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THE PHYSICS OF SUBCRITICAL SYSTEMS (1)

1. Introduction

The main lesson learnt so far in the four decades of pacific uses of atomic energy, after the two Geneva Conferences of 1955 and 1964, is that the accidents associated with nuclear energy posing major risk problems, i.e., greater than those perceived acceptable as an inevitable price to be paid for progress, are related to human factors. Such has been in fact the case with the events occurred at Three Mile Island, Chernobyl, and, more recently, Tokaimura. This latter accident, in particular, has broadened the perception of major possible risks outside the reactor system. Besides, although the quite positive experience with LWR reactors in the last decades cannot be forgotten, it is also perceived that a broader, world-wide nuclear energy expansion would pose a number of concerns related to the safety of reactor plants (power excursions, residual heat risk), as well as to those associated with the fuel flow (criticality accidents, fuel diversion, radiological risk, proliferation) and to the problems related to reprocessing (antiproliferation issues, fuel transportation risks). To find an answer to these issues in a long term scenario, the following objectives should be persued:

1. A nuclear energy reactor concept assuring optimal characteristics in relation to economy, safety, anti-proliferation, anti-diversion, radiological risk minimization, public acceptance;
2. A viable, smooth transition from today's nuclear energy production structure into that, at equilibrium, relevant to the new concept;
3. The possibility of closing the fission cycle, whenever a different important energy source is developed as a substitution, with minimal radio-toxicity from residual waste.

To answer these issues, a variety of different projects have been proposed in these years, among which we remind the molten salt reactor, the pebble-bed reactor and the encapsulated reactor (stemming from the well known IFR concept). Many of these systems are considered also in a subcritical configuration, i.e., as hybrid (ADS) concepts. A number of advantages (at short as well as long range terms) are claimed for this choice:

- an high degree of safety
- ability of significantly mitigating the waste stream
- ability to efficiently reducing the existing stocks of plutonium
- optimal use of uranium and thorium natural resources
- closure of the fission cycle.

The main justification for using ADS, rather than critical systems, appears however related to safety considerations, the distance from criticality conditions resulting equivalent, as will be shown, to an extra amount of delayed neutrons. This property, in particular, allows to consider them the best candidates as minor actinide (Am, Cu) incinerators, in consideration of the relatively small delayed neutron fraction associated with these elements.

In order to show the physical characteristics of these systems, we shall first describe the coupling of the reactor power with the accelerator. We shall then present a simple approach extending to ADS systems the reactivity balance approach for the fast reactor studies. Then, in Part 2, the concept of generalized reactivity applied to these system will be illustrated, together with some peculiarities of their physical behavior.

2. Reactor /Accelerator Coupling

The coupling of a subcritical reactor with an proton accelerator producing spallation neutrons in a target implies an efficiency loss, increasing with the increasing subcriticality. To illustrate this concept, let us consider for simplicity an homogeneous subcritical bare system in a one group approximation driven by a source of intensity S(r). The general solution of the flux may be written (Glasstone and Edlund, 1952), at aymptotic conditions,

$$\phi(\mathbf{r},t)=\frac{p}{K_\infty \Sigma_c}\sum_{\substack{n=1\\(n\ \text{disp.})}}^{\infty}\left[\frac{K_n S_n}{(1-K_n)}\right] f_n(\mathbf{r})\,. \qquad (2.1)$$

where f_n are the eigenfunctions, solutions of the equation

$$\nabla^2 f_n + B_n^2 f_n = 0\ ,$$

B_n^2 being the geometrical buckling, Σ_c is the macroscopic capture cross section of the core, p the resonance escape probability, K_n the multiplication coefficient associated with the n-th eigenfunction and S_n are the moments of the neutron source expansion. We assume for simplicity a source distribution corresponding to the first flux eigenfunction, or fundamental mode (so that moments S_n are zero for n>1). Multiplying by $\nu\Sigma_f$ and integrating over the whole core, we obtain the fission neutron source:

$$\int_{core}\phi(\mathbf{r})\nu\Sigma_f\, d\mathbf{r}=\frac{p\nu\Sigma_f}{K_\infty\Sigma_a}\frac{K_{eff}}{(1-K_{eff})}S_1\int_{core} f(\mathbf{r})d\mathbf{r}=\frac{K_{eff}}{(1-K_{eff})}\int_{core} S(\mathbf{r})d\mathbf{r}\ , \qquad (2.2)$$

where K_{eff} ($\equiv K_1$) is the fundamental eigenvalue.

