united nations
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international centre for theoretical physics

SMR 1302 - 18

WINTER SCHOOL ON LASER SPECTROSCOPY AND APPLICATIONS

19 February - 2 March 2001

Laser Cooling and Bose Einstein Condensation

Part 4 and 5

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These are preliminary lecture notes, intended only for distribution to participants.

LASER COOLING
and
Bose Einstein Condensation

M. Inguscio # 4, 5

EFFETTO DOPPLER: NEMICO DA BATTERE

DOPPLER EFFECT IS RELATIVISTIC

Resonance:

$$\omega_L \simeq \omega_0 \sqrt{1-\left(\frac{v}{c}\right)^2} + \underline{K}\cdot\underline{v}$$

$$\omega_L \simeq \underbrace{\omega_0 + \underline{K}\cdot\underline{v}}_{\text{I}} \; \underbrace{- \frac{\omega_0}{2}\left(\frac{v}{c}\right)^2}_{\text{II}}$$

"Spettroscopia n.l."

$$\text{II} \quad \omega_0 \frac{k_B T}{m c^2}$$

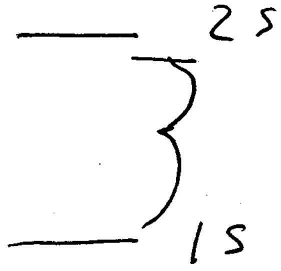
shifts ed asimmetrie
a temperature ambiente
dell'ordine di 10^{-12}

SHIFTS AND ASYMMETRIES

$\sim 10^{-12}$

Hänsch,
Garching

Hydrogen 1s – 2s



in a beam

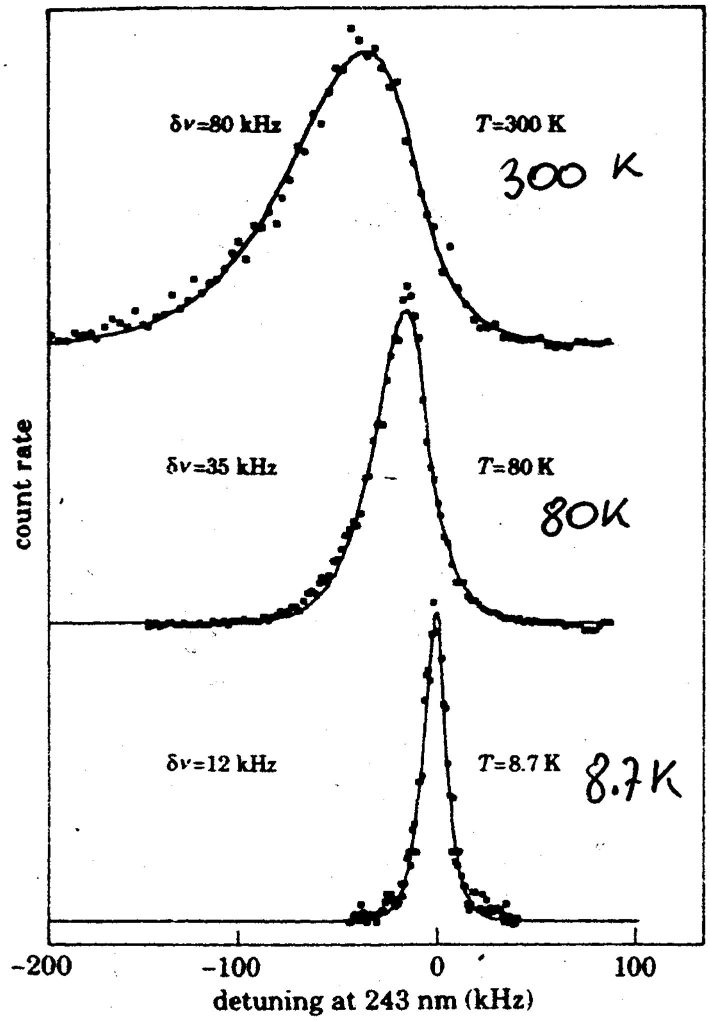
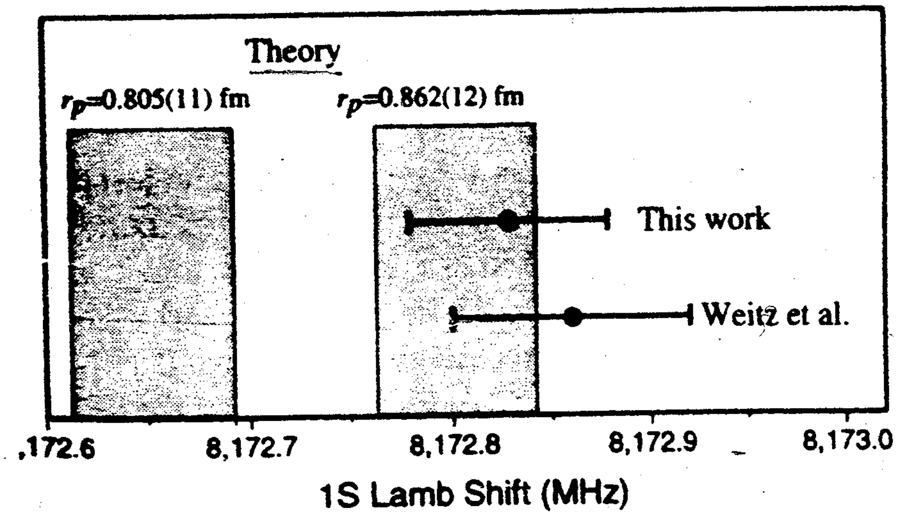


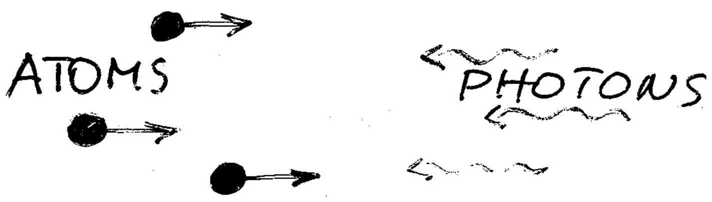
Fig. 4. – Doppler-free two-photon spectra of the $F = 1$ hyperfine component of the hydrogen $1S$-$2S$ transitions, recorded by coaxial excitation of an atomic beam at three different nozzle temperatures. At 8.7 K, the resolution reaches 1 part in 10^{11}.



PROTON
SIZE

GROUND STATE LAMB SHIFT

CRAZY IDEA THINKING OF STOPPING ATOMS WITH A PHOTON BOMBARDMENT ?



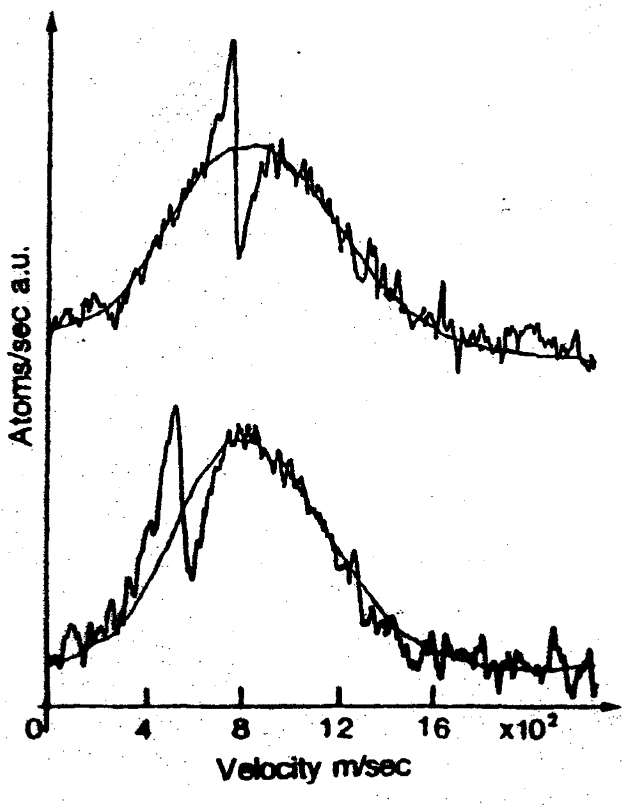


Fig. 2. Laser cooling observed in the absence of a magnetic field for two different frequency settings of the cooling laser beam.

←

low velocity

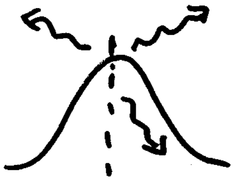
while they decelerate they also go out of resonance

OPTICAL COOLER

Energy conservation

the atom emits light *more energetic* than the one it has absorbed

absorption



emission

Emission centered while absorption at lower frequencies

Where does this energy come from?

From thermal motion kinetic energy...

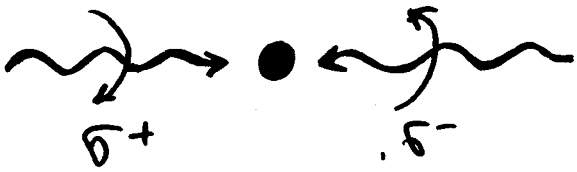
and temperature is a measure of kinetic energy

Cools but does not trap

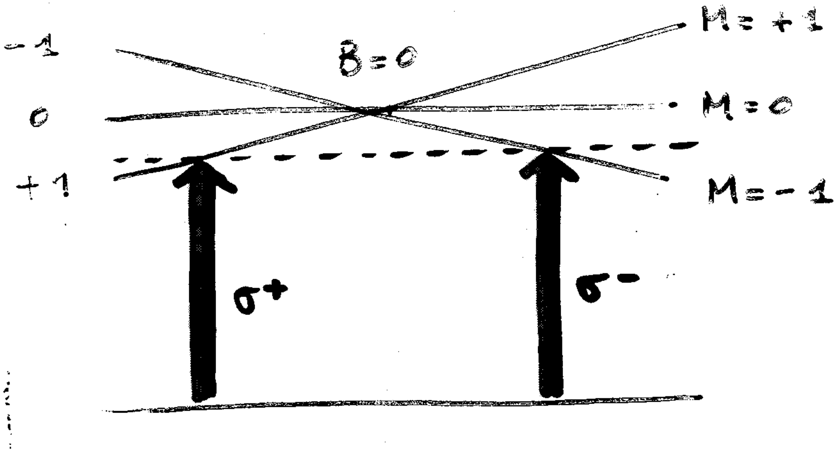
CONSERVATION LAWS : ANGULAR MOMENTUM

Conservation of angular momentum

MAGNETO - OPTICAL TRAP



magnetic field gradient



an atom at the right is resonant with the photon σ^- (not with the σ^+ photon because the M=+1 level is too high in energy)
→ the atom will absorbs more photons from the right moving towards the center.

Analogous for an atom at the left: it will absorbs photons from the left moving towards the center

$$\vec{F} = -\gamma \vec{v}$$

$$\vec{F} = -k \vec{z}$$

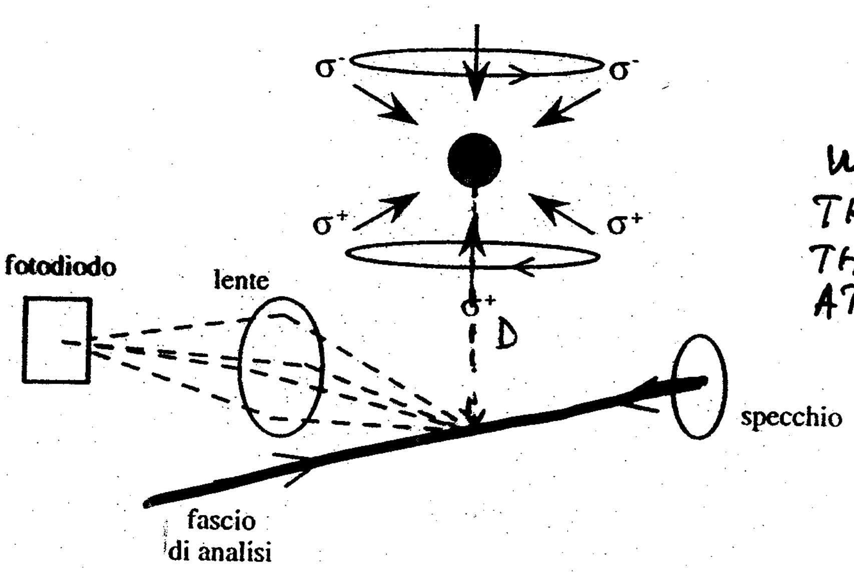
$$\longrightarrow \quad \vec{F} = -\gamma \vec{v} - k \vec{z}$$

