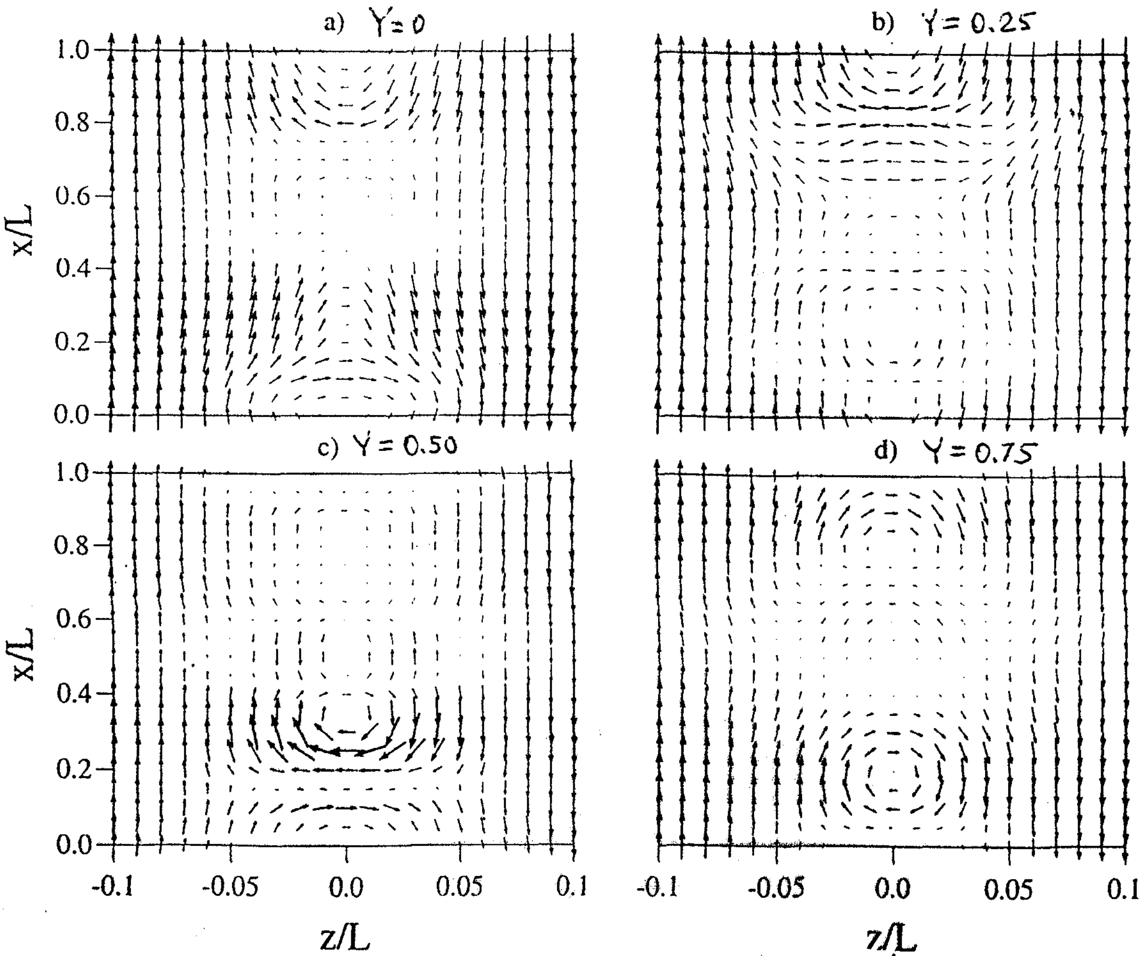
5) Simulation box

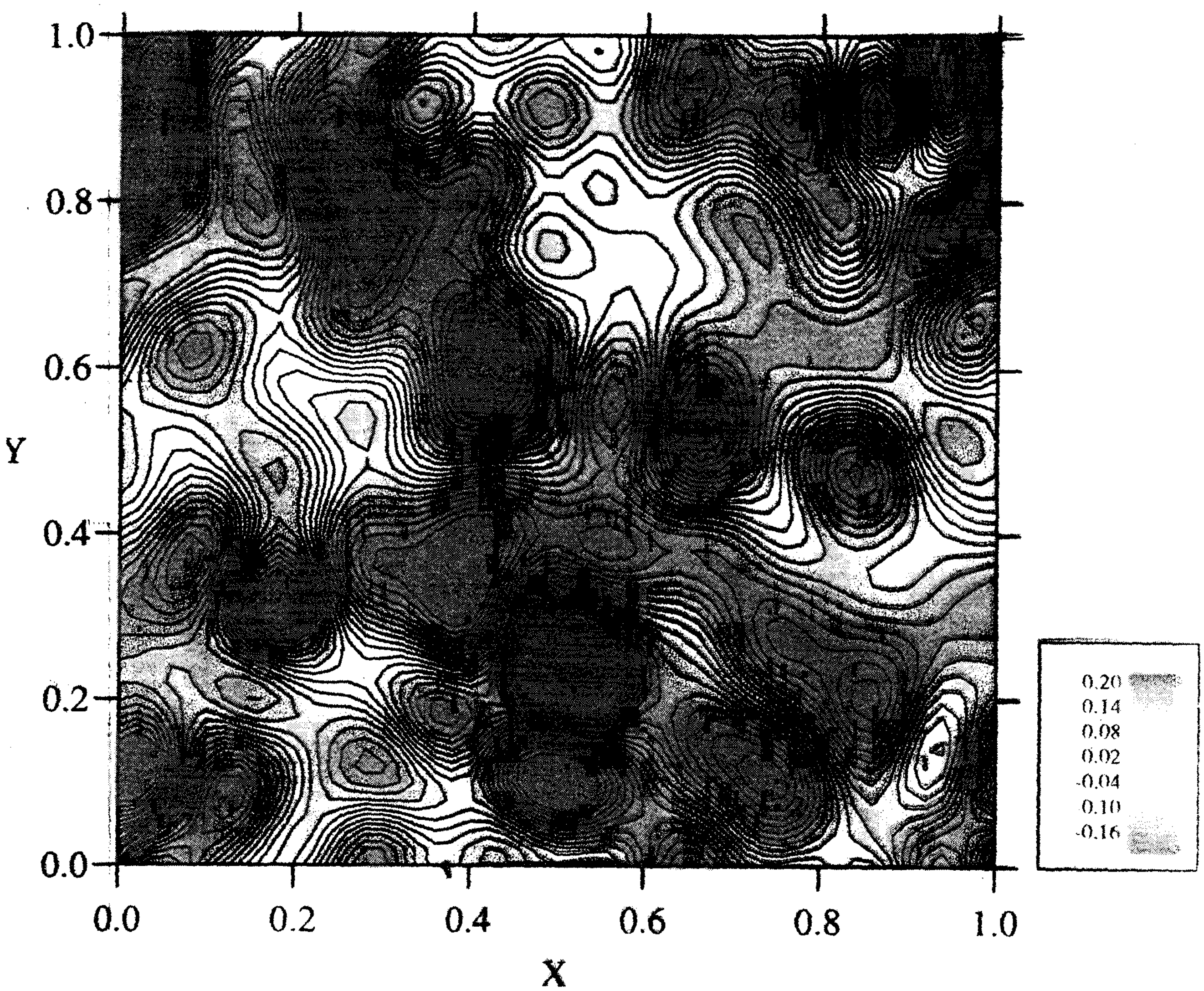