Expression (2.2) may be extended to any geometry and neutron energy group representation, so that we may write:

$$\textit{overall fission neutron source} = \int dE \int_{core}\phi(\mathbf{r},E)\nu\Sigma_f\, d\mathbf{r} = \frac{K_{eff}}{(1-K_{eff})}\int dE\int_{core} S(\mathbf{r},E)d\mathbf{r} \equiv \frac{K_{eff}\, s}{(1-K_{eff})} \qquad (2.3)$$

where

$$s=\int dE\int_{core} S(\mathbf{r},E)d\mathbf{r} \qquad (2.4)$$

is the overall extraneous neutron source.

Let us derive in the following a simple expression coupling the reactor power with that of the accelerator needed to produce an assigned reactor power, evidencing the dependence of this latter quantity from the subcriticality level and from other parameter associated with the accelerator.

Let's consider the accelerator power (in MW):

$$W_{acc} = cE_p/f_b \ , \qquad (2.5)$$

where c represents the current (in mA), E_p the proton energy (in GeV) and f_b the accelerator efficiency.

The neutron source results

$$s = n_{/mA}c = \frac{n_{/mA} f_b W_{acc}}{E_p} \ ,$$

$n_{/mA}$ representing the effective number of source neutrons produced per mA current.

The overall fission source (2.3) may then be written:

$$\int dE \int_{core} \phi(\mathbf{r}, E)\nu\Sigma_f \, d\mathbf{r} = \frac{n_{/mA} f_b W_{acc} K_{eff}}{E_p(1-K_{eff})} \qquad (2.6)$$

The quantity $n_{/mA}$ is an increasing function of the proton energy E_p.

A current of 1 mA corresponds to 0.625×10^{16} protons/sec. We shall denote as $p_{/mA}$ this quantity. Indicating with "m" the number of neutrons produced by each proton hitting the target (for example, of tungsten, or lead[1], and assuming a weight "g" for these neutrons[2], we have $n_{/mA} = gmp_{/mA}$.

The ratio W_{acc}/W_e may be seen as the fraction of electricity lost in a plant of electric power W_e ($=f_eW_t$). Substituting in (2.6) $\varphi\Sigma_f V_{core}$ with W_t/κ, where κ represents the units of energy (in MJ) per fission, it results:

$$\frac{W_{acc}}{W_t} = \frac{\nu E_p(1-K_{eff})}{K_{eff}\,\kappa f_b g m p_{/mA}} \qquad (2.8)$$

i.e.,

$$\frac{W_{acc}}{W_e} = \frac{\nu E_p(1-K_{eff})}{K_{eff}\,\kappa f_b f_e g m p_{/mA}} \ . \qquad (2.9)$$

Recalling equation (2.5), we may also write the expression relevant to the current intensity needed for an ADS systerm of assigned power and subcriticality level:

$$c = W_t \, \nu \, (1-K_{eff})/ K_{eff} \, \kappa g m p_{/mA} \ . \qquad (2.10)$$

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[1] The number of spallation neutrons (m) produced by a proton hitting a lead target with energy $E_{p,}$ in the energy range of interest, increases along the empirical expression (Andriamonje, et al., 1995):

$$m = 3.717 \times 10^{-5} E_p^2 + 3.396 \times 10^{-3} E_p - 0.367.$$

Recent indications give different values, lower by 10÷20%, for 1 GeV protons.

[2] So, to account of their importance (in relation to the system power) with respect to the average one of fission neutrons.

The power of the proton beam will be

$$W_{beam} = E_p W_t \nu (1-K_{eff})/ K_{eff} \kappa g m p_{/mA} \quad . \qquad (2.11)$$

It is generally assumed that the optimal proton energy E_p is of the order of 1 GeV. This is mainly suggested by the need of limiting as much as possible the demaging of the window through wich the proton beam accesses the multplying region[3].

Then, assuming k_{eff}=0.96, f_b = 0.5, f_e = 0.38, ν=2.7, m=33 e g= 1.2 and recalling that κ (energy units per fission) is of the order of 200 Mev (= 3.2 x 10^{-17} MJ), we obtain:

$$W_{acc}/W_e = 0.074 \;, \quad W_{acc}/W_t = 0.02 \;, \quad c = 0.014\, W_t \quad .$$

In this case, 7.4% of the electric power is absorbed by the accelerator. For a thermal power of 840 MW (corresponding to the dimensions of a PRISM like reactor), for a multiplication coefficient K_{eff}=0.96, the power absorbed results W_{acc}= 24 MW, corresponding to proton current of about 10 mA.

