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SMR 1398/2

WORKSHOP ON
NUCLEAR REACTION DATA AND NUCLEAR REACTORS:
PHYSICS, DESIGN AND SAFETY

25 February - 28 March 2002

Neutron Standards and Basic Microscopic Theory

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These are preliminary lecture notes, intended only for distribution to participants.

Neutron Standards and Basic Microscopic Theory

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February 25th 2002

Standard Neutron Cross Sections

Introduction to Potential Scattering

Neutron-Proton Scattering and N-N Interaction

R-Matrix Data Analysis

$^3He(n,p)^3H$ Reaction and Realistic Microscopic Models

^6Li - Neutron Scattering and Semi-Realistic Models

Conclusions

Standard Reactions

Light nuclei

$H(n,n)H$	1 keV – 20 MeV
$^{3}He(n,p)^{3}H$	thermal – 50 keV
$^{6}Li(n,t)^{4}He$	thermal – 1 MeV
$^{10}B(n,\alpha)^{7}Li$	thermal – 250 keV
$^{10}B(n,\alpha,\gamma)^{7}Li$	thermal – 250 keV
$C(n,n)C$	thermal – 1.8 MeV

Heavy nuclei

$^{197}Au(n,\gamma)$	0.2 – 2.5 MeV
$^{235}U(n,f)$	thermal – 20 MeV
$^{238}U(n,f)$	threshold – 20 MeV

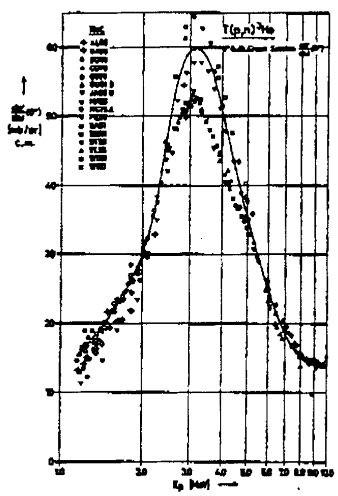
Theoretical treatment of light and heavy nuclei vastly different

light nuclei	heavy nuclei
few well defined open channels	many, unspecified
few (broad) resonances	many, narrow
center-of-mass motion important	unimportant
detailed microscopic models	bulk properties
"Few Nucleon regime"	"Nuclear matter"

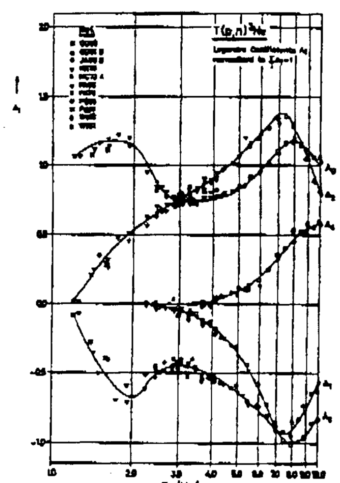
See typical examples

Neutron Production Cross Section $\mathrm{T(p,n)^3He}$

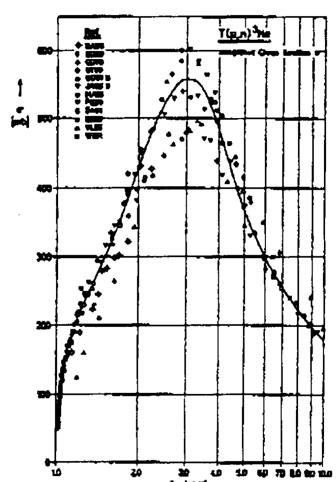
$T(p,n)^3He$. $\sigma(0°$ c.m.), **Legendre Coefficients**, σ(**total**)
Recommended and Experimental Values



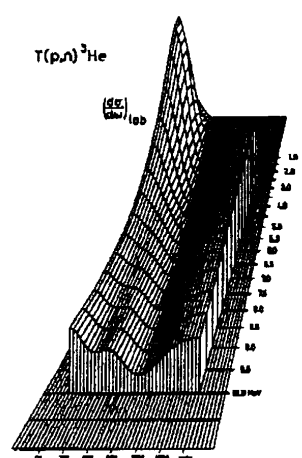
$\sigma(0°$ c.m.). Experimental and Recommended Values



Legendre Coefficients
Experimental and Recommended Values



σ(total). Experimental and Recommended Values



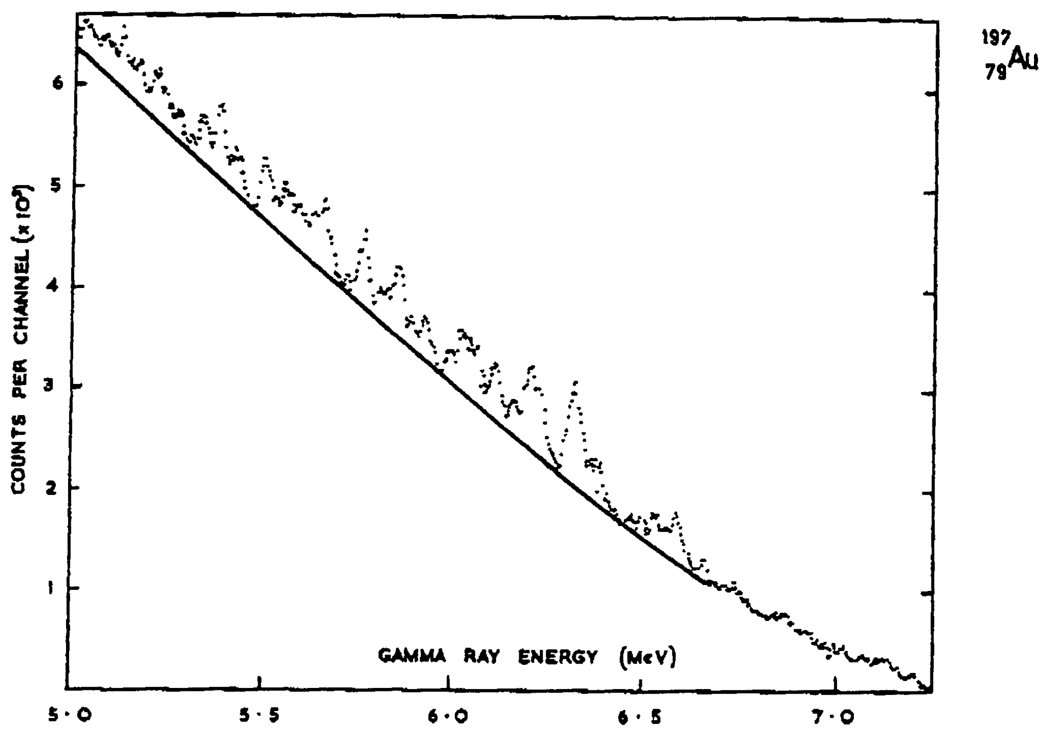
$\sigma(\theta, E_p)$. Recommended Values

Simple energy variation over large range

Nuclear Data Tables 11(1973)576

Neutron Capture in Gold

BIRD, ALLEN, BERGQVIST, AND BIGGERSTAFF



Spectrum for capture of 10 to 60 keV neutrons in gold. Target – 1.5 kg metal. Detector – Ge(Li) Escape peaks and background have been subtracted and a correction applied for neutron energy broadening of peaks (Al 68 b). See also Be62 a, Fi61.

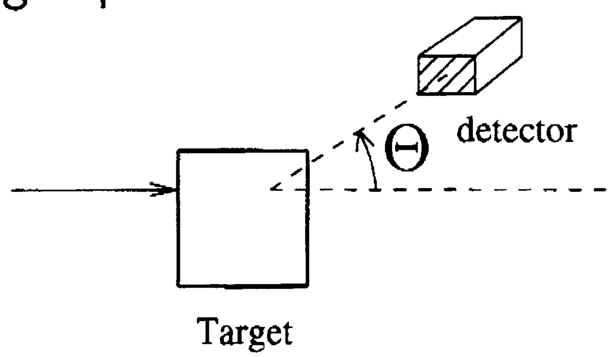
Complex energy variation in small intervall

Nuclear Data Tables 11(1973)518

Neutron - Proton Scattering
H(n,n)H

Nuclear Physics: proton and neutron elementary particles
$\Rightarrow$ considered pointlike

Scattering experiment:

