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Numerical Simulation of Dynamics of Tectonic Plates: Spherical Block Model

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Abstract

Modeling earthquakes plays an important role in investigation of different aspects of seismic risk. The present paper is devoted to the problem of numerical simulation of dynamics of a system of global tectonic plates on the sphere. The approach under consideration exploits the block models and assumes that the block structure is a part of spherical layer between two con-centric spheres, one of them (outer sphere) representing Earth's surface. The system of blocks moves as a consequence of prescribed motion of the boundaries and the underlying medium. Displacements of the blocks are determined so that the system is in quasistatic equilibrium state. Block interaction along the faults is viscous-elastic while the ratio of the stress to the pressure is below a certain strength level. When the level is exceeded for a part of some fault a stress-drop (a failure) occurs in accordance with the dry friction model. The failures represent earthquakes. As a result of numerical simulation a synthetic earthquake catalog is produced. Parallel algorithms allowing to perform modeling dynamics of rather large structures are outlined. Some preliminary results of simulation for systems of tectonic plates are presented. In particular, directions of block movement are indicated, character of interaction between blocks is studied, and space distribution of the strongest events is obtained. Some features of synthetic catalogs inherent in real ones are noted.

1 Introduction

The increasing vulnerability of our society is an alarming global tendency. Searching for economic efficiency without paying attention to possible risks often leads to clustering individual properties, production processes, buildings, infrastructure and population in risk prone areas. Essential catastrophic losses due to recent natural and anthropogenic hazards require new risk-based approaches to economic development, including catastrophic risk estimation and management [2, 5]. In determining the vulnerability of a region, one needs to know the design of each structure and infrastructure, for which specific mitigation measures could be utilized, as well as their location in relation to possible hazards. It is also necessary to characterize patterns of possible disasters, their geographical locations and timing.

A lack of reliable empirical data stipulates the necessity of catastrophe modeling. Advances in computer and mathematical modeling make it possible to simulate a variety of different scenarios of catastrophes using data from historical evidence, scientific facts and experts' estimates. Catastrophe modeling incorporates generators of scenarios with appropriate data, enabling to evaluate losses for particular locations, companies or regions.

The present paper continues investigations started in [20] where the main attention was focused on some aspects of risks due to earthquakes. Earthquakes represent local catastrophic natural events of a great destructive force. So far these phenomena are not well understood from a physical standpoint, they are uncontrolled and unpredictable with a sufficient accuracy. Relaible statistical and phenomenological analysis of earthquakes is rather difficult since existing observation data cover only relatively short time intervals (about one hundred years or even less) in comparison with the duration of tectonic processes responsible for the seismic activity, therefore the patterns of the earthquake occurrence identifiable in a real catalog may be only apparent and may not repeat in the future, thus excluding any statistical tests. In this connection, mathematical models of lithosphere dynamics represent important tools for study of the earthquake preparation process. An adequate model should reproduce premonitory patterns determined empirically before large events and can be used to suggest and to investigate new patterns that might exist in real catalogs. In contrast with real catalogs, a synthetic one resulting from numerical simulation may contain information on a seismic flow for a very long time interval. This allows to hope for obtaining more precise estimations of the characteristics of seismic flow [21].

Although there is no adequate theory of the seismo-tectonic process, various properties of the lithosphere, such as spatial heterogeneity, hierarchical block structure, different types of non-linear rheology, gravitational and thermodynamical processes, fluid migration and stress corrosion, are probably relevant to the properties of earthquake sequences. The qualitative stability of these properties in different seismic regions suggests that the lithosphere can be modeled as a large dissipative system that does not essentialy depend on the particular details of the specific processes active in a region. For the detailed review of the most important directions of modeling seismic processes, see [7].

This work deals with modeling lithosphere dynamics by means of block models which exploits the hierarchical block structure of the lithosphere proposed in [1] and treats the seismic process in rather abstract way, in order to reproduce general universal properties of seismicity, first of all, the Gutenberg-Richter frequence of occurence law, migration of events, seismic cycle and so on [11]. The main principles of these models were formulated in [6, 7, 8], detailed description being given, for instance, in [8, 9, 17].

In existing block models, a seismically active region is represented as a system of absolutely rigid blocks forming a layer with a fixed thickness between two horizontal planes. Lateral boundaries of blocks consist of segments of comparatively thin, weak, less consolidated fault zones such as lineaments and tectonic faults intersecting the layer with arbitrary dip angles. In the seismotectonic process major deformation and most earthquakes occur in such fault zones. The system of blocks moves as a consequence of action of outside forces applied to it. The motion may be described by three parameters (so called two-dimensional model) as well as by six ones (three-dimensional model). The system is supposed to be in quasistatic equilibrium state. As the blocks are absolutely rigid, all deformations take place in the fault zones and at the block bottoms. In the model the strains are accumulated in fault zones. This reflects strain accumulation due to deformations of plate boundaries. Of course considerable simplifications are made in the model, but they are necessary to understand the dependence of earthquake flow on main tectonic movements in a region and its lithosphere structure. This assumption is justified by the fact that for the lithosphere the effective elastic moduli in the fault zones are significantly smaller than those within the blocks. The interaction between the blocks and with the underlying medium is viscous-elastic ("normal state") while the ratio of the stress to the pressure is below a certain strength level. When this level is exceeded in some part of a fault plane a stress-drop ("a failure") occurs in accordance with the dry friction model. The failures represent earthquakes. Immediately after the earthquake and for some time, the corresponding parts of the faults are in "creep state". This state differs from the normal one because of the faster growing of inelastic displacements and lasts until the stress falls below a given level. As a result of the numerical modeling a synthetic earthquake catalog may be produced. In problems of risk estimation the procedure outlined above acts as an artificial generator of possible scenarios of catastrophic events [20].

It should be noted that the two-dimensional block model is developed in details. Models approximating dynamics of lithosphere blocks of real seismoactive regions were built on its basis [17, 19]. It was used for studying dependence of properties of seismic flow on geometry of faults and given motions [13]. Three-dimensional model [15, 18] is a generalization of two-dimensional model. In contrast with the latter, which admits displacements of blocks only along the plane between them, it is intended for accounting vertical component of displacements. Three additional degrees of freedom were introduced for this purpose. Therewith, the three-dimensional model being at the moment under development keeps a number of constraints inherent in the two-dimensional one. Particularly, in both models, a block structure is located between two horizontal planes. Besides, while trying to simulate motion of a system of global plates with flat block models, it turned out that significant distortions take place; this evidences advisability of consideration of the block structure on a sphere [16]. It obviously makes sense to introduce the spherical model for modeling motion of a system of namely global tectonic plates, while the impact of sphericity is negligible in case of a separate seismoactive region due to its relative smallness.

In this work, being the continuation of [4], the emphasis is on the spherical modification and numerical algorithms. Some results of simulation of dynamics of different systems of tectonic plates are presented.

2 Brief description of the model

Let us describe basic constructions and ideas of the approach used for creating the spherical modification of the block model.

2.1 Block structure geometry, block movement

A spherical layer of a depth H bounded by two concentric spheres is considered. The outer sphere is treated as the Earth's surface and the inner one is treated as the boundary between the lithosphere and the mantle. A block structure is a limited and simply connected part of this layer. Partition of the structure into blocks is defined by faults intersecting the layer. Each fault is a conic surface characterized by the following two properties. First, the line of the fault on the Earth's surface is an oriented arc of a big circle. And second, the plane tangent to the fault surface in a point of this line has a dip angle α with the Earth's surface. In case of such a definition of a fault, angle α (measured to the left of the fault line) has the same value in all points of the fault on the Earth's surface. Then geometry of a block structure is described by a system of lines of fault intersection with the outer sphere embounding the layer, and by the dip angles. Common points of faults on the outer and inner spheres are called vertices. Fragments of faults limited by corresponding pairs of adjacent vertices are called segments. Intersections of blocks with limiting spheres are spherical rectangles, those on the inner sphere are called bottoms. It is supposed that the block structure may be bordered by boundary blocks which are adjacent to boundary segments. Another possibility is to consider the structure covering the whole Earth's surface (without boundary blocks). This is the principal distinction of the spherical model from the others.