$$m\ddot{z} + \gamma \dot{z} + kz = 0$$

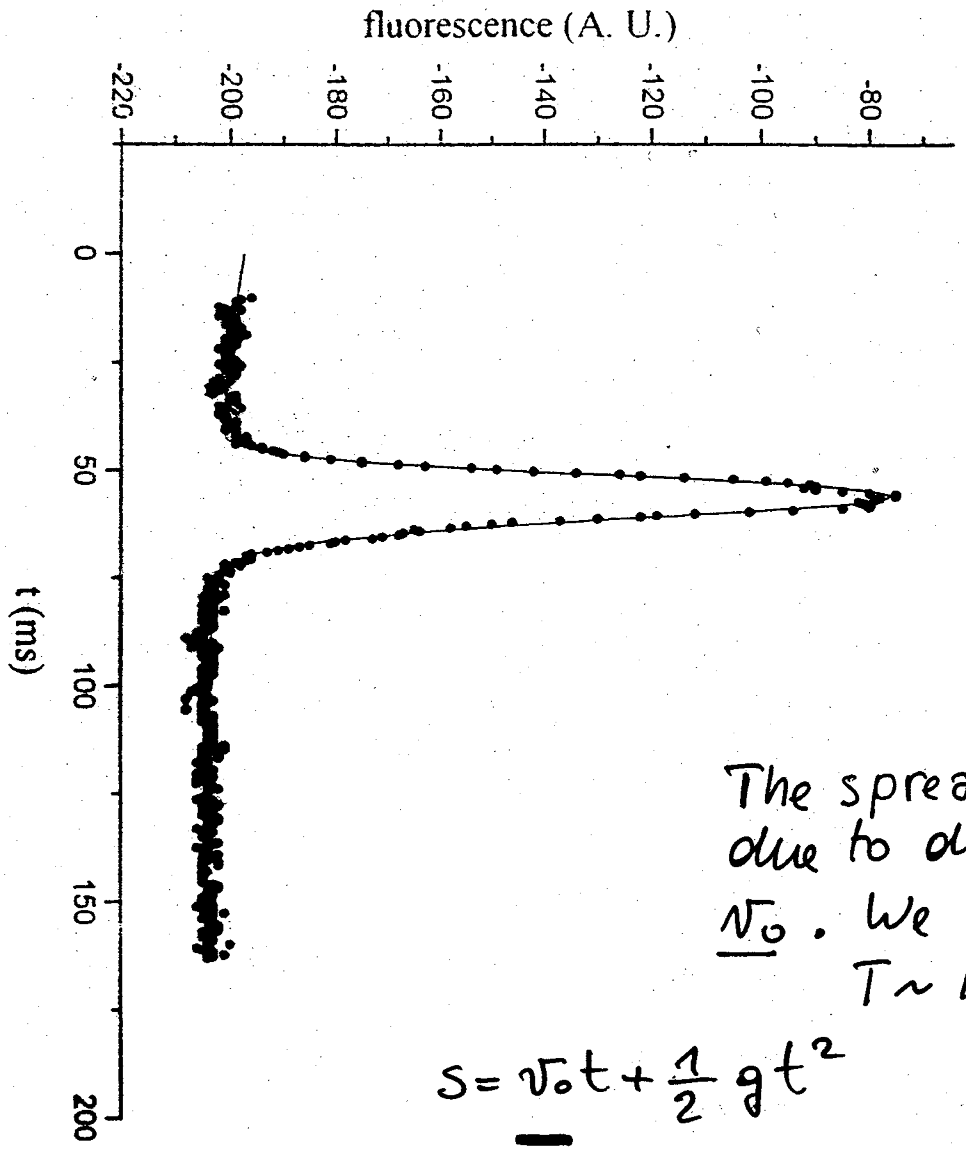
famous damped oscillator

TEMPERATURE

BUT THEN THEY FALL BECAUSE OF GRAVITY



WE CAN MONITOR THE PASSAGE OF THE FALLING ATOMS

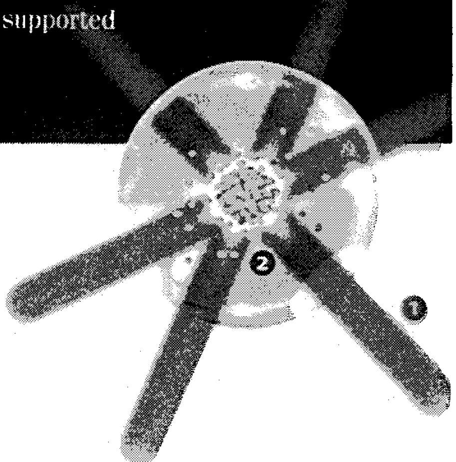


The spread is due to different $\underline{v_0}$. We measure $T \sim 10^{-6}$ K

$$s = \underline{v_0} t + \frac{1}{2} g t^2$$

An ambitious scientific programme

The PHARAO project opens up new scientific horizons from atomic physics to general relativity and cosmology. PHARAO is at the crossroads of state-of-the-art technologies in the fields of optics, laser manipulation of atoms and time and frequency metrology. French laboratories, which have been supported by CNES since 1991 are international leaders in these fields.

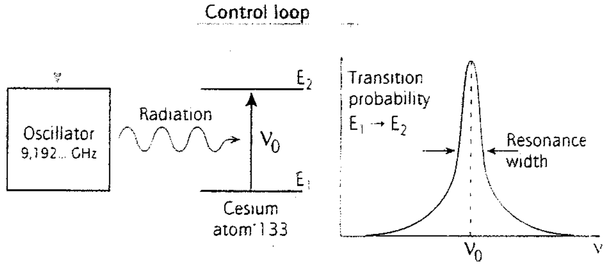


The "optical molasses" typically contains 100 millions of ultra-cold atoms.

Laser cooling*

Six laser beams ❶ cross inside a vacuum chamber containing a cesium vapour. If the laser's frequency is tuned below the resonance frequency of an optical transition of the atoms, the latter are subject to radiation pressure in the opposite direction to their velocity. The thermal agitation of the atoms is then dramatically reduced and the chilled atoms are literally trapped in an optical molasses ❷. At equilibrium, the temperature of the atomic gas is close to 1 microKelvin. This corresponds to a residual agitation velocity of 1 cm per second, i.e. ten thousand times less than at ambient temperature. These low velocities increase observation times to several seconds.

(*) research performed in Pr. Claude Cohen-Tannoudji's laboratory (Nobel Prize in Physics 1997).



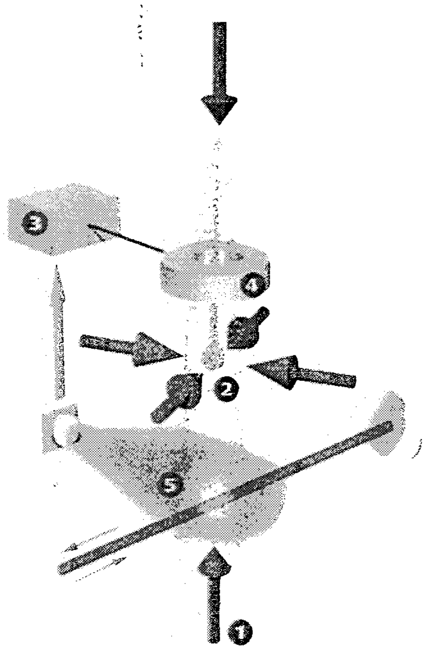
Definition: a second is equal to 9 192 631 770 periods of the radiation which corresponds to the transition between the two hyperfine levels of Cesium 133's ground state.

The principle of an atomic clock

The principle of an atomic clock is to lock an oscillator ❸ of frequency ν to the atomic resonance frequency ν_0. Heisenberg's uncertainty principle shows that the greater the interaction time T of the atoms with the radiation emitted by the oscillator, the narrower the resonance.

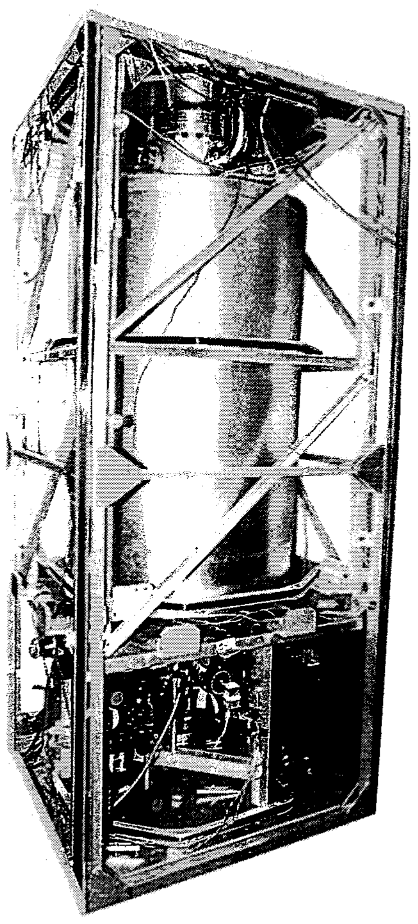
Two key points determine the ultimate performances of an atomic clock: a narrow resonance and a high signal-noise ratio.

Laser cooling allows an interaction time 100 to 1 000 times greater than for a conventional Cesium clock.



The atomic fountain

On the ground, the cold atoms are used in a vertical configuration called the "atomic fountain". The laser-cooled atoms are launched upwards. They follow a ballistic trajectory for about one second during which they are subject to two successive micro-wave ❹ interactions, one when travelling upwards, the other when travelling downwards (Ramsey's method). The transition probability is measured by fluorescence ❺.



BNM-LPTF's fountain: in operation since January 1995, it has a relative stability of $1.3\ 10^{-13}\ \tau^{-1/2}$ (where τ is the measurement time in seconds). With an accuracy of $2\ 10^{-15}$, this is currently the world's most accurate clock.

Cesium fountain

Precision $2 \cdot 10^{-15}$ (10^{-10} sec/day)

1000 times longer interaction time

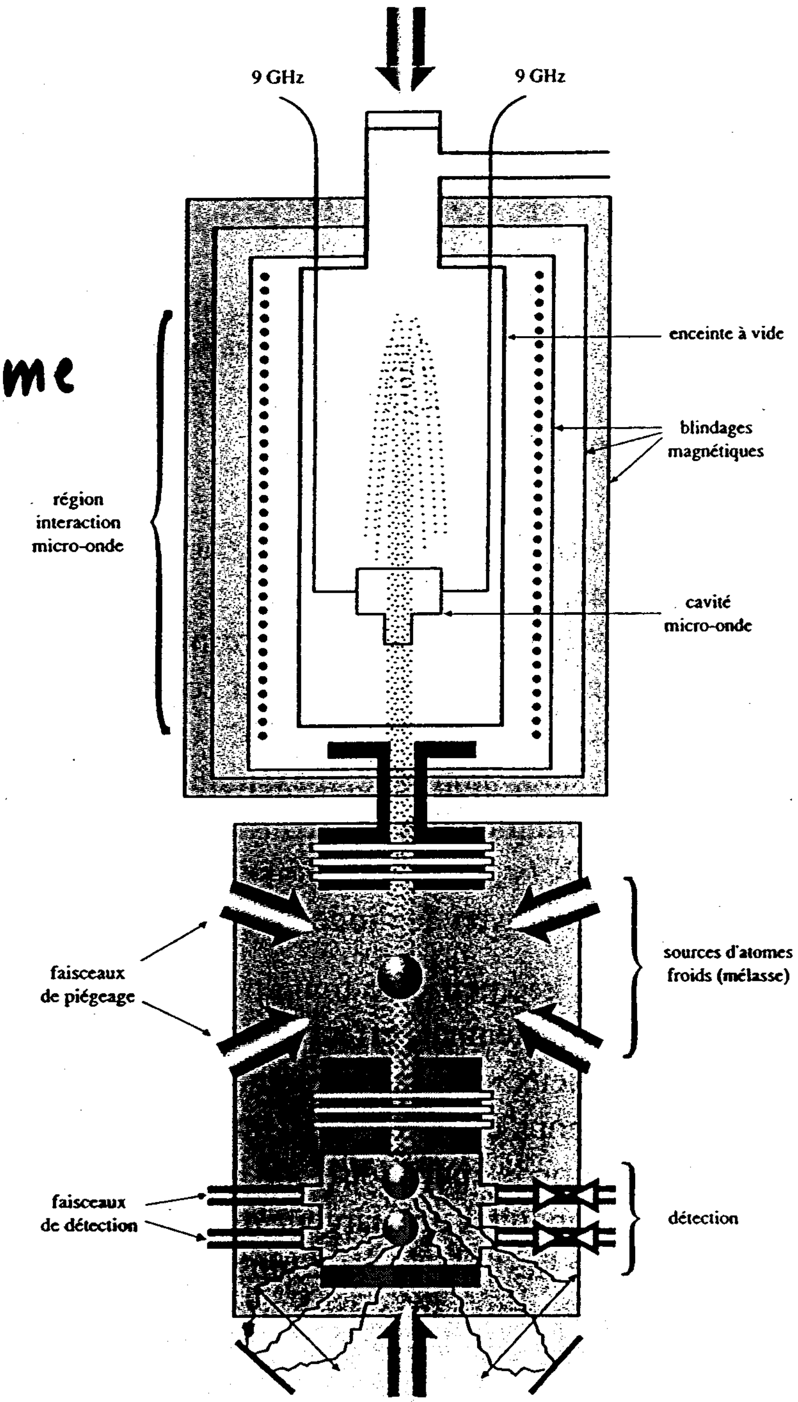
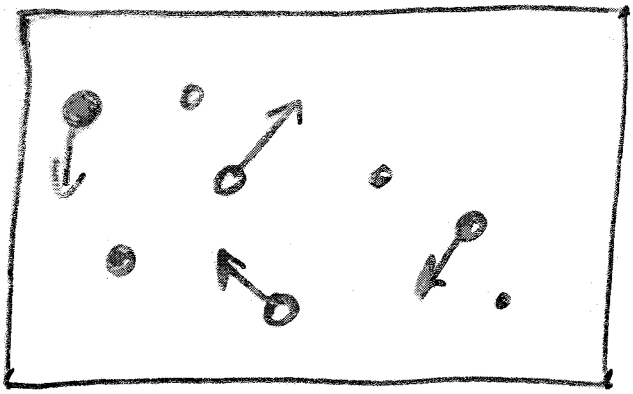