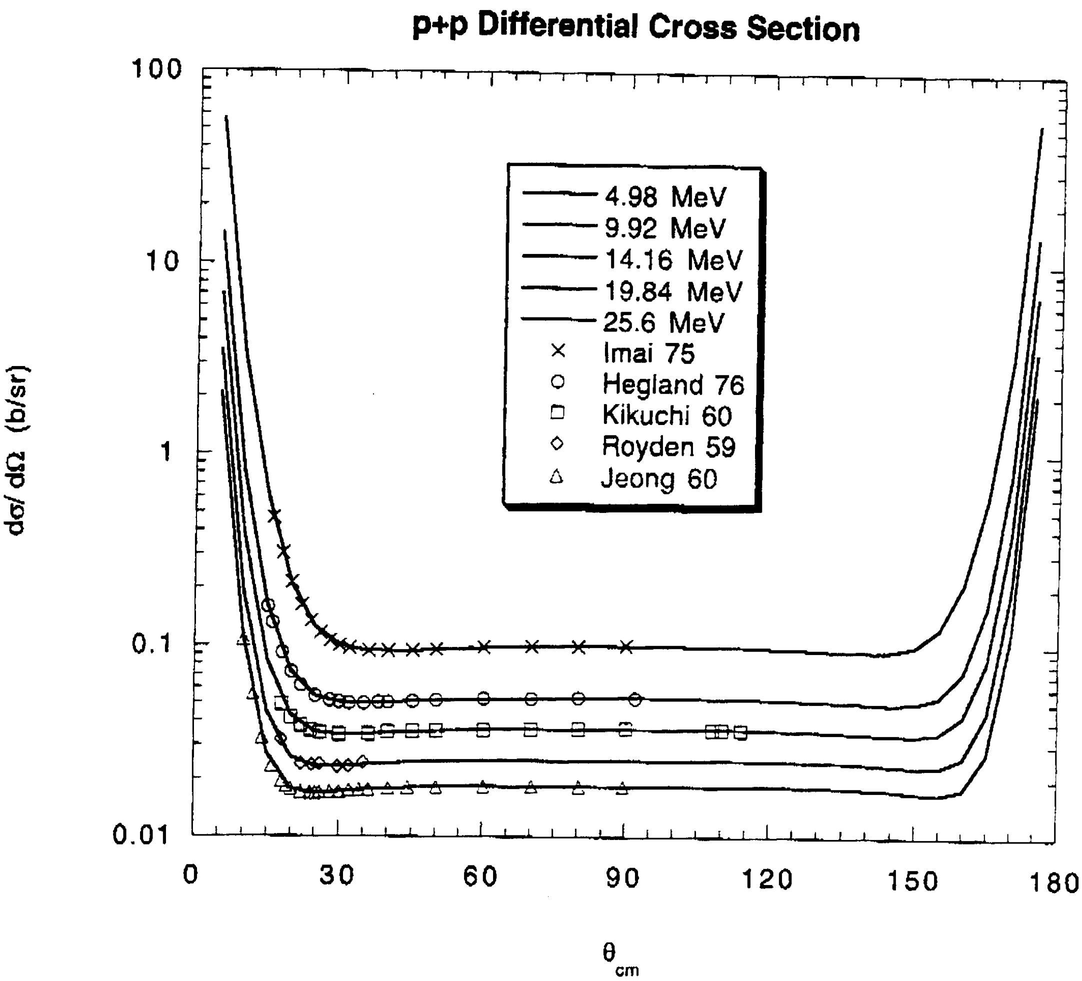


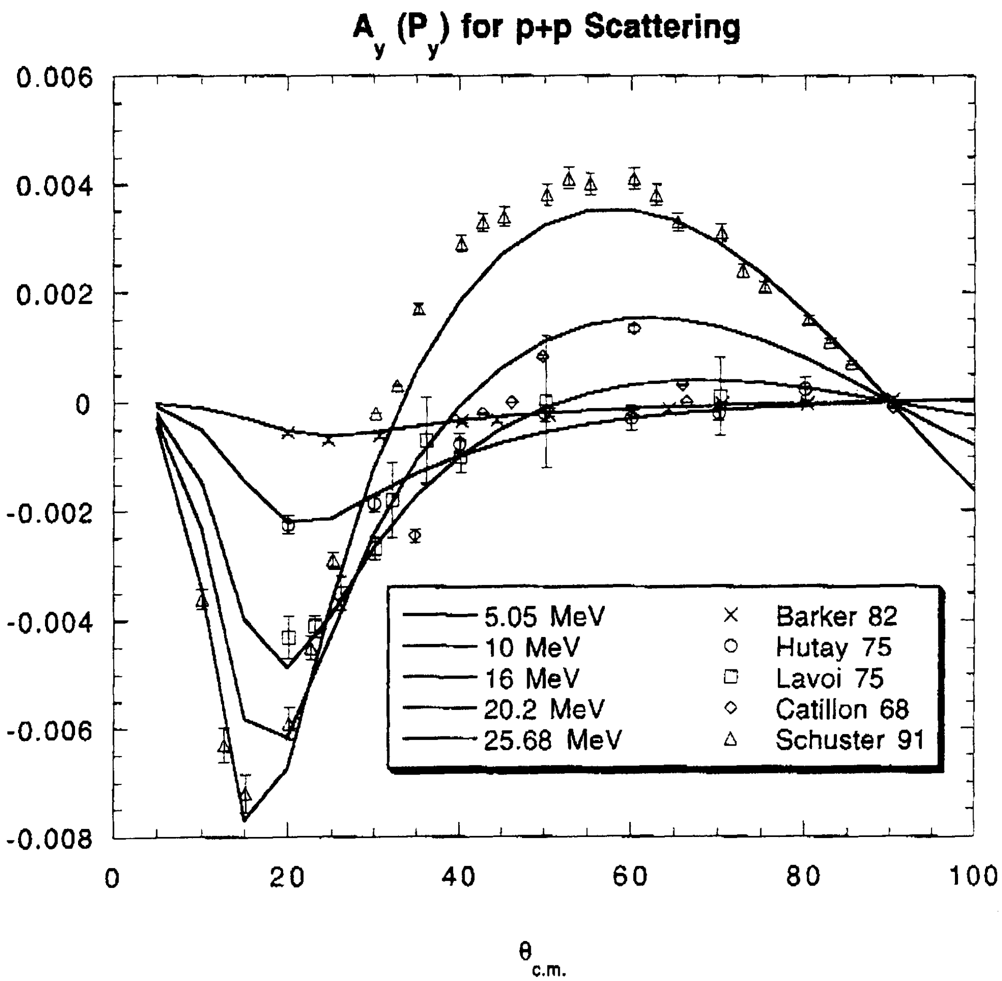
incoming neutron impinging on proton target
$\hat{=}$ incoming wave

scattered neutron measured in detector
$\hat{=}$ scattered wave

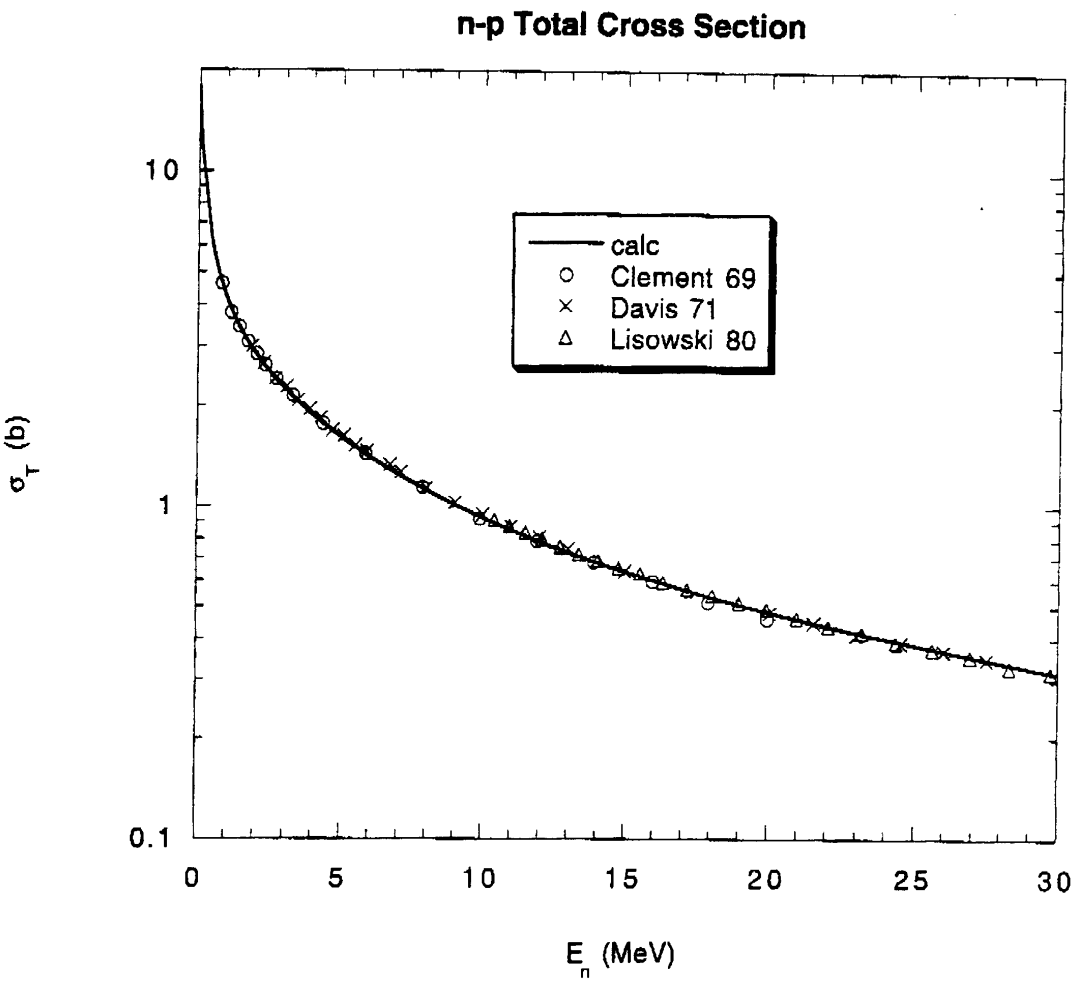
Theory: center-of-mass system
relative and center-of-mass coordinates
center-of-mass moves with constant velocity
$\Rightarrow$ can be separated trivially