Since $\kappa = (\mathrm{MJ})/\mathrm{fission}$, it is also, denoting with "e" the elementary electron charge (in coulomb)

$$10^3 \kappa p_{/mA} = \frac{\kappa}{\dfrac{e/\sec}{1(\mathrm{proton})/\sec}} \equiv \frac{\kappa}{e} = \frac{(\mathrm{MJ})/\mathrm{fission}}{(\mathrm{MJ})/\mathrm{MeV}} = (\mathrm{MeV})/\mathrm{fission} = \varepsilon_f \quad (\cong 200 \;\; \mathrm{MeV})$$

and then in the preceding expressions the product $\kappa p_{/mA}$ may be replaced by $10^{-3}\varepsilon_f$ ($\cong 0.2$).

3. Balance of Reactivity

Extensive studies on the ADS behavior under incidental conditions are presently made for verifying their claimed advantage, under the safety point of view, with respect to the critical reactors. An extensive analysis of these systems was made in the late 80's at ANL (Wade, 1986), specifically with respect to the Integrated Fast Reactor (IFR). A synthetic, quite effective deterministic method was used based on a "balance of reactivity" approach. We can rewrite a similar methodology in relation to the ADS, making rather use of a "balance of power" approach. This would allow to estimate the behavior of the ADS systems at abnormal conditions. The balance of reactivity approach consists in writing a quasi-static balance of reactivity (ρ):

$$\rho = (P-1)A + (P/F-1)B + \delta T_{in} C + \delta\rho_{ext} = 0 \qquad (3.1)$$

where

P and F are power and coolant flow (normalized to unity at operating conditions),
δT_{in} is the change from normal coolant inlet temperature T_{in},
C is the inlet temperature reactivity coefficient,
(A+B) is the reactivity coefficient experienced in going to full power and flow from zero power isothermal at constant coolant inlet temperature,
B is the power/flow reactivity coefficient,
$\delta\rho_{ext}$ is an external reactivity insertion.

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[3] The number of spallation neutrons per incident proton increases with the energy E_p (see note 1), and, then, increasing it , the proton current correspondingly decreases for producing the same neutron source intensity.

In Eq. (3.1) it is assumed that convergence (criticality) has been reached asymptotically. There are circumstances in which this is not physically possible, as in presence of a scram intervention, implying a strong negative reactivity insertion (in this case $\delta\rho_{ext} = -|\Delta\rho_{scram}|$). This occurrence may be identified, since in these cases the resulting values δT_{in} or P/F, in LOHS and LOF events, respectively, with scram intervention, loose physical sense (being negative).

In the following, we consider the same problem starting, rather than from a balance of criticality approach (suitable for critical systems), from a balance of power one (seemingly, more suitable for subcritical ones). To note that the formulation proposed shows to be quite general, being applicable to subcritical, as well as to critical systems.

4. Balance of Power

Whereas in a critical reactor the equilibrium condition after a (limited) incidental transient corresponds to a new criticality state, in an ADS system in a similar circumstance, due to the external source presence, and assuming that the incidental transient does not lead to criticality (an intrinsic prerequisite for this system safety), any subcritical steady state condition may be generally achieved. The following equations are then proposed, at equilibrium,

$$\rho = (P-1)A + (P/F - 1)B + \delta T_{in} C + \delta\rho_{ext} \qquad (4.1)$$

$$P = \alpha \frac{s_n + \delta s_n}{1 - K_{eff}\left(1 + \frac{\rho}{K_{eff}}\right)} \quad , \qquad (4.2)$$

K_{eff} being the multiplication coefficient of the subcritical system before the accidental event, ρ the reactivity associated with the deviation of the multiplication coefficient from K_{eff}, and δs_n a change of the external neutron source s_n, induced by a change δi of the accelerator current i.

With the power unit definition (at nominal conditions)

$$P_o = \alpha \frac{s_n}{1 - K_{eff}} = 1 \qquad (4.3)$$

we have

$$\alpha = \frac{1 - K_{eff}}{s_n} \qquad (4.4)$$

Eq.(4.2) then may be written

$$P = \frac{1 - K_{eff}}{s_n} \frac{s_n + \delta s_n}{1 - K_{eff}\left(1 + \frac{\rho}{K_{eff}}\right)} \quad . \qquad (4.5)$$

Substituting expression (4.1) for reactivity ρ and indicating by $\overline{\rho}$ the subcriticality $(1-K_{eff})$, we obtain

$$P[\bar{\rho}-(P-1)A-(P/F-1)B-\delta T_{in}C-\delta\rho_{ext}]-\bar{\rho}(1+\frac{\delta i}{i})=0 \qquad (4.6)$$

where we have set $\frac{\delta i}{i}$, i.e., the fractional change of the accelerator current, in place of $\frac{\delta s_n}{s_n}$.

This equation is quite general. In fact, if the external source term $\bar{\rho}(1+\frac{\delta i}{i})$ vanishes (since for a critical system it is K_{eff}=1), we obtain again Eq.(3.1).

