

the
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SMR 1302 - 3

WINTER SCHOOL ON LASER SPECTROSCOPY AND APPLICATIONS

19 February - 2 March 2001

SPECTROSCOPY OF COLLIDING COLD ATOMS

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These are preliminary lecture notes, intended only for distribution to participants.

Spectroscopy of colliding cold atoms

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Program:

Lect. I - Basic Concepts

- General Introduction
- Cooling atoms with laser beams
- Trapping atoms
- Examples of applications

Lect. II - Cold collisions in the presence of light

- Exoergic collisions: homonuclear
- Exoergic collisions: heteronuclear
- New model at low intensity limit

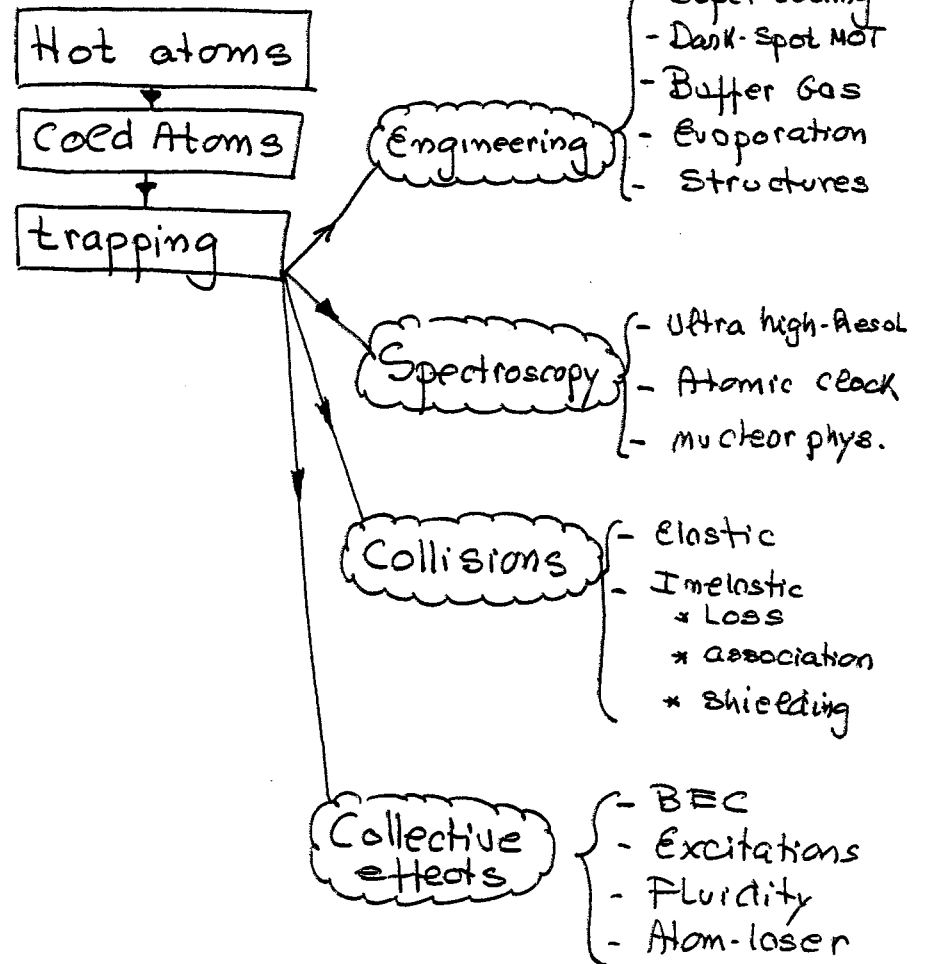
Lect. III - Cold collisions and chemical reaction

- Photoassociative spectroscopy
- Photoassociative ionization
- Shielding collisions with photons
- New ideas

Extra - Bose Einstein-Condensation

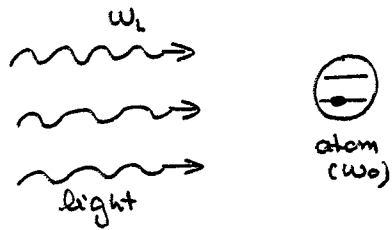
I - Basic Concepts

I.1) Introduction



⇒ Cold atoms have expanded the frontiers of atomic and molecular physics

* Radiation Pressure



Dipole Interaction

$$\Omega = -\frac{\vec{\mu} \cdot \vec{E}_0}{\hbar}$$

* Spontaneous force

- absorption $\Delta \vec{p} = \hbar \vec{k}$
- emission $\langle \vec{p} \rangle = 0$

$$\vec{F}_s = (\text{absorption rate}) \hbar \vec{k} = \hbar \vec{k} \frac{1}{2} \left(\frac{s}{s+1} \right)$$

$s \equiv$ saturation parameter

* Dipole force

- Induced dipole interacting with field gradient

$$\vec{F}_d = -\frac{1}{2} \frac{\nabla \Omega^2}{\Omega^2} \hbar \Delta \omega_L \left(\frac{s}{1+s} \right)$$

$\Delta = \omega_L - \omega_0$ (detuning)

\vec{F}_d is conservative; \vec{F}_s is not

$\Rightarrow \vec{F}_s$ can be used to cool atoms

\vec{F}_d can confine atoms

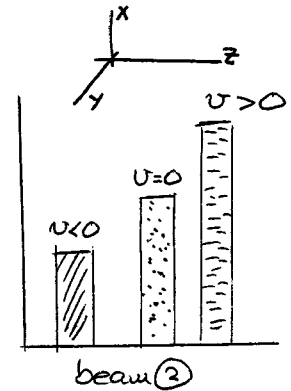
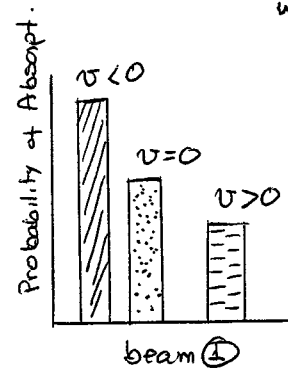
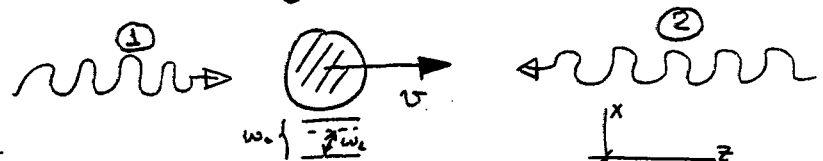
How to cool?