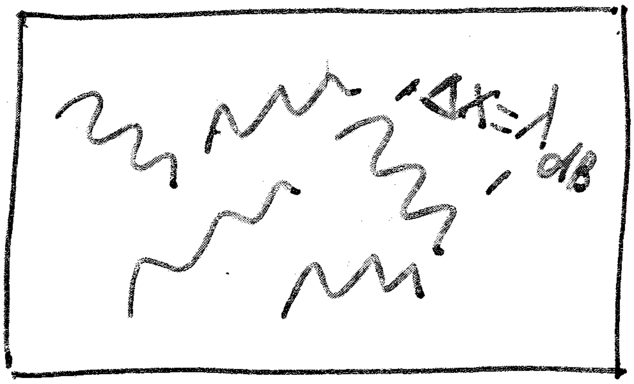
Fig. 1. — Schéma de la fontaine à atomes de césium fonctionnant au BNM-LPTF. Les premiers résultats ont donné une exactitude de $3 \cdot 10^{-15}$.

ATOMIC GAS



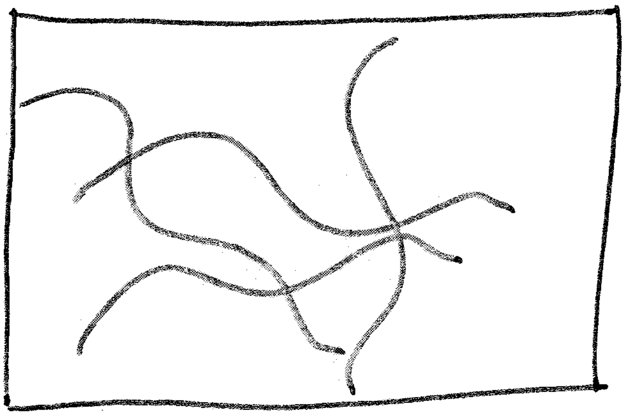
Room temperature
(300 K)

$v \simeq$ Km/sec



Ultralow temperature
($\sim 10^{-6}$ K)

$v \simeq$ cm/sec



$T \sim 10^{-9}$ K ? $v \simeq 0$

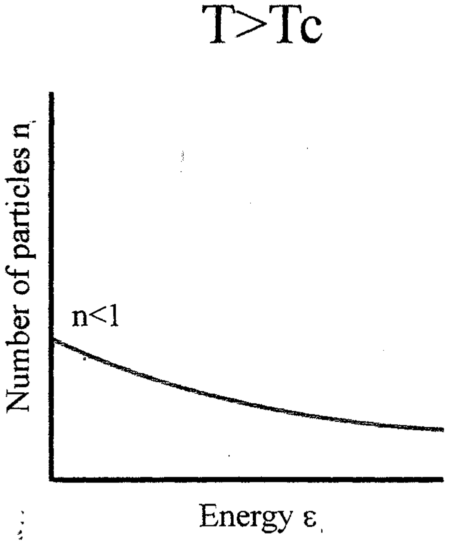
Even lower T:

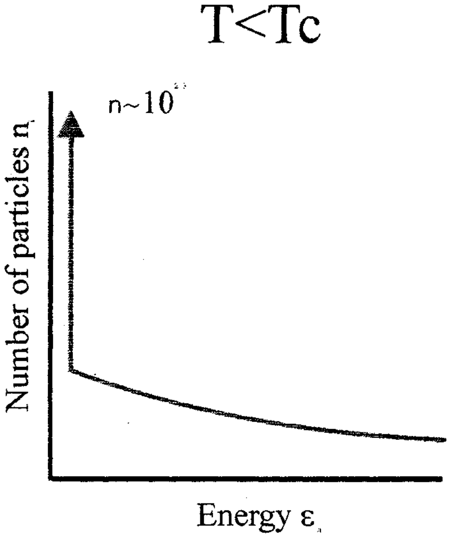
Atoms lose their
individuality

$$\lambda_{dB} = \frac{h}{M \cdot v}$$

THE BOSE-EINSTEIN STATISTICS

$$n_i = \frac{1}{e^{(\xi_i-\mu)/k_BT} - 1}$$


T>Tc


T<Tc

- The new statistics at low temperature saturates

$$T_c \sim \frac{\hbar^2}{K_B \cdot m} n^{2/3}$$

- A phase transition (without interactions) at a critical temperature Tc.
- For T<Tc the number of condensed atoms is (homogeneous gas):

$$N = N_0\left[1-\left(\frac{T}{T_C}\right)^{3/2}\right]$$

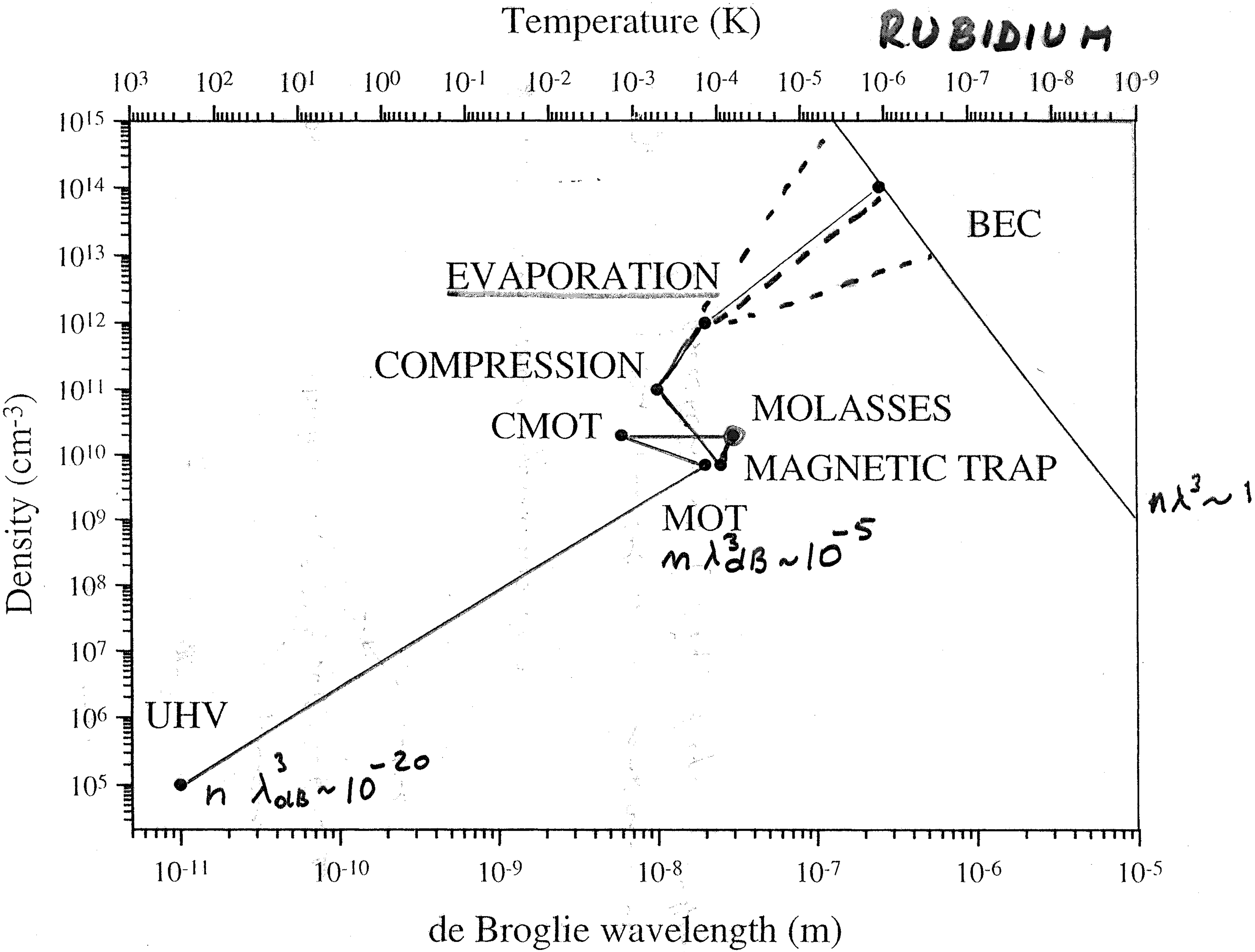
degeneracy at $\rho = n \cdot \lambda_{dB}^3 \sim 1$

Gas at room temperature (T~300 K)
and in good vacuum ($n=10^5$ cm^{-3})

$$\rho = n\lambda^3_{dB} \sim 10^{-20} \quad !$$

- Must be dilute (interactions, avoid "trivial" phase transitions to liquid and solid)

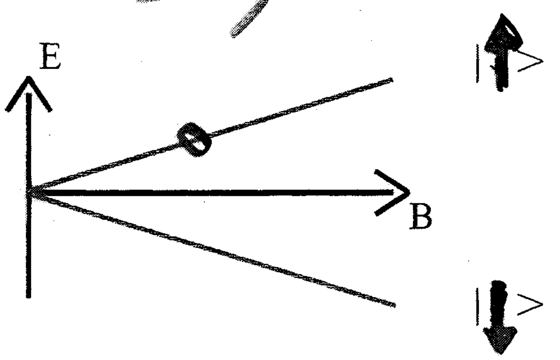
typically we need $T<10^{-6}$ K at $n\sim10^{14}$


RUBIDIUM
Temperature (K)
10^3 10^2 10^1 10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9}
Density (cm^{-3})
10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^9 10^8 10^7 10^6 10^5
BEC
EVAPORATION
COMPRESSION
MOLASSES
CMOT
MAGNETIC TRAP
MOT
$n\lambda^3 \sim 1$
$n \lambda_{dB}^3 \sim 10^{-5}$
UHV
$n \lambda_{dB}^3 \sim 10^{-20}$
10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5}
de Broglie wavelength (m)