Eq.(4.6) may be solved with respect to P. We easily obtain

$$P=\frac{\bar{\rho}+(A+B)-\delta T_{in}C-\delta\rho_{ext}-\sqrt{[\bar{\rho}+(A+B)-\delta T_{in}C-\delta\rho_{ext}]^2-4\bar{\rho}(A+\frac{B}{F})(1+\frac{\delta i}{i})}}{2(A+\frac{B}{F})} \qquad (4.7)$$

An important quantity to be analyzed is the coolant output temperature T_{out}. If by ΔT_c we denote the coolant temperature rise at nominal full power/flow ratio, the coolant outlet temperature change δT_{out} is defined by the expression

$$\delta T_{out}=\delta T_{in}+(\frac{P}{F}-1)\Delta T_c \quad . \qquad (4.8)$$

4.1 Loss Of Heat Sink (LOHS)

In this case the inlet temperature T_{in} increases while the coolant flow remains constant. A dynamic study should be done to analyze the heat balance evolution. However, some qualitative considerations can be made.

Consider two possibilities :

- <u>The current shuts off.</u>

The coolant flow remains unchanged, while P→0. It is found, from Eq.(4.6)*, having set δi=-i (and $\delta\rho_{ext}$=0),

$$\delta T_{in}=\frac{A+B}{C}+\frac{\bar{\rho}}{C} \qquad (4.9)$$

which corresponds to the analogous expression for the IFR system, with at right side of the (negative) $\frac{\bar{\rho}}{C}$ term in place of $\frac{|\Delta\rho_{scram}|}{C}$, relevant to the (negative) reactivity insertion with the scram intervention.**

* In cases like this, in which the external source term $\bar{\rho}/(1+\delta i/i_o)$ vanishes, in Eq. (4.6) the power P multiplying the terms in square parenthesis is dropped. In fact these terms correspond to the asymptotic overall reactivity, which, in case of equilibrium (criticality) convergence, should vanish. As said previously, if the criticality is not attainable, the δT_{in} value would result negative, i.e., out of physical sense.

** In Wade's formulation this term , for a critical reactor, would result

Since, as power decreases, the outlet temperature T_{out} collapses into T_{in}, we can also write, recalling Eq.(4.8),

$$\delta T_{out} = \delta T_{in} - \Delta T_c = [\frac{(A+B)+\bar{\rho}}{C\Delta T_c} - 1]\Delta T_c \qquad (4.10)$$

So, for a subcritical system, there is a reduction of δT_{out} with the decreasing of (A+B)/C (usually, a prevailing term) and the increasing (in absolute value) of the negative term $\bar{\rho}/C$.

- <u>The current fails to be shut-off</u> (LOHSWS, Loss of Heat Sink Without current Shut-off)
From Eq.(4.7) it can be shown that in this case the power is sustained down to a lower limit proportional to $\bar{\rho}$. The integrated energy, if not adequately absorbed by the system heat capacity, may lead to unacceptable temperature levels. It is then essential in this case that some intrinsic device is introduced which stops the insertion of external neutrons. In general, it can be said that, in relation to this event, a relatively small value of $\bar{\rho}$, i.e., a relatively small subcriticality level (and, correspondingly, a relatively small neutron source s_n), is desirable to limit the intensity of the asymptotic power and, consequently, the value of the outlet temperature (before a corrective intervention takes place).

4.2. Current-related Transient of Over-Power (TOC)

For an ADS a TOC event (analogous to the TOP event of the IFR) may be defined as a current increase (Δi_{TOC}) at nominal operation level. This change may correspond, for instance, to the reserve of current for compensating reactivity loss with burn-up. The coolant flow F remains unchanged .

Short term (T_{in} unchanged)

Eq.(4.7) in his case becomes

$$P = \frac{\bar{\rho} + (A+B) - \sqrt{[\bar{\rho}+(A+B)]^2 - 4\bar{\rho}(A+B)(1+\frac{\Delta i_{TOC}}{i})}}{2(A+B)} \qquad (4,11)$$

Assuming that $\Delta i_{TOC}/i$ is a small quantity with respect to unity, we obtain

$$P = 1 - \frac{\bar{\rho}}{(A+B)-\bar{\rho}}\frac{\Delta i_{TOC}}{i} \qquad (4.12)$$

The quantity

$$-\bar{\rho}\ \frac{\Delta i_{TOC}}{i} \qquad (4.13)$$

may be viewed as the reactivity loss $-\Delta\rho_{TOC}$ ($\Delta\rho_{TOC}$ being a positive quantity), during reactor operation and life, to be compensated by the current reserve margin to which Δi_{TOC} corresponds. Eq.(4.12) then can also be written

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$$\delta T_{in} = \frac{A + |\Delta\rho_{scram}|}{C},$$

For comments on this, see Section 2.

$$P = 1 - \frac{\Delta\rho_{TOC}}{(A+B) - \bar{\rho}} \quad , \qquad (4.14)$$

which is quite similar to the expression relevant to the IFR for the corresponding TOP event [to which it corresponds exactly if we set K_{eff} equal to unity, as may be easily verified from Eq.(4.1)].