- deceleration z-axis
- diffusion x, y,
- Doppler effect can select absorption

Ex: Laser deceleration of an atomic beam

* Two light beams ($\Delta < 0$) + One atom

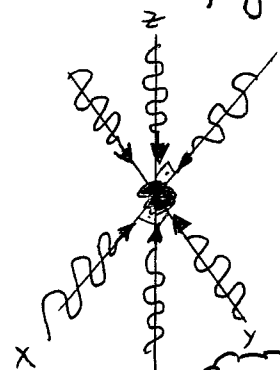


Net result:

* force opposed to velocity
 $m \frac{dv}{dt} = -\alpha v$

* Viscous force $\Delta < 0$; Energy remove \Rightarrow Optical Molasses

- 3D configuration:



- * 6 laser beams create a 3D viscous medium
- * No confinement
- * Atoms cool down

$$m \frac{d\vec{v}}{dt} = -\alpha \vec{v}$$

$\alpha = \alpha(\Delta, \Omega)$

Damping comes from spontaneous force
 → depends on spontaneous emission

→ Random motion

→ Heating

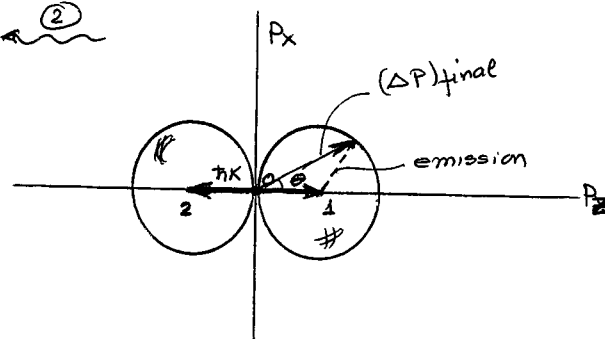
Balance between cooling and heating
 takes to a limit \Rightarrow Doppler
 Temperature

* Equation for atom Kinetic Energy

$$\frac{dK_i}{dt} = v_i F_i + H_i$$

$$K_i = \frac{1}{2} m v_i^2 \quad i = x, y, z$$

2 laser beams in z-direction



* After one cycle of absorption-emission
 atom can end up in any point of
 a surface in p-space composed
 of two spheres of radius $\hbar k$

* Averaged momentum variation

$$\langle (\Delta p_x)^2 \rangle = \langle (\Delta p_y)^2 \rangle = \frac{(\hbar k)^2}{3}$$

$$\langle (\Delta p_z)^2 \rangle = \frac{4}{3} (\hbar k)^2$$

* Absorption - Emission rate = $\frac{\Gamma}{2} \cdot S$ (per beam)

$$S = \frac{\frac{1}{2} \Omega^2}{\Delta^2 + \frac{1}{4} \Gamma^2}$$

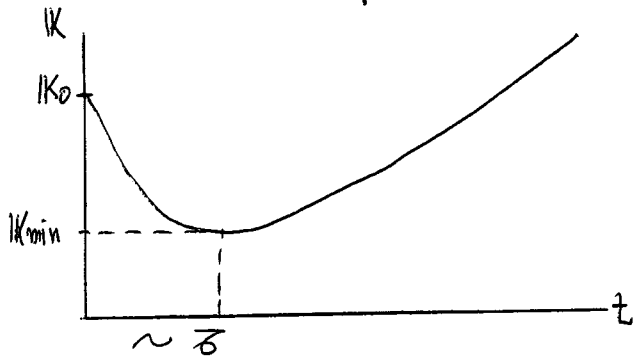
* Heating rate

$$H_i = \underbrace{2}_{\substack{\uparrow \\ \text{2 beams}}} \frac{\Gamma}{2} S \frac{\langle (\Delta p_i)^2 \rangle}{2m}$$

$$\frac{dK_i}{dt} = v_i F_i + H_i = +v_i^2 \alpha + H_i = +2 \alpha \frac{K_i}{m} + H_i$$

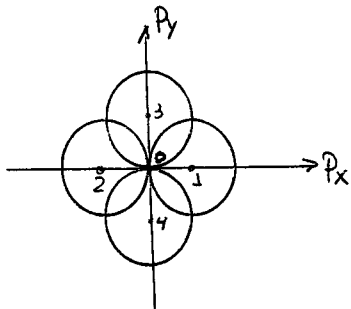
$$K(t) = \sum K_i = K_0 + (e^{-t/\bar{\tau}} - 1) K_{0z} + \beta [t - \bar{\tau}(e^{-t/\bar{\tau}})]$$

$$\bar{\tau} = \frac{(\Delta^2 + \frac{1}{4}\Gamma^2)m}{4\Gamma\hbar k^2 |\Delta|}$$



Heating dominates \Rightarrow needs to cool all directions

6 laser beams

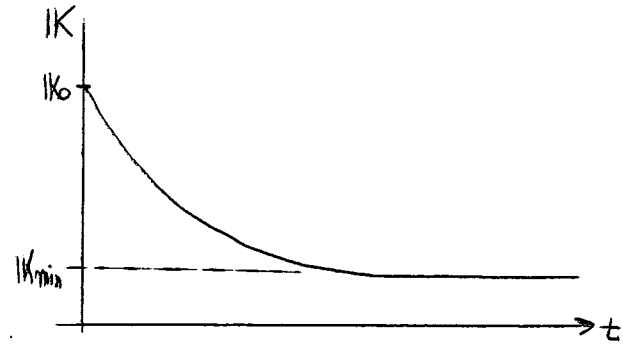


$$\langle (\Delta p_i)^2 \rangle = \frac{2}{3} (\hbar k)^2$$

$$\frac{dK}{dt} = 2\Gamma\hbar k^2 s \frac{2\Delta}{(\Delta^2 + \frac{1}{4}\Gamma^2)} \frac{K}{m} + 3\Gamma s \frac{(\hbar k)^2}{m}$$