$\Rightarrow$relative motion $\hat{=}$ scattering by fixed potential

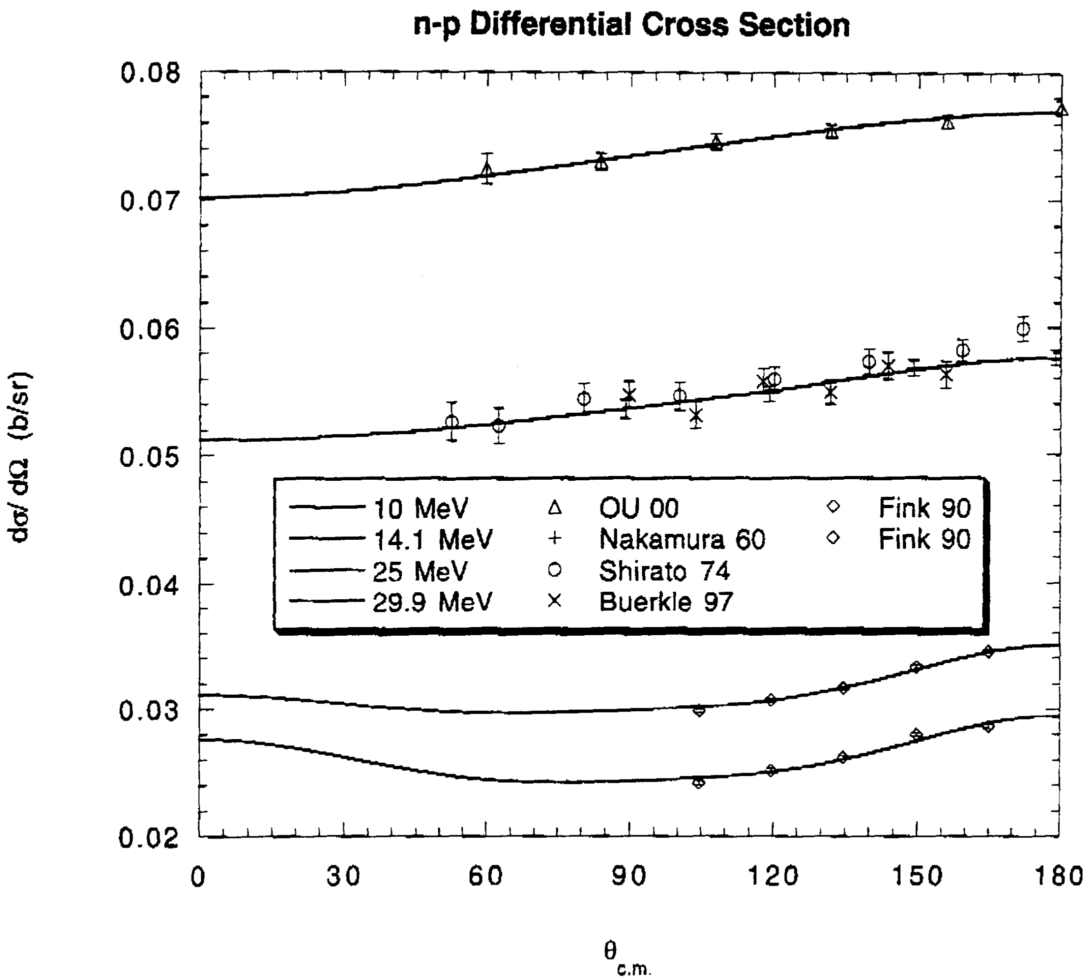

p+p Differential Cross Section
100
10
1
0.1
0.01
0
30
60
90
120
150
180
dσ/ dΩ (b/sr)
θ cm
4.98 MeV
9.92 MeV
14.16 MeV
19.84 MeV
25.6 MeV
Imai 75
Hegland 76
Kikuchi 60
Royden 59
Jeong 60



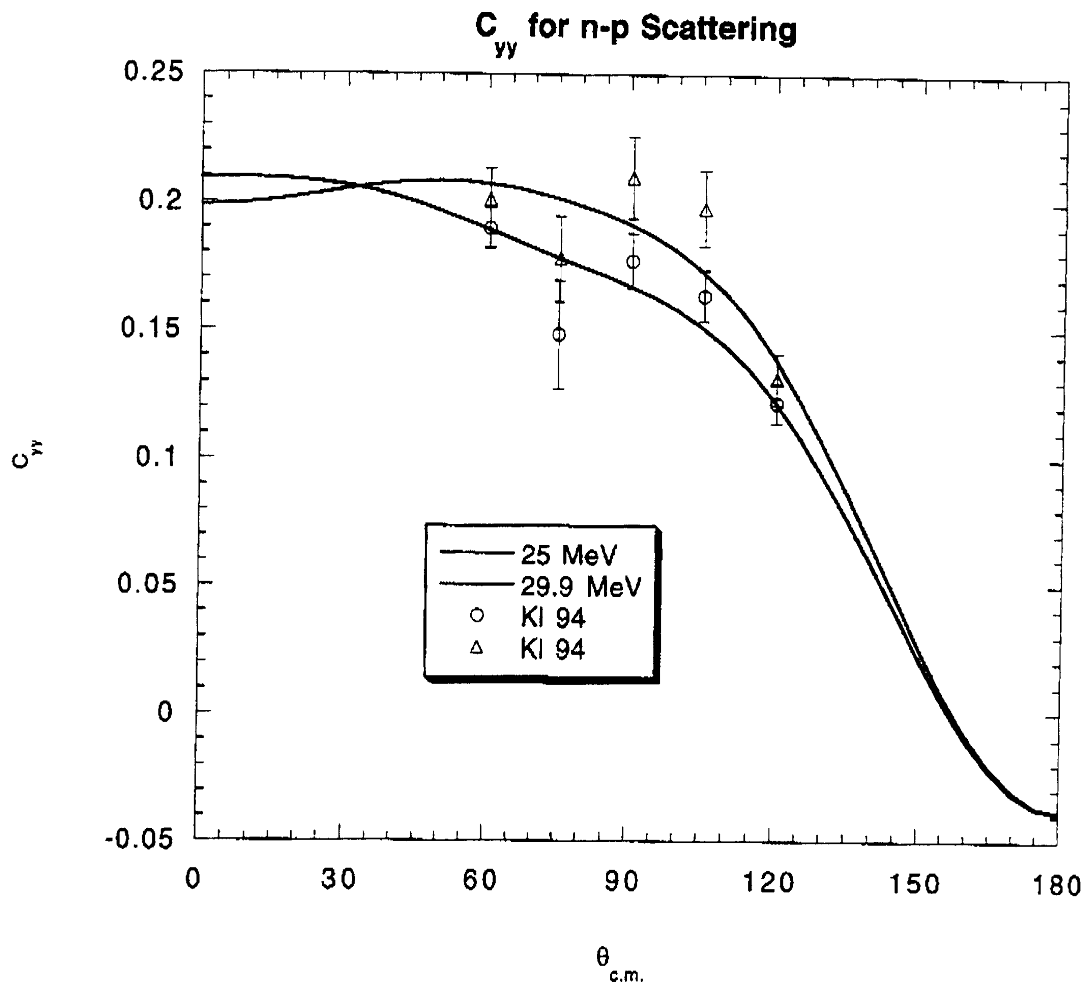
A_y (P_y) for p+p Scattering
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
0
20
40
60
80
100
5.05 MeV
10 MeV
16 MeV
20.2 MeV
25.68 MeV
Barker 82
Hutay 75
Lavoi 75
Catillon 68
Schuster 91
$\theta_{c.m.}$