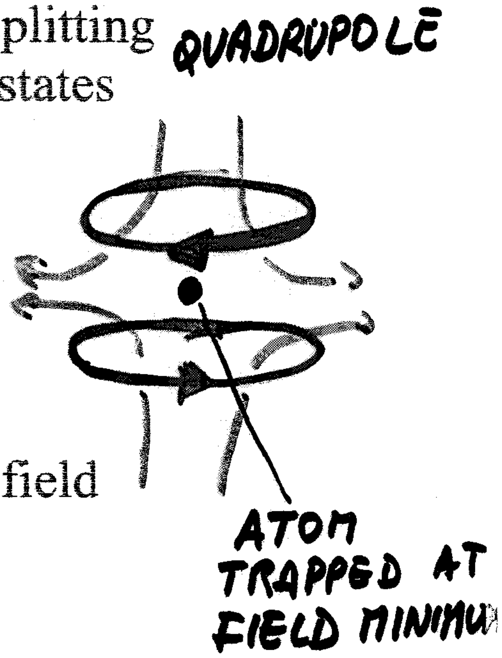
aa

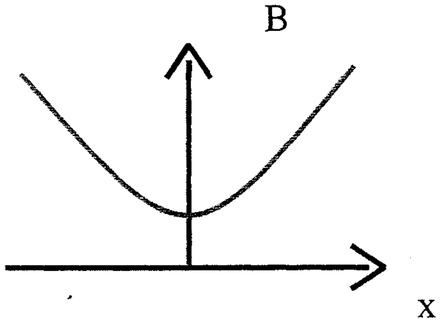
MAGNETIC TRAPPING

$E = -\vec{\mu} \cdot \vec{B}$



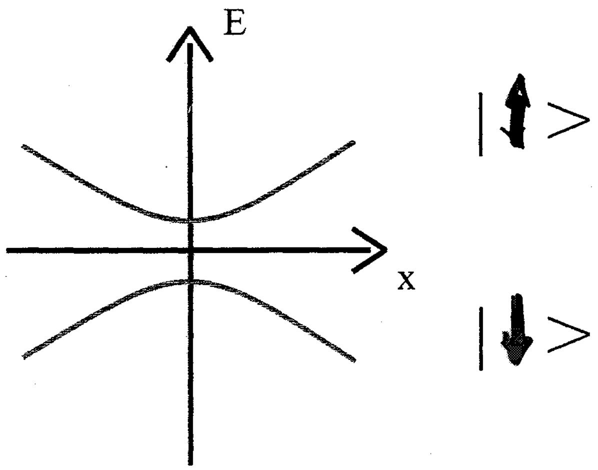
Zeeman splitting of spin states



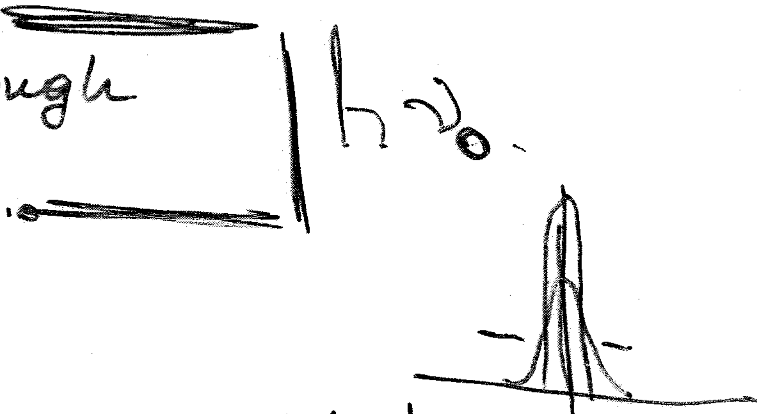


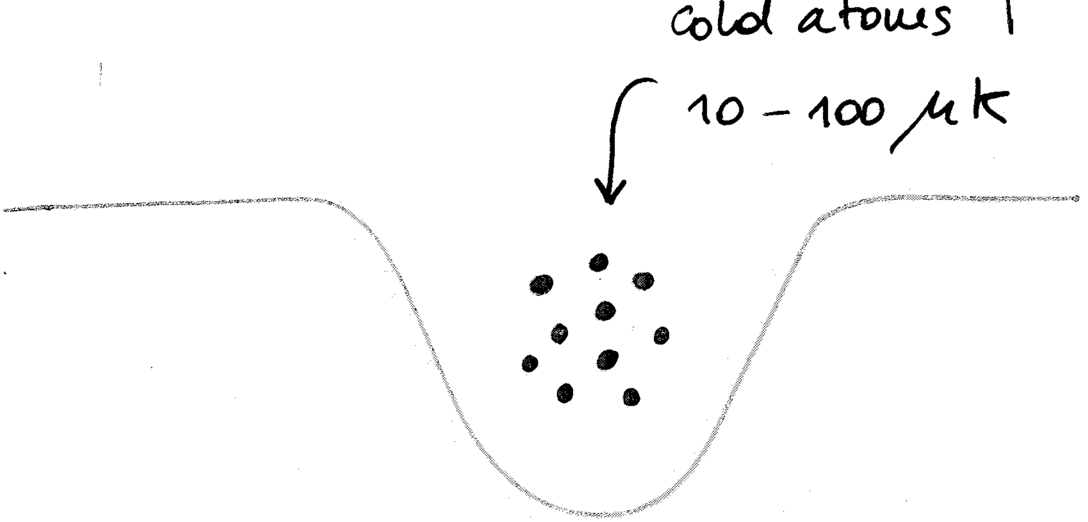
Trapping field

$B_{min}=0$ must be avoided (Majorana spin-flip)



atoms are cold but not enough



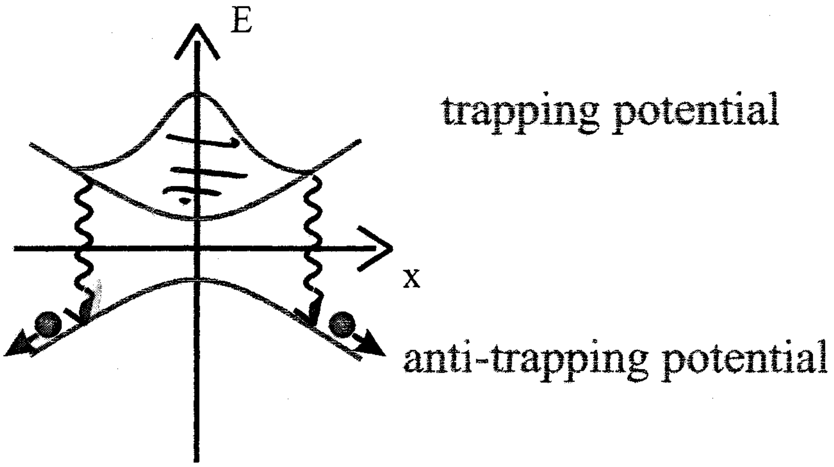


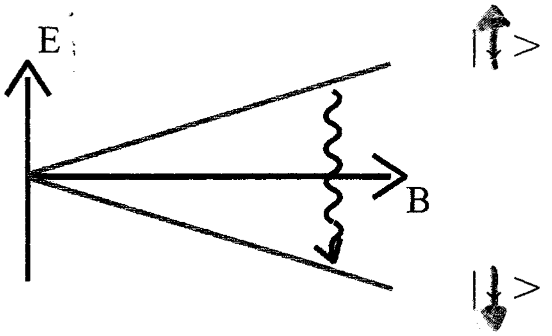
MAGNETIC TRAP

we put the atoms in a magnetic "cup"

. . . like coffee in a cup, they are cooled down by EVAPORATION

EVAPORATIVE COOLING





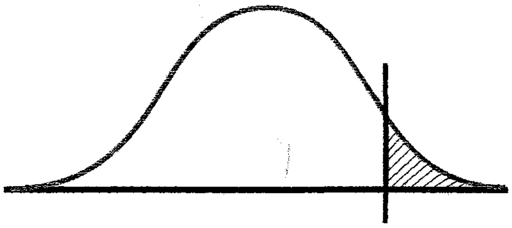
rf drives spin-flip at a specific value of |B|

$$\nu = \Delta E / h$$

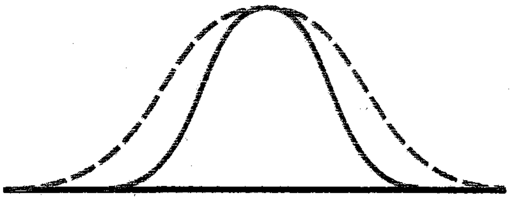
Atoms with energy$\gg$ <E> are ejected by the rf-induced spin-flip. Sweeping the rf frequency to lower values the temperature goes down.

RUNAWAY EVAPORATION

velocity distribution (Maxwell-Boltzmann)



rf cut



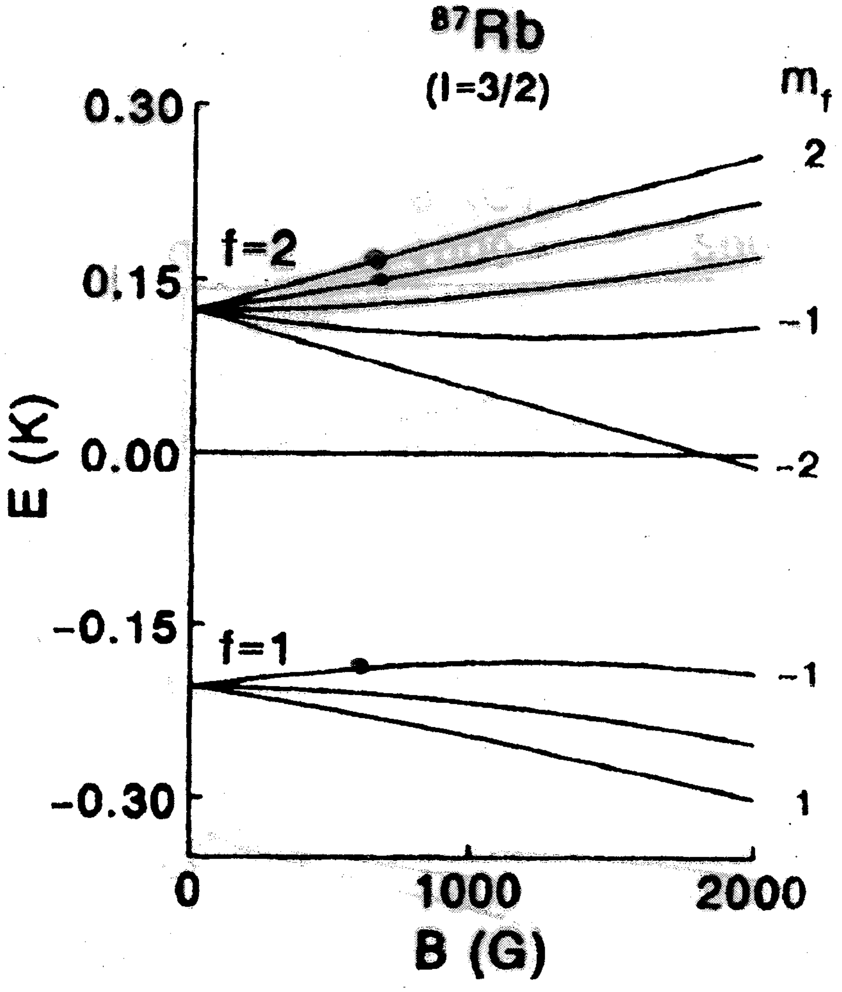
thermalization
(elastic collisions)

magnetic trap lifetime τ

elastic collision rate $(n\sigma v)^{-1}$

$n\sigma v\ \tau > 200 \Rightarrow$ runaway evaporation

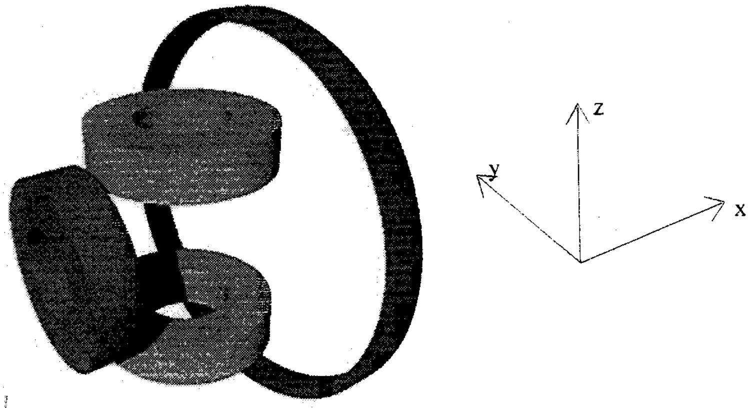
we need interactions !