We can then write, recalling Eq.(4.8),

$$\delta T_{out} = -\frac{\Delta\rho_{TOC}}{(A+B) - \bar{\rho}} \ \Delta T_c \ . \qquad (4.15)$$

As with the IFR, given a reactivity margin ($\Delta\rho_{TOC}$) to be accommodated as a current reserve, also for an ADS system a large value (in absolute terms) of the sum (A+B) would then be desirable. The subcriticality condition also significantly helps under this respect.

Long term (P ® 1)

δT_{in} gradually increases until an adequate subcriticality is reached. It is found, from Eq.(4.6),

$$\delta T_{out} (\equiv \delta T_{in}) = -\frac{\bar{\rho}}{C} \frac{\Delta i_{TOC}}{i} \equiv -\frac{\Delta\rho_{TOC}}{C} \ , \qquad (4.16)$$

an expression quite similar to that relevant to the TOP event of IFR (as may be easily found from Eq.(4.1), with $\delta\rho_{ext}$=$\Delta\rho_{TOP}$). A large value of C would be in this case desirable.

4.3. Loss of Flow (LOF)

With this event the inlet temperature T_{in} is assumed not to change while the coolant flow will coasts down to natural circulation. A dynamic study should also here be done to analyze the heat balance evolution. However, some qualitative consideration can be made.

We again consider two possibilities:

- <u>The current is shut off</u>.

In this case P→0. From Eq. (4.6)* we obtain , at long term,

$$\frac{P}{F} = 1 + \frac{A}{B} + \frac{\bar{\rho}}{B} \qquad (4.17)$$

which looks like the analogous expression for the IFR case with $\frac{\bar{\rho}}{C}$ term in place of $\frac{|\Delta\rho_{scram}|}{C}$, relevant to the (negative) reactivity insertion with the scram intervention.**

* In this case, in analogy with what said for the LOHS with scram event, if the equilibrium convergence is not attainable, the P/F value would result negative, i.e., out of physical sense.

** In Wade's formulation this term , for a critical reactor, would result

Then

$$\delta T_{out} = (\frac{A}{B} + \frac{\bar{\rho}}{B})\Delta T_c \tag{4.18}$$

In this case, a small $\frac{A}{B}$ and a large $\frac{\bar{\rho}}{B}$ (intended in absolute value since this is a negative quantity) would be desirable. To notice in this case that the extra term $\frac{\bar{\rho}}{B}\Delta T_c$ helps reducing the δT_{out} value with respect to the IFR case.

At short term, at which dynamic effects make the system depart from equilibrium and which need be taken into proper account, considerations similar to those expressed with respect to the IFR can be made, in particular those relevant to the pump coast-down time (τ). With the ADS, however, the situation would also in this circumstance be alleviated by a large $\frac{\bar{\rho}}{B}$ (absolute) value.

- The current fails to be shut-off (LOFWS, Loss of Flow Without current Shut-off).

In this case the power is sustained down to a lower limit. As with the LOHSWS case, the integrated energy, if not adequately absorbed via natural circulation, may lead to unacceptable temperature levels. It is then essential also for this case that some intrinsic device is introduced which stops the insertion of external neutrons.

For very small coast-down values of the coolant flow F_{NC} (of the order of 1% of the nominal flow), from Eq.(4.7) we would obtain

$$P \sim \sqrt{-\frac{F_{NC}}{B}\bar{\rho}} \tag{4.19}$$

In a real system, F_{NC} would be of the order of 10% of the nominal flow. Eq.(4.7), rather than Eq.(4.19), should therefore be used. The dependence on the subcriticality level $\bar{\rho}$, however, remains. It can then be said that also in relation to a LOFWS event a relatively small $\bar{\rho}$ value (and, correspondingly, a relatively small neutron source s_n), and a large value (in absolute terms) of B are desirable to limit the intensity of the asymptotic power, and, consequently, the outlet temperature (before a corrective intervention takes place).

At short range, the problem associated with the pump coast-down time (τ), is aggravated for an ADS with respect to an IFR by the presence of the persistent external source which may be viewed as an amplification of the delayed neutron holdback problem.