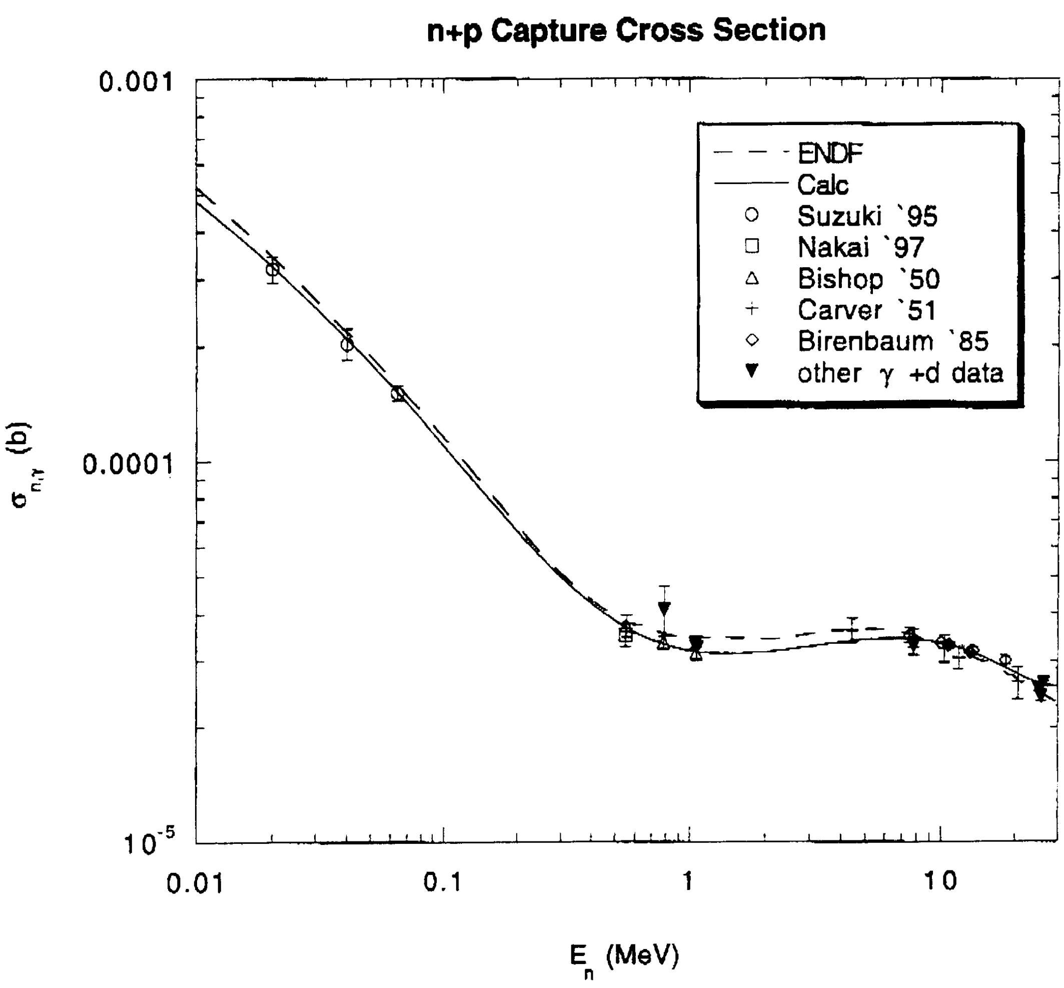
n-p Total Cross Section
calc
Clement 69
Davis 71
Lisowski 80
10
1
0.1
0
5
10
15
20
25
30
σT (b)
En (MeV)



n-p Differential Cross Section
0.08
0.07
0.06
0.05
0.04
0.03
0.02
dσ/ dΩ (b/sr)
0
30
60
90
120
150
180
θ c.m.
10 MeV
14.1 MeV
25 MeV
29.9 MeV
OU 00
Nakamura 60
Shirato 74
Buerkle 97
Fink 90
Fink 90



C_{yy} for n-p Scattering
C_{yy}
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
30
60
90
120
150
180
$\theta_{c.m.}$
25 MeV
29.9 MeV
KI 94
KI 94


n+p Capture Cross Section
0.001
0.0001
10^{-5}
$\sigma_{n,\gamma}$ (b)
0.01
0.1
1
10
E_n (MeV)
ENDF
Calc
Suzuki `95
Nakai `97
Bishop `50
Carver `51
Birenbaum `85
other γ +d data

Potential Scattering

Potential of finite range $\hat{=}$ interaction region
Description: Incoming free wave
modified in interaction region
scattered particles outside interaction region
free again

Dimensions: neutron beam formed far from target
dimension of beam much larger than H-atom
detector far from target
Conservation of probability:
Incoming beam
total flux through successive planes orthogonal to beam constant
$\Rightarrow$ Incoming beam $\hat{=}$ plane wave $\propto e^{i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}$
Scattered particles
total flux through increasing spheres around potential constant
surface of spheres $\propto$ radius r squared
$\Rightarrow$ Scattered particles $\hat{=}$ spherical wave $\propto \frac{e^{ikr}}{r}$

Ansatz for total wave function $\psi(\vec{r}) = e^{i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} + f(\Theta)\frac{e^{ikr}}{r}$

Differential cross section $= \frac{\text{scattered flux through area dA}}{\text{incoming flux}}$

$$d\sigma(\Theta, \phi) = |f(\Theta)|^2 d\Omega$$

Partial Wave Expansion

Potential rotationally invariant
$\Rightarrow$ conservation of angular momentum
$\Rightarrow$ angular momentum unchanged during scattering
$\Rightarrow$ consider each angular momentum separately

Spherical harmonics $Y_{lm}(\Theta, \phi)$ are eigenfunctions of orbital angular momentum operator
$\Rightarrow$ form complete set of functions for spherical angles Θ, ϕ
$\Rightarrow$ Any function $f(\Theta, \phi)$ can be expanded in spherical harmonics
Simplest spherical harmonic
$Y_{00}(\Theta, \phi) = const = 1/\sqrt{4\pi}$

Ansatz for total scattering wave function

$\Psi(r, \Theta, \phi) = \sum_{l,m} u_l(r)/r Y_{lm}(\Theta, \phi)$

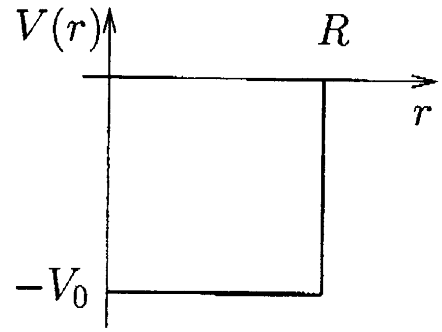
u_l obeys radial Schrödinger equation

$$\left[\frac{d^2}{dr^2} + \frac{2mE}{\hbar^2} - \frac{2mV(r)}{\hbar^2} - \frac{l(l+1)}{r^2}\right] u_l(r) = 0$$

Probability interpretation and continuity of flux
$\Rightarrow$ wave function u_l and derivative $du_l(r)/dr$ continous
regular at origin: $u_l(0) = 0$

Example: No potential
$\ell = 0$ solutions
general solution
standing waves: $\sin kr, \cos kr$
(Riccati-)Bessel functions $j_l(kr), n_l(kr)$
In- and outgoing waves:e^{-ikr}, e^{ikr}
Hankel functions $h_l^{(1)}(kr), h_l^{(2)}(kr)$
regularity $\Rightarrow$ only $\sin kr$ allowed

Scattering Phase Shift



Example:

square-well potential, $\ell = 0$

$$V(r) = \begin{cases} -V_0 & r \leq R \\ 0 & r > R \end{cases}$$

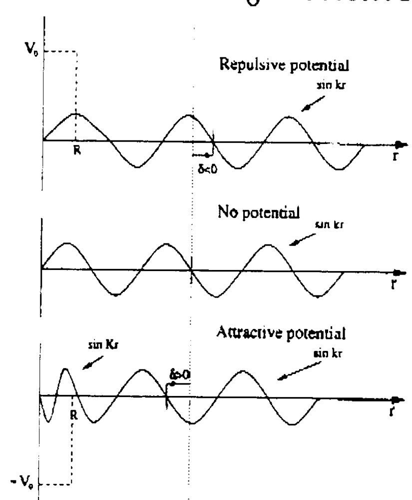
wave function

$$u_0(r) = \begin{cases} \sin Kr & r \leq R \\ A \sin kr + B \cos kr & r > R \\ \propto \sin(kr + \delta_0) & \end{cases}$$

with $k^2 = 2mE/\hbar^2$ and $K^2 = 2m(E + V_0)/\hbar^2$

for r = R logarithmic derivative of u_0 continous

$\Rightarrow \tan \delta_0 = B/A$



$\Rightarrow$ weak potential: attractive $\delta > 0$

repulsive $\delta < 0$

phase shift δ is function of energy

S-Wave Scattering

No potential

$$\psi_0 = \underbrace{\frac{\sin kr}{kr}}_{free} = \underbrace{\frac{1}{2ikr}(e^{ikr}}_{outgoing} - \underbrace{e^{-ikr})}_{incoming} \quad wave$$

with potential, outside potential

$$\psi_0 = N\frac{\sin(kr+\delta_0)}{kr} = \frac{1}{2ikr}(S_0 e^{ikr} - e^{-ikr})$$

outgoing wave is modified, incoming must not

$$\psi_0 = \frac{N}{2ikr}(e^{ikr+i\delta_0} - e^{-ikr-i\delta_0})$$

$\Rightarrow N = e^{i\delta_0}$ and $S_0 = e^{2i\delta_0}$

$$\psi_0 = \frac{e^{i\delta_0}}{2ikr}\left(e^{ikr+i\delta_0} - e^{-ikr-i\delta_0}\right)$$

$$= \underbrace{\frac{1}{2ikr}\left(e^{ikr} - e^{-ikr}\right)}_{free \quad wave} + \underbrace{\frac{e^{ikr}}{2ikr}\left(e^{2i\delta_0} - 1\right)}_{scattered \quad wave \quad \psi_{sc}}$$

$$\psi_{sc} = \frac{e^{ikr}}{kr} e^{i\delta_0} \sin \delta_0 = \frac{e^{ikr}}{r} f_{l=0}(\Theta)$$

