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On the Predictability of Characteristic Earthquakes from Recurrence Times

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ON THE PREDICTABILITY OF CHARACTERISTIC EARTHQUAKES FROM RECURRENCE TIMES

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1 INTRODUCTION

Plate tectonics provides a mechanism for the tectonic loading and relaxation process that define the seismic cycle. It has also been widely reported (Scholz, 1990) that earthquakes tend to occur once and again at the same fault, defining what is known as *earthquake recurrence*, that is, "the time between subsequent rupturing on a given segment of fault, hence to the period of the loading cycle" (Scholz, 1990).

Past observations have been used to predict the next event as, for example, for the case of Parkfield series (Bakun and Lind, 1985), with no success, through a recurrence model (Shimazaki and Nakata, 1980). This prediction was severely criticized by Savage (1993) from statistical grounds. In these cases, observations were used along with some assumptions on the stress behavior (recurrence model) or statistical distributions (Savage, 1993), but no any physical model was appealed. Sornette and Knopoff (1997) warned that when dealing with statistics, the final result is depending on the assumed probability distribution.

Independently, with the underlying hypothesis of the recurrence of seismic cycles, Hagiwara (1974) and Rikitake (1975) developed a method for the estimation of the probability of occurrence of the next earthquake, also based on the observation of previous activity, in which the main hypothesis consist in assuming that the crustal rupture time follows a Weibull distribution, widely used in probabilistic control with respect to the failure-time of buildings and factory products. Under this assumption the mean return period and its associated standard deviation can be computed, along with the hazard rate. This approach has been used recently by Rikitake (1999) to reevaluate the probability of a great earthquake to recur in the Tokai district, Japan.

A problem that concerns all analysis based on the analysis of short, non-periodic and possibly non-stationary time series is whether or not they are representative of the whole process: which is the realibility of the obtained prediction? Paleoseismology can provide a few more recurrence times, but the problem remains the same. The aim of the present study is to analyze the robustness of the predictions by repeating Rikitake's procedure with a numerically generated chaotic time series, for which all parameters are well controlled. As a physical model to simulate the recurrence time of earthquakes we have used the dripping faucet model (Shaw, 1984) in the chaotic regime. The inter-event time between successive drops can be assimilated to the recurrence time between characteristic earthquakes, and its mass can be assimilated to its magnitude.

2 RIKITAKE'S METHOD

Hagiwara (1974) suggested that the statistical distribution of ultimate crust strain may be expressed by a Weibull distribution. This approach has been followed by Rikitake (1975, 1976) to estimate the probability of a series of great earthquakes occurred in a given seismogenic zone.

Following Rikitake (1976), let us denote a cumulative probability for the recurrence of a great earthquake during a period between 0, when the last earthquake occurred, and t by $F(t)$. Writing

$$R(t) = 1 - F(t) \tag{1}$$

and assuming a Weibull distribution $\lambda(t) = Kt^m$, $K > 0$ and $m > -1$, the reliability $R(t)$ is found as

$$R(t) = \exp\left[-\int_0^t \lambda(t)dt\right] = \exp[-Kt^{(m+1)}/(m+1)] \tag{2}$$

where K and m are the parameters to be determined from observations from which the mean return period and its standard deviation can be readily determined. To estimate the governing parameters K and m, taking the double natural logarithm of $1/R$ one obtains

$$\ln\ln(1/R) = \ln[K/(m+1)] + (m+1)/\ln t, \tag{3}$$

From this equation K and m can be obtained from actually observed data by least squares.

Still following Rikitake (1976), in practice one counts the frequency of return period (n_i) for each time range Δt suitable chosen. The probability of a return period falling in a range between $i\Delta t$ and $(i+1)\Delta t$, (i = 0, 1, 2, ...) is obtained as n_i/N for which N is the total number of data. The cumulative probability is then obtained as

$$F = \sum_{i=0}^{i} n_i/N \tag{4}$$

so that R can be readily obtained from eq. (1). With R thus calculated, $\ln(\ln R)$. *vs.* $\ln t$ plots are made by adopting an appropriate time interval, and a straight line can be fit to data, from which k and m are estimated.

The mean return period $E(t)$ is given by

$$E(t) = \left(\frac{K}{m+1}\right)^{\frac{-1}{m+1}} \Gamma\left(\frac{m+2}{m+1}\right) \tag{5}$$

and its associated standard deviation σ as

$$\sigma = E(t)\left[\Gamma\left(\frac{m+3}{m+1}\right) - \Gamma^2\left(\frac{m+2}{m+1}\right)\right]^{1/2} / \Gamma\left(\frac{m+2}{m+1}\right) \tag{6}$$

On the condition that no earthquake occurs in the time range between 0 and t, the probability of having an earthquake between t and $t+s$ can be computed from the cumulative probability $F(t)$ as a conditional probability FS(t), defined as

$$FS(t) = [F(t+s) - F(t)]/[1 - F(t)] \tag{7}$$

3 THE DRIPPING FAUCET MODEL

In the framework of a discussion about predictability in physical systems, Shaw (1984) presents as a paradigmatic example a very well known one by insomniacs, that of a leaking faucet. From an experimental point of view, water from a tank is measured as it passes through an adjustable brass nozzle. Depending on the flow rate of water, the drop rate can be periodic, quasi-periodic or chaotic. Upon substitution of the word *water* by the word *stress*, the analogy between recurrence of earthquakes and recurrence of drops is total. To simulate the dripping faucet Shaw (1984) designed a very simple mathematical model: a mass, representing the drop, grows linearly in time, stretching a spring that represents the force of surface tension. When the spring reaches a certain length the mass is suddenly reduced, representing a drop detaching, by an amount dependent on the speed of the mass when it reaches the critical distance. We thus have driven nonlinear oscillator, the nonlinearity arising from the sudden change in mass, and with position, velocity and mass providing the three variables required for the occurrence of chaotic behavior in a system evolving in continuous time.