4.4. CIT (Chilled Inlet Temperature)

A chilled inlet temperature, or overcooling event (the inverse of a LOHS), inducing a negative change δT_{in} of the coolant inlet temperature, may occur if a steam-line rupture overcools the secondary coolant which in turn overcools the primary core inlet temperature. At constant pump flow the resulting reactivity increase is

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$$\frac{P}{F} = 1 + \frac{A + |\Delta\rho_{scram}|}{B},$$

For comments on this, see Section 2.

compensated by a power increase with resultant core temperature rise increase. From Eq.(4.7), since $C\delta T_{in}$ is a small (positive) quantity with respect to unity, we may write, assuming the accelerator current intensity maintains constant,

$$P = 1 - \frac{C\delta T_{in}}{(A+B) - \overline{\rho}} \tag{4.20}$$

and then, recalling Eq.(4.8),

$$\delta T_{out} = \left(1 - \frac{C\Delta T_c}{(A+B) - \overline{\rho}}\right)\delta T_{in} \tag{4.21}$$

which compares with the similar expression for the IFR system. In our case the core outlet temperature results then reduced by a large power coefficient (A+B), a small inlet temperature coefficient C (both in absolute terms) and, as expected, by a relatively large $\overline{\rho}$ value.

4.5. IOR (Insertion of Reactivity)

The asymptotic power following an accidental reactivity insertion $\delta\rho_{ext}$, during normal operation, is likewise obtained from Eq.(4.7). Since $\delta\rho_{ext}$ may be assumed small with respect to unity, we may write, assuming that the accelerator current intensity is maintained constant,

$$P = 1 - \frac{\delta\rho_{ext}}{(A+B) - \overline{\rho}} \tag{4.22}$$

which is quite similar to the expression relevant to the CIT event, with $\delta\rho_{ext}$ in place of $C\delta T_{in}$.

Then

$$\delta T_{out} = -\frac{\delta\rho_{ext}\Delta T_c}{(A+B) - \overline{\rho}} \ , \tag{4.23}$$

Also for this case, then, the core outlet temperature would result reduced by a large power coefficient (A+B) (in absolute terms) and a relatively large $\overline{\rho}$ value.

5. General Conclusions

For an ADS system, the following general conclusions can be drawn:

1 - A large negative power coefficient (A+B) would be required for reducing TOC (at short term), CIT and IOR accidents, whereas, inversely, small ones would be needed for limiting the consequences of LOHS events with current cut-off. A trade-off between these two contradictory requirements need to be found, if a relatively large value for the (absolute) value of C is not available.

2 - A small current reserve is desirable (so that $\frac{\Delta i_{TOC}}{i}$ is small), for to reducing TOC accidents (at small and long terms). This may be achieved (in a system assumed without reactor life control elements) by compensating the burn-up criticality swing by an adequate internal conversion ratio and burnable neutron poisoning

3 - A small A/B value, to reduce the consequences of a LOF accident with current cut-off. Since for other effects there are contradictory requirements on these coefficients, some trade-off between coefficients A and B requirements in this case need also to be found.

4 - Some intrinsic safety mechanism should be introduced into the system, to stop the current beam and prevent LOHSWS and LOFWS events.

In Appendix, the results are shown of a study [Gandini, et al., 1999] in which the above approach has been adopted for some quantitative consideration relevant to ADS systems safety. A comparison is also illustrated between the Russian fast critical reactor BREST (Orlov and Slessarev, 1988, and Adamov, 1994) and an ADS with similar characteristics, in order to evidence the respective peculiarities at accidental conditions.

APPENDIX

As an example of application of the power balance approach described above, the Russian lead cooled fast reactor BREST (Orlov and Slessarev, 1988, and Adamov, 1994) and, for a relative comparison, corresponding ADS systems with various degrees of subcriticality have been considered. The reactivity effects and other relevant characteristics are presented in the Table 1.

Table 1. Effects and coefficients of reactivity for the BREST reactor.

Lead density variation in the reactor α_{Pb}	+ 0.19 pcm/°C
Radial core expansion α_R	- 0.67 pcm/°C
Assembly plate expansion $\alpha_G \approx 2\alpha_R$[Wade]	-1.4 pcm/°C
Axial fuel elements expansion α_E	- 0.11 pcm/°C
Doppler effect at nominal fuel temperature α_D	- 0.43 pcm/°C
Temperature effect of reactivity $\Delta\rho_{TER}$	- 20 pcm
Power effect of reactivity $\Delta\rho_{PER}$	- 150 pcm
Neptunium effect of reactivity $\Delta\rho_{Np}$	- 100 pcm
Change of isotopic composition due to burnup $\Delta\rho_{FBE}$	+ 30 pcm
Operational reactivity margin $\Delta\rho_{0P}$	+ 40 pcm
Total reactivity margin $\Delta\rho_{TOC}$	+ 340 pcm
Effective fraction of delayed neutrons β_{eff}	360 pcm
Prompt neutron lifetime	$8.2\ 10^{-7}$ sec