Partial cross section: $\frac{d\sigma}{d\Omega} = |f_{l=0}(\Theta)|^2 = \frac{\sin^2 \delta_0}{k^2}$

Energy Dependence of Scattering Phase Shifts

Physical quantities contain phase shifts only in expressions like $e^{2i\delta}$ or $e^{i\delta}\sin\delta$

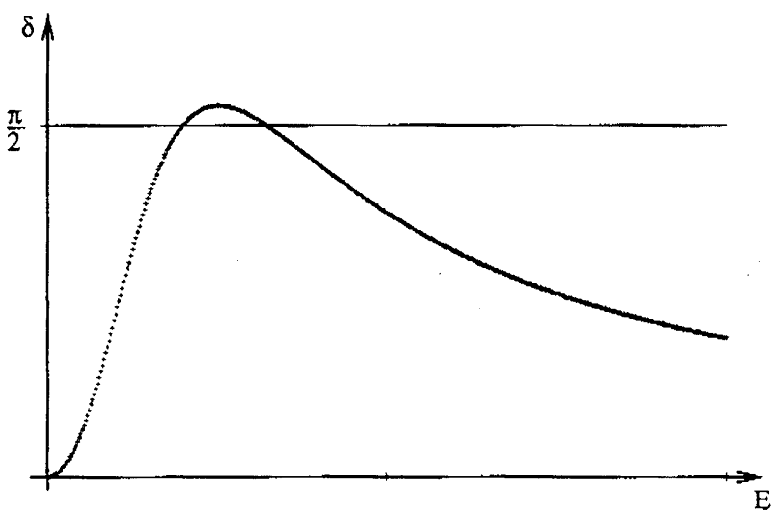
$\Rightarrow$ multiples of π do not matter for δ

Standard choice $\delta(Energy = \infty) = 0$

Levinson theorem:

$\delta(E=0) - \delta(E=\infty) =$ number of bound states $\times\pi$

Typical behaviour



Example: P-Wave Phase Shifts

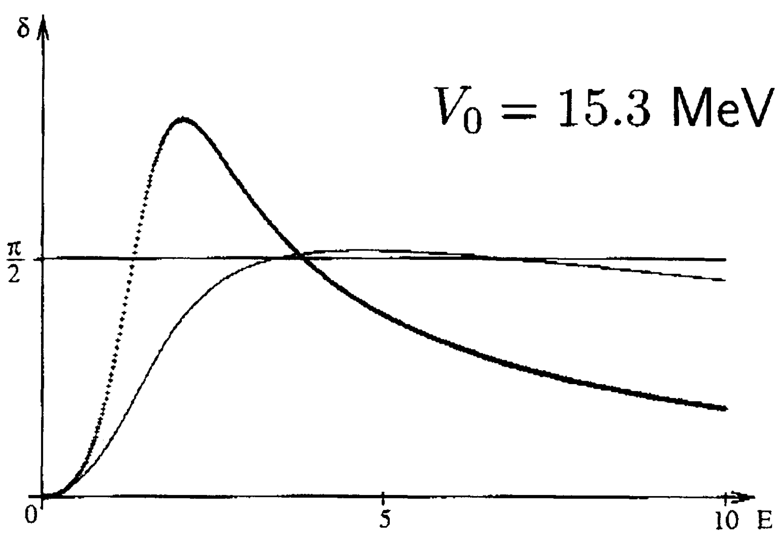
Exercise:

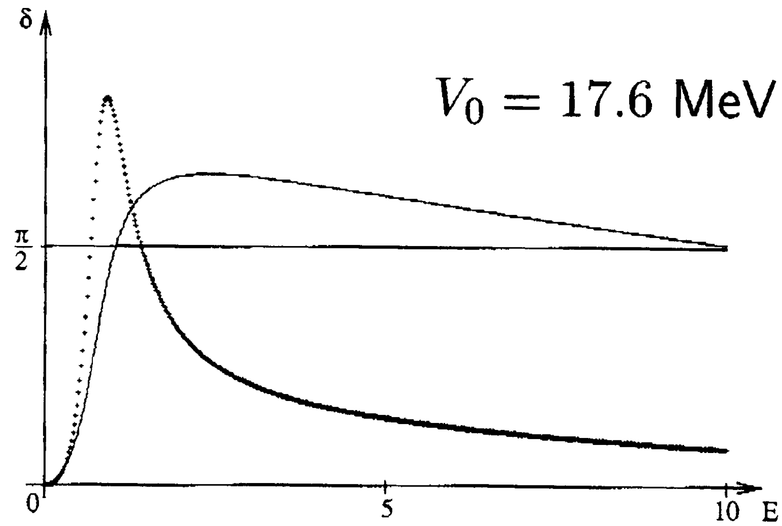
Spherical square well potential, range R = 3.14fm

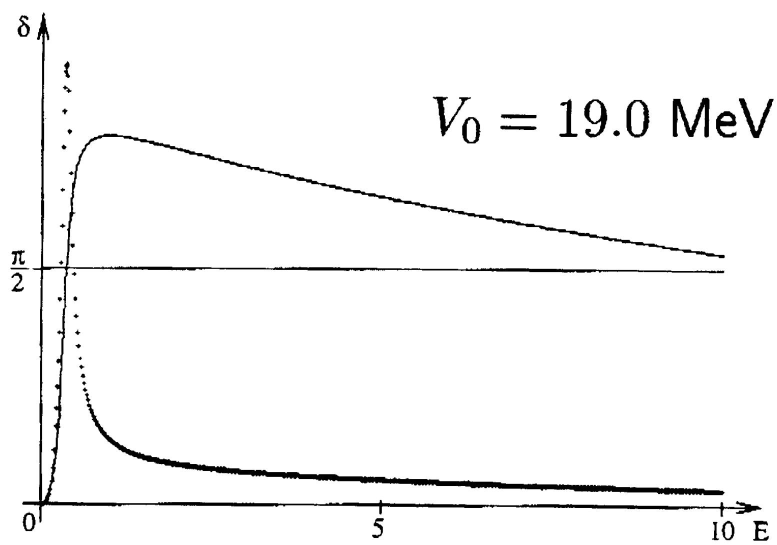
energy range considered 0 - 14 MeV

plot $\delta_{l=1}$ and $\sigma_1 = 4\pi 3 \sin^2 \delta / k^2$ as function of energy for V_0 = 15.3 , 17.6 , and 19.0 MeV

(M = 1000 MeV/c^2, $\hbar c \approx$ 200 MeV fm) $j_1(4.49) \approx 0$







Note: Only narrow resonances have their maximum at $\delta = \frac{\pi}{2}$

Neutron - Proton - Scattering

Nuclear Physics: neutron and proton elementary particles
What do we know about their interaction ?
n + p form bound state deuteron $J^\pi = 1^+$
no n + n, or p + p bound state known
n + p state with spin zero unbound
$\Rightarrow$ NN interaction spin-isospin dependent

Simplest deuteron model: attractive potential, spin S = 1, no orbital angular momentum
However, deuteron has quadrupol moment
$\Rightarrow$ aspherical shape needed
$\Rightarrow$ need higher orbital angular momenta of positive parity
$\Rightarrow$ D -wave component $\ell = 2$, coupled channels

S - wave: $(l = 0, S = 1)^{J=1^+}$
D - wave: $(l = 2, S = 1)^{J=1^+}$
$\Rightarrow$ need non-central tensor force

General Nucleon-Nucleon Force

$V_{NN}(\vec{r}) = V_{central}(r)$

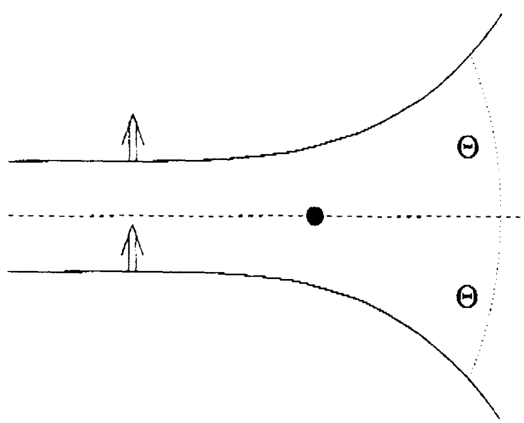
no direction distinguished, can be different for parallel spins ↑↑ and anti-parallel spins ↑↓

$+\ V_{tensor}(r)\left[Y_2(\hat{\vec{r}})\,[\vec{\sigma_1}\vec{\sigma_2}]^2\right]^0$ spins parallel ↑↑

$\propto \vec{r}\cdot\vec{\sigma_1}\quad \vec{r}\cdot\vec{\sigma_2}\quad - \quad \frac{1}{3}r^2\quad \vec{\sigma_1}\cdot\vec{\sigma_2}$