Quantitatively this model can be written as

$$\frac{d}{dt}\left(m\frac{dy}{dt}\right) = mg - ky - b\frac{dy}{dt} \tag{8}$$

where $m(t)$ is the mass of the drop, g the gravity, k the spring constant, b the friction constant and y the displacement, subject to the boundary conditions

$$\begin{aligned} \dot{m}(t) &= ct. \\ \Delta m &\propto \left.\frac{dx}{dt}\right|_{x=x_0} \end{aligned} \tag{9}$$

More elaborate quantitative models have been developed since the original work of Shaw, but this one suffices for the present purpose.

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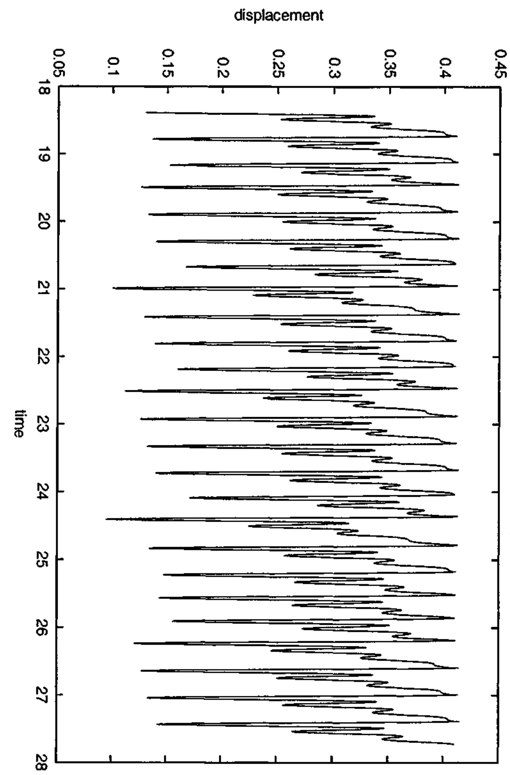