Coolant (lead) parameters:

- inlet temperature - 420 °C,
- outlet temperature - 540°C,
- coolant normal heating ΔT_C = 120°C.
- difference between average fuel and average coolant temperature T_f = 500°C

One can use the following expressions to calculate coefficients A, B and C (Wade, 1986):

$A = (\alpha_D + \alpha_E) T_f = - 0.75\ \beta_{eff}$
$B = (\alpha_D + \alpha_E.+ \alpha_{Pb} + \alpha_R)\Delta T_C/2 = - 0.17\ \beta_{eff}$
$C = (\alpha_D + \alpha_E.+ \alpha_{Pb} +\alpha_G) = -0.0049\ \beta_{eff}/°C$

where α_G is the temperature coefficient relevant to the fuel supporting plate.

TOC event

Since one of the possible use of ADS is that of transmutating (incinerating) TRU fuel, systems with solid fuels may be assumed to have a significant burnup reactivity swing, due to the limited breeding available in this case. So, in the example considered, the value $\Delta\rho_{TOC} = 2\beta_{eff}$ has been assumed.
In a TOP event scenario relevant to a critical reactor it is assumed that all rods run out, this introducing a positive reactivity instantly, whereas in a TOC event scenario relevant to an ADS the accelerator produces the maximum proton current instantly, the coolant flow inlet temperature remaining fixed in both cases at short/intermediate state. All this causes a rise of the power and, then, of theoutlet temperature. As time goes on, the inlet temperature starts to rise because the plant cannot absorb the amount of heat produced. In the ideal case, the inlet temperature would increase enough to reduce the power back to its initial level. This corresponds to an *asymptotic* state.

Table 2 and 3 present the results relevant to different levels of subcriticality for ADS and for critical reactors with similar parameters for the TOC event at short/medium term and asymptotic terms, respectively, assuming that $F=F_o$.

We note, in particular, that the rise of the outlet temperature in the asymptotic case does not depend on the level of subcriticality, if the alteration, rather than in terms of current change, is given in terms of equivalent reactivity.

At the beginning of a TOC transient (short/intermediate state), the ADS system considered has an acceptable temperature rise, compared with the corresponding critical. As expected, the outlet temperature is lower for the lowest K_{eff} condition.

Considering asymptotic states, one can conclude that all systems (critical reactors or ADS) may be subject to an excessive temperature rise in correspondence with large $\Delta\rho_{TOC}$ values, of the order of $\Delta\rho_{TOC}=\beta_{eff}$ or higher.

Table 2. TOC parameters ($\Delta\rho_{TOC} = 2\beta_{eff}$) at short/intermediate

<table>
<tr><th>ADS</th><th>Critical reactor</th></tr>
<tr><td>$P = 1 - \frac{\Delta\rho_{TOC}}{(A+B) - \bar{\rho}}$
$\delta T_{out} = -\frac{\Delta\rho_{TOC}}{(A+B) - \bar{\rho}} \Delta T_c$</td><td>$P = 1 - \frac{\Delta\rho_{TOC}}{(A+B)}$
$\delta T_{out} = -\frac{\Delta\rho_{TOC}}{(A+B)} \Delta T_c$</td></tr>
<tr><td>$\bar{\rho} = 10\beta_{eff}$
$P = 1.18$
$\delta T_{out} = 20°C$
$\mathbf{T_{out} = 560°C}$</td><td rowspan="3">$P = 3.2$,
$\delta T_{out} = 260°C$
$\mathbf{T_{out} = 800°C}$</td></tr>
<tr><td>$\bar{\rho} = 5\beta_{eff}$
$P = 1.32$,
$\delta T_{out} = 40°C$
$\mathbf{T_{out} = 580°C}$</td></tr>
<tr><td>$\bar{\rho} = 2\beta_{eff}$
$P = 1.60$
$\delta T_{out} = 75°C$
$\mathbf{T_{out} = 615°C}$</td></tr>
</table>

Table 3. Asymptotic parameters for TOC

ADS	Critical reactor
$\delta T_{out} = -\frac{\bar{\rho}}{C}\frac{\Delta i_{TOC}}{i} \equiv -\frac{\Delta\rho_{TOC}}{C}$	$\delta T_{out} = -\frac{\Delta\rho_{TOC}}{C\Delta T_C}\Delta T_C$
All values $\bar{\rho}$ $P \approx 1$ $\delta T_{out} = 405°C$ $\mathbf{T_{out} = 945°C}$	$P \approx 1$ $\delta T_{out} = 405°C$ $\mathbf{T_{out} = 945°C}$

LOHS-WS event

If the process of secondary heat exchange is arrested, in a critical reactor the inlet temperature starts increasing. The negative reactivity effect induced by the inlet temperature rise is compensated by the positive one relevant to the power decrease, up near zero-level. Assuming $F = F_o$, then $T_{in} \rightarrow T_{out}$.