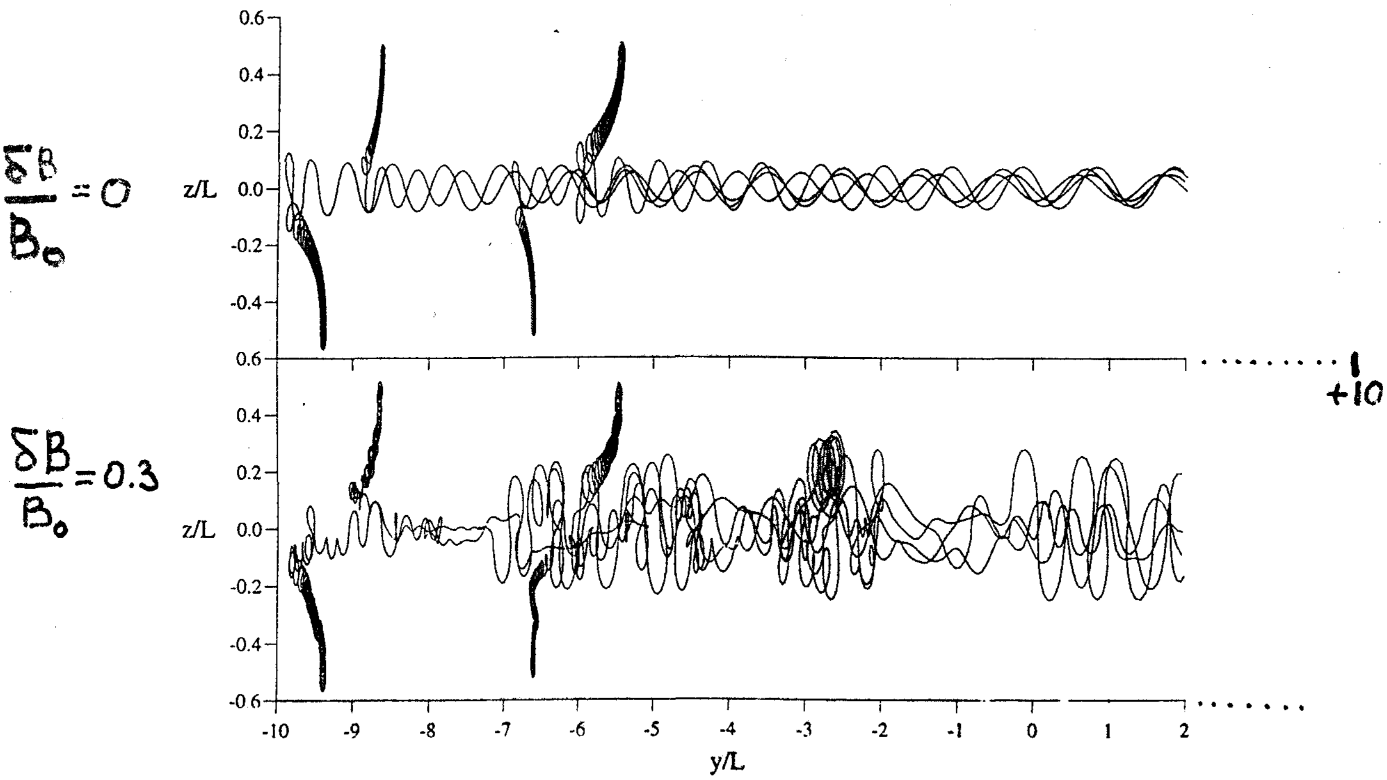
Structure of magnetic field



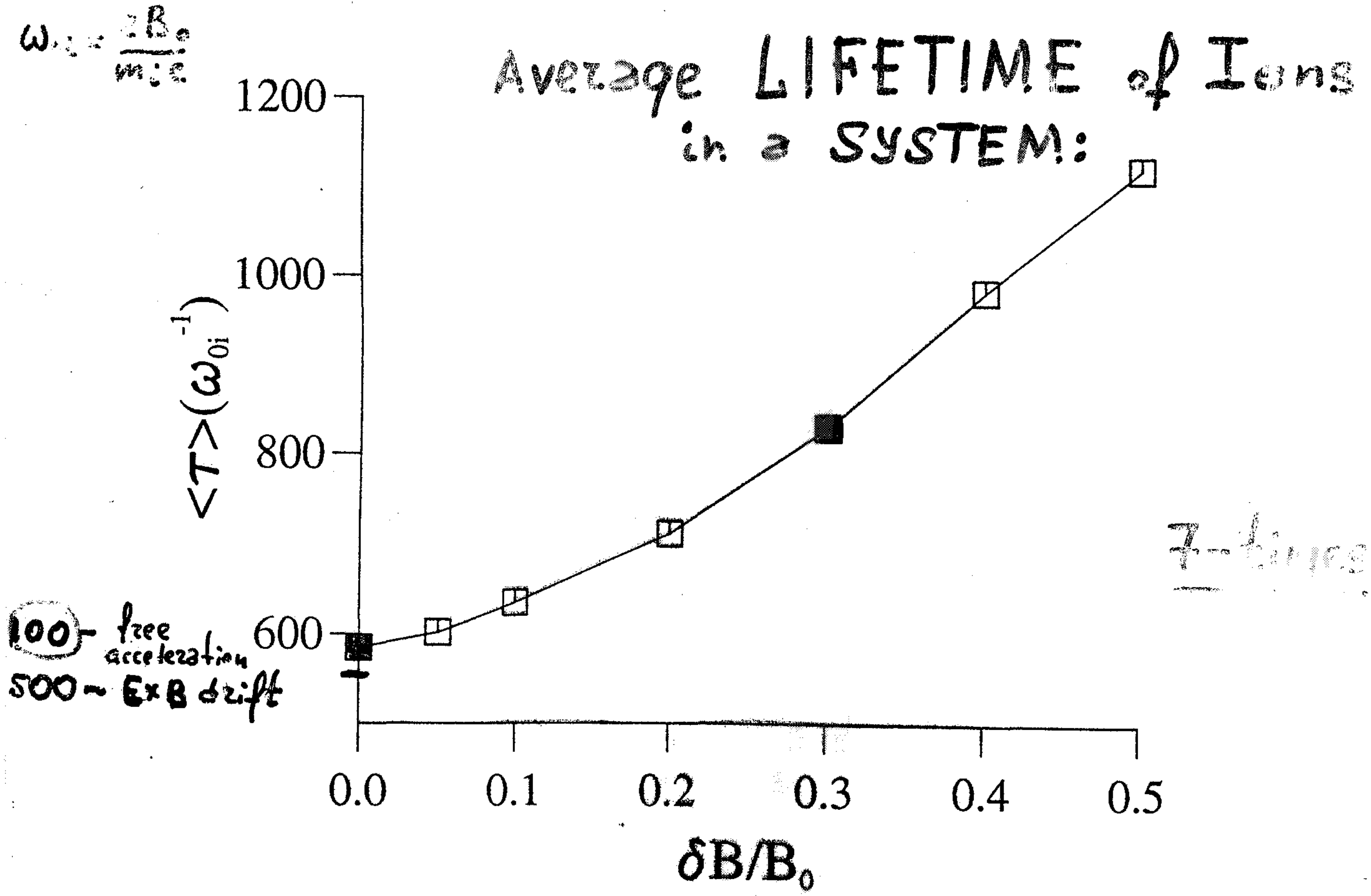



1.0