FIGURE 1

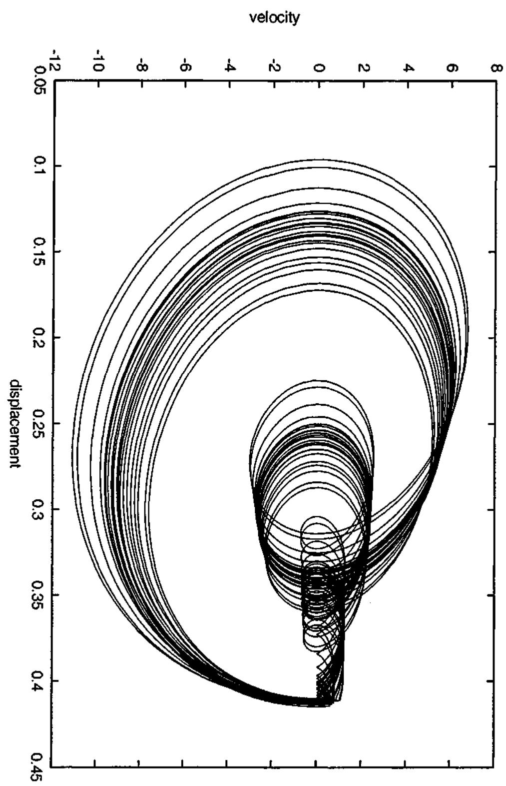


FIGURE 2

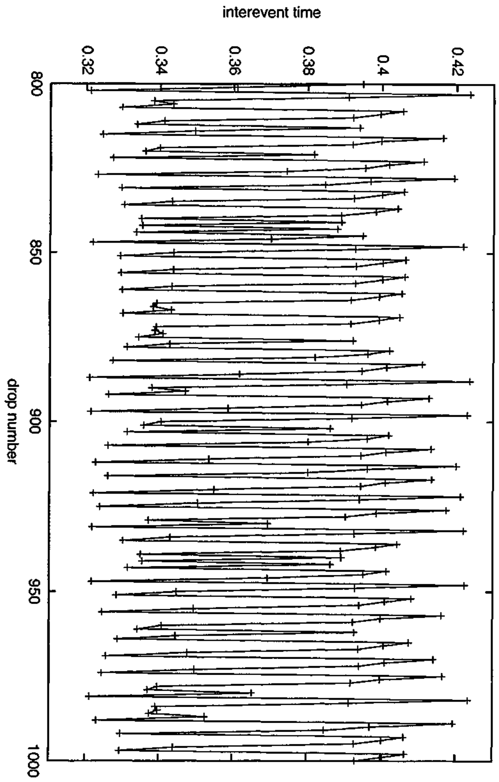


FIGURE 3

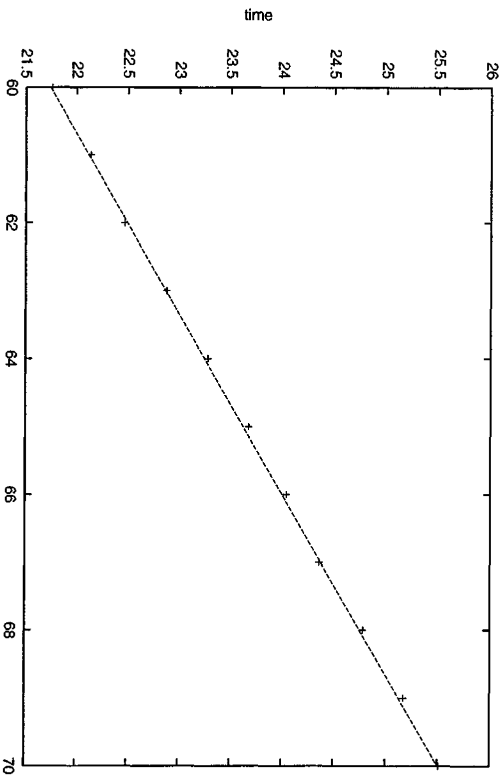


FIGURE 4

FIGURE 5

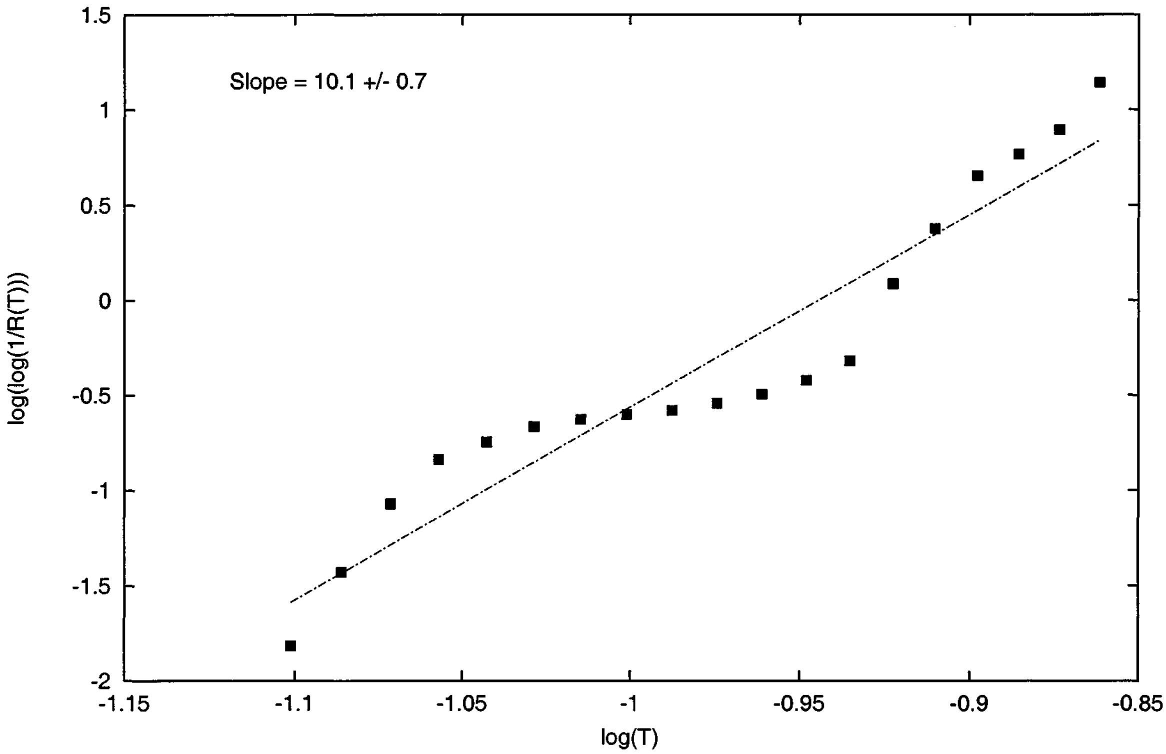
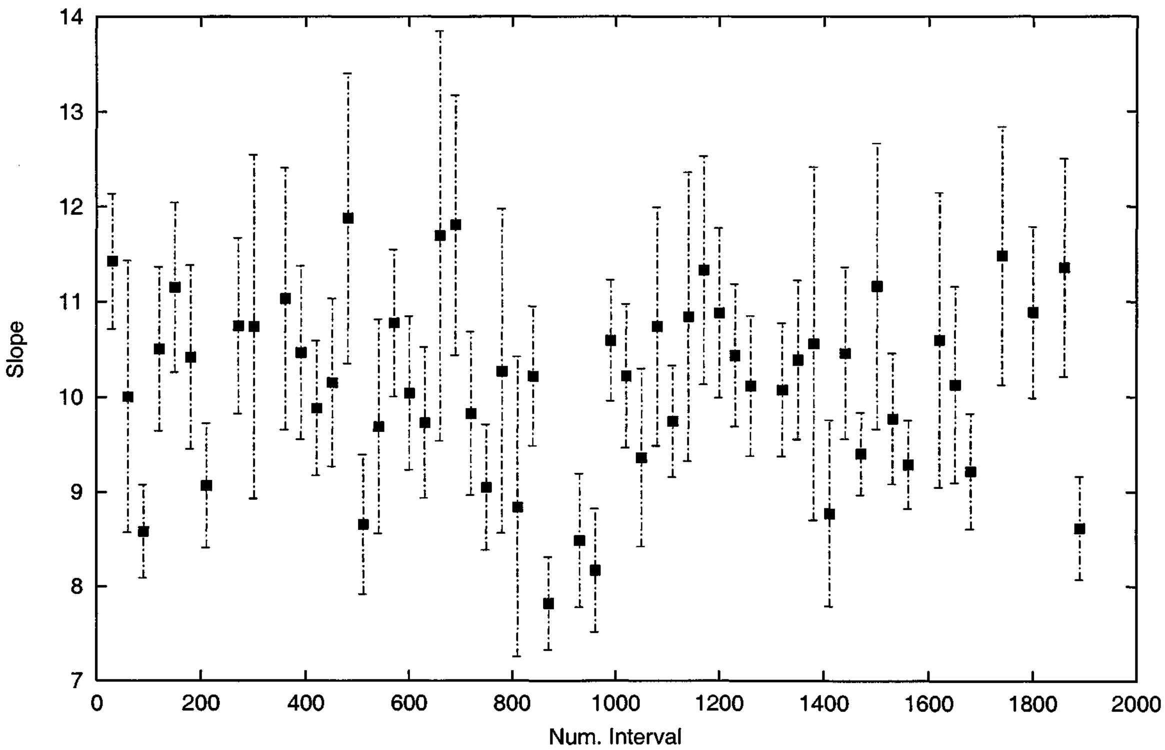