The blocks are assumed to be absolutely rigid. All block displacements are supposed to be infinitely small, compared with block sizes. Therefore, the geometry of the block structure does not change during the simulation, and the structure does not move as a whole. The gravitation forces are not essentially changed because of the blocks displacements and, since the block structure is in quasi-static equilibrium state at the initial time moment, it is natural to assume that the gravity does not cause movements of the blocks.

All vertices on the outer sphere are defined by geographic coordinates (latitude φ, and longitude ψ) in a spherical coordinate system linked to the Earth's center (we call it "System-O"). In spherical modification based on the 3D model, all blocks (including boundary blocks) have six degrees of freedom.

The displacement of each block consists of the translation and the rotation components. The translation component is determined by translation vector (x, y, z). The rotation component is described by means of three special angles γ, β, λ to immovable rectangular coordinate system, (X, Y, Z), with origin at the mass center of the block, point C, which has coordinates (φ_C, ψ_C, R_C). The X axis is directed along the parallel; the Y axis is directed along meridian, the Z axis is directed along the Earth's radius outwards. Denote this system "System-C". Let us assume that the coordinate system with axes X_1, Y_1, Z_1 is strictly connected with the mass center of the block (it coincides under the absence of block displacements with the immovable system with axes X, Y, Z, in which we consider all movements of the block). The scheme of rotation of the block and of corresponding system (X_1, Y_1, Z_1) with respect to system (X, Y, Z) is presented in Fig. 1. The first angle γ is defined as the angle of rotation of axes Y and Z around axis X providing fulfillment of the following condition: if axis Z_2 is the intersection of planes XOZ_1 and YOZ, then axis Z should be mapped into axis Z_2, at that $Y \to Y_2$. The second angle β is defined as the angle of rotation of axes X and Z_2 around axis Y_2 providing transformation of axis Z_2 into axis Z_1 (it is possible since Z_1 belongs to XOZ_2), at that $X \to X_2$. And the third angle λ is defined as the angle of such rotation of axes X_2 and Y_2 around axis Z_1 that $X_2 \to X_1$, $Y_2 \to Y_1$.

According to the definition of the rotation angles, components Δ_x, Δ_y and Δ_z of displacement at a block's point on the sphere with geographic coordinates (φ, ψ) have the following form in System-C:

$$\begin{aligned} \Delta_x &= x - \hat{Y}\lambda + \hat{Z}\beta, \\ \Delta_y &= y + \hat{X}\lambda - \hat{Z}\gamma, \\ \Delta_z &= z - \hat{X}\beta + \hat{Y}\gamma, \end{aligned} \tag{1}$$

where $(x,\ y,\ z)$ is the block's shift; $(\hat{X},\ \hat{Y},\ \hat{Z})$ are coordinates in System-C of the vector, which is directed from the mass center of the block to point $(\varphi,\ \psi)$; angles $(\gamma,\ \beta,\ \varphi)$ are supposed to be small.

Note that in this modification, blocks can leave the spherical surface (as they have six degrees of freedom).

The model uses dimensionless time. When interpreting the results, some realistic value (e.g., 1 year) should be given to one unit of dimensionless time.

2.2 Viscose-elastic interaction between blocks. Quasi-static equilibrium equations

All the values of the components of translation vector and the angles of rotation are found from the condition that the sum of all forces acting on the block and of total moment of these forces have to be zero (at every moment of time the structure is supposed to be in a quasistatic equilibrium state). The interaction of the blocks with the underlying medium takes place on the inner sphere. The movements of the boundaries of the block structure (the boundary blocks) and of the underlying medium are assumed to be an external action on the structure. The rates of these movements are considered to be known. Motion is described as a rotation on the sphere, i.e. position of axis of rotation and angle velocity are given.

Since the depth of the spherical layer is significantly less than block structure dimensions, we consider only points belonging to a fault line on the Earth's surface, while computing numerical characteristics of block interaction. So, it is assumed that all characteristics are described only by coordinates $(\varphi,\ \psi)$ and do not depend on the depth H.

Let us consider a point with coordinates $(\varphi,\ \psi)$ belonging to some fault separating blocks with numbers i and j, block i being leftward, and block j being rightward. Denote $\vec{e}_t$ unit vector tangent to the fault line at this point and directed along the fault. Let it have coordinates $\vec{e}_t = (e_1, e_2, 0)$ in rectangular coordinate system with origin at point $(\varphi,\ \psi)$ and axes introduced analogously to those of system-C (we call this system "system-P"). Let us define vector $\vec{e}_l = (-e_2 \cos\alpha, e_1 \cos\alpha, -\sin\alpha)$, which lies on the plane tangent to the fault's surface at the given point and is perpendicular to vector $\vec{e}_t$ (here α is the dip angle of the fault). Introduce also vector $\vec{e}_n = (-e_2 \sin\alpha, e_1 \sin\alpha, -\cos\alpha)$, which is perpendicular to the mentioned plane. Let right triple $(\vec{e}_t, \vec{e}_l, \vec{e}_n)$ define a rectangular coordinate system with origin at point $(\varphi,\ \psi)$, "system-T". Let $(\Delta_x,\ \Delta_y,\ \Delta_z)$ be the vector of relative displacement of blocks at point $(\varphi,\ \psi)$ in system-P. Components of the displacement on the plane tangent to the fault's surface at this point in system-T are correlated with Δ_x, Δ_y and Δ_z by the following:

$$\Delta_t = \Delta_x e_1 + \Delta_y e_2, \quad \Delta_l = -\Delta_x e_2 \cos\alpha + \Delta_y e_1 \cos\alpha - \Delta_z \sin\alpha,$$

$$\Delta_n = -\Delta_x e_2 \sin\alpha + \Delta_y e_1 \sin\alpha + \Delta_z \cos\alpha.$$

The elastic force per unit area $(f_t,\ f_l,\ f_n)$ applied to the point of the fault is defined by

$$f_t = K_t(\Delta_t - \delta_t), \quad f_l = K_l(\Delta_l - \delta_l), \quad f_n = K_n(\Delta_n - \delta_n). \tag{2}$$

Here, δ_t, δ_l, δ_n are corresponding inelastic displacements, evolution of which is described by the equations

$$\frac{d\delta_t}{dt} = W_t f_t, \quad \frac{d\delta_l}{dt} = W_l f_l, \quad \frac{d\delta_n}{dt} = W_n f_n. \tag{3}$$

The coefficients K_t, K_l, K_n, W_t, W_l, and W_n in (2) and (3) may be different for different faults.

Now, let us calculate components of relative displacement, Δ_x, Δ_y Δ_z, with the use of formulas (1). We obtain

$$\Delta_x = \Delta_x^i - \Delta_x^j, \quad \Delta_y = \Delta_y^i - \Delta_y^j, \quad \Delta_z = \Delta_z^i - \Delta_z^j, \tag{4}$$

where $(\Delta_x^i,\ \Delta_y^i,\ \Delta_z^i)$ and $(\Delta_x^j,\ \Delta_y^j,\ \Delta_z^j)$ are vectors of displacement (in system-P) of point $(\varphi,\ \psi)$ as a point of blocks i and j respectively. Now, in order to obtain components of these vectors, one should multiply the displacements in system-C (defined by (4)) by the matrix of transformation from system-C, corresponding to the block, to system-P. Due to unwieldiness of these computations, they are omitted here. Let us note only that in such a way, one can find displacements both for points on any fault and on the block bottom.

In system-P connected with point $(\varphi,\ \psi)$ of the block bottom, the elastic force per unit area, $(f_x^u,\ f_y^u,\ f_z^u)$, has the form:

$$f_x^u = K_u(\Delta_x^u - \delta_x^u), \quad f_y^u = K_u(\Delta_y^u - \delta_y^u), \quad f_z^u = K_u^n \Delta_z^u, \tag{5}$$

where δ_x^u, δ_y^u are corresponding inelastic displacements, evolution of which is described by the equations:

$$\frac{d\delta_x^u}{dt} = W_u f_x^u, \quad \frac{d\delta_y^u}{dt} = W_u f_y^u. \tag{6}$$

It is assumed that there is no inelastic displacement in vertical direction (along z-axis). The coefficients K_u, K_u^n and W_u in (5) and (6) may be different for different blocks. Vector $(\Delta_x^u, \Delta_y^u, \Delta_z^u)$ of relative displacement of the block and the underlying medium at point $(\varphi,\ \psi)$ considered in system-P is defined by (1) and (4) analogously to the case of finding displacement of a fault point.