0.8
0.6
0.4
0.2
0.0
Y
0.0
0.2
0.4
0.6
0.8
1.0
X
0.20
0.14
0.08
0.02
-0.04
0.10
-0.16

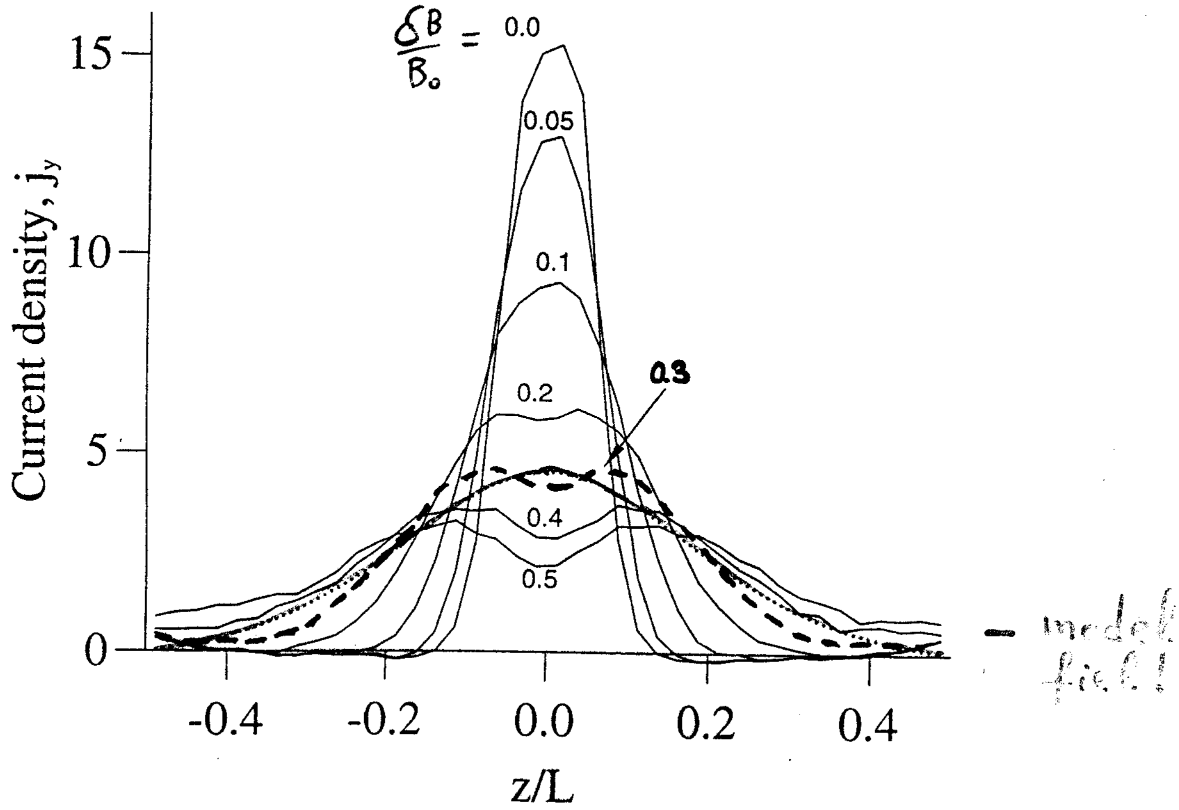
Injection scheme: $2\cdot10^5$ particles randomly distr. at XY-planes ($z=\pm0.5L$). V: Maxwellian