(\$\Delta < 0\$)

$$K(t) = (K_0 - K_{min}) e^{-t/\bar{\tau}} + K_{min}$$



$$K_{min} = \frac{3\hbar(\Delta^2 + \frac{1}{4}\Gamma^2)}{4|\Delta|} \quad (\text{Doppler Limit})$$

$$\bar{\tau} = \frac{(\Delta^2 + \Gamma^2/4)m}{4\Gamma\hbar k^2 |\Delta| s}$$

Since \$K_{min}\$ depends on \$\Delta \to\$ optimization

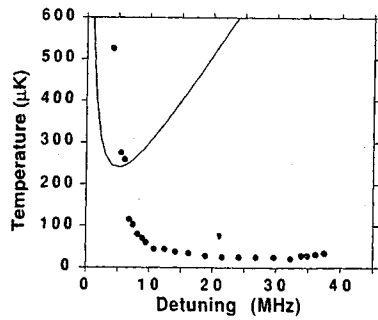
$$\frac{dK_{min}}{d\Delta} = 0 \Rightarrow$$

$$(K_{min})_{min} = \frac{3}{4} \hbar \Gamma$$

$$\Delta = -\Gamma/2$$

System	Na	Cs	K	Rb	Li
Doppler Limit	237 \$\mu\$K	125	137	141	140

* Measurements of $T_{min}(\Delta)$



→ Much lower value can be obtained
 Explanation: Multi-level nature of the atoms together with optical pumping

Consider a $J = 1/2 \rightarrow J' = 3/2$ cooling transition

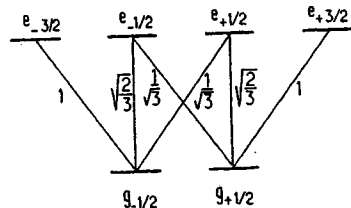
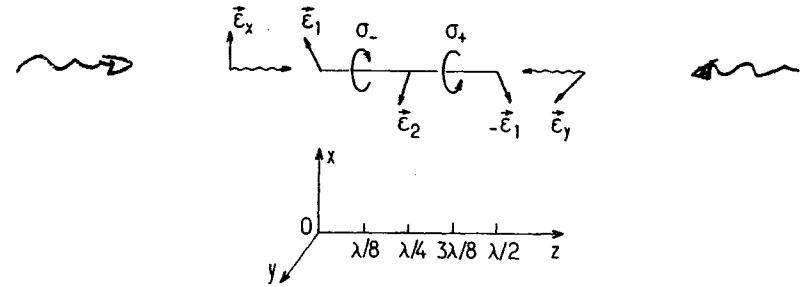


Fig. 2. Atomic level scheme and Clebsch-Gordan coefficients for a $J_g = 1/2 \rightarrow J_e = 3/2$ transition.

→ Light polarization can accumulate population in a certain level

→ Consider polarization as follows:



* There is polarization variation as a function of position

- $g_{-1/2}$ state higher population when σ^-
- $g_{+1/2}$ state higher populated when σ^+

* Since interaction is also dependant of polarization \rightarrow Stark shift of levels ($\sim \frac{h \Omega^2}{\Delta}$) oscillates with position. Δ

Combination of light shift and population

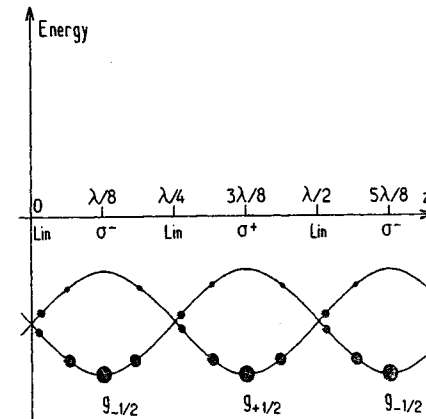
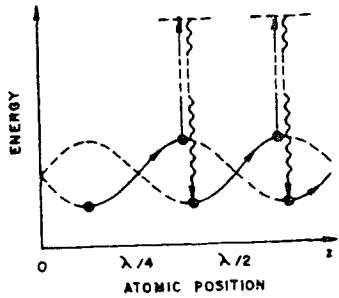


Fig. 3. Light-shifted energies and steady-state populations (represented by filled circles) for a $J_g = 1/2$ ground state in the lin \perp lin configuration and for negative detuning. The lowest sublevel, having the largest negative light shift, is also the most populated one.

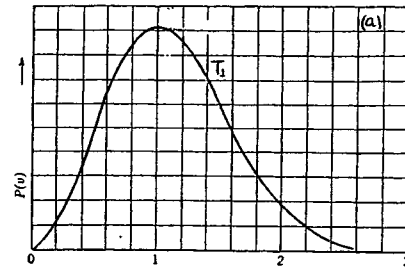
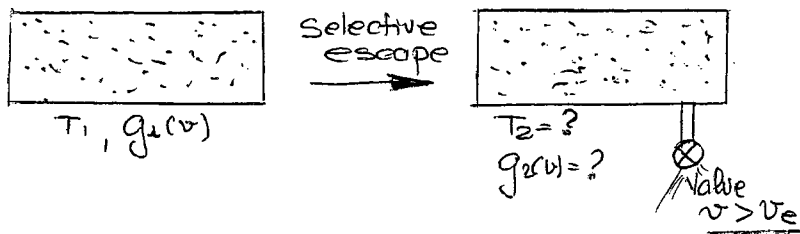
→ Atoms moving in this system will suffer the effects of both:
light-shift and optical pumping
⇒ Sisyphus cooling



* New limit ⇒ $(kT)_{min} \sim \frac{\hbar \Omega^2}{\Delta}$
→ good agreement with experim.

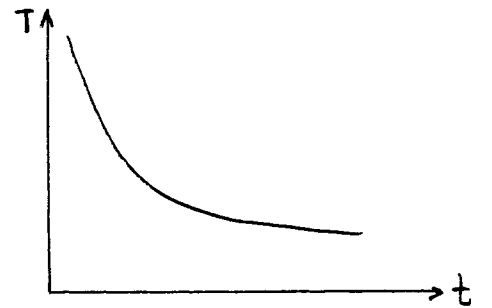
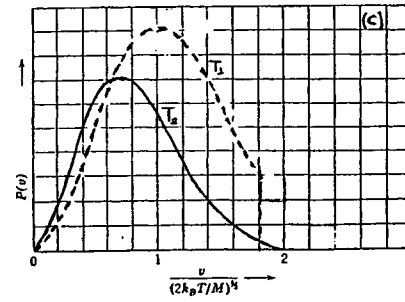
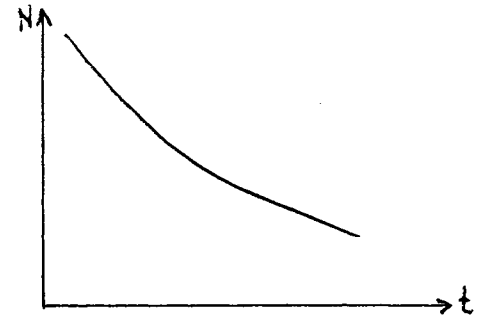
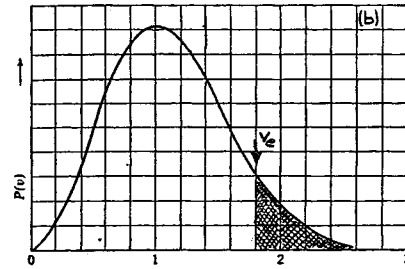
Evaporative Cooling