As mentioned above, components of translation vectors of the blocks and angles of their rotation around the mass centers of the blocks are found from the condition that the total force and the total moment of forces acting on each block (written in system-C corresponding to the block) are equal to zero. This is the condition of quasistatic equilibrium of the system and at the same time the condition of minimum energy.

It is important that dependence of forces and moments on displacement and rotation of blocks is linear. Therefore, the system of equations for determination of these values is linear. It can be obtained in the following form:

$$A\boldsymbol{w} = \boldsymbol{b}. \tag{7}$$

Here, components of unknown vector $\boldsymbol{w} = (w_1, w_2, \dots, w_{6n})$ are the components of translation vectors of blocks and the angles of their rotation (n is the number of blocks), i. e. $w_{6m-5} = x_m$, $w_{6m-4} = y_m$, $w_{6m-3} = z_m$, $w_{6m-2} = \gamma_m$, $w_{6m-1} = \beta_m$, $w_{6m} = \lambda_m$ ($m = 1, 2, \dots, n$). The elements of matrix A ($6n \times 6n$) and vector $\boldsymbol{b}$ ($6n$) are determined from rather complicated formulas, which are deduced from (1)–(6) with transformation of forces and moments to system-C. For brevity sake, these formulas are omitted in this paper. It should be noted that matrix A does not depend on time and its elements are defined only once, at the beginning of calculations. The components of vector $\boldsymbol{b}$ depend on time, explicitly, because of the movements of the underlying medium and of the block structure boundaries and, implicitly, because of the inelastic displacements.

2.3 Discretization

In computational purposes, time discretization is performed by introducing a time step Δt. The state of the block structure under consideration is determined at discrete time moments $t_i = t_0 + i\Delta t$ ($i = 1, 2, \dots$), where t_0 is the initial time. The transformation from the state at t_i to the state at t_{i+1} is made as follows: (a) new values of inelastic displacements δ_x^u, δ_y^u, δ_t, δ_l, δ_n are calculated from equations (3) and (6); (b) translation vectors and the rotation angles at t_{i+1} are calculated for the boundary blocks and the underlying medium; (c) components of $\boldsymbol{b}$ in system (7) are found, and this system is used to determine the translation vectors and the rotation angles for the blocks.

For calculation of various curvilinear integrals, one should discretize (split to cells) spherical surfaces of block bottoms and fault segment arcs. Therewith, values of forces and inelastic displacements are supposed to be equal in all points of a cell. Note that according to the assumption, segments are not subject to discretization by depth (which is negligible small, compared with block and segment sizes); we assume that in calculations for faults, one can use characteristics of cells belonging to fault lines on the Earth's surface.

2.4 Earthquake and creep

At every time t_i, we calculate the value of the quantity κ by the following formula

$$\kappa = \frac{\sqrt{f_t^2 + f_l^2}}{P - f_n}, \tag{8}$$

where P is the parameter, which may be interpreted as the difference between the lithostatic and the hydrostatic pressure (P has the same value for all faults).

For each fault the three levels of κ are fixed

$$B > H_f \geq H_s.$$

It is assumed that the initial conditions for numerical simulation of block structure dynamics satisfy the inequality $\kappa < B$ for all cells of the fault segments. If, at some time t_i, the value of κ in some cell of a fault segment reaches the level B, a failure ("earthquake") occurs. By failure we mean slippage during which the inelastic displacements δ_t, δ_l, δ_n in the cell change abruptly to reduce the value of κ to the level H_f. Note that this procedure for 3D models essentially differs from the analogous procedure for 2D model. The new values of the inelastic displacements in the cell are calculated from

$$\delta_t^e = \delta_t + \gamma^e \xi_t f_t, \quad \delta_l^e = \delta_l + \gamma^e f_l, \quad \delta_n^e = \delta_n + \gamma^e \xi_n f_n, \tag{9}$$

where δ_t, δ_l, δ_n, f_t, f_l, f_n are the inelastic displacements and the components of elastic force vector per unit area just before the failure. The coefficients $\xi_t = K_l/K_t$ ($\xi_t = 0$ if $K_t = 0$) and $\xi_n = K_l/K_n$ ($\xi_n = 0$ if $K_n = 0$) account for inhomogeniety of displacements along the fault plane (in different directions) and normal to it (they reflect the assumption that the same value of the elastic force per unit area results in different values of rates of changing different inelastic displacements). The coefficient γ^e is given by

$$\gamma^e = \frac{\sqrt{f_t^2 + f_l^2} - H_f(P - f_n)}{K_l\sqrt{f_t^2 + f_l^2} + K_n H_f \xi_n f_n}. \tag{10}$$

It follows from (2), (8)–(10) that after calculation of the new values of the inelastic displacements and the elastic forces the value of κ in the cell is equal to H_f. Here, the following facts should be noted. After the calculation according to (2), (9), the signs of the elastic forces should be the same as just before the failure. Therefore, the case when $(1 - K_n\xi_n\gamma_e) < 0$ (and the sign of f_n changes) s to be considered in its own right as well as the case when $(1 - K_l\gamma_e) < 0$ (and the signs of f_l and f_t change). It may be proved that the second situation is possible only if $f_n < 0$. In the both cases we assume

$$\delta_n^e = \Delta_n, \quad \gamma^e = \frac{\sqrt{f_t^2 + f_l^2} - H_f P}{K_l\sqrt{f_t^2 + f_l^2}}.$$

After calculations described above for all the failed cells, the new components of vector $\boldsymbol{b}$ are computed, and from the system of equations (7) the translation vectors and the angles of rotation for the blocks are found. If for some cell(s) of the fault segments, $\kappa > B$, the whole procedure is repeated. When for all cells of faults it becomes $\kappa < B$, calculation is continued by usual scheme.

Different times could be attributed to the failures occuring on different steps of the procedure: if the procedure consists of p steps, the time $t_i + (j-1)\delta t$ can be attributed to the failures occuring at jth step, and the value of δt should be selected to satisfy the condition $p\delta t < \Delta t$.

The cells of the same fault plane, in which failure occurs at the same time, form a single earthquake. The parameters of the earthquake are defined as follows: (a) the origin time is $t_i + (j-1)\delta t$; (b) the epicentral coordinates and the source depth are the weighted sums of the coordinates and depths of the cells involved in the earthquake (the weight of each cell is given by its area divided by the sum of areas of all cells involved in the earthquake); (c) the magnitude is calculated by the formula proposed in [22]:

$$M = 0.98 \log_{10} S + 3.93, \tag{11}$$

where S is the sum of areas of cells included in the earthquake measured in km^2. The use of this formula seems to be reasonable due to the following speculations. The magnitude of earthquakes can be defined by using the difference between the energy of the system before and after an earthquake, which can be treated as the strain energy E released through an earthquake. According to [13] in the block models there is the linear dependence between E and S, that can be explained by the fact that the energy released through an earthquake depends mainly on the total area of the fault plane involved in the event. Depth of earthquake in the considered modification is not defined.

Immediately after the earthquake, it is assumed that the failured cells are in the creep state. It means that, for these cells, in equations (3), which describe the evolution of inelastic displacements, the parameters W_t^s ($W_t^s > W_t$), W_l^s ($W_l^s > W_l$), and W_n^s ($W_n^s > W_n$) are used instead of W_t, W_l, and W_n. They may be different for different faults. The failured cells are in the creep state as long as $\kappa > H_s$, while when $\kappa \leq H_s$, the cells return to the normal state and hereinafter W_t, W_l, and W_n are used in (3).

Thus, a synthetic earthquake catalog is produced as a result of numerical simulation.

3 Parallel algorithm for numerical simulation

Computational experiments [14] show that the block models of lithosphere dynamics (especially 3D modifications) are quite time and memory consuming on sequential computers that does not allow to simulate dynamics of complicated structures with large number of blocks and small enough step of space discretization. In addition, considering a structure on a sphere significantly complexifies computations.