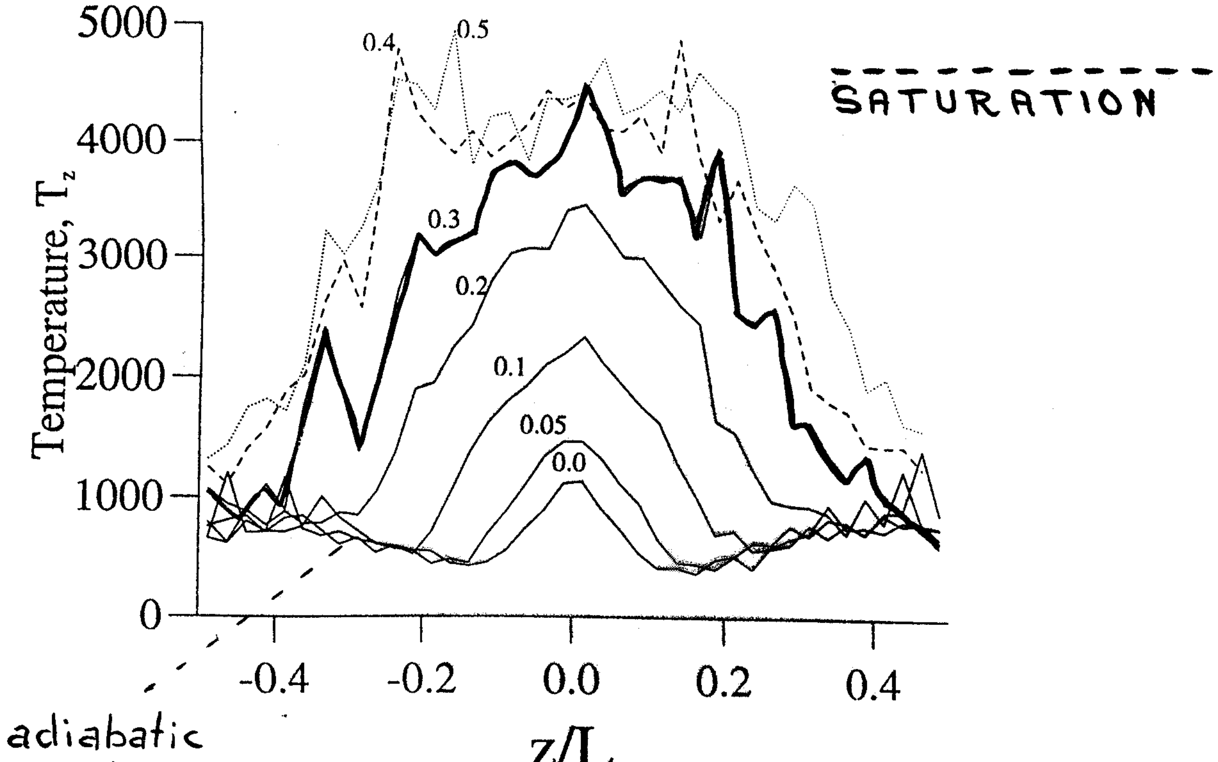


Average LIFETIME of Ions in a SYSTEM:



$I_y \approx$ const





PRESSURE BALANCE AND ENERGY BUDGET

~1D $\qquad P_{zz} + \frac{B_x^2}{8\pi} \approx \text{const} \qquad \left(\frac{\delta B}{B_0} \approx 0.3\right)$

contrary to B_n ≈const case contibution of stresses due to off-diagonal terms is negligible