$+V_{spin-orbit}(r)\quad \vec{L}\cdot(\vec{\sigma_1}+\vec{\sigma_2})$

distinguishes left-right scattering of polarized nucleons



Observable: polarization (measured in exit channel)
analysing power (prepared in initial channel)

measures interference of different partial waves
⇒ sensitive to small components

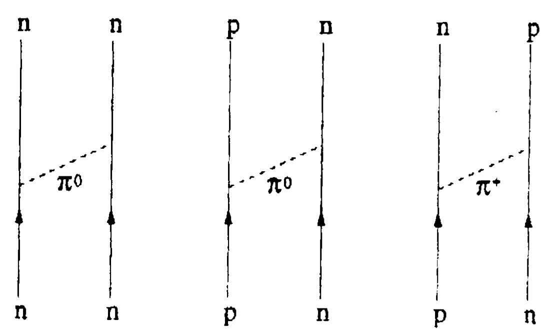
Theoretical Approach

Quantum - Chromo - Dynamics: THE theory of strong interactions also governs nuclear physics

However, no solutions known till know

Instead of QCD degrees of freedom, quarks and gluons, use hadron degrees, nucleons and mesons

Meson-Exchange-Models
long range part dominated by lightest meson, the pion
Model parameters fitted to deuteron properties and many scattering data



radial dependence of short range phenomenologic
⇒ Paris-, Argonne-, Nijmegen- ... potential
radial dependence of short range given by heavy mesons
⇒ Bonn-, Moscow-, ... potential

These potentials, so called realistic NN-potentials, reproduce pp and np data within error bars, due to fit

See following figures for NN-data

Idea: use these potentials to describe reactions of heavier nuclei

S-Wave Scattering

No potential

$$\psi_0 = \underbrace{\frac{\sin kr}{kr}}_{free} = \underbrace{\frac{1}{2ikr}(e^{ikr}}_{outgoing} - \underbrace{e^{-ikr})}_{incoming\quad wave}$$

with potential, outside potential

$$\psi_0 = N\frac{\sin(kr+\delta_0)}{kr} = \frac{1}{2ikr}(S_0 e^{ikr} - e^{-ikr})$$

outgoing wave is modified, incoming must not

$$\psi_0 = \frac{N}{2ikr}(e^{ikr+i\delta_0} - e^{-ikr-i\delta_0})$$

$$\Rightarrow N = e^{i\delta_0} \qquad \text{and} \qquad S_0 = e^{2i\delta_0}$$

$$\psi_0 = \frac{e^{i\delta_0}}{2ikr}\left(e^{ikr+i\delta_0} - e^{-ikr-i\delta_0}\right)$$

$$= \underbrace{\frac{1}{2ikr}\left(e^{ikr} - e^{-ikr}\right)}_{free\quad wave} + \underbrace{\frac{e^{ikr}}{2ikr}\left(e^{2i\delta_0} - 1\right)}_{scattered\quad wave\quad \psi_{sc}}$$

$$\psi_{sc} = \frac{e^{ikr}}{kr}e^{i\delta_0}\sin\delta_0 = \frac{e^{ikr}}{r}f_{l=0}(\Theta)$$

Partial cross section: $\frac{d\sigma}{d\Omega} = |f_{l=0}(\Theta)|^2 = \frac{\sin^2\delta_0}{k^2}$

R-Matrix Data Analysis

Idea: Separate configuration space in two parts
I: Interaction region of finite channel radius a
II: asymptotic space (no interaction, except point Coulomb)
Solution in II known, free incoming and outgoing waves, $I - S(E)O$
Hamiltonian not hermitian in finite space
Choose boundary condition B, derivative zero
Instead $(H - E)u_E = 0$ use
$(H + \mathcal{L}_\mathcal{B} - E)u_E = \mathcal{L}_\mathcal{B} u_E$
$u_E = \underbrace{(H + \mathcal{L}_\mathcal{B} - E)^{-1}}_{BlacksfunctionG} \quad \mathcal{L}_\mathcal{B} u_E$
Eigenfunctions of H in I u_λ form complete set
expand in I G and u_E in terms of u_λ
$\Rightarrow u_E(r) = G(r,a) a \frac{du_E}{dr}|_a$ and
$G(r,a) \propto \sum_\lambda \frac{u_\lambda(r) u_\lambda(a)}{E_\lambda - E}$

Define R-Matrix
$R = \frac{u_E(a)}{a\frac{du_E}{dr}|_a} = G(a,a) = \sum_\lambda \gamma_\lambda^2/(E_\lambda - E)$

Connection to Scattering Matrix

In asymptotic region II $u_E \propto I - S(E)O$
logarithmic derivative continuous at channel radius a

$$\underbrace{R = \frac{u_E}{au'_E}|_a}_{internal} = \underbrace{\frac{I - SO}{a(I' - SO')}|_a}_{asymptotic}$$

$$\Rightarrow S(E) = \frac{I(a)}{O(a)} \quad \frac{1 - L_I R}{1 - L_O R}$$

with $L_O = \frac{aO'(a)}{O(a)} = \left(\frac{aI'(a)}{I(a)}\right)^* = (L_I)^*$
In simple case: L_O positive imaginary
$\Rightarrow$ poles of S in lower half-plane
Data analysis:
Use finite number of R-matrix pole positions (energies E_λ) and residues (reduced width amplitudes γ_λ) to reproduce data

Physical Constraints in R-Matrix Theory

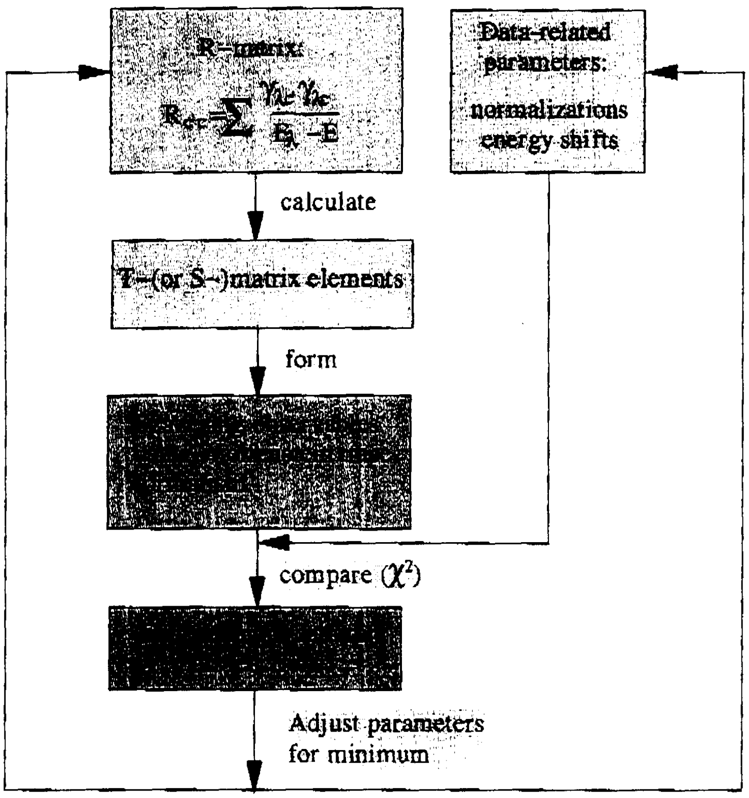
Unitarity ($SS^\dagger = S^\dagger S = \mathbf{1}$)
comes from R real (hermitian), constraints
S-matrix elements for different reactions

Built-In Symmetries of Strong Interaction
Conservation of angular momentum and parity
Time - reversal invariance due to R, S symmetric

Approximate Symmetry
Charge independence (charge symmetric)

Truncation of nuclear partial wave series
Due to finite channel radius and
Coulomb/angular-momentum barrier

Energy Dependent Analysis Code



Capabilities and Features

1) Accomodates general (spins, masses, charges) two-body channels

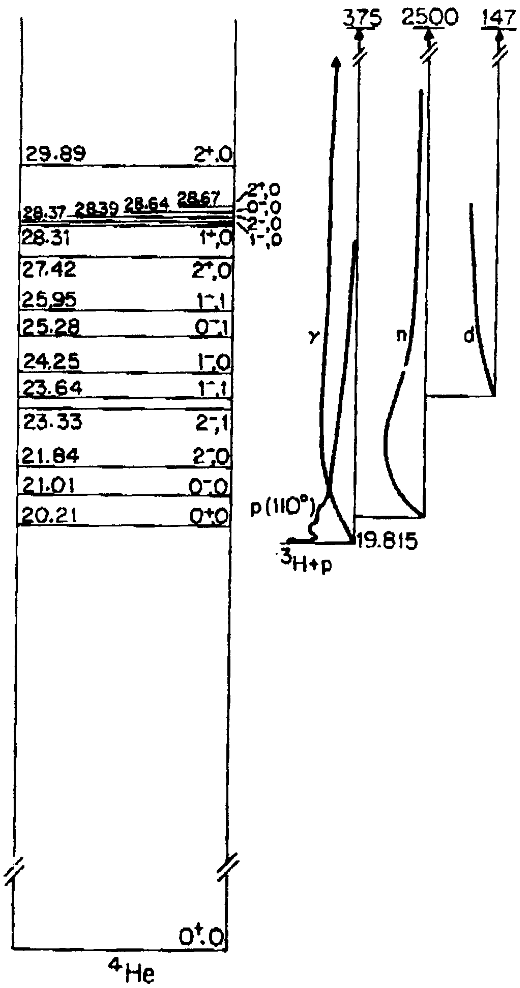
2) Uses relativistic kinematics and R-matrix formulation

3) Calculates general scattering observables for 2→ 2 processes

4) Has rather general data-handling capabilities

5) Uses modified variable-metric search algorithm that gives parameter covariances at a solution.