However, the approach applied to modeling admits sufficiently effective parallelization of calculations on a multiprocessor machine, and namely this fact makes real passing to a system of tectonic plates in the global scale (with the use of real geophysical and seismic data) and to the spherical geometry. The principal features of the parallel software created are the following: 1) multiprocessor machines are applied to calculations; 2) personal computers are used to prepare input data and to visualize output data [3].

The variant of parallel program for the spherical block model was realized by the scheme "master-worker" ("processor farm") on working stations basing on microprocessors Alpha-21164 (533MHz, 256Mb) at the Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences (Ekaterinburg, Russia). For compatibility of the program code with different platforms (in the sense of fast transition, ideally, by means of simple recompiling), the special library MPI ("Message Passing Interface") was used, and the parallel algorithm was designed in such a way that the unique loading module was formed for all processors. The block-scheme of this algorithm is presented in Fig. 2–4. Let us give necessary explanations.

In the beginning of the work the number of processor, which the program has loaded to, is detected (zero processor becomes the master). Then the information on a block structure is loaded, and auxiliary calculations (space discretization, calculation of matrix A) are performed. It is important that a part of calculations performed only by the master requires less time expenditures. At every time step the most time-consumable procedure is calculation of values of forces and inelastic displacements in all cells of space discretization of block bottoms and fault segments. Since these calculations may be performed independently from each other, they are shared between all processors analyzing their own portions of cells. The exchange of information between processors at every time step is realized according to the following scheme (see Fig. 3). The master calculates new values of block, boundary block (if necessary) and underlying medium displacements, then necessary parameters are transferred to the workers. Recalculated values of the right-hand part of system (7) are returned to the master, then the next time step is carried out. For processing the situation treated as an earthquake, the scheme is slightly complicated, since in this case the master should ask all the workers until cells of segments in the critical state exist. The time of calculations on each processor is much more than the time of exchange. Therefore rather high useful loading of each processor is achieved.

For testing the dependence of time of solving the problem on the number of processors and comparing with sequential algorithm, the following values were analyzed: acceleration coefficient $S_r = T_1/T_r$ and effectiveness coefficient $E_r = S_r/r$, where T_r is the time of program performance on multiprocessor computer with r processors, T_1 is the corresponding time for sequential algorithm. Note that T_r is the sum of pure time of calculations and expenditures for necessary exchanges. It is appeared that S_r is slightly less than r, consequently, E_r is close to 1, and the parallelization effectiveness is rather high and it insignificantly decreases with increasing the number of processors in action (in correspondence with the parallelization scheme).

The scheme described in this section was applied to simulation of dynamics of different block structures: both model and approximations of real regions. Service procedures [3] give to a user possibilities of specification of a block structure by graphic or numeric way, visualization of obtained sequence of earthquakes, creation and processing of synthetic catalogs of earthquakes in standard 20 byte format etc. In the next section we present some results of modeling obtained by means of parallel program.

4 Some numerical results

Taking into account that spherical geometry is reasonable to introduce for studying dynamics of a system of global plates, one can define the following goals of modeling:
- creation of a global image of instant cinematics of the largest tectonic plates in the known system of "hot spots" [10];
- modeling of subduction and spreading belts, study of character of interaction between plates at their boundaries;
- analysis of vertical component of plate motions;
- estimation of spatial distribution of epicenters of strong earthquakes in the world scale;
- simulation of spatial and time migration of earthquakes;
- ascertainment of mechanisms of plate motion (for instance, plate's abilities to transmit stress through long distances or necessity of additional sources).

It should be noted that the tasks listed above are formulated "as a prospect".

4.1 South American seismic region

At the first stage, modeling of a rather small subsystem of plates was begun. The structure includes South America, Caribbean, Cocos, and Nazca plates (Fig. 5). Other, surrounding, plates (North America, Africa, Antarctica, and Pacific) are treated as boundary blocks moving by known laws [10]. This region is chosen because it includes various types of plate boundaries with quite contrast motions and high seismic activity. The structure under consideration has 4 blocks, 33 vertices, 36 faults (and segments), and 4 boundary blocks. Dip angles of faults at boundary South America/Nazca equal 50°, other faults have dip angles of 90°.

Discretization was defined by the following values of steps: by time— 0.01, by space— 3 km. for segments and 1/3° for block bottoms. The largest block bottom was split into 40 000 cells. The following model values for coefficients in formulas (2)–(6) were used in the basic variant: for all faults— $K_t = K_l = K_n = 0.01$, $W_t = W_l = W_n = 0.01$, for all blocks— $K_u = 10$, $K_u^n = 20$, $W_u = 0.1$. The parameters of movement of the underlying medium and boundary blocks were taken from [10].

As results of computation, the program returns quantitative characteristics of block displacements, which may be treated as velocities (in cm/yr.), and relative displacements of points belonging to fault segments separating blocks (these displacements give notion on qualitative character of interaction between tectonic plates). Obtained data were compared with real ones, and behavior of boundary points showed that model zones of subduction and spreading correspond to observed ones (Fig. 5). This may be treated as a promising result. However, it seems to be early to discuss any quantitative characteristics of such processes.

A synthetic catalog of earthquakes was also obtained as a result of the experiment. The following its characteristics were studied: frequency-magnitude plots, spatial distribution of epicenters, clustering phenomenon, and some other features.

The catalog covers a period of 200 units of dimensionless time and contains 418786 events with magnitude of 6.1 through 8.9, calculated by formula (11). It follows that the magnitude values obtained exceed real ones.

Since modeling earthquakes in the system of tectonic plates is preliminary, we give only some features inherent in the synthetic catalog without carrying out its analysis in detail. Clustering (grouping) of events may be seen both for separate segments and for the whole structure (Fig. 6), and main shocks may be indicated in the groups. The pattern of seismicity repeats qualitatively in a certain interval of dimensionless time (which depends on the fault), periods of post-seismic relaxation and stress accumulation are also seen [11]. One can observe the phenomenon of the earthquake migration along faults (we mean the temporary sequence). Spatial distribution of events shows that, although the model earthquakes occur at nearly all segments of the structure, there are some faults where the first model events happen and a significant part of all synthetic seismicity is concentrated (these spots are marked on Fig. 5). These faults correspond to main seismoactive zones (Nazca/South America and Nazca/Pacific boundaries). Frequency-magnitude dependence plots for the synthetic and real subcatalogs are shown on Fig. 7. Note that the curve built for the real data has a bit smaller inclination than the curve built for the observed data.

An experiment was performed in order to answer the question: which external stress source (motion of which boundary blocks) has the strongest influence on synthetic seismicity occurring in the considered subsystem of plates? The following three variants were analyzed: (1) African plate is motionless; (2) both Africa and Pacific have non-zero velocities (taken from [10]); (3) Pacific is motionless. Three corresponding synthetic catalogs obtained for a time period of 20 units of dimensionless time were compared. It is found that under motionless Pacific plate, seismic activity of the structure is lower than under motionless

Table 1: Levels of activity of seismic boundaries of the plate system under studying

Plate boundary	Boundary length, km	Number of events			Density of seismic moment		
		N_1	N_2	N_3	D_1	D_2	D_3
sam-ant	6267	403	481	483	316	340	329
afr-sam	12827	1320	1288	1294	1174	851	841
car-nam	3818	462	457	443	323	335	319
car-coc	1750	1768	1794	1805	28	27	28
car-sam	2975	145	144	138	505	556	483
car-naz	850	483	490	523	2990	2772	1451
coc-pac	2821	2526	2537	1411	199	197	13
coc-naz	2700	1427	1423	1470	521	514	457
naz-sam	6083	3042	3160	3173	15023	13125	12024
naz-pac	7627	1975	1952	1326	9602	9476	5768

Africa plate, influence of which being relatively small. In case (1), number of events is 13567, magnitude varying from 6.3 up to 9.06; in case (2), number of events is 13744, magnitude varying from 6.3 up to 8.9; in case (3), number of events is 12084, magnitude varying from 6.3 up to 9.06.