⁸⁷Rb
(I=3/2)
m_f
2
−1
−2
−1
1
f=2
f=1
0.30
0.15
0.00
−0.15
−0.30
E (K)
0
1000
2000
B (G)

with L. RICCI - TRENTO

MAGNETIC TRAP
(Ioffe-Pritchard type)



$$B=B_0+(B''/2)x^2+[B'^2/(2B_0)](y^2+z^2)$$

$$V=-\mu\cdot\mathbf{B}$$

$$V(x,y,z)=m/2\,\omega^2[(y^2+z^2)+\lambda^2x^2]$$

$$\lambda=\omega_x/\omega_r$$

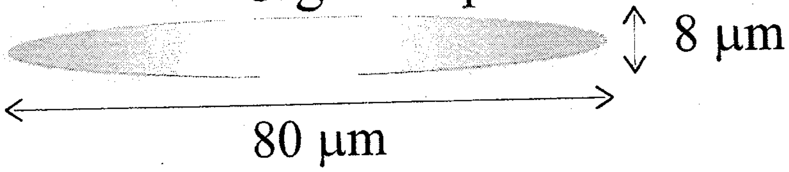
$$\omega_x=\sqrt{\mu B''/m}$$

$$\omega_y=\omega_z=\omega_r=\omega_x=\sqrt{(\mu B'^2)/(mB_0)}$$

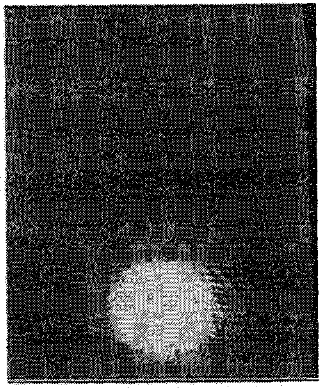
typical values

$$@\ I=240\ A \Rightarrow \begin{array}{l} B_0=2G \\ \omega_x=2\pi\cdot 12.6\ Hz \\ \omega_r=2\pi\cdot 160\ Hz \end{array}$$

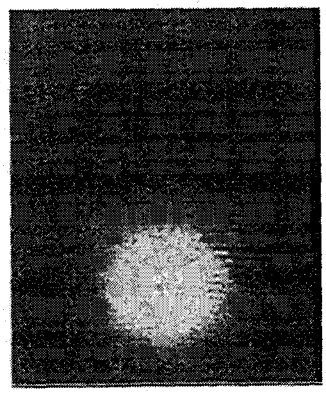
Cigar shaped



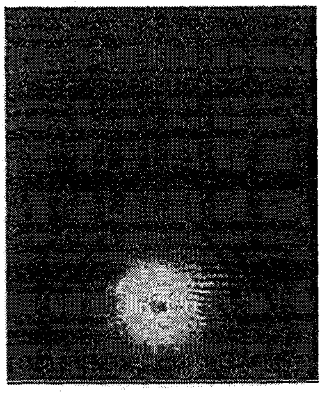
BEC TRANSITION



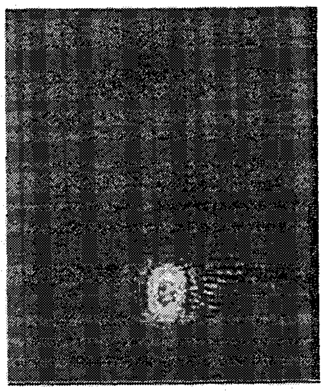
0.94 MHz



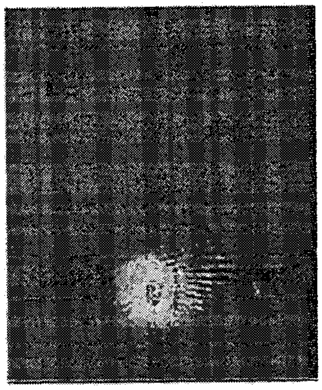
0.93 MHz



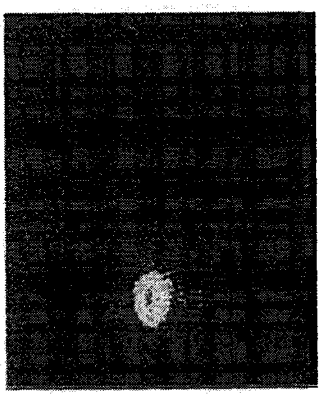
0.92 MHz



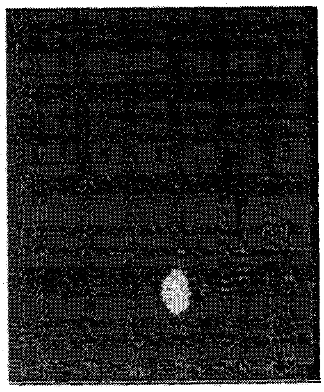
0.91 MHz



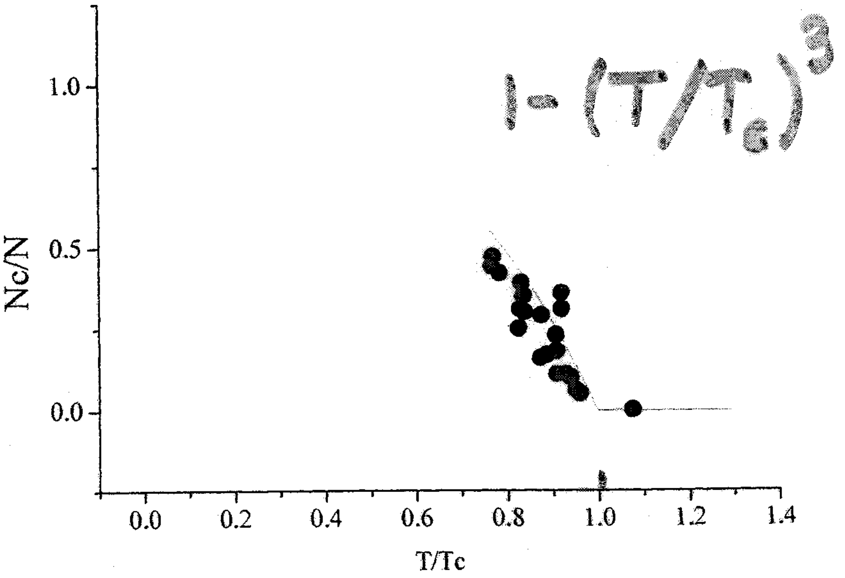
0.90 MHz



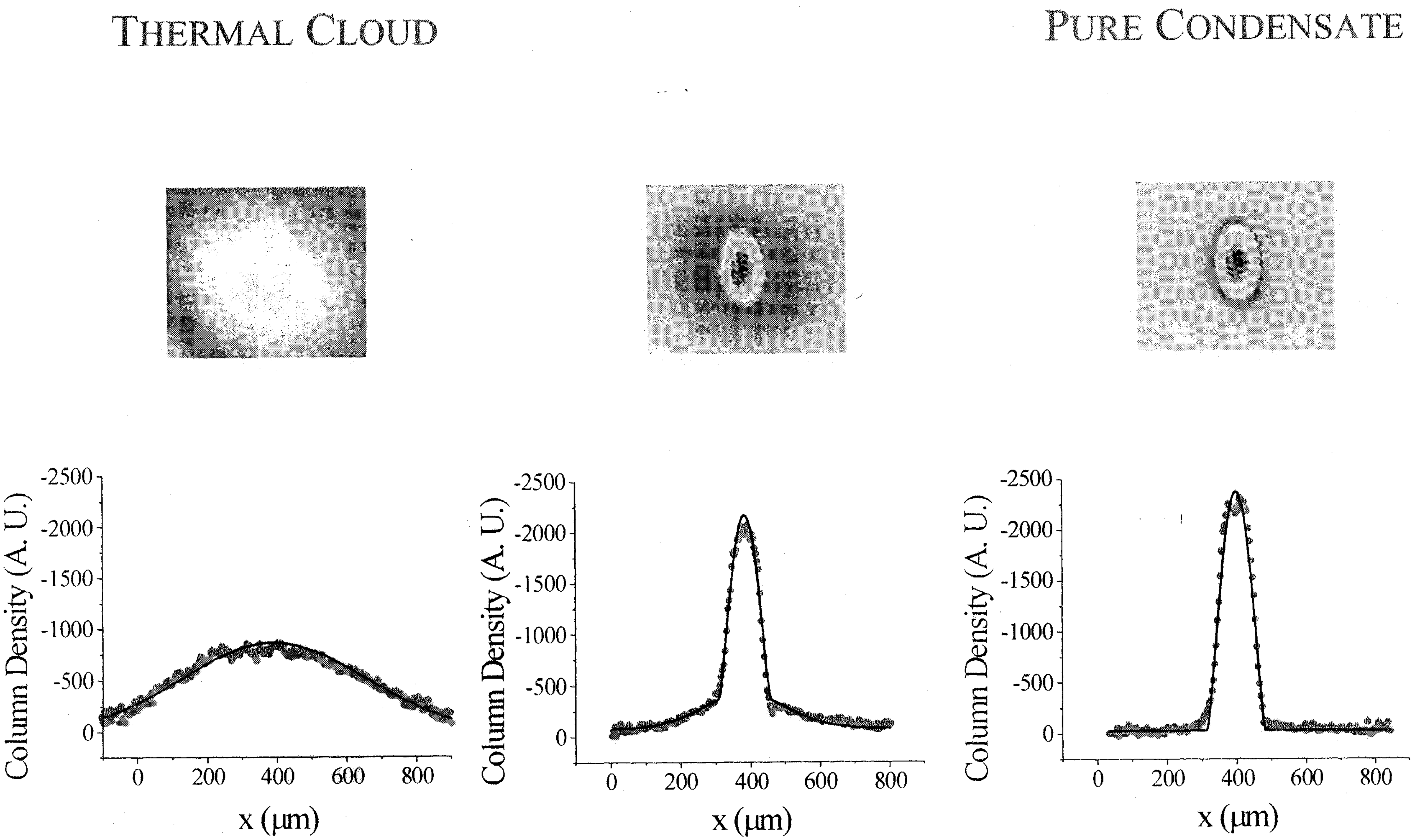
0.89 MHz

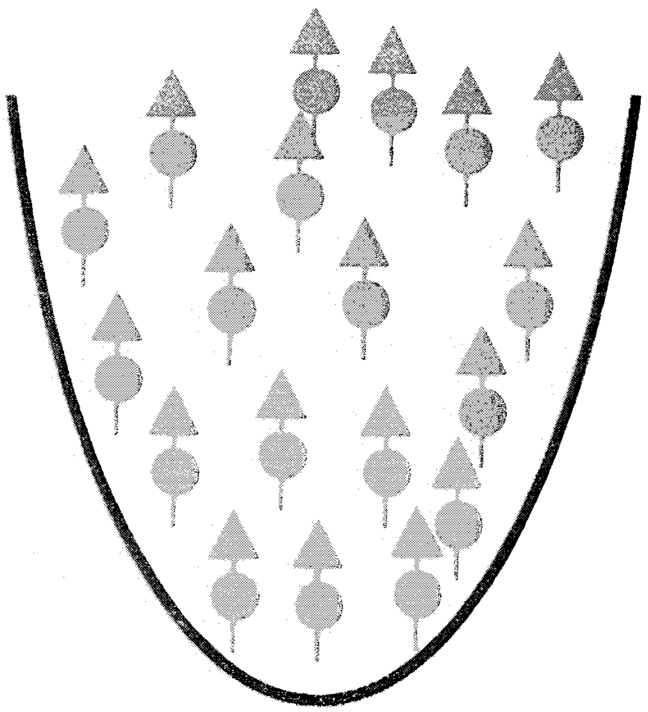


0.88 MHz



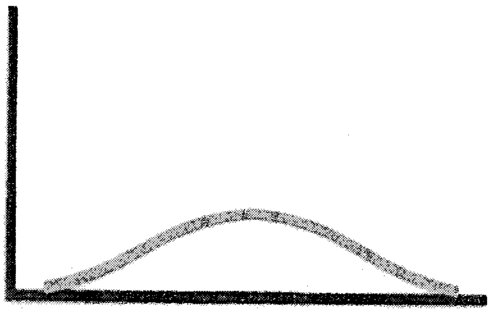
CONDENSED FRACTION


THERMAL CLOUD
PURE CONDENSATE
Column Density (A. U.)
-2500
-2000
-1500
-1000
-500
0
0
200
400
600
800
x (μm)
Column Density (A. U.)
-2500
-2000
-1500
-1000
-500
0
0
200
400
600
800
x (μm)
Column Density (A. U.)
-2500
-2000
-1500
-1000
-500
0
0
200
400
600
800
x (μm)



$T > T_c$

Gaussian distribution



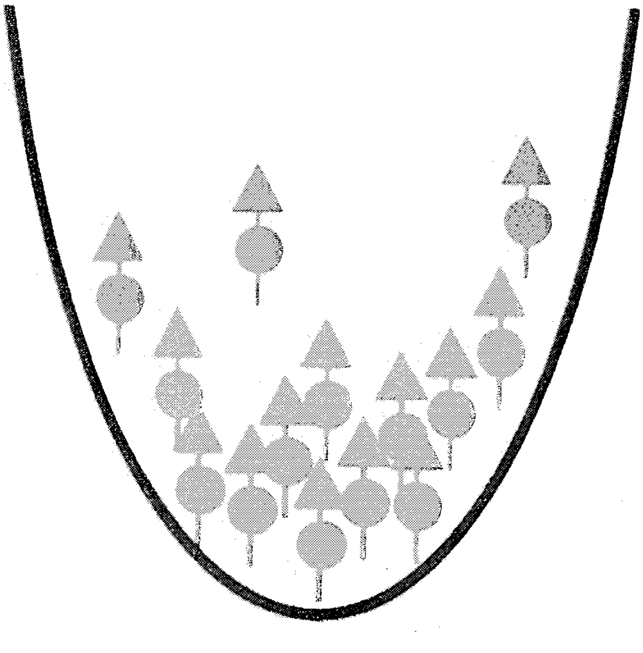
$$\omega_{ho} = (\omega_x \omega_y \omega_z)^{1/3}$$