FIGURE 6



Likelihood method

$$L = \prod_i f\{x,\theta\}$$

$$L = \prod_i \alpha\beta x_i^{\beta-1}\exp(-\alpha x_i^{\beta})$$

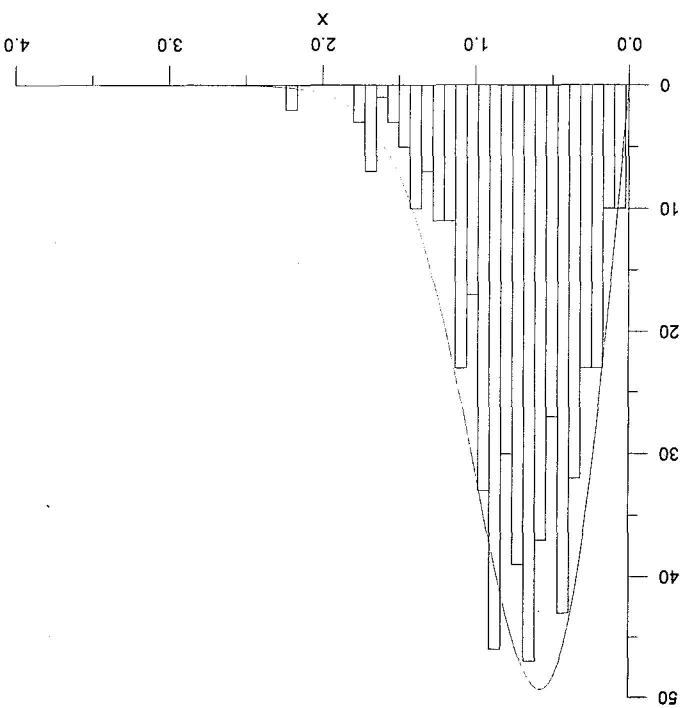
$$\ln L = \ln \prod_i \alpha\beta x_i^{\beta-1}\exp(-\alpha x_i^{\beta})$$

$$= \sum_i \ln \alpha\beta x_i^{\beta-1} + \sum_i(-\alpha x_i^{\beta})$$

$$= N\ln\alpha\beta + (\beta-1)\sum_i^N \ln x_i - \alpha\sum_i^N x_i^{\beta}$$

$$\frac{\partial \ln L}{\partial \alpha} = 0 \quad \frac{\partial \ln L}{\partial \beta} = 0$$

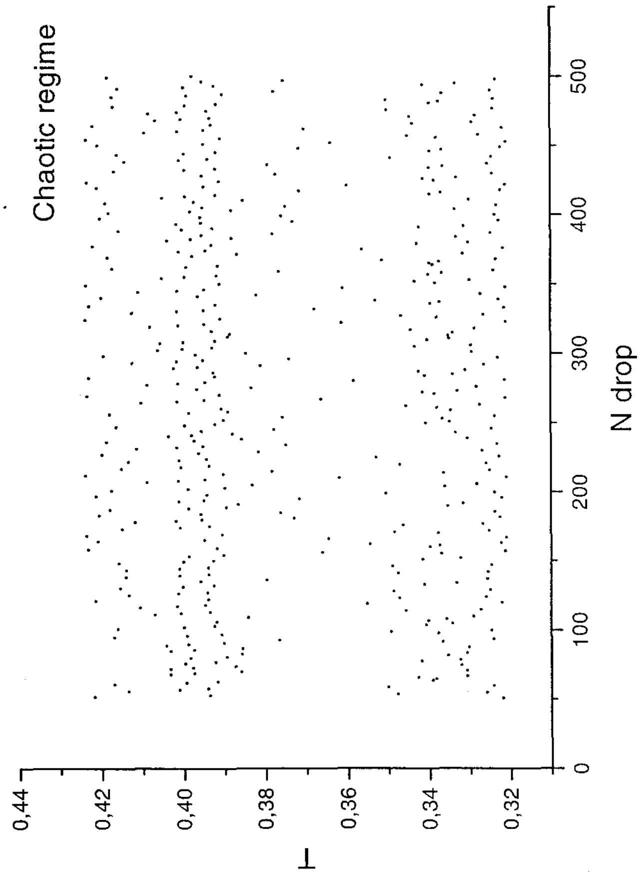
$$\frac{\partial \ln L}{\partial \alpha} = 0 \Rightarrow \alpha = \frac{N}{\sum_i x_i^{\beta}}$$