Table 1 reflects levels of activity of different seismic boundaries of the plate system under study. The following notations of the plates are used: *sam*, South America; *ant*, Antarctica; *afr*, Africa; *car*, Caribbean; *nam*, North America; *coc*, Cocos; *pac*, Pacific. Length of a boundary is measured in kms., (N_1, N_2, N_3) and (D_1, D_2, D_3) are number of events and seismic moment per unit length of boundary (measured in 10^{10} N) for the three variants of motion of boundary blocks respectively. Total seismic moment for a boundary is the sum of moments of all earthquakes occurred on the boundary. The following formula was used for calculations [12]:

$$\log_{10} M_0 = 1.5M + 9.14,$$

where M_0 is the seismic moment of the earthquake; M is the magnitude.

The characteristic under consideration (seismic moment density) is maximum (up to 15.023×10^{13} N) on the boundary South America/Nazca (in reality: an active subduction zone). Boundaries Pacific/Nazca and Caribbean/Nazca (spreading zones) are of a smaller, but quite significant, values of the ratio: up to 9.602×10^{13} N and 2.99×10^{13} N respectively. Among all cases, maximum density of seismic energy per unit length of boundary is observed under motionless Africa plate. It is slightly less under motion of both plates and minimal under motionless Pacific plate. Let us try to explain this fact on the example of considering boundary South America/Nazca by means of the following qualitative speculations.

1) Due to sphericity of the structure (see Fig. 8), motion of Africa plate causes occurrence of force $\vec{F}_A$, which is nearly parallel to the section of the fault on boundary Nazca/South America. This force acts upon South America plate and has such components on n- and l-axes of boundary Nazca/South America (System-T) that decrease the subduction motion and increase compression on the fault, i. e. it decreases the value of κ (8).

2) Motion of Pacific plate causes occurrence of force $\vec{F}_P$, which is nearly perpendicular to the section of the fault on boundary Nazca/South America. Hence, this force has almost no impact on the subduction motion, but it increases extension on the fault along n-axis, i. e. increases value of κ (8).

Therefore, seismic activity is higher in the second variant. Let us emphasize that, first, abovementioned explanations may be true only thanks to spherical shape of the structure and, second, strict substantiation of this fact requires additional calculations and argument. It seems that investigations in this direction are perspective in connection with modeling the movement of plates. Fast improvement of regional and global geodynamical models based on modern technologies makes this problem more actual. It should be noted that the motions are more often considered not only in relative but also in absolute coordinate systems that allows to pass from analysis of plate cinematics to studying driving forces.

4.2 Global system of tectonic plates

Numerical modeling of dynamics of the global system of tectonic plates covering almost all Earth's surface was started from the variant where the largest blocks were treated as boundary ones. Namely, the structure contains the following plates as internal blocks: South America, Nazca, Cocos, Carribean, Africa, Arabia, Somaly, India, Phillipines, Australia, and the following ones as boundary: North America (1), Euroasia (2), Antarctica (3), Pacific (4). The structure has 10 blocks, 141 vertices, 150 faults (and segments), and 4 boundary blocks. Dip angles of faults at boundaries with clearly observed subduction (for example, South America/Nazca) equal 50°, other faults have dip angles of 90°.

Discretization was defined by the following values of steps: by time— 0.01, by space— 3 km. for segments and 2/3° for block bottoms. The largest block's bottom was split into 25 000 cells, the longest segment— into 1000 cells. As for the subsystem of plates considered in the previous section, the following values for coefficients in formulas (2)–(6) were used in the basic variant: for all faults— $K_t = K_l = K_n = 0.01$, $W_t = W_l = W_n = 0.01$, for all blocks— $K_u = 10$, $K_u^n = 20$, $W_u = 0.1$. The parameters of movement of the underlying medium and boundary blocks were taken from [10].

Several variants were computed for a time period of 20 units of dimensionless time (to obtain more quickly model events, we essentially decrease all thresholds for stress at faults). The qualitative information on interaction of plates and on the most active seismic boundaries was obtained. By means of relative displacements of boundary points (for example, at such characteristic places as boundaries South America/Nazca, Pacific/Nazca, South America/Africa, India/Euroasia, around Philippines and so on) the qualitative character of interaction between tectonic plates along plate boundaries was established. In addition, spatial distribution of the strongest model events was obtained. All this information is presented in Fig. 9, where the divergent (spreading), convergent (subduction) and transform (sliding) plate boundaries are marked. The principal similarity in location of the zones in question was discovered when comparing the model and real characteristics. The analysis of space distribution of epicenters of model events brought to light the most active seismic boundaries, including South America/Nazca, Pacific/Nazca, India/Euroasia, south-east, east, north-east and especially north part of Australia and Philippines. Activity is extremely low at the boundaries (among others): India/Australia, east of Africa.

Dependencies of structure's dynamics and distribution of model events on the character of boundary blocks movements and numerical values of parameters were analyzed. The following variants were considered: (0) all boundary blocks have non-zero velocities; (1)-(4) corresponding boundary block is motionless.

We obtained that (approximately) the largest activity is in case (2), the smallest— in case (4), the same— in other cases.

Then we start to apply the model to a closed structure of tectonic plates on a sphere (without extraction of boundary blocks). It should be noted that the possibility of considering the structure without boundary blocks is the peculiarity of the spherical model (in comparison with plane modifications). This variant is much more processor time and memory consuming than previous ones, and so far there were no attempts to simulate dynamics of similar block structures.

The closed system includes all plates mentioned above as internal and boundary plates and, in addition, plate Juan de Fuca. Thus, there are 15 blocks, 186 vertices and 199 faults (segments) in the structure. Discretization was defined by the following values of steps: by time— 0.01, by space— 9 km. for segments and 1° for block bottoms. The largest block bottom was split into 90 000 cells, the longest segment— into 300 cells.

The first numerical experiments with such system, on the one hand, showed the identity of simulation results for the same region (see Fig.10) treated as a part of different structures (with boundary blocks and without them) and, on the other hand, revealed some new properties need to be extra analyzed (for example, the difference between vertical components of block movements). But it is the subject of future investigations.

5 Conclusion

Some preliminary results of modeling dynamics of systems of large-scale blocks with spherical geometry are presented. Qualitative characteristics of plate motion and of character of their interaction are obtained. Synthetic catalogs, which have some "real" features, were created. The spherical modification of block model shows some phenomena, which may occur only due to sphericity of a structure. This allows to hope to discover new factors causing seismic activity of regions. Results of numerical modeling are supposed to be used during generation of possible seismicity scenarios for seismic regions.