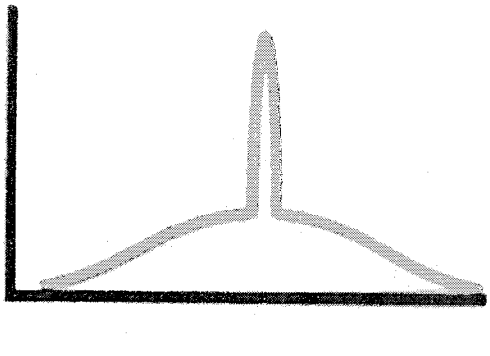
$$T_c = \frac{\hbar \omega_{ho}}{K} 0.94 N^{1/3}$$

$$\sim 90\,nK \; (N \sim 10^5)$$

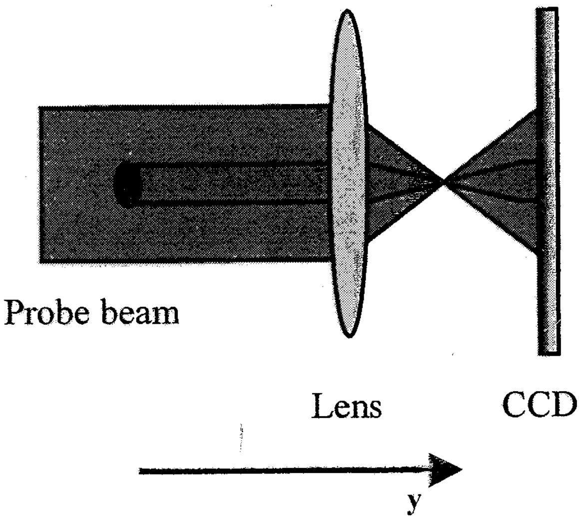
$T < T_c$

Ground-state peak





Absorption imaging



Column density

$\tilde{n}(x,z) = \int n(x,y,z)dy$

The column density is fitted with the assumption that n is the sum of two distributions

Gaussian (uncondensed fraction)

- temperature of the cloud

$$k_B T = m\omega_x^2\sigma_x^2 \quad \text{(equipartition principle)}$$

+

Inverted Parabola (condensed fraction) which is the solution of the G-P equation in the Thomas-Fermi approximation

WE OBTAIN

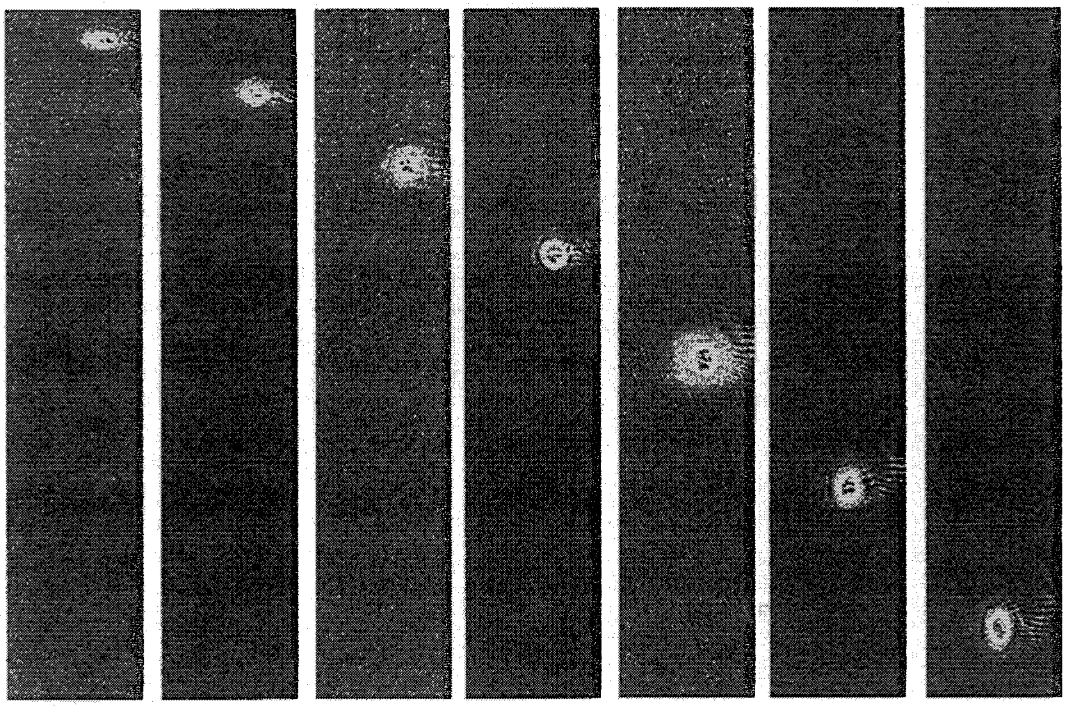
- Number of atoms
- Dimensions of the condensate (R_x, R_z) ⇒ A.R. oscillations
- Coordinates of the center of mass (C_x, C_z) ⇒ sloshing

Typical values:

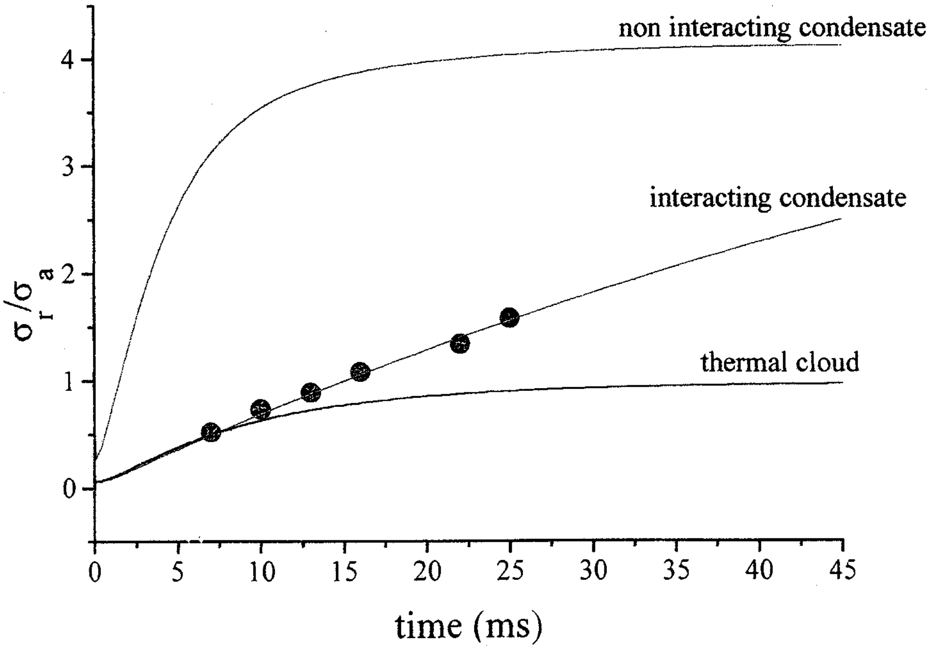
F=2 $N \sim 10^5$ $R_x \cong 40\ \mu m$, $R_z \cong 4\ \mu m$

F=1 $\sim 10^6$

EVOLUTION OF THE ASPECT RATIO DURING THE EXPANSION

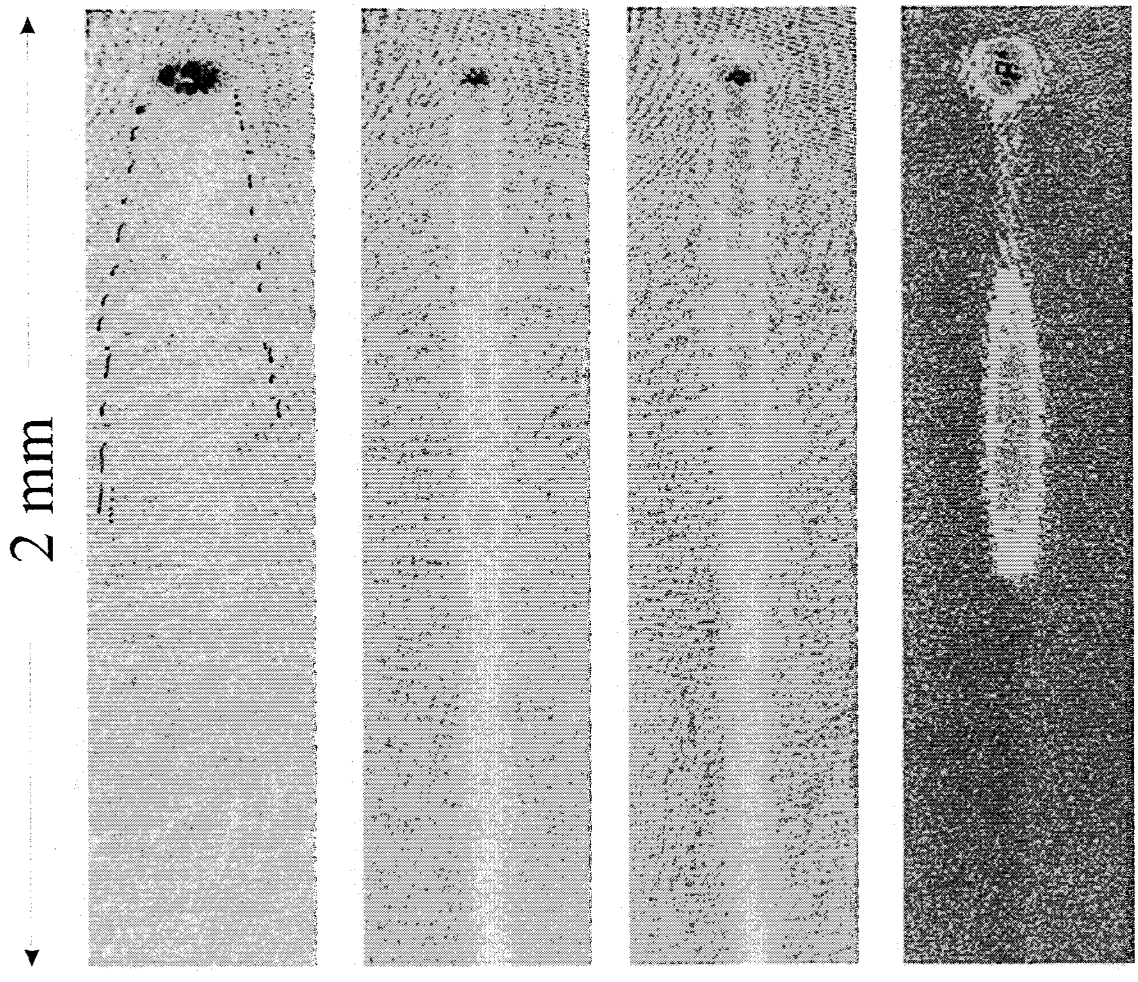


7ms 10ms 13ms 16ms 19ms 22ms 25ms



CW
ATOM LASER

@ LENS.UNIFI.IT



$T > T_c$

MINARDI, FORT, MADDALONI, INGUSCIO
in "BEC and ATOM LASERS"
COND-MAT/0002171

RAMSEY TYPE INTERFEROMETER WITH BOSE EINSTEIN CONDENSED ATOMS

An assembly of Bose Einstein condensed atoms has a definite phase?

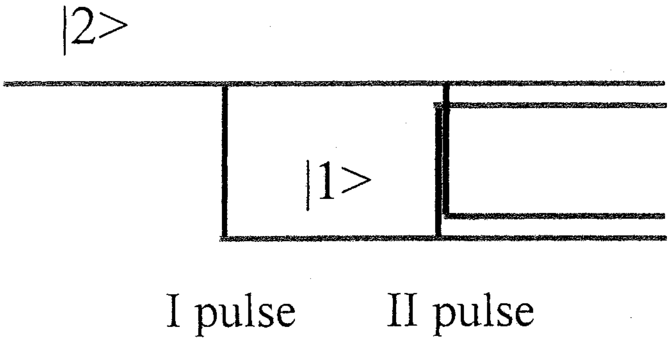
$$\Phi(r,t) = \sqrt{n(r,t)}e^{iS(r,t)}$$

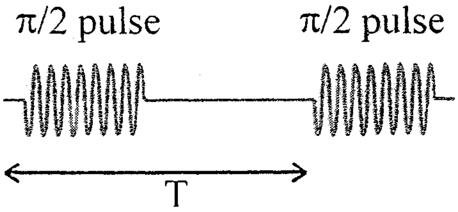
macroscopic wavefunction with a definite phase

How does the phase evolve?