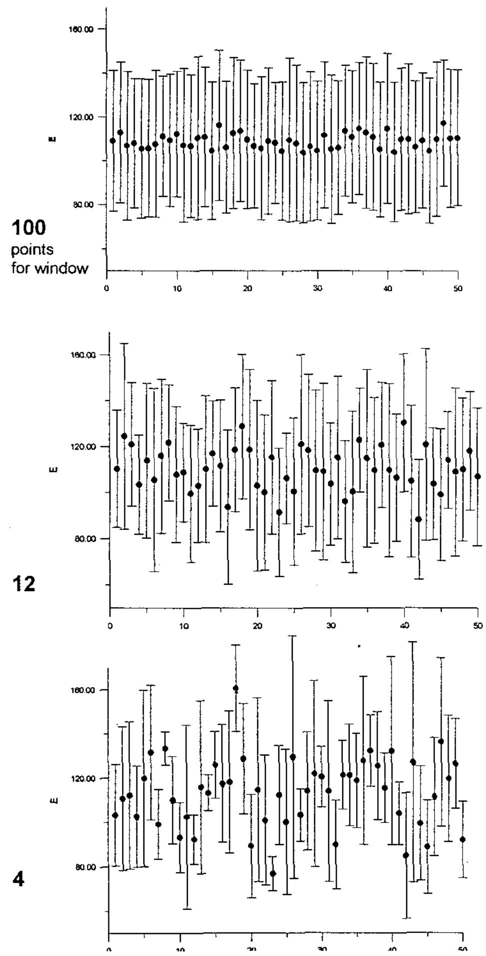
Zone	Rikitake's results				Likelihood				Rikitake			
	k	m	E	sig	k	m	E	sig	k	m	E	sig
Hokkaido-kurile	7.2E-8	2.9	85.3	24.6	3.5E-8	3.01	91.7	25.6	7.2E-2	-0.7	4E3	4E4
Aleutian-Alaska	3.1E-5	9.5	27.2	8.9	4.9E-5	2.03	33.9	12.2	0.18	-0.5	15.3	36.9
Central america	2.3E-16	9.5	34.5	3.6	1.0E-17	10.4	35.6	3.76	0.10	-0.8	2E4	8E5
Northern south Ame.	8.0E-8	0.58	46.3	30.0	4.9E-4	1.08	48.36	24.3	0.10	-0.8	2E6	2E7
Southern South Ame.	1.1E-7	4.3	100	22.5	3.5E-5	1.39	92.6	41.2	4.6E-2	-0.9	2E11	1E13

Adjusted parameters from Likelihood and Rikitake's method.

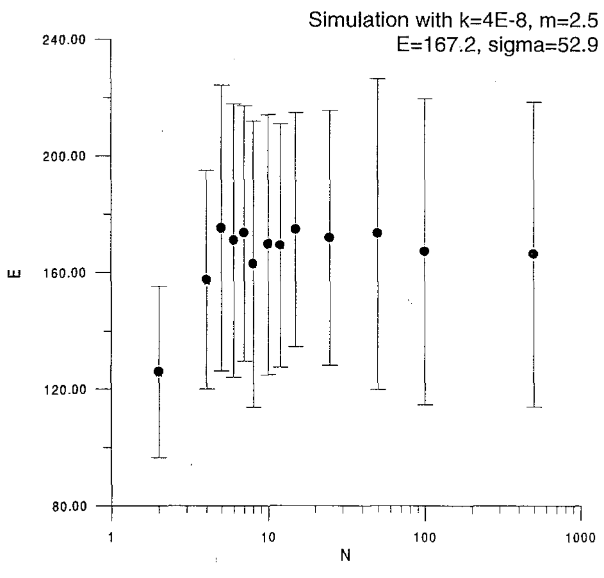
k	m	k(like)	m(like)	k(Riki.)	m(Riki.)
3	1	3.09	1.01	2.89	0.80
5e-5	2.8	4.6e-5	2.82	0.01	0.61
8e-9	1.5	6.9e-9	1.52	0.003	-0.31
100	9	103.01	9.07	8.89	0.83
45	5	46.2	5.05	7.07	0.86