For an ADS system, there is no equilibrium in the outlet coolant temperature. The outlet temperature is increasing constantly because the ADS power cannot approach zero level, notwithstanding significant feed-backs. The power is sustained down to a lower limit proportional to $\bar{\rho}$. This means that the lower the K_{eff} value, and the higher the neutron source has been chosen, the higher will be the rate of the asymptotic outlet temperature increase.

This means that core intrinsic characteristics do not allow to achieve a deterministic safety level, in case of failure of the proton beam stop device.

In Table 4 the results relevant to the coolant asymptotic temperatures for LOHS-WS accidents are shown.

Table 4. LOHS-WS asymptotic parameters

ADS	**Critical reactor**
No equilibrium of outlet coolant temperature	$\delta T_{out} = \left(\frac{A + B}{C \Delta T_c} - 1 \right) \Delta T_c$
For all K_o $\mathbf{T_{out} > 1000°C}$ $\mathbf{W \rightarrow W_{asympt}}$	$\delta T_{in} = 190°C$ $\delta T_{out} = 70°C$ $\mathbf{T_{out} = 610°C}$ $\mathbf{W \rightarrow 0}$

LOF-WS event

With this event the inlet temperature T_{in} is assumed not to change while the coolant flow will coasts down to natural circulation.

The consequent raising power to flow ratio induces an increase of the core average temperature, this in turn inducing a negative reactivity feedback. This negative reactivity is compensated by a positive one induced by the power reduction. Asymptotically, a natural circulation flow F_{NC} will be established.

For preliminary quantitative analysis, one can take $F_{NC} \approx 0.15\ F_0$ at nominal core thermal parameters.

Table 5 presents the evaluation of power change as well as the outlet lead temperature growth for ADS at different levels of subcriticality and for the corresponding critical reactor. The results show that the asymptotic temperature level for ADS is unacceptable.

<u>CIT-WS event</u>

With this event an inlet temperature decrease of $100°C$ has been assumed. In Table 6 the results are given relevant to the power change as well as the outlet coolant temperature increase for ADS at different levels of subcriticality and for the corresponding critical reactor.

As far as this type of accident is concerned, the ADS and the corresponding critical system have comparable behaviors.

Table 5. LOF-WS asymptotic parameters ($F_{NC} = 0.15F_0, P_0/F_0 = 1$)

<table>
<tr><th>ADS</th><th>Critical reactor</th></tr>
<tr><td>Power decreasing with $\bar{\rho}$
[See Eq.(8)]</td><td>$P = 1 + \frac{B\left(\frac{F_{NC}}{F_o} - 1\right)}{B + A\frac{F_{NC}}{F_o}}$
$\delta T_{out} = \left(\frac{B\left(\frac{F_{NC}}{F_o} - 1\right)}{B + A\frac{F_{NC}}{F_o}}\right)\Delta T_C$</td></tr>
<tr><td>$\bar{\rho} = 10\beta_{eff}$
$P = 0.93$
$\delta T_{out} = 625°C$
$\mathbf{T_{out} = 1165°C}$</td><td rowspan="3">$P = 0.48$,
$\delta T_{out} = 260°C$
$\mathbf{T_{out} = 800°C}$</td></tr>
<tr><td>$\bar{\rho} = 5\beta_{eff}$
$P = 0.87$
$\delta T_{out} = 575°C$
$T_{out} = \mathbf{1115°C}$</td></tr>
<tr><td>$\bar{\rho} = 2\beta_{eff}$
$P = 0.78$
$\delta T_{out} = 505°C$
$\mathbf{T_{out} = 1044°C}$</td></tr>
</table>

Table 6. CIT-WS asymptotic parameters

ADS	Critical reactor
$P = 1 - \frac{C\delta T_{in}}{(A+B) - \overline{\rho}}$ $\delta T_{out} = (1 - \frac{C\Delta T_c}{(A+B) - \overline{\rho}})\delta T_{in}$	$P = 1 - \frac{C\delta T_{in}}{(A+B)}$ $\delta T_{out} = (1 - \frac{C\Delta T_c}{(A+B)})\delta T_{in}$
All K_{eff} and $\overline{\rho}$ $P \approx 1.05$ $\delta T_{out} < 10°C$ **T_{out} = 550°C**	$P \approx 1.5$, $\delta T_{out} = 35°C$ **T_{out} = 575°C**

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