6 Acknowledgments

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Figures

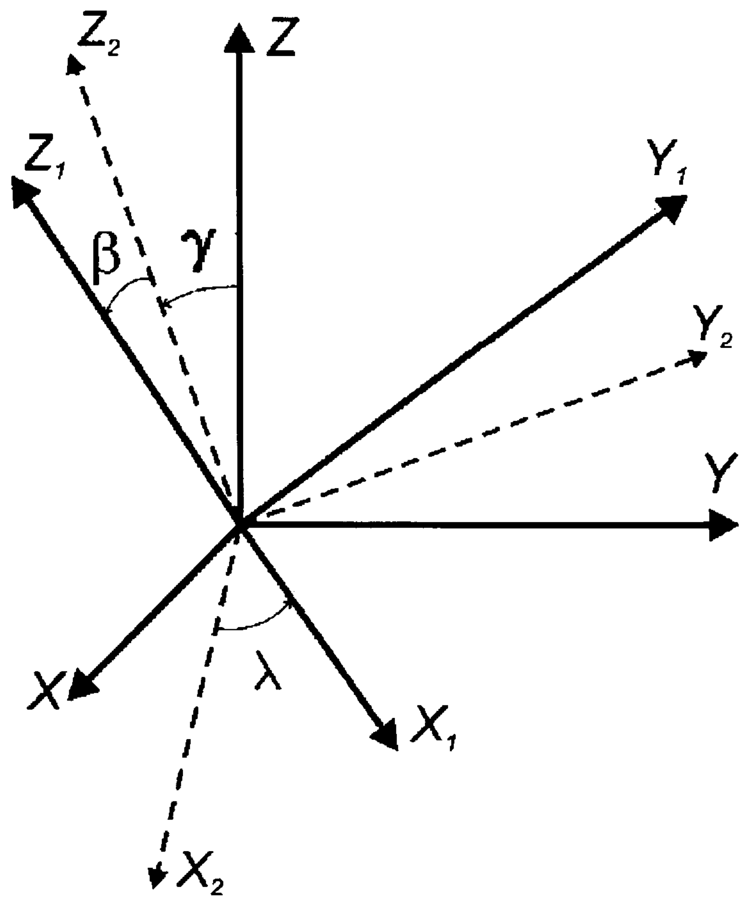

Fig. 1. Definition of rotation angles γ, β, and λ

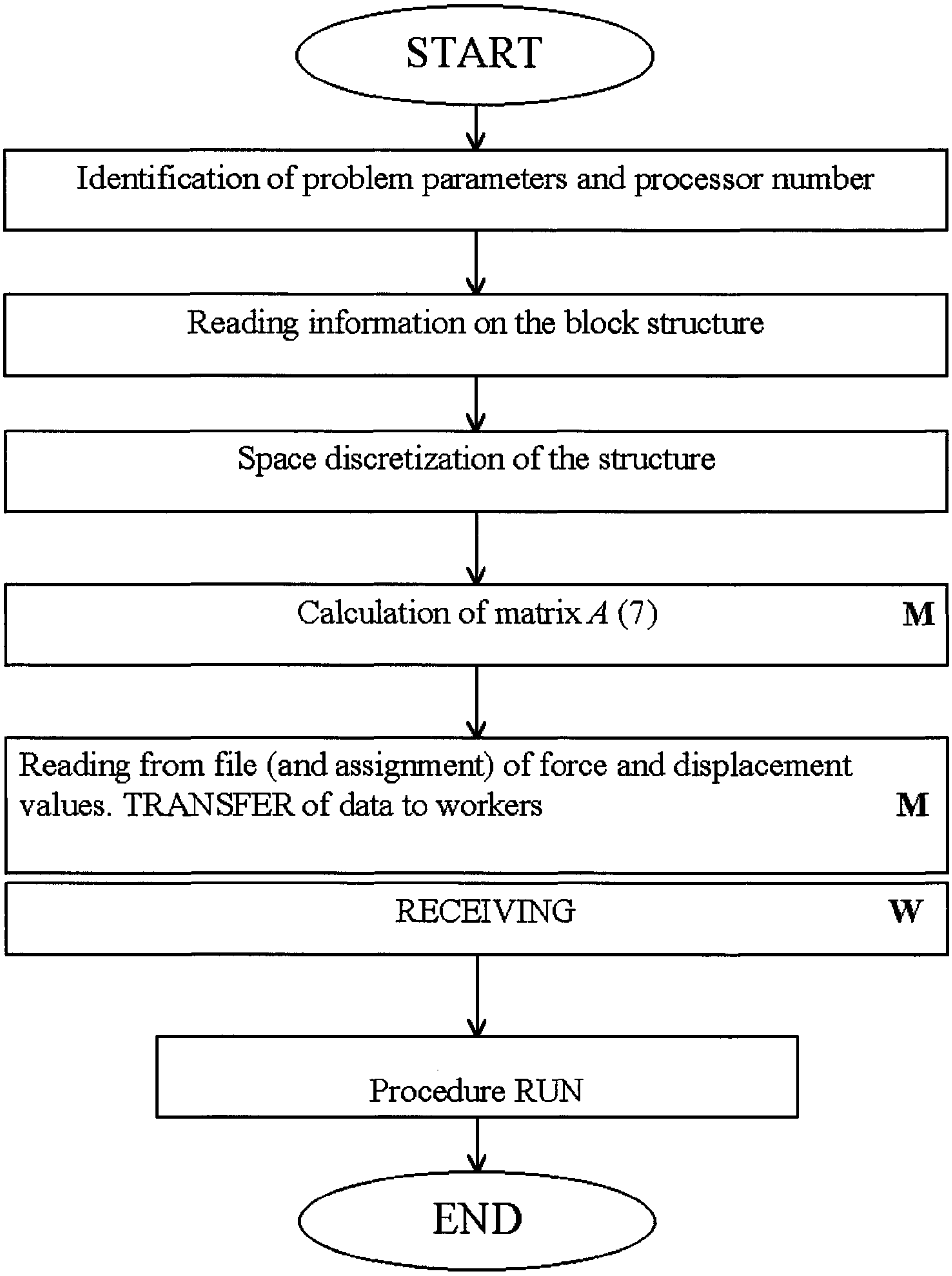

Fig. 2. Scheme of parallelization of the block model. Notation: operations carried out only by master are marked by "M", only by workers— by "W".