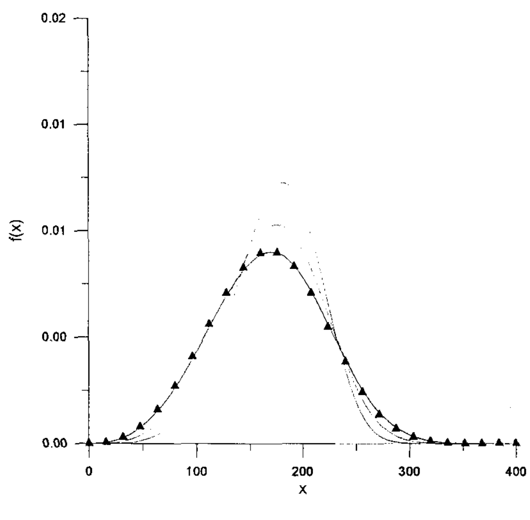


Window displacement

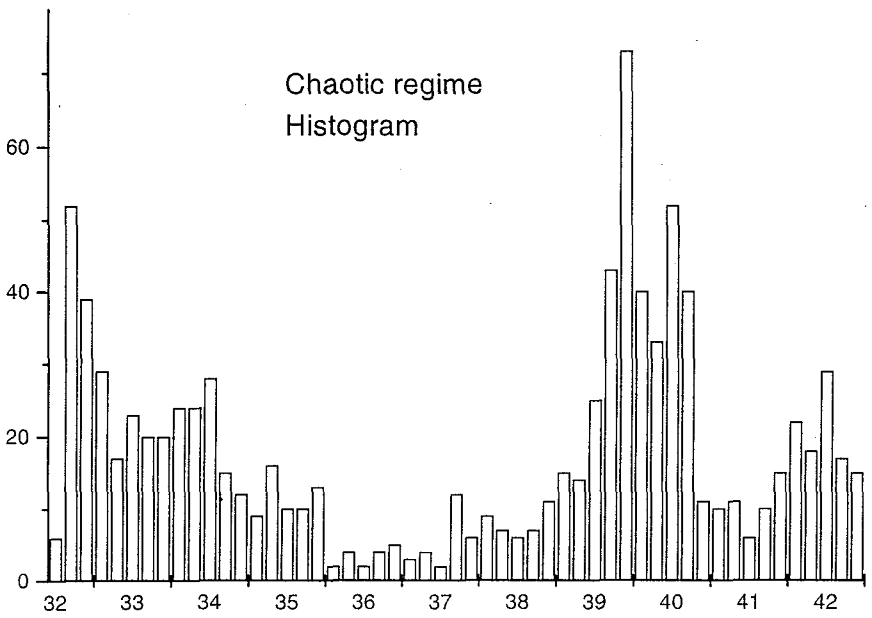


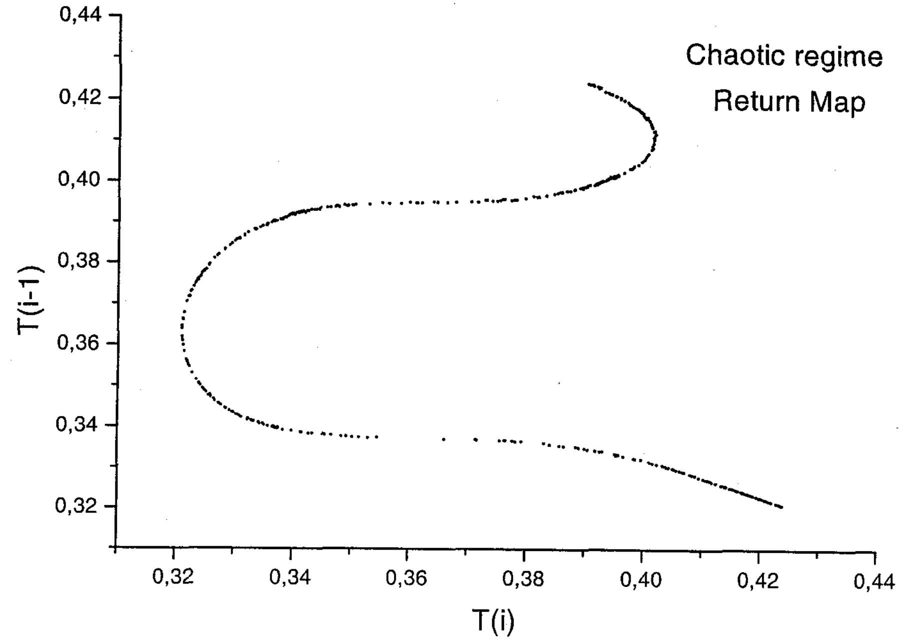
Expected values obtained with different number of points.