- * Does not involve laser but is very important
- * Imagine a confined gas



$$\int_0^{v_e} g_1(v, T_1) dv = \int_0^{\infty} g_2(v, T_2) dv$$

$T_2 < T_1$
⋮



⇒ Ultra-low temperatures can be obtained
 $T \sim 10^9 K$ is possible

Trapping

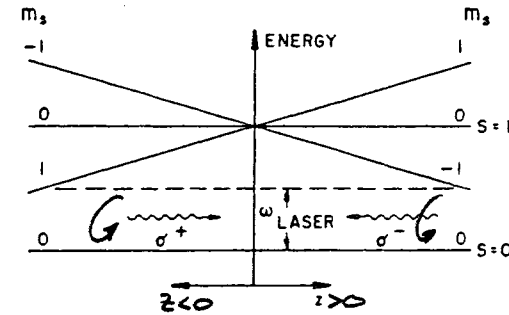
- * To trap is necessary first to cool
- * Trapping \rightarrow restoring force to equilibrium position
force \Rightarrow
 - * From light
 - * static B-field
 - * Oscillating fields

Magneto-Optical Trap (MOT)

* Use of internal structure of atoms to produce a spatial selection rule

* Consider:

- \rightarrow Atom with ground state $S=0$
- \rightarrow Excite state $S=1$
- \rightarrow Magnetic field with linear gradient (profile)
- \rightarrow Counter propagating lasers with circular polarization of opposite helicity. ($\Delta < 0$)



- * Selection rules
 - $\sigma^+ \rightarrow \Delta m_s = +1$
 - $\sigma^- \rightarrow \Delta m_s = -1$
- * Atom moving $z > 0$; transition $0 \rightarrow -1$ closer to resonance
 σ^- stronger \Rightarrow Force to the left
- * Atom moving $z < 0$; transition $0 \rightarrow +1$ closer resonant,
 $\rightarrow \sigma^+$ stronger \Rightarrow Force to the right.
- * Atom at $z = 0$, both transitions are equal \rightarrow No net force
 $\Delta < 0 \rightarrow$ damping force
 $B(z) \rightarrow$ restoring force

$$m \frac{d\vec{v}}{dt} = -\alpha \vec{v} - \kappa \vec{r}$$

* Using radiative force

$$\frac{\alpha}{2\pi} = \frac{16 \Omega_0^2 |\Delta| (k/\hbar)}{(1+2\Omega_0^2)^2 \left[1 + \frac{4\Delta^2}{1+2\Omega_0^2}\right]^2}$$

$$\frac{K}{2\pi} = \frac{16 \Omega_0^2 |\Delta|}{(1+2\Omega_0^2)^2 \left[1 + \frac{4\Delta^2}{1+2\Omega_0^2}\right]^2} \frac{d\omega}{dz}$$

→ System is normally over damped
Good!!!!

→ Capture velocity is small
BAD!!!!

$$v_c \sim 12-20 \text{ m/s}$$

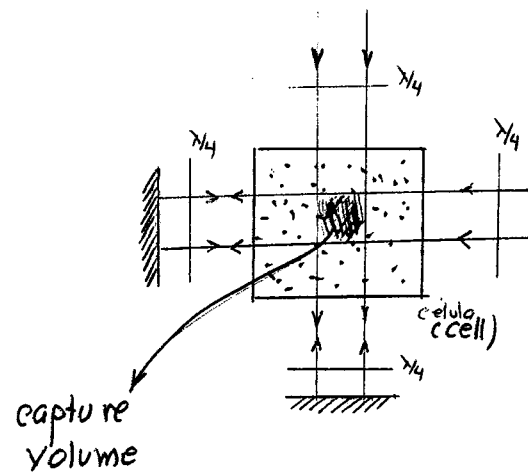
Room temperature ($v_{mp} \sim 500 \text{ m/s}$)
only very low tail of Maxwell distribution
can be captured

* First demonstration of MOT was
done with decelerated atomic beam

* Would be possible to capture
from vapor (Hot) phase?

YES, if you can accumulate the
atoms for long time

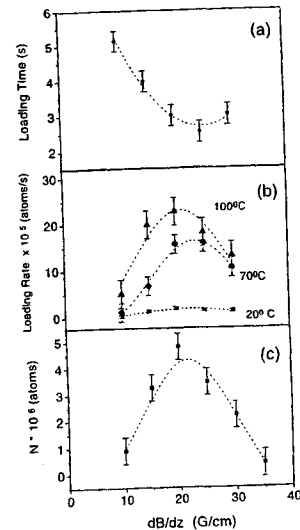
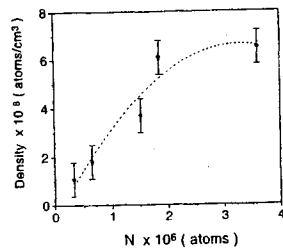
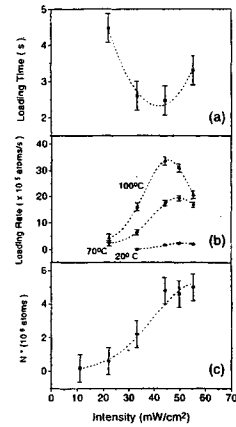
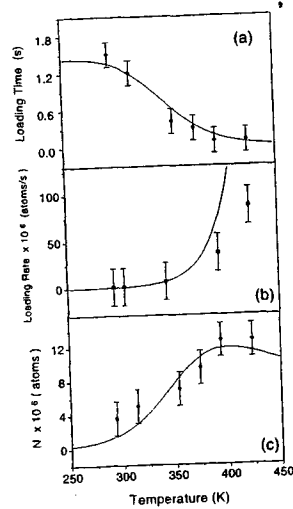
Capture → thermalize → Capture →



How it looks like?