^{4}He Level Scheme



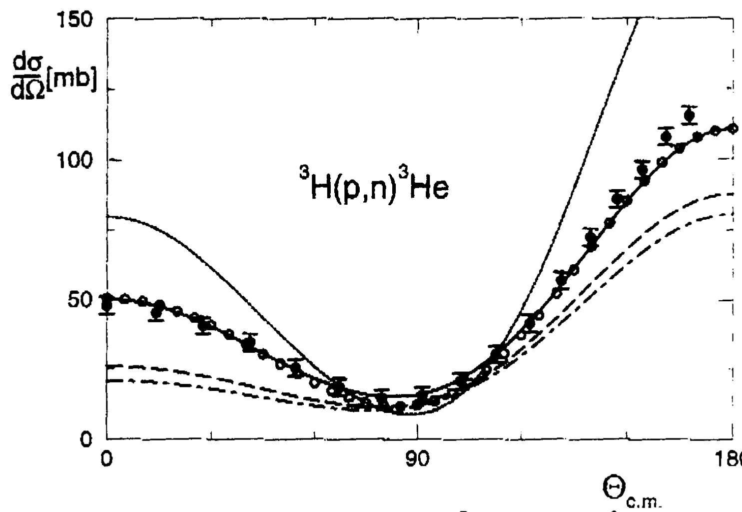
physical channels: proton - triton, neutron - ^{3}He, deuteron - deuteron

several channel spins possible

⇒ coupled channels

The Reaction $^3\mathrm{He(n,p)t}$

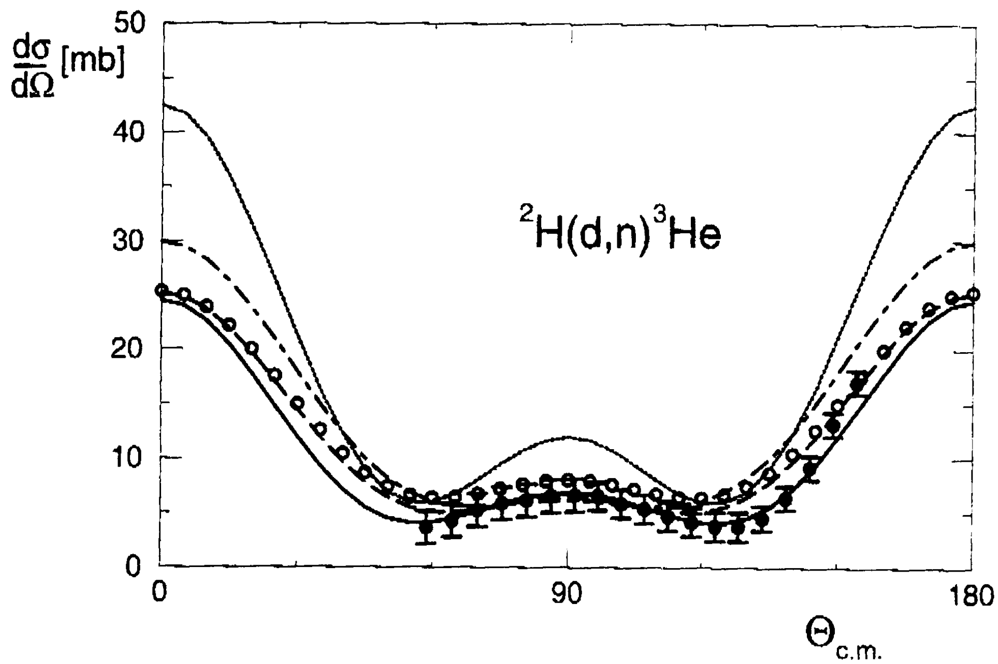
Due to open channels always coupled channels
Four-body problem, difficult to calculate
huge computer power neccessary
only qualitative agreement with data
bit prior information into R-matrix analysis
many parameters neccessary
e.g. $\ell = 2$: 36 channels; $\ell = 3$: 51 channels



Differential cross section of the reaction $^3\mathrm{H}(p,n)^3\mathrm{He}$ calculated for $E_{cm} = 3.0$ MeV. The data are for 4.101 MeV protons from Perry. The full line represents the R-matrix analysis, the dashed one the full calculation, the dot-dashed one the small calculation, and the dotted one the semi-realistic calculation. The open circles denote the full calculation with the $^3P_2$ matrix element replaced by the corresponding R-matrix one.

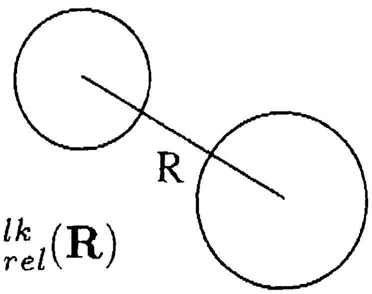
Difficulties of Analysis

Many coupled channels⇒ many parameters
Resonances very broad ⇒ not too well defined
background problematic
not enough data
inconsistent data
How to treat break-up channels



Differential cross section for the reaction $^2\mathrm{H}(d,n)^3\mathrm{He}$ calculated for $E_{cm} = 2.11$ MeV. Data are for 4.0 MeV deuterons from Schulte. The labeling is as before.

Resonating Group Model Ideas



Composite system

RGM Ansatz $\Psi_l = \sum_{k=1}^{chan} \psi_{chan}^{k} \cdot \chi_{rel}^{lk}(\mathbf{R})$

Variation $\langle \delta\Psi_l \mathcal{A} | \mathrm{H} - \mathrm{E} | \Psi_l \rangle = 0$

Channel function $\psi_{chan} = [Y_L(\hat{\mathbf{R}}) \otimes [\phi_1^{j_1} \otimes \phi_2^{j_2}]^{S_c}]^J$

Ansatz $\psi = \psi_{chan}(\sum_i b_i \cdot \mathrm{Gaussian})$ (bound state)

or $\chi_{rel}^{lk}(R) = \delta_{lk} \cdot F_k(R) + a_{lk} \cdot \tilde{G}_k(R) + \sum_i b_{lki} \cdot$ Gaussian (scattering state)

Variational parameters a_{lk} and $b_{(lk)i}$

Decompose Hamiltonian

$$H - E = H_1 - E_1 + H_2 - E_2 +$$
$$\sum_{\substack{i\in 1\\ j\in 2}} V_{ij} - V_{Coul} +$$
$$T_R + V_{Coul} - (E - E_1 - E_2) =$$
$$H_1 - E_1 + H_2 - E_2 + V_{short} + H_R - \tilde{E}$$

with $\mathcal{A} \cdot (H_i - E_i)\phi_i = 0$ and $(H_R - \tilde{E})F/G = 0$

$\Rightarrow$ All integrals shortranged

Note: Relative thresholds fixed by $\tilde{E}$

Resonating Group Model Technicalities

⇒ Expand all Functions including F and G

- in terms of Gaussians
- times solid spherical harmonics
- times monomials in R^2

⇒ All individual integrals analytically calculable,
provided potential is of Gaussian form including differential operators
All Operators allowed which occur in Argonne and Bonn (r-space) potentials

- Correct center of mass motion
- No limit on number of channels
- No limit on number of nucleons
- Up to 6 clusters, i.e. up to 6 orbital angular momenta

⇒ Allow for distortion of fragments via different ϕ and/or different decompositions of the system

Three- and more-body channels approximately treated via two-body channels

Fragment wave functions ϕ_1 and ϕ_2 must be strongest bound in given model-space
⇒ Relative thresholds can only be changed by increasing dimension of model-space or other potential

Realistic NN-interactions versus effective ones

Examples: Bonn, Argonne-14, Argonne-18

Start from deuteron p - n
S,D-wave 2 configurations

binding due to tensor force

proceed via $^3\mathrm{H}/^3\mathrm{He}$ N - N - N
S,P,D,F-waves S,P,D,F 37 configurations

to $^4\mathrm{He}$
Some hundred configurations

present limit $^6\mathrm{Li}$
Some thousand configurations

All nuclei $\mathrm{A} \geq 3$ underbound due to missing three-nucleon force

Larger systems: Use effective NN-forces with reduced repulsive core

$\Rightarrow$ nuclei $\mathrm{A} \leq 4$ bound via central force alone, just one configuration
$\Rightarrow$ higher orbital symmetry
$\Rightarrow$ much simpler wave functions, nuclei up to $\mathrm{A} = 12$ accessible

Effective Interactions versus Realistic ones

Effective interactions

- simple wave functions
- comparatively fast calculations, e. g. ^{10}B - neutron scattering
- parameter studies possible
- model space dependence unclear
- severe overbinding possible
- limited energy range, $E_{\mathrm{threshold}} + \approx 25$ MeV