enthalpy $W=U+P_{yy}$

z=0
y=20L $\qquad Q_y + \left(\frac{1}{2} n\left(T_x + 3T_y + T_z\right) + \frac{nmV_y^2}{2}\right) V_y \approx \frac{\left(qnV_y\right)E_o L_y}{2}$

$\frac{\delta B}{B} \approx 0.3$ heat flux<5%, Enthalpy~90%, directed energy-5-8%

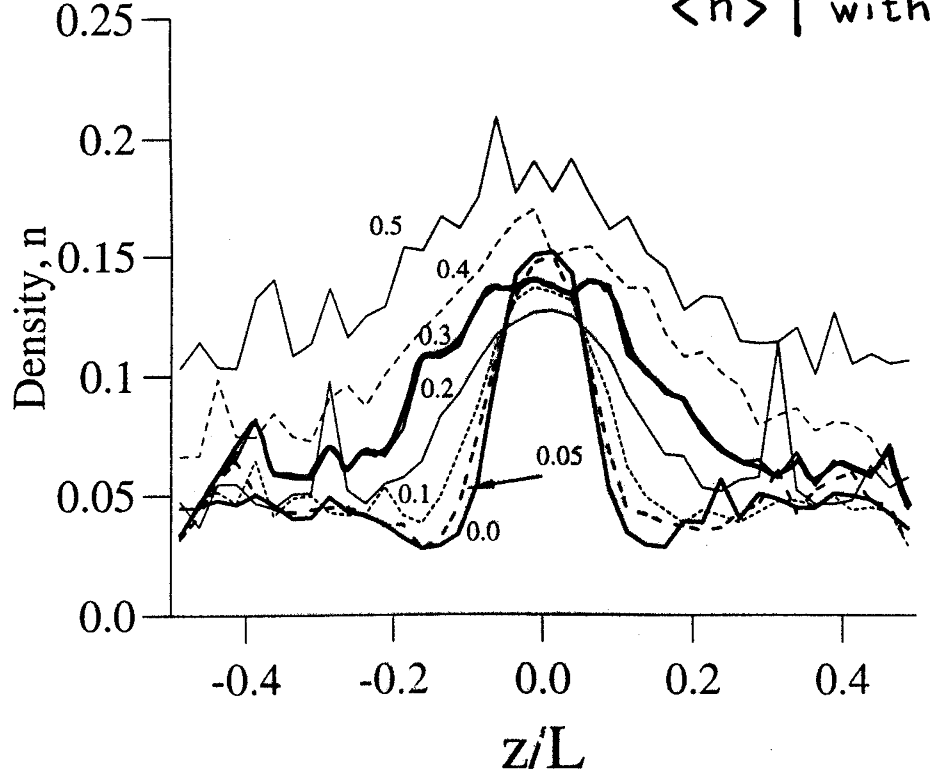
heating is almost isotropic $T_{\parallel} \gtrsim T_{\perp}$