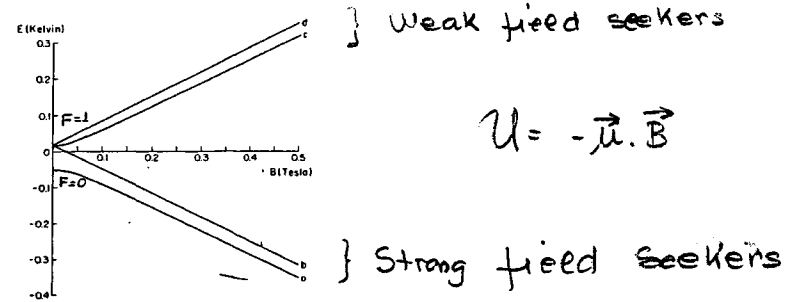
* MOT operation depends of several parameters \rightarrow needs optimization

* MOT always produce a mixture of atoms in both states (background and excited)

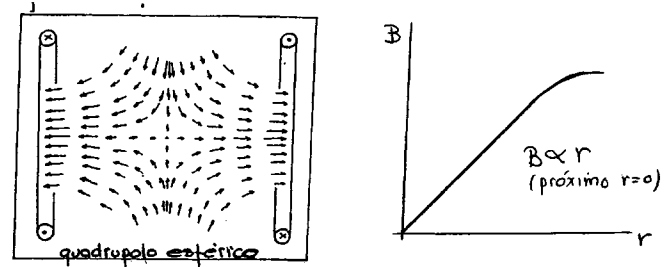


Magnetic Trapping

* Possible due to spin-field interaction



* Producing minimum of B-field \Rightarrow confinement of atoms

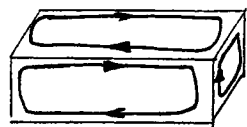


\Rightarrow Atoms can escape near $B=0$

trapping States $\xrightarrow{t_0}$ untrapping States

"Majorana Flops"
rate $\sim \frac{10^{-3}}{R^2}$ (mm³/s)

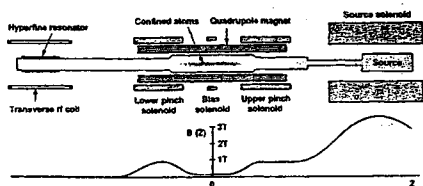
* Minimum of B-field with $B \neq 0$



transversal \Rightarrow quadrupole
longitudinal \Rightarrow disc solenoids



- * Solution at MIT (1987)
- * Experiments with hydrogen

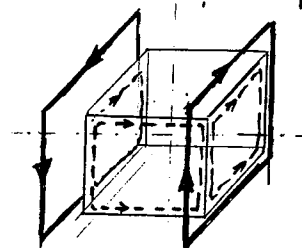


* Pure magnetic trap are not very deep.
Atoms must be already cold.



$$\begin{aligned} (1000\text{K}) \quad v &= 1000\text{m/s} \rightarrow \Delta B = 2 \cdot 10^7\text{G} \\ (10\text{K}) \quad v &= 100\text{m/s} \rightarrow \Delta B = 2 \cdot 10^6\text{G} \\ (10^{-1}\text{K}) \quad v &= 10\text{m/s} \rightarrow \Delta B = 2 \cdot 10^5\text{G} \\ (10^{-3}\text{K}) \quad v &= 1\text{m/s} \rightarrow \Delta B = 2 \cdot 10^3\text{G} \end{aligned}$$

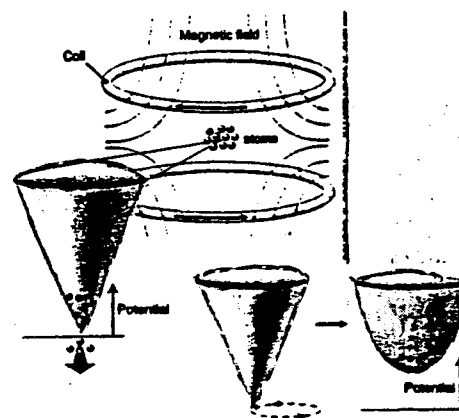
* TOP (time Orbiting Potential)
(avoid escape $B=0$)
- Composition of spherical quadrupole with oscillating field



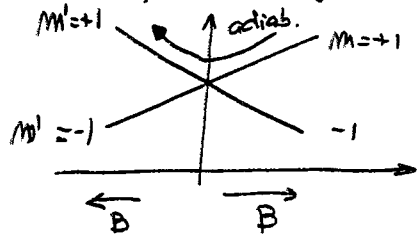
$$U = \mu [x B' + B_0 \cos \omega_b t] \hat{x} + \mu [y B' + B_0 \sin \omega_b t] \hat{y} - z y B' \hat{z}$$

For $\omega_t < \omega_b$
(oscillation of atoms < oscill. field)

$$\bar{U} = \frac{\omega_b}{2\pi} \int_0^{2\pi/\omega_b} U(t) dt = -\mu B_0 - \frac{\mu B'}{4B_0} (r^2 + 8z^2)$$



* Loss of atoms by "Majorana Flop"
 - take place when spin does not follow field adiabatically



* This occurs when rate of change in the field (seen by the atom) is bigger than Larmor freq.

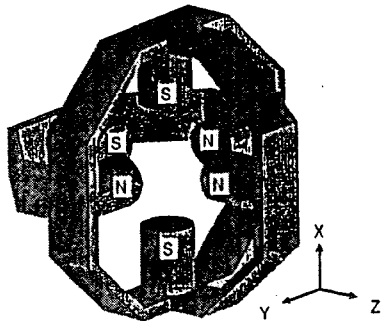
* Consider atom travelling at \underline{R} from the center ($B=0$). At this position

$$\left. \begin{aligned} \omega_L &= \frac{\mu (RB')}{\hbar} \\ \text{Rate of field Variation} &= \frac{v}{R} \end{aligned} \right\} \frac{v}{R} > \frac{\mu RB'}{\hbar}$$

transition Region $R < \sqrt{\frac{v\hbar}{\mu B'}}$

Rate of loss = flux of atoms at this region

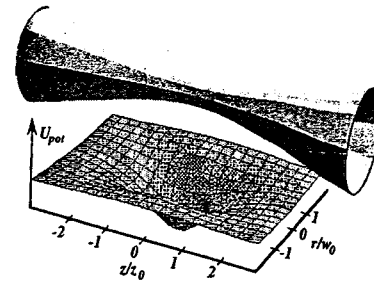
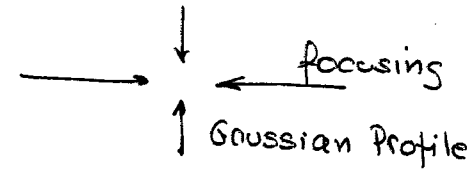
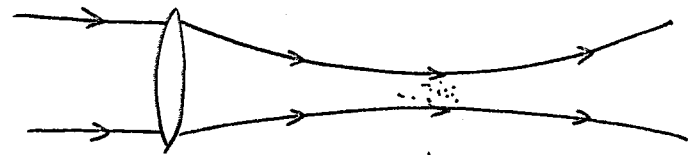
Others:



FORT: Use dipolar force

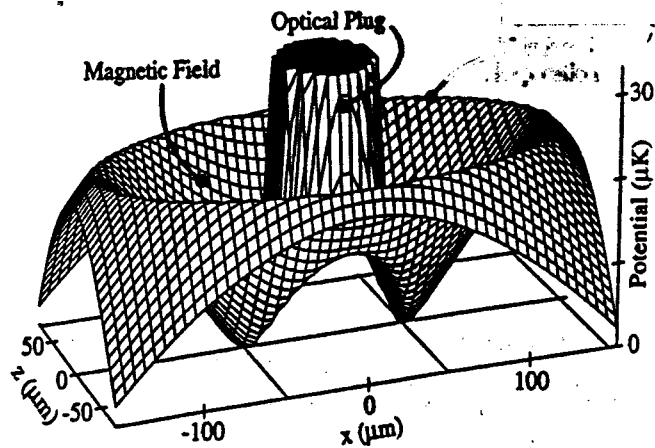
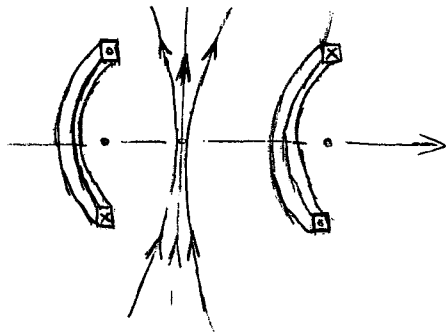
$$F_d \propto -\Delta \cdot R^2$$

$\Delta < 0 \rightarrow$ attraction towards maximum intensity

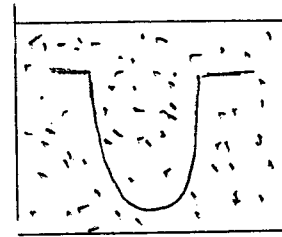


"Optical Plug"

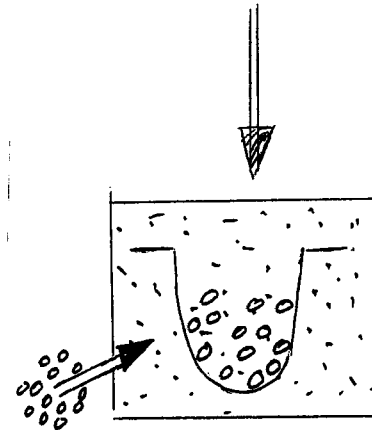
- * Another alternative to avoid $B=0$
- * Dipole force to create a repulsive barrier around $B=0$
 $\Delta > 0$, repel atoms



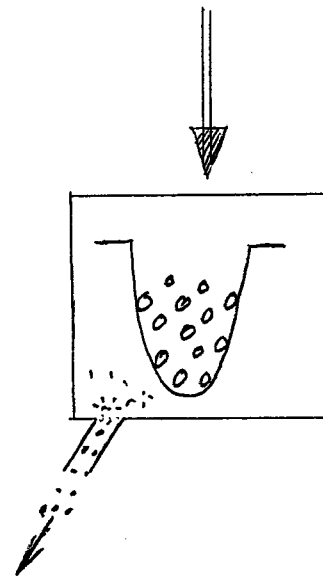
Cooling by a buffer-gas



Magnetic trap
 +
 untrapped cold gas (He)



Injection of trapable
 species \rightarrow thermalizing
 \rightarrow trapping
 [Ni, Co, Er, Eu, Ho, Gd... CaM.]



Promotion of buffer

$$\Delta T = \frac{T_{\text{buffer}} - T_0}{(M+m)^2 / 2Mm}$$

A few applications

Ultra-high resolution laser spectroscopy

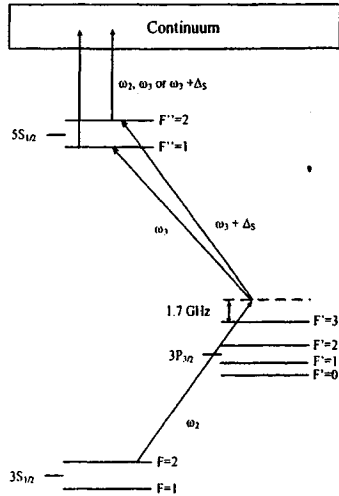


Fig. 2. Energy levels diagram of sodium, showing the transitions relevant for the measurement.

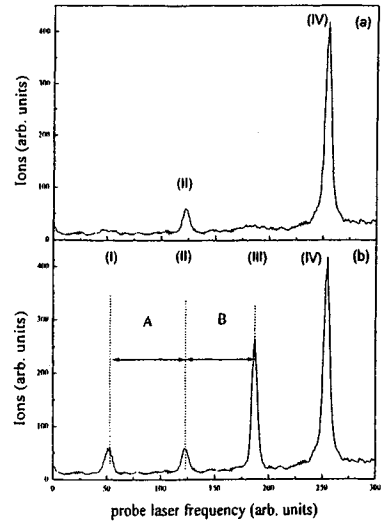


Fig. 3. Ion spectrum obtained during the "probe cycle" (a) without and (b) with the sideband in the probe laser frequency. See text for details.