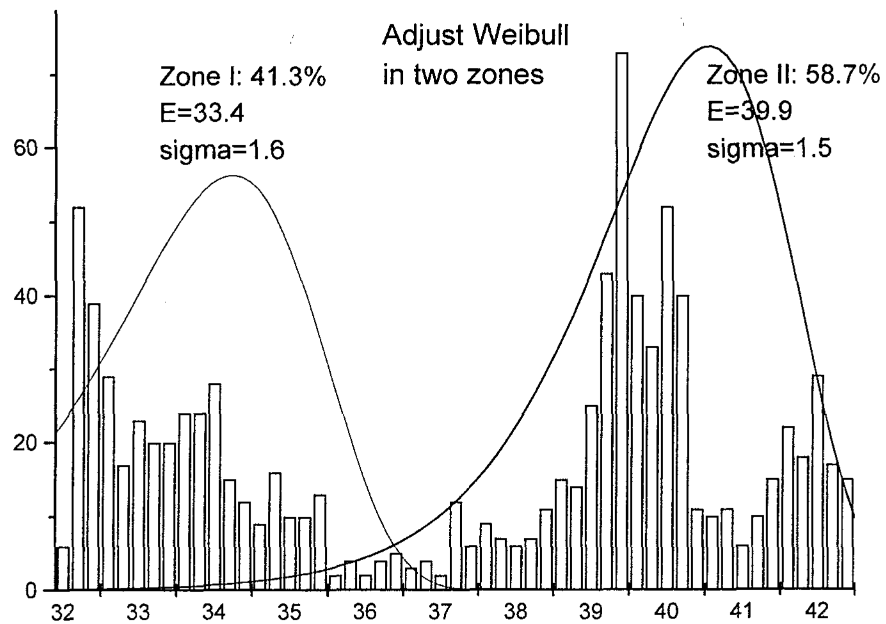


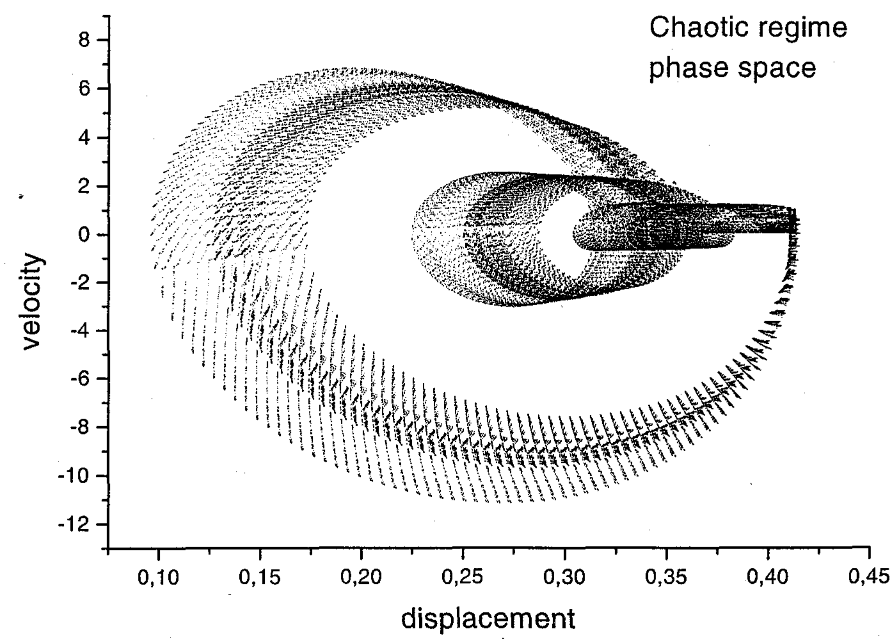


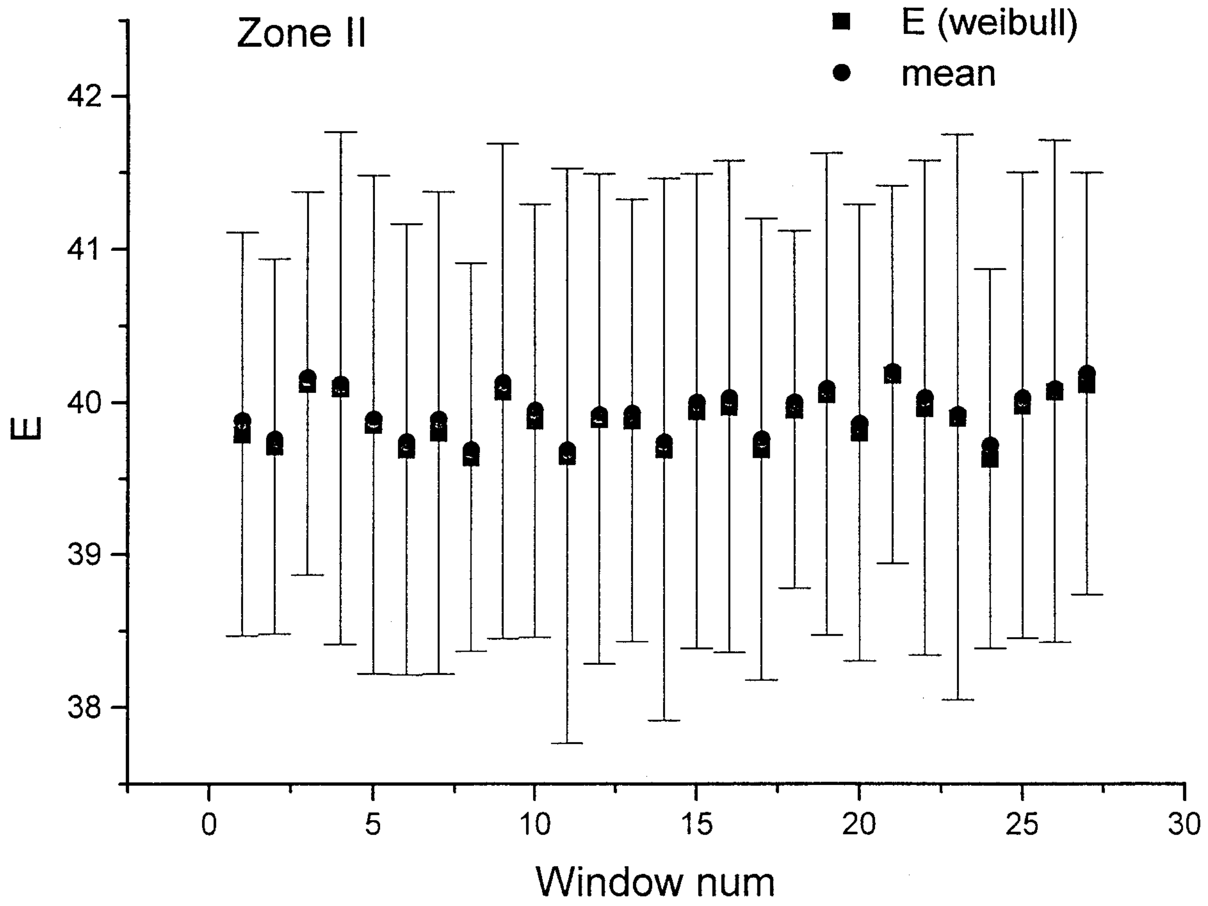
Points num	k	m	Points num	k	m
500	**3.7e-8**	**2.51**	10	7.7e-10	3.29
100	3.5e-8	2.52	8	1.7e-8	2.69
50	2.12e-8	2.60	7	2.57e-10	3.49
25	3.2e-10	3.45	**5**	**2.69e-9**	**3.01**
15	**2.2e-11**	**3.98**	4	8.11e-11	3.81
12	1.61e-10	3.61	2	1.7e-10	3.88