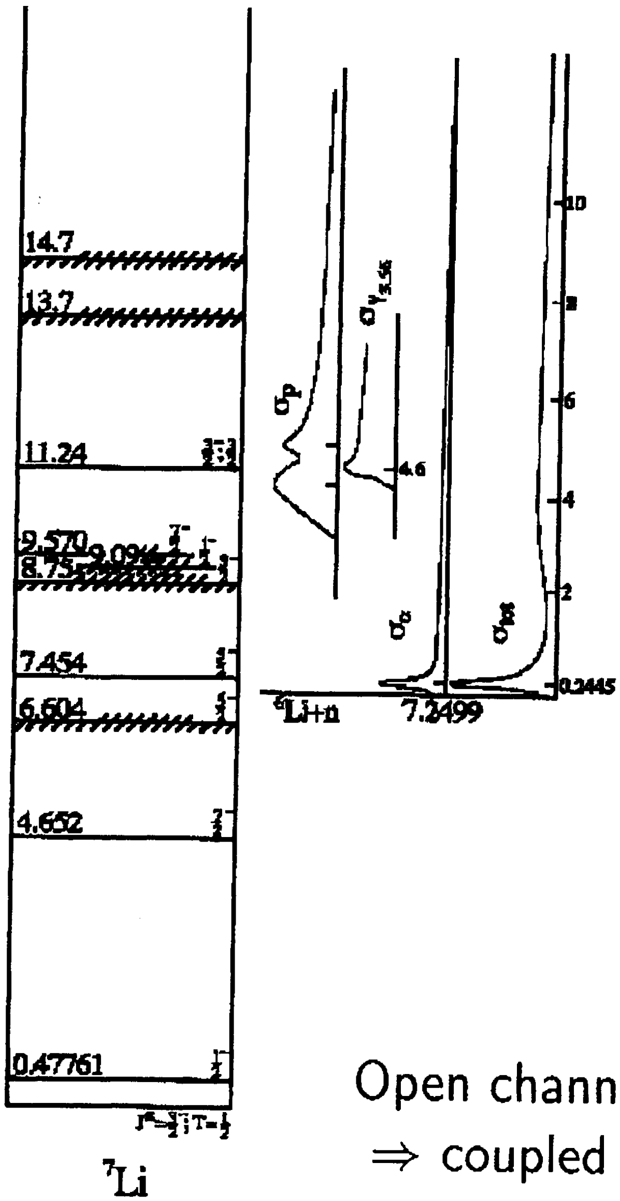
Realistic interactions

- complicated wave functions
- tedious long lasting calculations
- model spaces increase rapidly with A, limit around A = 6
- parameterfree calculation
- calculation improves with increasing model space
- no overbinding possible
- large energy range, up to pion threshold

Strategy

Study small system A
Blackuce model space till qualitative change
Use this model space as input for A + 1 system

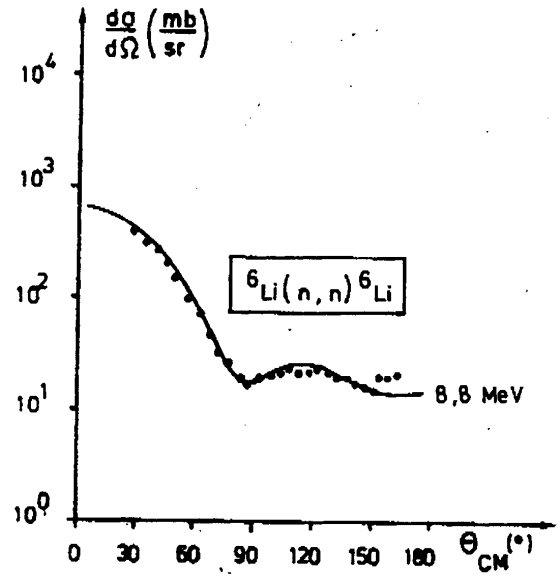
$^{6}\mathrm{Li}(\mathrm{n},\mathrm{t})\alpha$ Reaction



Open channel α - triton
$\Rightarrow$ coupled channels

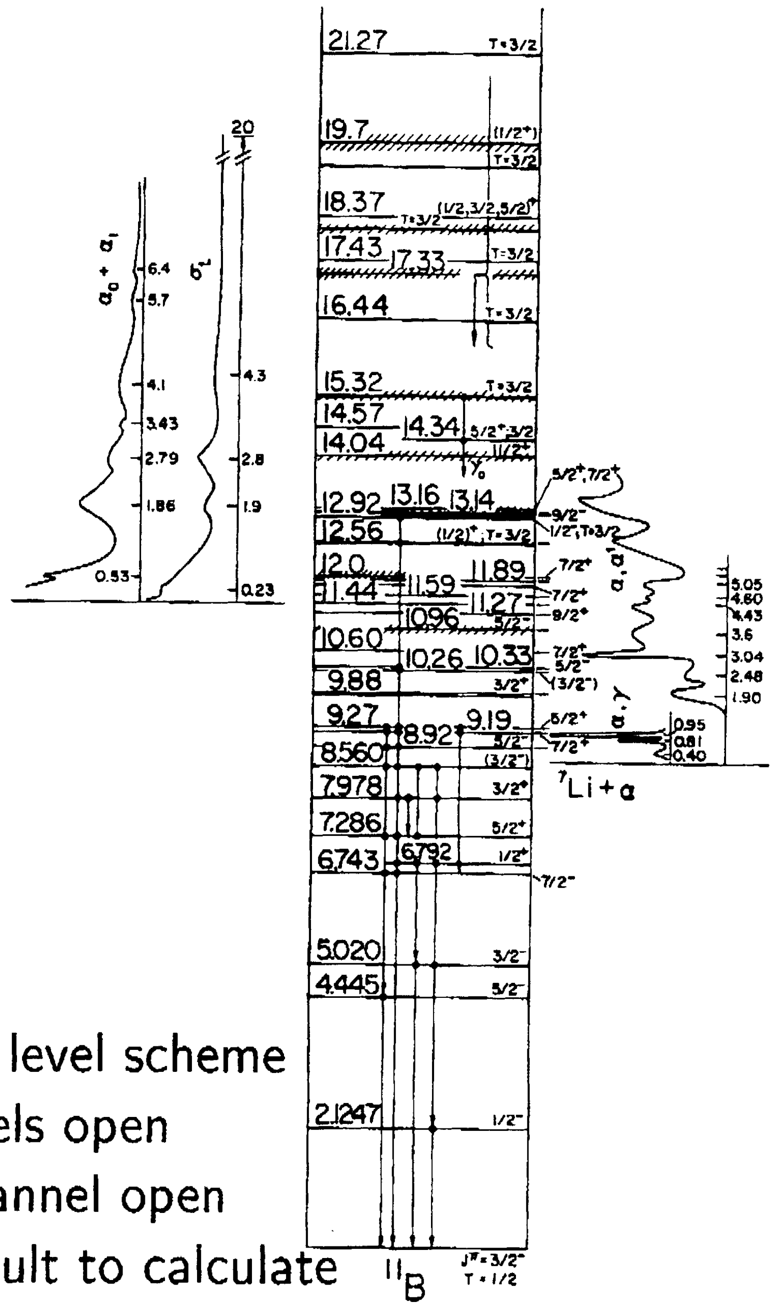
Well defined, narrow $\frac{5}{2}^{-}$ resonance in relevant energy range

Typical Resonating Group Result



M. Herman 1985 unpublished
see also 'Use of the Optical Model for ...
Neutron Cross-Sections ...', NEADC-222 'U'
page 77, OECD Paris 1986

$^{10}B(n,\alpha)^{7}Li$ Reaction



- Complicated level scheme
- Many channels open
- Break-up channel open
- ⇒ very difficult to calculate
- Data analysis tedious

General Nucleon-Nucleon Force

$V_{NN}(\vec{r}) = V_{central}(r)$

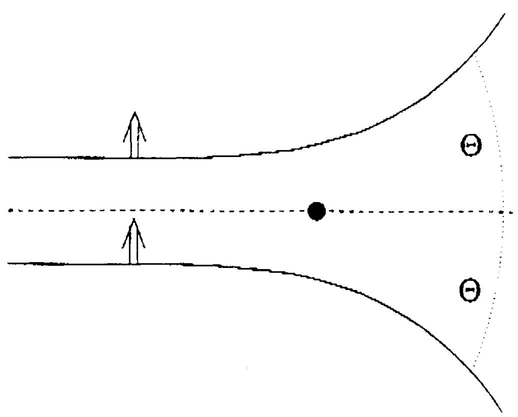
no direction distinguished, can be different for parallel spins $\uparrow\uparrow$ and anti-parallel spins $\uparrow\downarrow$

$+ \; V_{tensor}(r) \left[Y_2(\hat{\vec{r}}) \, [\vec{\sigma_1}\vec{\sigma_2}]^2 \right]^0$ spins parallel $\uparrow\uparrow$

$\quad \propto \vec{r} \cdot \vec{\sigma_1} \quad \vec{r} \cdot \vec{\sigma_2} \quad - \quad \frac{1}{3} r^2 \quad \vec{\sigma_1} \cdot \vec{\sigma_2}$

$+ V_{spin-orbit}(r) \quad \vec{L} \cdot (\vec{\sigma_1} + \vec{\sigma_2})$

distinguishes left-right scattering of polarized nucleons



Observable: polarization (measured in exit channel)
analysing power (prepared in initial channel)

measures interference of different partial waves
$\Rightarrow$ sensitive to small components