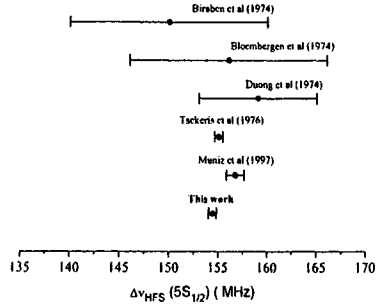
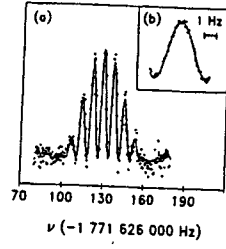
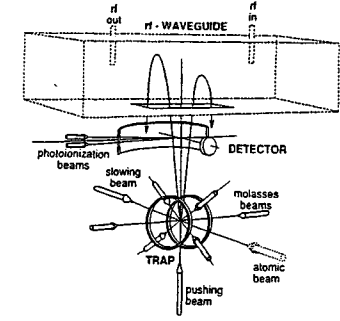
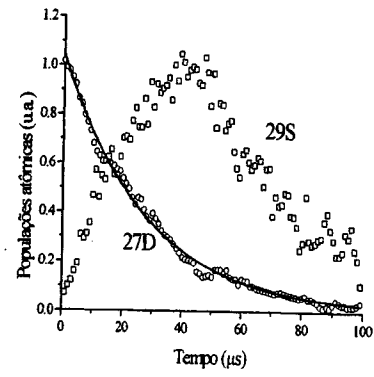


Fig. 5. Graphical comparison between our result and previous measurements for the hyperfine splitting of the Na 5S level. References are in the text.

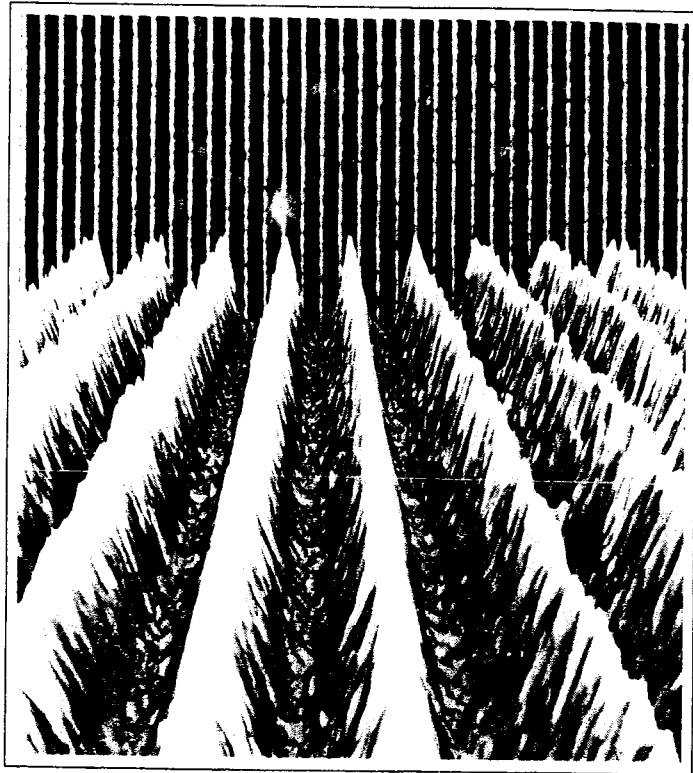
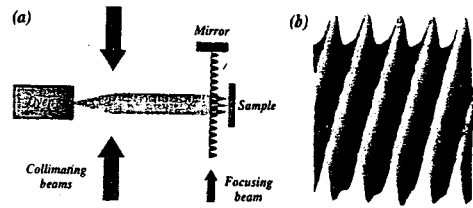
* Atomic Clock "Fountain"



* Rydberg Atoms



Atomic Lithography



* Optical Lattices

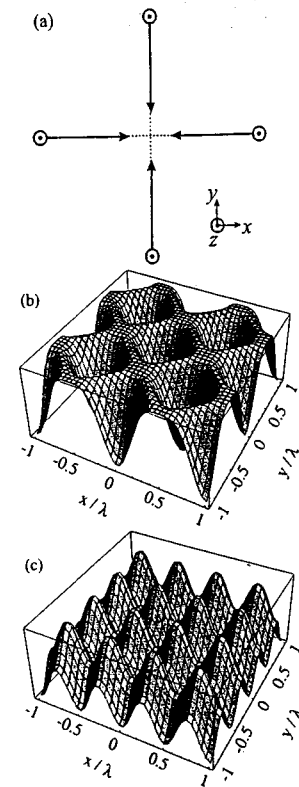
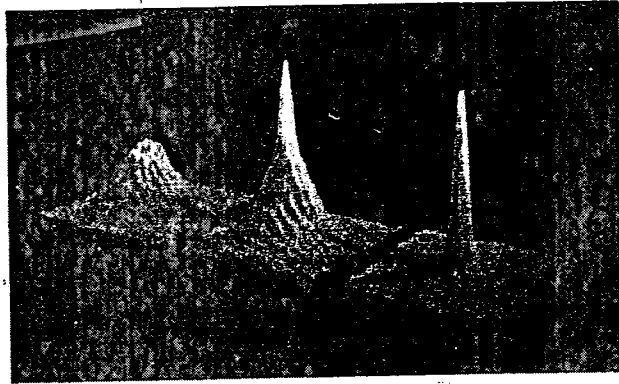


Fig. 3. (a) Beam configuration generating a phase-dependent 2D periodic lattice. The figure is drawn with $k_1 \perp k_2$. (b) Dipole potential for red detuned laser beams and $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0$. (c) Dipole potential for red detuned laser beams and $\phi_1 = \phi_3 = \pi/2$, $\phi_2 = \phi_4 = 0$.

Introduction to Bose-Einstein Condensation

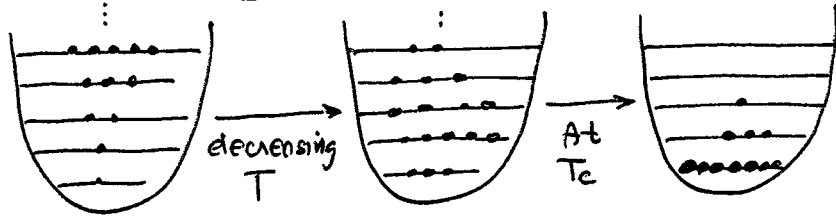


- * Sistema where $\lambda_{DB} \sim M^{-1/3}$
 → coherence and special macroscopic properties

$$\lambda_{DB} = \frac{h}{\sqrt{2\pi M kT}}$$

Cooling particles is possible to reach the quantum degenerated regime.

Cold Atoms are good candidates



Macroscopic occupation of ground state ⇒ change in the macroscopic behavior

To understand BEC, let's review the basic concepts.