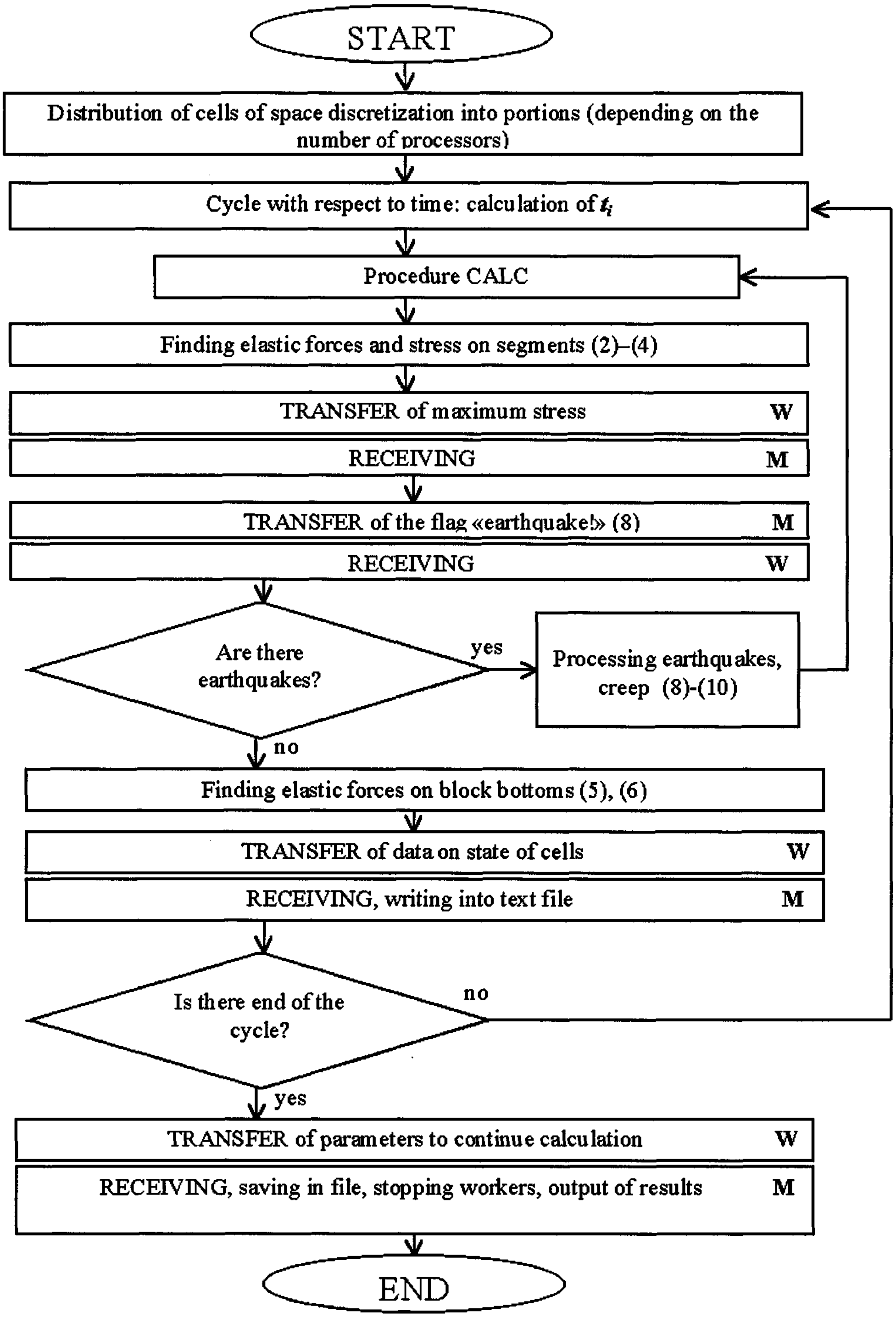

Fig. 3. Procedure RUN

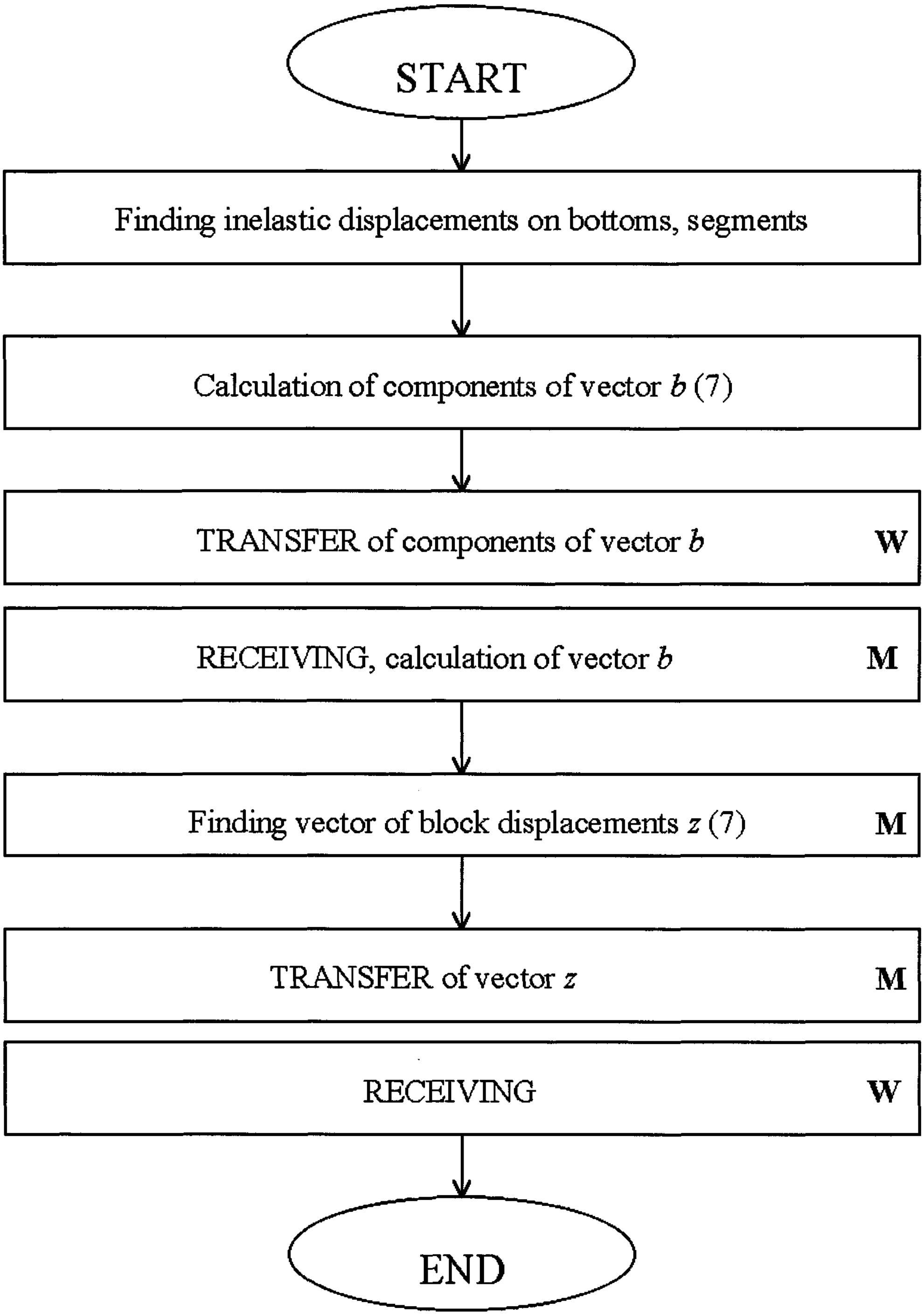

Fig. 4. Procedure CALC

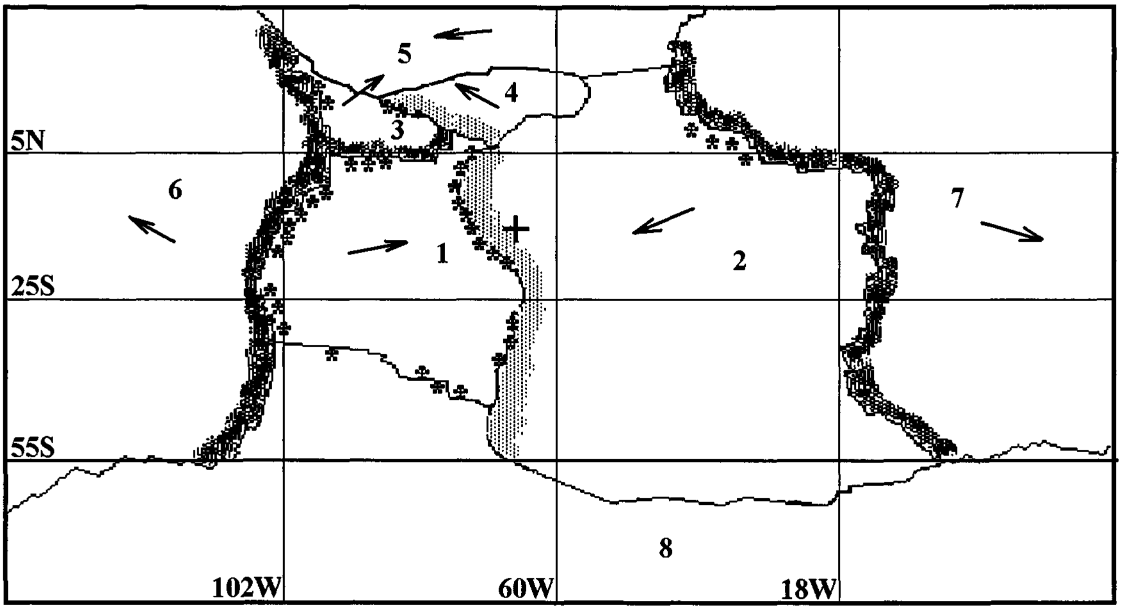

Fig. 5. Results of simulation of plate motion and spatial distribution of strong earthquakes: the directions of model plate motion (arrows), subduction zones (light shading), spreading zones (dark shading), epicenters of model events (asterisks). Numbers stand for the plates: 1 — Nazca, 2 — South American, 3 — Cocos, 4 — Carribean, 5 — North American, 6 — Pacific, 7 — Africa, 8 — Antarctica. Symbol "+" marks the segment with evidently detecting clustering of model events.

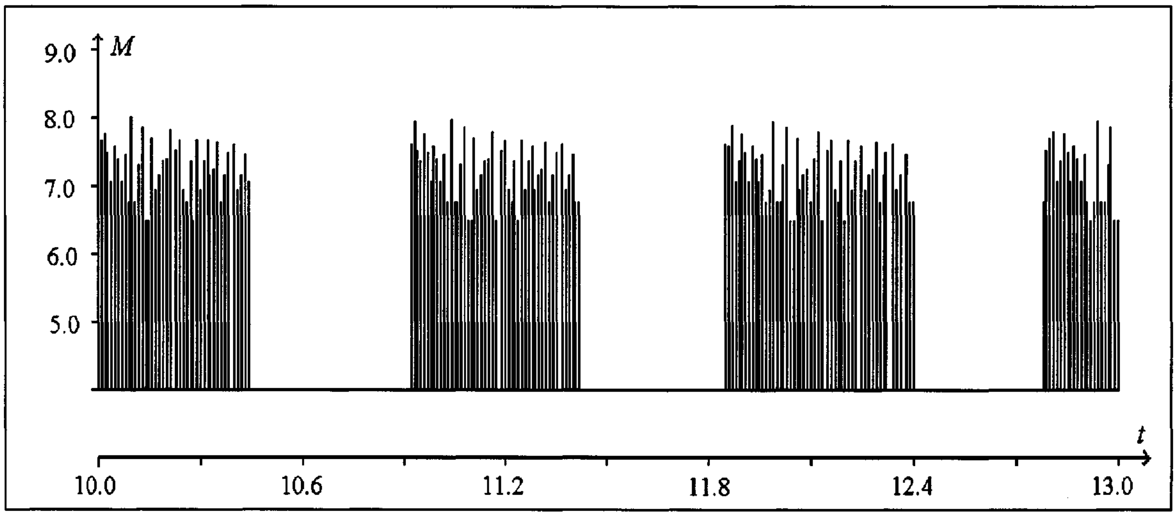

Fig. 6. The dependence of magnitude of model earthquakes on time for the segment marked by symbol "+" in Fig. 5.