Interferometers measure the relative phase between two BECs

Ramsey interferometer





initial state

$$|2\rangle$$

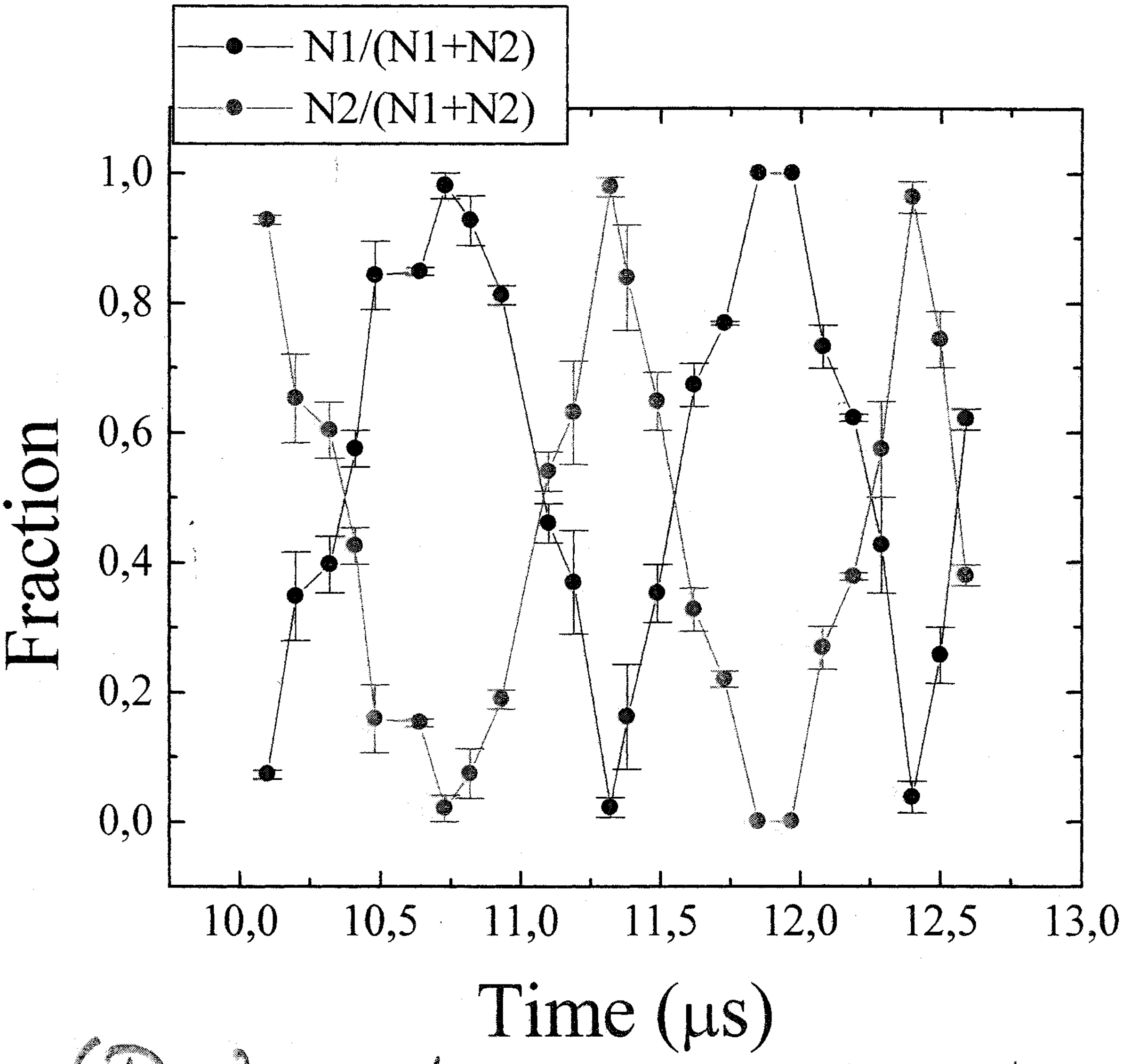
I pulse (“splitter”)

$$\frac{i}{\sqrt{2}}|1\rangle e^{-\frac{iE_1}{\hbar}t}+\frac{1}{\sqrt{2}}|2\rangle e^{-\frac{iE_2}{\hbar}t}$$

II pulse (“recombiner”)

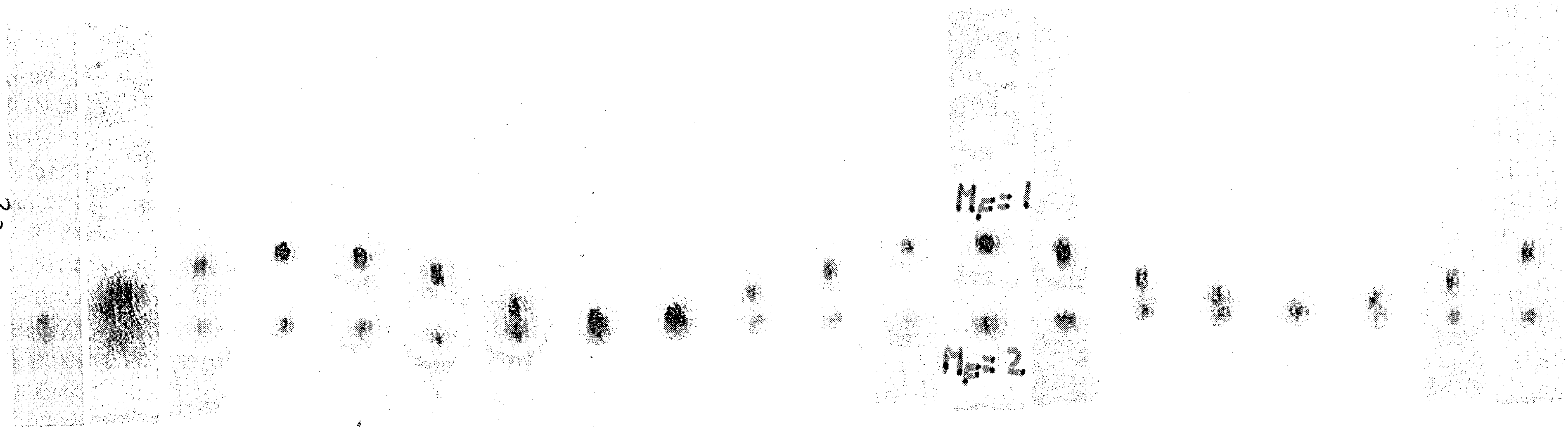
$$\frac{i}{\sqrt{2}}\left(e^{i\omega T}+1\right)|1\rangle+\frac{1}{\sqrt{2}}\left(1-e^{i\omega T}\right)|2\rangle \qquad \omega=(E_2-E_1)/\hbar$$

RAMSEY TYPE OSCILLATIONS



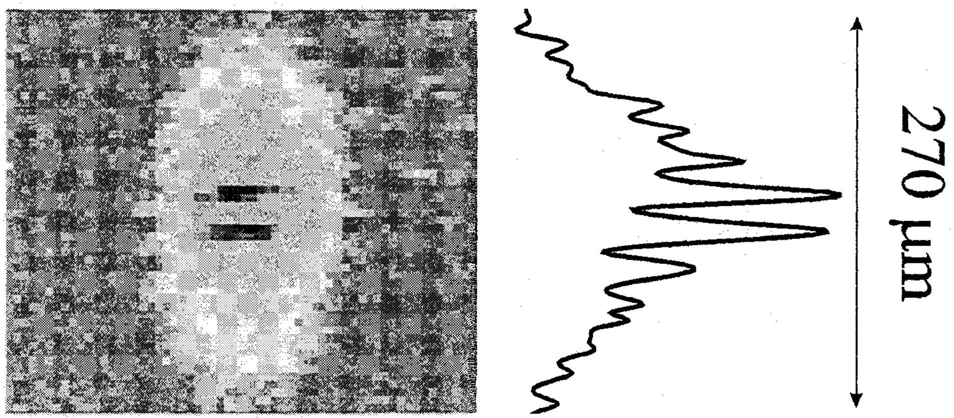
(Delay between the two pulses)

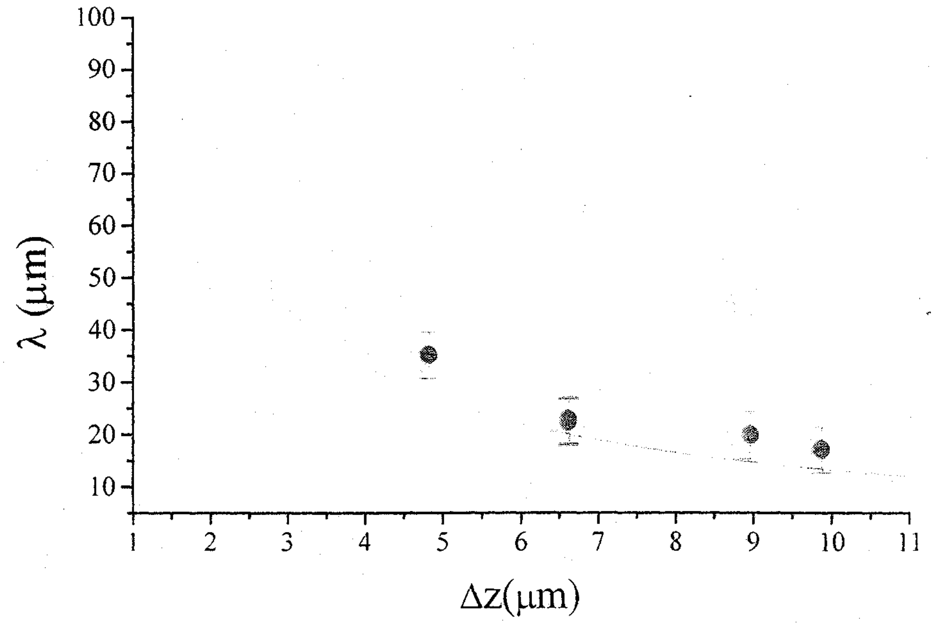
Two-Component Condensate



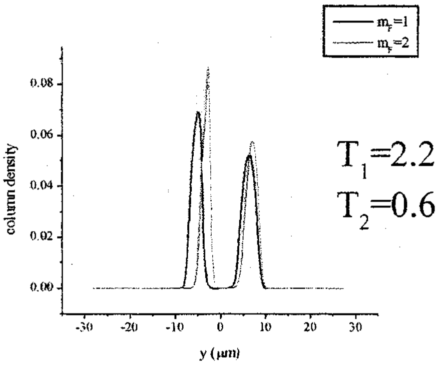
t = (26 + n 1) ms +20 ms (expansion)

$\Delta y > 2R$

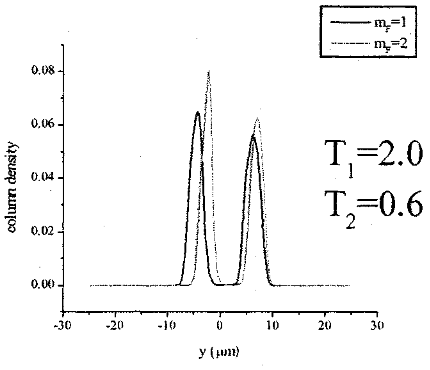




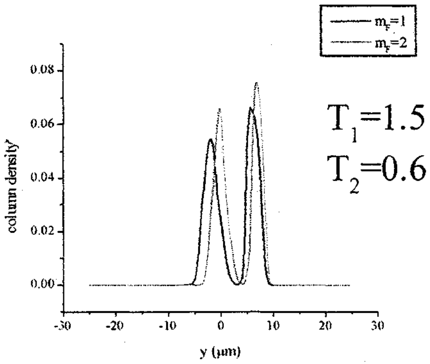
Column density – two coupled Gross Pitaevskii Equations



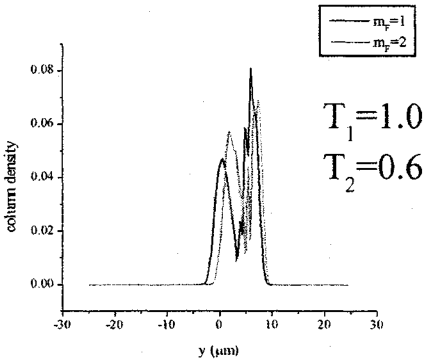
T_1=2.2 ms
T_2=0.6 ms



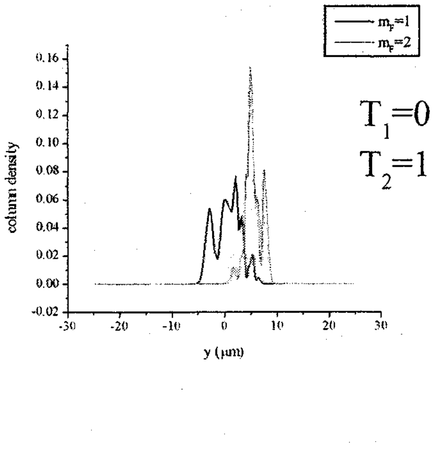
T_1=2.0 ms
T_2=0.6 ms



T_1=1.5 ms
T_2=0.6 ms



T_1=1.0 ms
T_2=0.6 ms



T_1=0.5 ms
T_2=1.6 ms

$$\langle \Psi_2 | \Psi_1 \rangle < 1$$

- spatial separation between the two condensates → decrease of the overlap region
- spatial modulation of the relative phase accumulated between the two pulses

evolution of the condensate phase

$$\Psi \equiv |\Psi| e^{i\Phi}$$

$$\Phi(r,t) = \underbrace{\alpha_x(t)x^2 + \alpha_y(t)y^2 + \alpha_z(t)z^2}_{\text{mean field expansion}} + \underset{\substack{\uparrow \\ \text{center of mass motion}}}{\beta(t)z}$$

numerical simulations:

$$\Phi_2 = const.$$

$$\Phi_1 \cong \frac{m}{\hbar}\left(v_0(t) + \delta v(t)\right)z$$

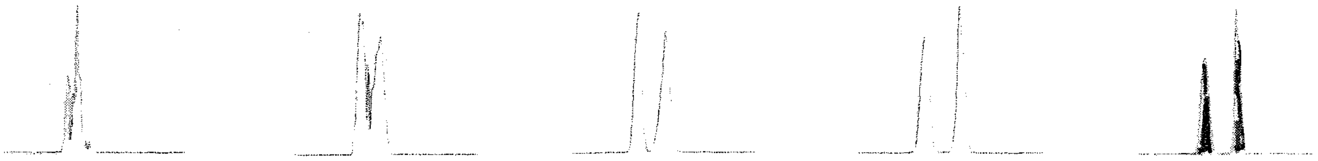
$v_0(t) = \dfrac{g}{2\omega_{r1}} sin(\omega_{r1} t) \approx \dfrac{g}{2} t$ velocity acquired during the fall in the trapping potential