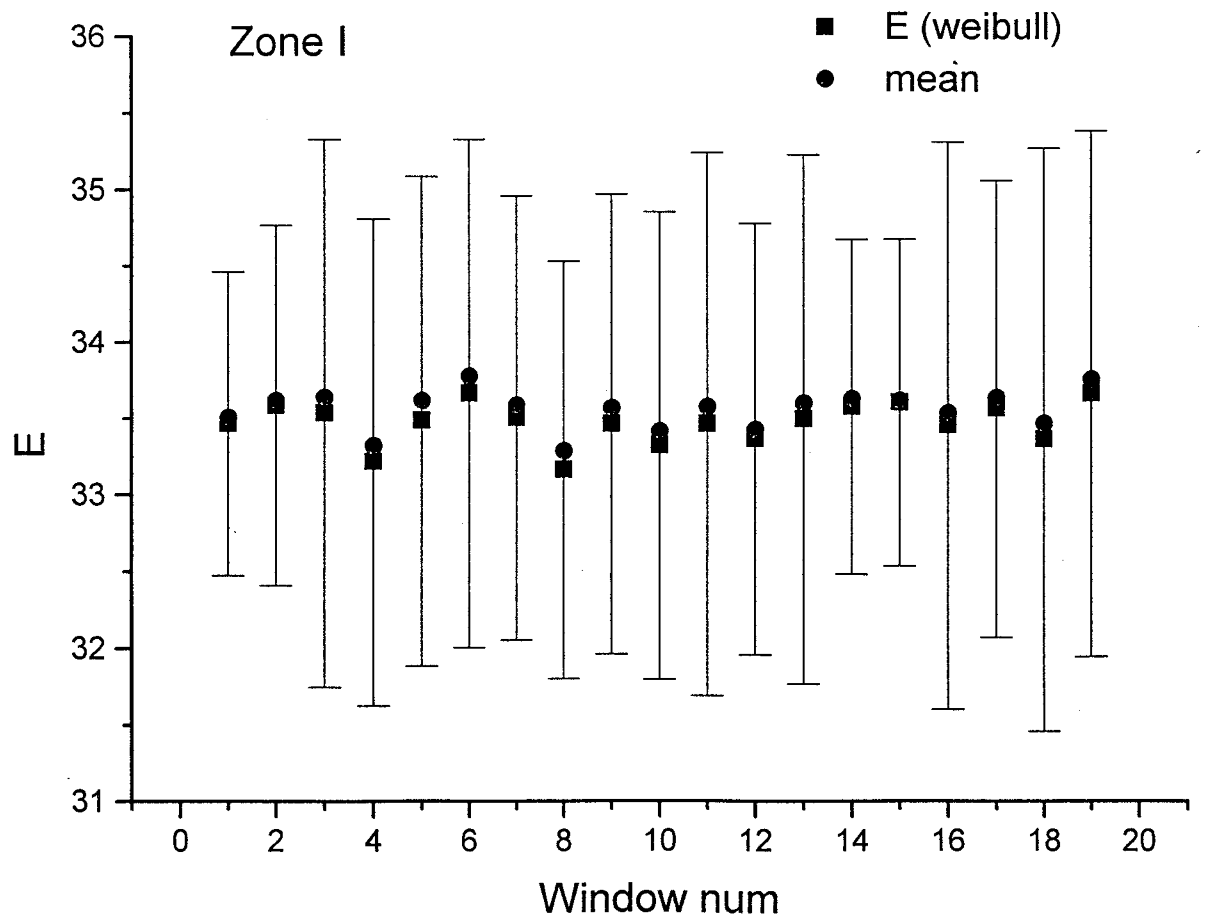

Chaotic regime
Histogram


Chaotic regime
Return Map
T(i-1)
T(i)


Adjust Weibull
in two zones
Zone I: 41.3%
E=33.4
sigma=1.6
Zone II: 58.7%
E=39.9
sigma=1.5


Chaotic regime
phase space
velocity
displacement


Zone II
E (weibull)
mean
42
41
40
39
38
E
0
5
10
15
20
25
30
Window num


Zone I
E (weibull)
mean
36
35
34
33
32
31
E
0
2
4
6
8
10
12
14
16
18
20
Window num