- * Consider a gas of bosons confined by a potential $U(\vec{r})$

- * The occupation of each state

$$M_\epsilon = \frac{1}{e^{(\epsilon - \mu)/kT} - 1}$$

$\mu \equiv$ chemical potential (function of N, T, U, \dots)

$$N = \sum_{\text{states}} M_\epsilon$$

$$E = \sum_{\text{states}} \epsilon M_\epsilon$$

⇒ Cont. approx. of states → density of states

$$g(\epsilon) = \frac{2\pi (2M)^{3/2}}{h^3} \int_{V(\epsilon)} \sqrt{\epsilon - U(\vec{r})} d^3r$$

Equations:

$$N = N_0 + \int_0^\infty M_\epsilon g(\epsilon) d\epsilon$$

$$E(T) = \int_0^\infty \epsilon M_\epsilon g(\epsilon) d\epsilon$$

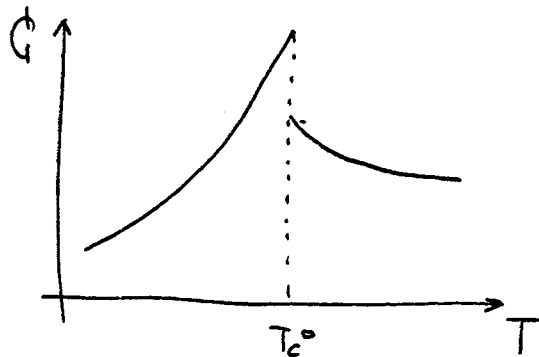
$$\Rightarrow \rho = \left. \frac{\partial E}{\partial T} \right|_N \quad ; \quad \frac{N_0}{N} = \text{condensated fraction}$$

Potential	T_c	$N_0/N (T < T_c)$	$C(T_c^-)/Nk$	$\Delta C(T_c)/Nk$
$U(z) = \begin{cases} \epsilon_1(z/a), & z > 0 \\ \infty, & z < 0 \end{cases}$	$\left[\frac{h^3 N}{1.45k^{3/2}(2\pi M)^{1/2}} \right]^{2/3} \left[\frac{\epsilon_1}{a} \right]^{2/3}$	$1 - \left[\frac{T}{T_c} \right]^{3/2}$	6.88	3.35
$U(z) = \epsilon_1(z/a)^2$	$\left[\frac{3h^3 N}{\sqrt{2}Sk^2\pi^4 M^{3/2}} \right]^{1/2} \left[\frac{\epsilon_1}{a^2} \right]^{1/4}$	$1 - \left[\frac{T}{T_c} \right]^2$	4.38	0
3D box	$\left[\frac{h^3 N}{2.612k^{3/2}(2\pi M)^{1/2} V} \right]^{2/3}$	$1 - \left[\frac{T}{T_c} \right]^{3/2}$	1.92	0
$U(r) = \epsilon_1(r/a)^2$	$\left[\frac{Nh^3}{1.202\pi^3 k^3 (2M)^{3/2}} \right]^{1/3} \left[\frac{\epsilon_1}{a^2} \right]^{1/2}$	$1 - \left[\frac{T}{T_c} \right]^3$	10.82	6.57
$U(z, \rho) = \epsilon_1(z/a)^2 + \epsilon_2(\rho/b)^2$	$\left[\frac{Nh^3}{1.202\pi^3 k^3 (2M)^{3/2}} \right]^{1/3} \left[\frac{\epsilon_1}{a^2} \right]^{1/6} \left[\frac{\epsilon_2}{b^2} \right]^{1/3}$	$1 - \left[\frac{T}{T_c} \right]^3$	10.82	6.57

Harmonic Oscillator:

$$KT_c^0 = 0.941 \hbar \omega N^{1/3}$$

$$\frac{N_0}{N_0} = 1 - \left(\frac{T}{T_c^0} \right)^3$$



* With interaction

$$H(\text{1 particle}) = \frac{P^2}{2M} + U(r) + \underbrace{\gamma \rho(r)}_{\text{Interaction}}$$

$$* U_{\text{eff}}(r) = U(r) + \gamma \rho(r)$$

$\rho(r)$ = can be calculated in first approx.

$$\gamma = \frac{4\pi \hbar^2 a}{M} \quad a = \text{scattering length}$$

$$\rho(r) \cong \frac{1}{\lambda_0^3} \exp\left(-\frac{U(r)}{kT}\right)$$

Considering H.O., $a > 0$

$\Delta T_c \rightarrow$ correction

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c^0} \right)^3 - \frac{4a}{\lambda_0} \left(\frac{T}{T_c^0} \right)^{7/2} + \dots$$

\Rightarrow Interactions must be treated in a more appropriated way (Last lecture)

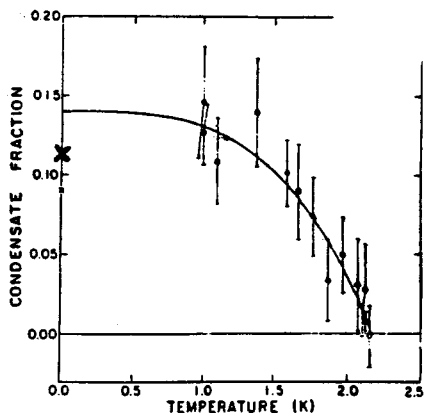
$$H = -\frac{\hbar^2}{2M} \nabla^2 + U(r) + A|\rho|^2$$

$$A = (N-1) \frac{4\pi \hbar^2 a}{M}$$

Exemple (^4He)

Consider as ideal gas $T_c^0 = 3.15\text{K}$
 * Superfluidity at 2.17K.

* Measurement of $\bar{p}=0$ population
 (Phys. Rev. Lett. 49, 279 (1982))



* Consider interaction

$V = 27.6 \text{ cm}^3$

$N = 6 \cdot 10^{23}$

$a = 1.9 \text{ \AA}$ (London, 1939) \rightarrow

$\mu a = 1.26$

$T_c \approx 2.1\text{K}$

Lista publicações pelo Grupo de
São Carlos em técnicas de
aprimoramento e aplicações
(nao incluindo cursos livres)

- Phys. Rev. A 47, 4583 (1993)
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- Phys. Rev. A 60, 259 (1999)
- Phys. Rev. A. 59, 3101 (1999)
- Phys. Rev. A. 59, 3101 (1999)
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