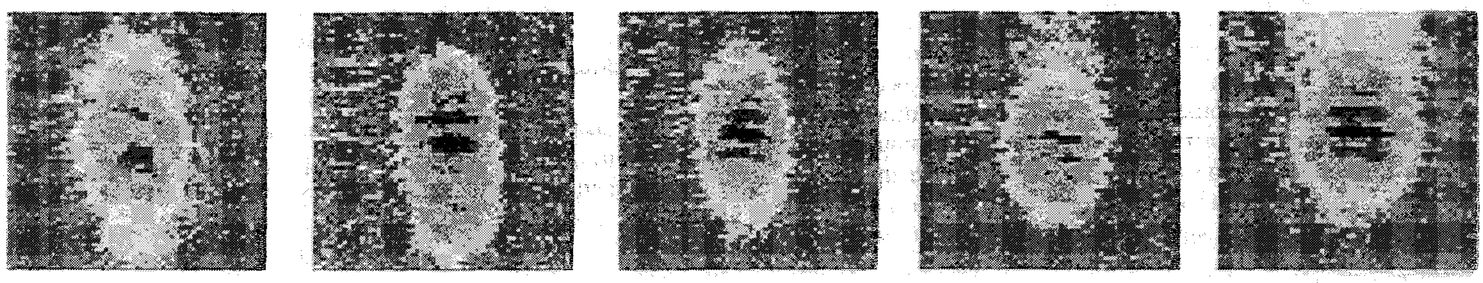
$\delta v(t) \approx 0$ velocity due to the mutual repulsion between the condensates

$$\Downarrow$$

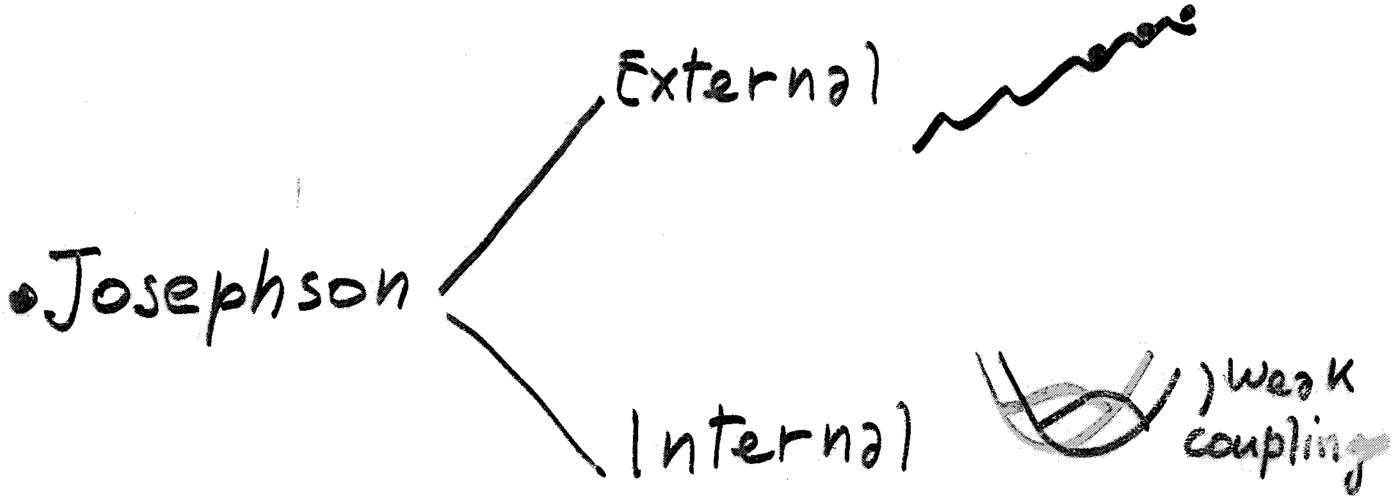
$$\lambda_v \cong \frac{2h}{mgt}$$

$t > \dfrac{2h}{mgR} \approx 200\,\mu s$	$\Rightarrow$	the relative phase between the two condensates is no more spatially uniform





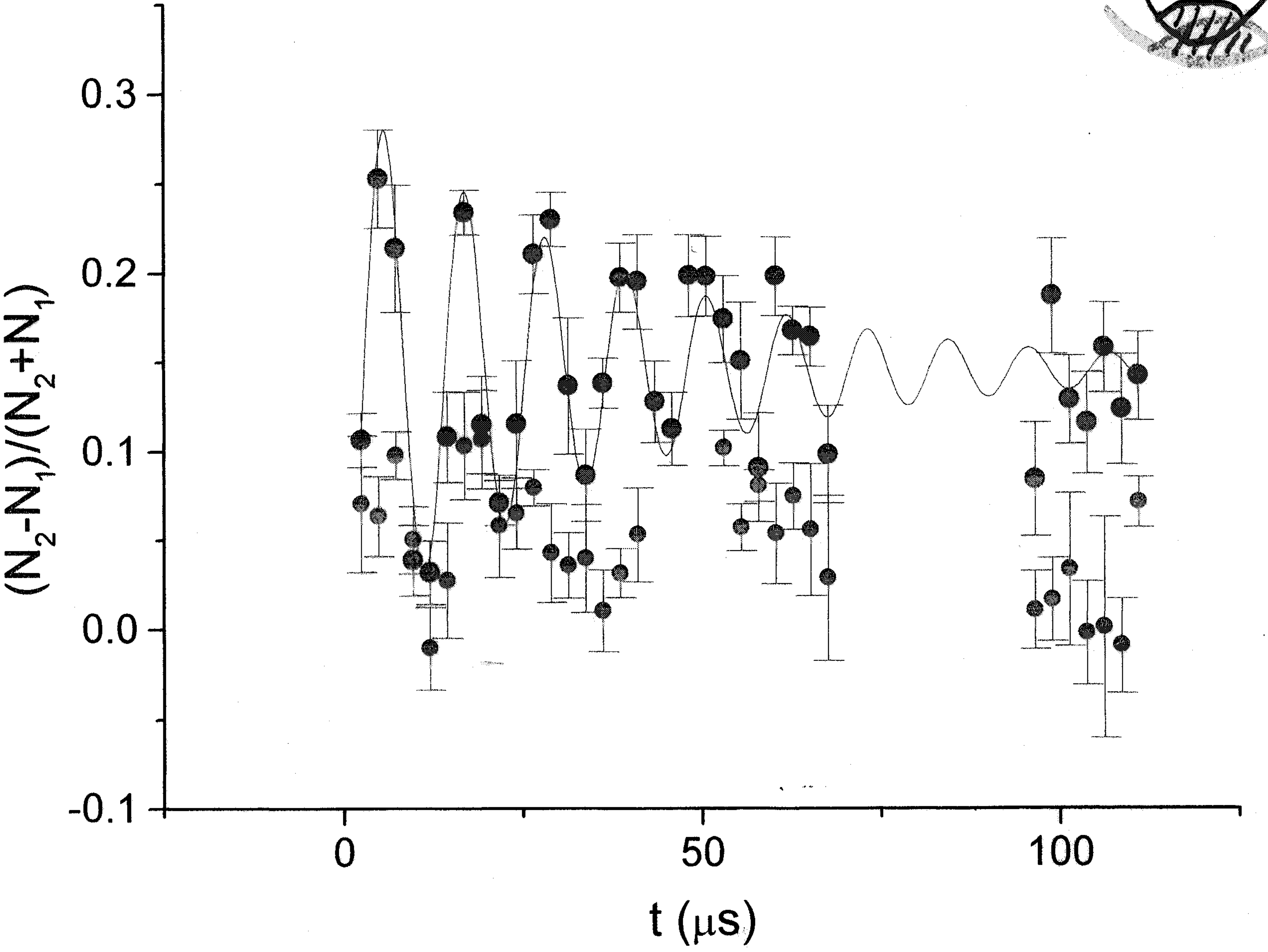
MORE AT LARGE



- Josephson
  - External
  - Internal

-
-

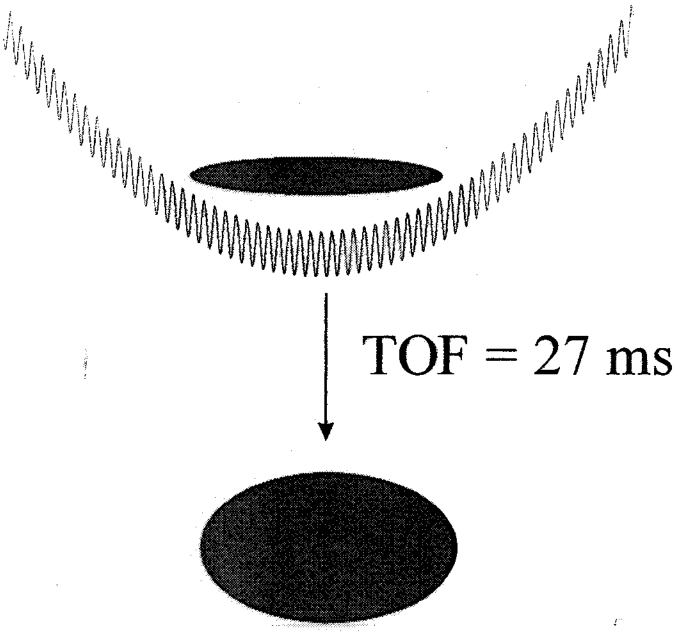
Quantum Macroscopic Oscillations



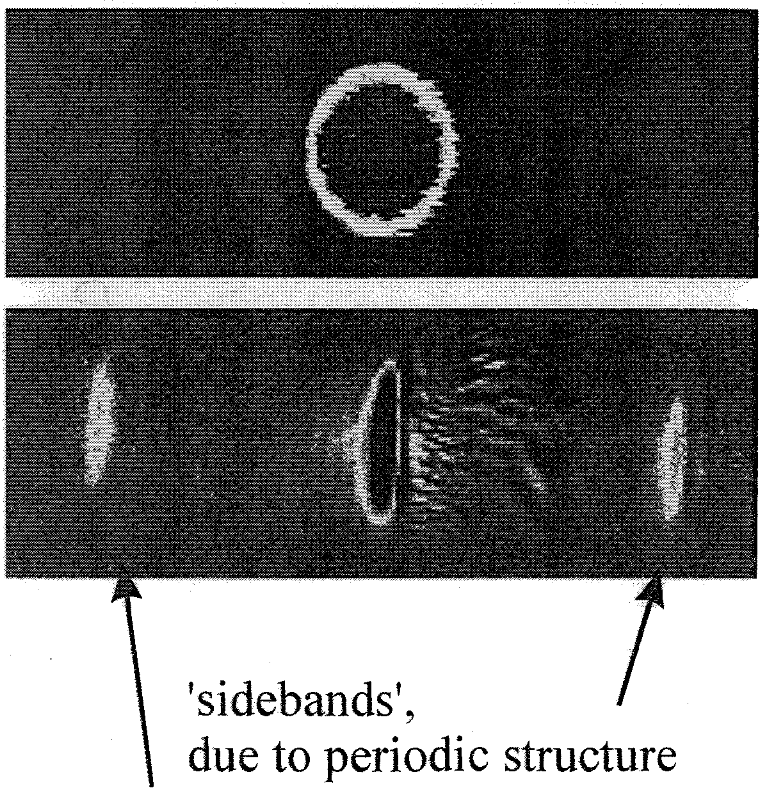
BEC in an optical lattice

Studies of the properties of the BEC:

Free expansion of the ground state



TOF = 27 ms



absorption images of the atomic distribution

'sidebands',
due to periodic structure
of the ground state

'optical analogies':
phase-locked modes of a laser, diffraction from grating