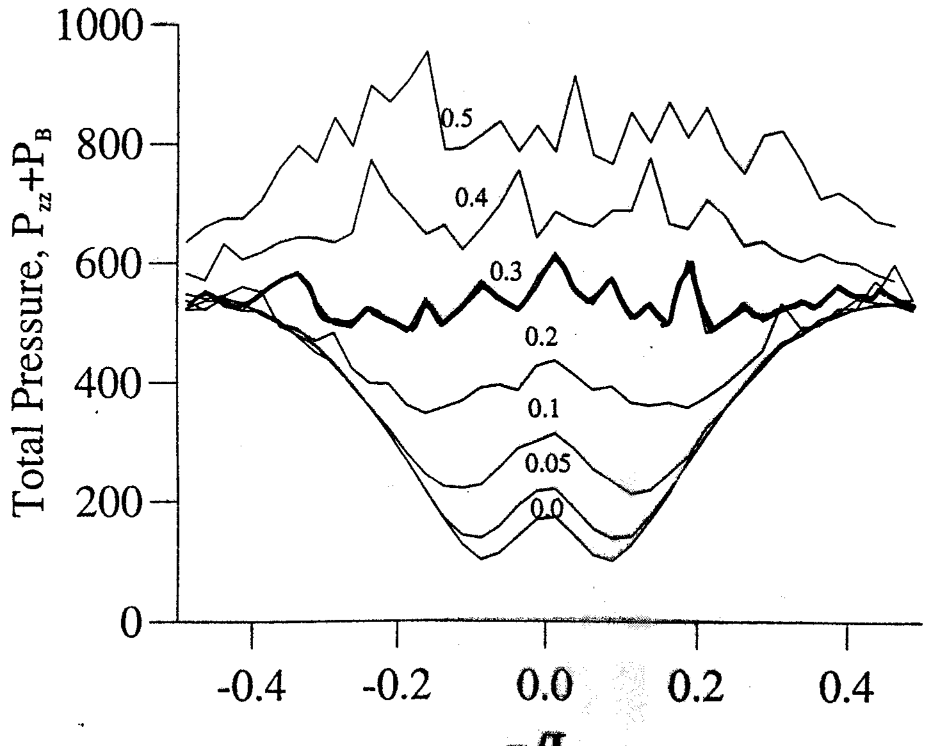
$$T_{max} \approx \frac{1}{5}\left(qE_0 L_y\right) \approx 8000 \frac{mV_E^2}{2} (\approx 5\text{keV})$$