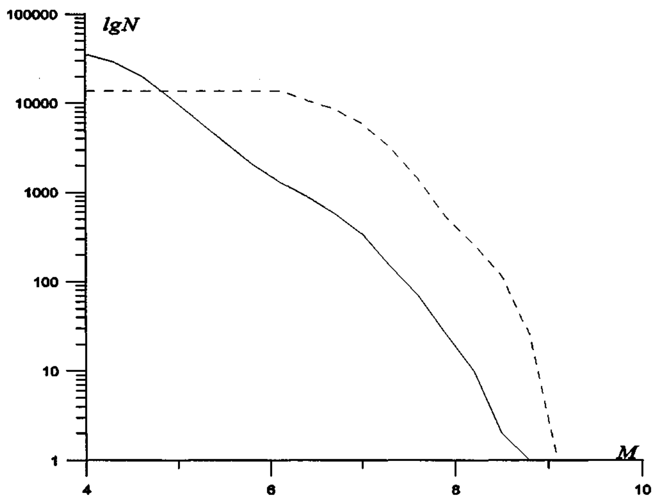

Fig. 7. Frequence-of-occurrence curves constructed for the real catalog (solid line) and synthetic catalog (dashed line); N is accumulated number of earthquakes, M is magnitude.

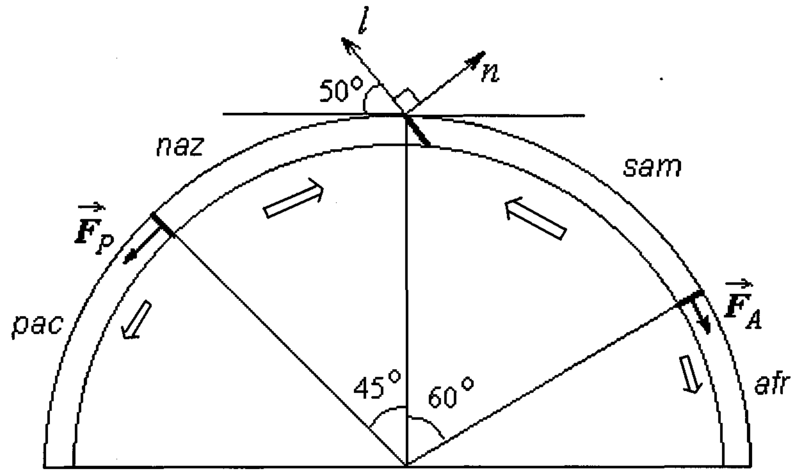

Fig. 8. Scheme of the impact of motion of Pacific and Africa plates on seismicity at the boundary Nazca/South America.

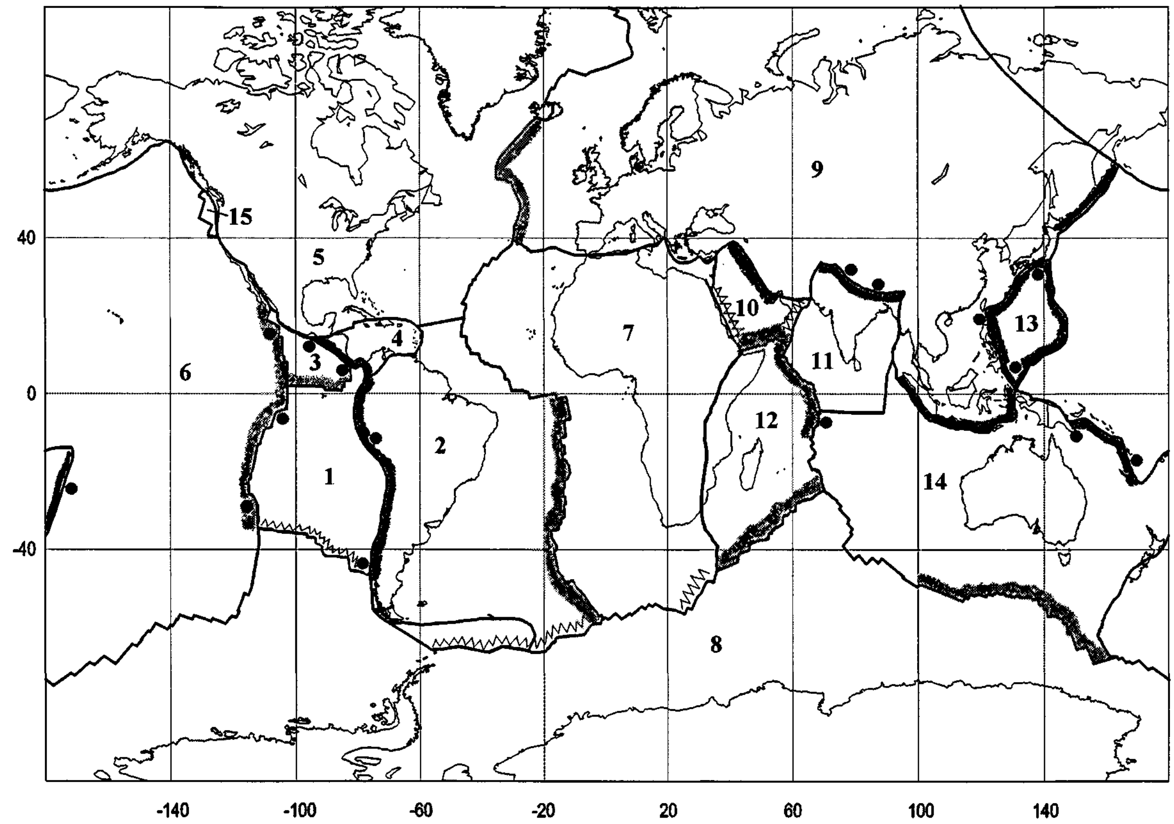

Fig. 9. Results of simulation of the character of plate boundaries and spatial distribution of the strongest earthquakes: divergent plate boundaries (spreading, light shading), convergent plate boundaries (subduction, dark shading), transform plate boundaries (sliding, toothed shading), epicenters of model events (circles). Numbers stand for the plates: 1 - Nazca, 2 - South America, 3 - Cocos, 4 - Carribean, 5 - North America, 6 - Pacific, 7 - Africa, 8 - Antarctica, 9 - Eurasia, 10 - Arabia, 11 - India, 12 - Somalia, 13 - Philippine, 14 - Australia, 15 - Juan de Fuca.

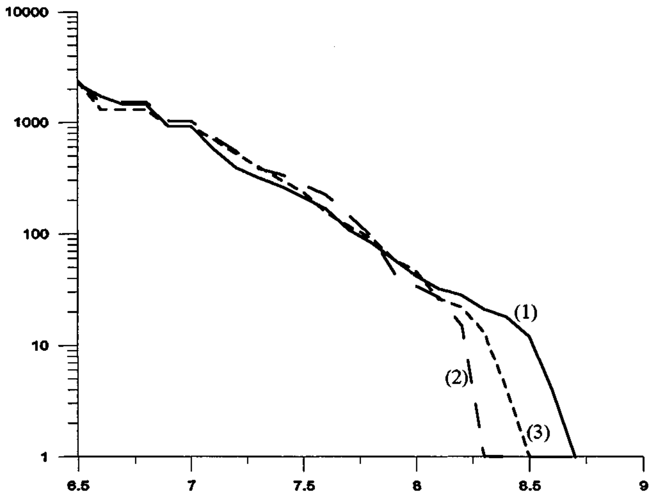

Fig. 10. Frequency-of-occurence curves constructed for three synthetic subcatalogs for the spherical rectangle region (longitude belongs to [85°, 60°], latitude belongs to [40°, 0°]). Numbers stand for the curves: (1)— for the structure with 4 blocks, (2)— for the structure with 10 blocks, (3)— for the structure with 15 blocks (without boundary ones); N is accumulated number of earthquakes, M is magnitude.