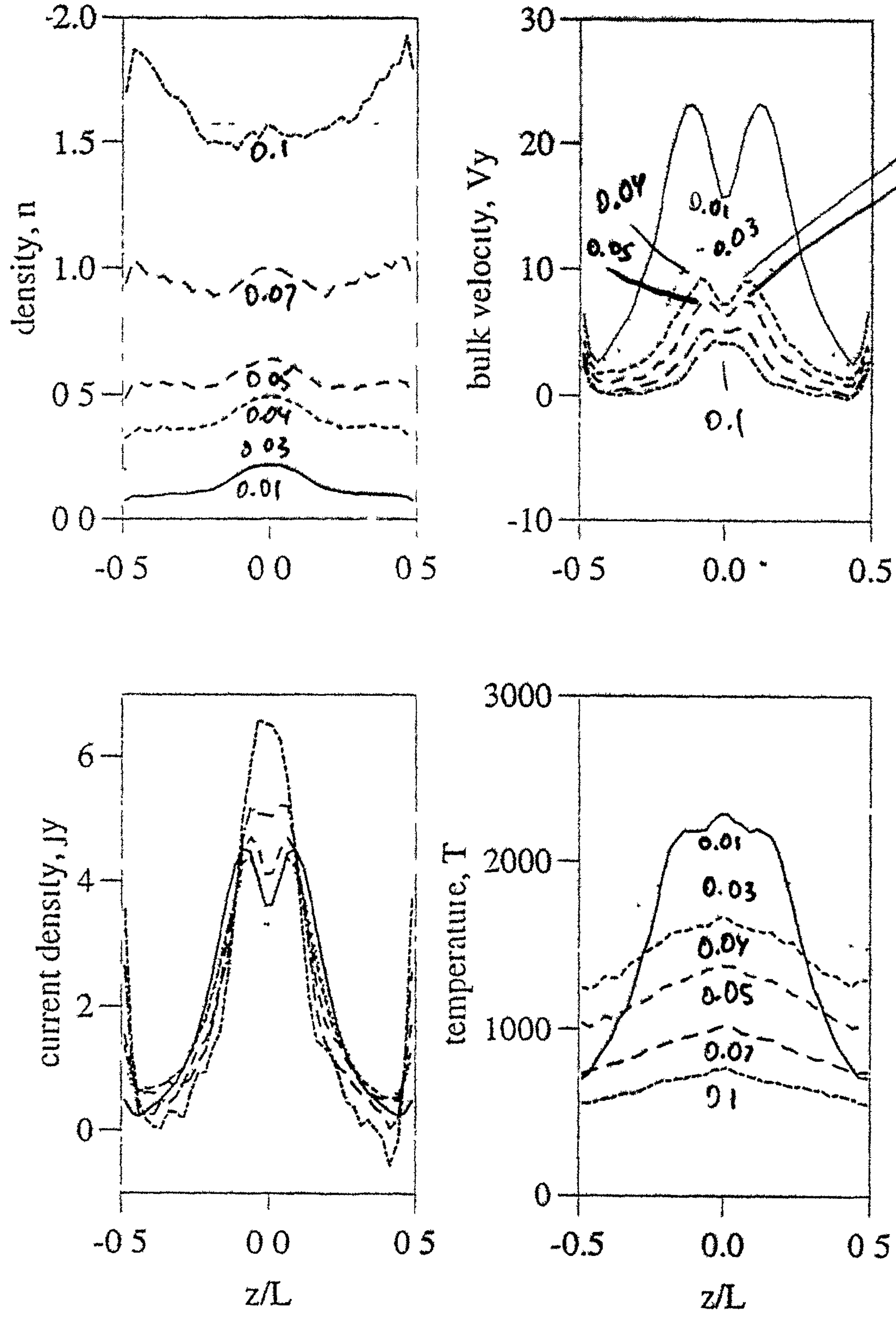
Proper sharing of energy between heating and current maintainance is achieved

only for $\frac{\delta B}{B}$ ~0.3

$\langle n \rangle \uparrow$ with $\frac{\delta B}{B_0} \uparrow$







SPLITTING FOR $\frac{\delta B}{B} = 0.3$

$T_{max} \sim b_z^{-1}$

$\frac{\delta B}{B_0} = 0.3 \quad b_z = 0.01 \ldots 0.1$

CONCLUSIONS - II

The thickness of the current sheet rapidly grows with the increase of the level of fluctuations $\delta B/B_o$.

For $\delta B/B_o \sim 0.3$ enough scattering is produced by δB to inflate the current sheet. Particle current ~ corresponds to the $B_{ox}(z)$.

For $\delta B/B_o \geq 0.3$ double humped current structure is formed (in accordance with Geotail measurments).

Temperature variation across current sheet is very essential $P(z)=n(z)\,\mathbf{T(z)}$.

For $\delta B/B_o \geq 0.2$ main part (90%) of electric energy is converged to the randomization of ion distribution and quasi isotropic heating up to 5-10 keV. Only 5% - 8% of potential drop goes to acceleration and current maintainance.

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