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NUCLEAR REACTION DATA AND NUCLEAR REACTORS:
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Statistical Theory of Nuclear Reactions

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These are preliminary lecture notes, intended only for distribution to participants.

Chapter 3.1.4

STATISTICAL THEORY OF NUCLEAR REACTIONS

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PART 3 SCATTERING IN NUCLEAR PHYSICS
Topic 3.1 Nuclear Physics

Contents

Nucleons, light or heavy nuclei with energies of up to several megaelectronvolts per nucleon, strike a nucleus. How do we account for the ensuing reaction processes (angular distributions, loss of kinetic energy and angular momentum, production of secondary particles)? The statistical theory developed to answer these questions forms the content of this chapter.

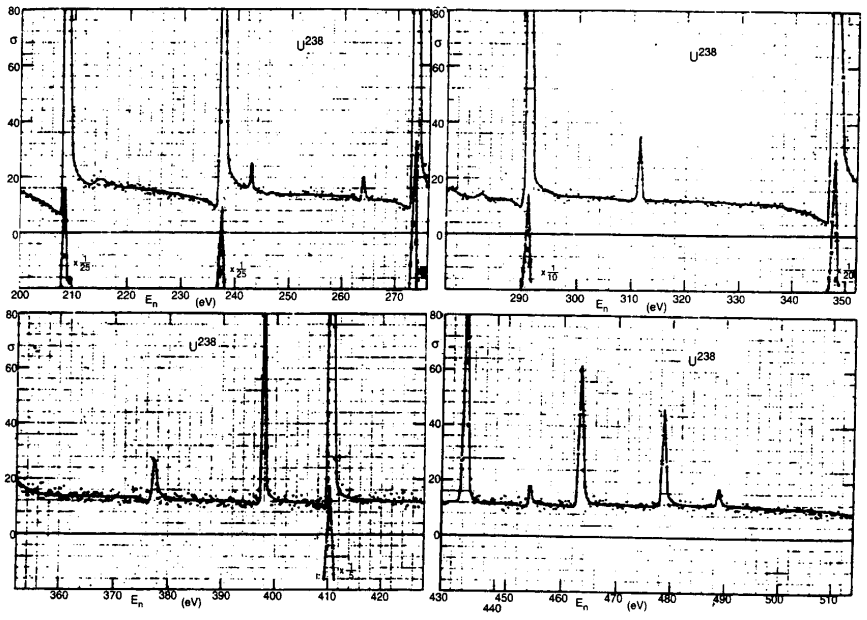
§1. Why a Statistical Theory?

Reaction processes induced by nucleons or nuclei on medium-weight or heavy target nuclei proceed either via a direct reaction or via the formation of a long-lived intermediate complex. The amplitudes for both processes are coherent if they lead to the same final states. The direct reaction typically involves only few nucleons or, more generally, few degrees of freedom. It allows for a theoretical description in terms of simple models. The direct contribution to elastic scattering, for instance, is calculated from a single-particle Schrödinger equation with a complex optical-model potential. The imaginary part describes absorption due to both direct inelastic processes that deplete the elastic channel and the formation of the long-lived intermediate complex. Theoretical models for formation and decay of the latter require an understanding of quasi-bound states of the "compound nucleus" formed by coalescing the target nucleus and the incident particle. It is here that the statistical approach applies. I show this now for the simplest case, elastic scattering of slow neutrons by medium-weight and heavy nuclei.

Figure 1 shows the total cross section for the scattering of neutrons on ^{238}U versus neutron energy E_n. The energy is in the 100-eV range and, thus, very small compared to the typical nuclear scale of 1 MeV. The data are taken using neutron time-of-flight spectroscopy. This limits the total number K of resonances seen in such data to typical values of $K \sim 200$ or so. The resonances displayed in Fig. 1 all have spin 1/2 and positive parity. Their average spacing d is about 10 to 20 eV, and their average total width Γ is on the order of 1 eV. A multilevel resonance analysis of the data yields values for the resonance energies ϵ_μ and for the widths Γ_μ, with μ running from 1 to K.

The resonances are quasi-bound excited states in the compound nucleus ^{239}U with an excitation energy of roughly 8 MeV, given by the binding energy of the last neutron in ^{239}U. The resonances form the simplest example of the intermediate complex referred to above and are called "compound-nucleus resonances". If we could count levels of the same spin and parity in ^{239}U from the ground state up, these resonances would carry running numbers in the range 10^6 or so. (Incomplete spectroscopic knowledge prevents us from actually doing this counting exercise, and the figure 10^6 is a rough estimate only. It is based on the nearly exponential increase of the nuclear level density with excitation energy. This increase accounts for the small value $d \sim 10$ eV displayed by the data.) At such high running numbers, dynamical models for nuclear spectra like the nuclear shell model cannot be used; the dimension of the matrices involved and the numerical accuracy required are too large. Instead, a statistical approach is highly successful. It has its root in ideas formulated by Bohr (1936) and

Figure 1 The total cross section for the scattering of slow neutrons by ^{238}U versus energy. Taken from Garg *et al.* (1964).



was developed by Wigner; see Porter (1965). For a recent review, see Guhr *et al.* (1998). We display this approach in its simplest form and consider the $1/2^+$ states in ^{239}U as bound states, disregarding their coupling to the neutron channel and to other channels. We return to this coupling later.

The nucleus is a time-reversal invariant system. Therefore, the Hamiltonian matrix $H_{\mu\nu}$ in Hilbert space for states $|\mu\rangle$ and $|\nu\rangle$ of fixed spin and parity can be chosen real and symmetric,

$$H_{\mu\nu} = H_{\nu\mu} = H^*_{\mu\nu}. \tag{1}$$

We denote the dimension of $H_{\mu\nu}$ by N and consider the limit $N \to \infty$. We consider an ensemble $\mathcal{E}$ of Hamiltonian matrices of the form (1), specified in terms of a statistical distribution of the matrix elements $H_{\mu\nu}$ and referred to as a random-matrix ensemble. We ask: Is it possible to derive statements about the distribution of eigenvalues and eigenfunctions for this ensemble? If so, and if the predictions obtained from such a "statistical" approach do agree in all respects with properties of the resonance energies ϵ_μ and resonance wave functions deduced from the data, we can simplify the calculation of the compound-nucleus contribution to nuclear scattering amplitudes: We would consider the energies ϵ_μ with $\mu = 1, \ldots, K$ as a typical subsequence of eigenvalues of a member of the ensemble $\mathcal{E}$. We would accordingly replace the actual nuclear Hamiltonian in the formal expression for the scattering matrix by the ensemble $\mathcal{E}$, and we would calculate the mean cross section by averaging over the ensemble. This average should then be compared with the average of the measured cross section taken over a sufficiently large energy interval containing many compound-nucleus resonances. We could proceed similarly for the calculation of the variance and/or of higher moments of the cross section. This, in a nutshell, is the programme of the statistical theory of nuclear reactions. The approach, if successful, predicts mean values and higher moments of observables but *not* the actual cross section in its full energy dependence as shown in Fig. 1. Such a prediction is possible only if all the energies ϵ_μ are known. This knowledge is forfeited when one uses a random-matrix model.

Implementation of this programme requires the following steps. First, the ensemble $\mathcal{E}$ must be specified, and it must be shown that the statistics of the resonance energies ϵ_μ do indeed follow the predictions of the ensemble. This step leads to the Gaussian orthogonal ensemble of random matrices (GOE) and is sketched in the remainder of this section. Next, the scattering matrix must be written in such a way that the Hamiltonian governing the quasi-bound states appears explicitly. This step amounts to the formulation of a theory of resonance reactions, see §2. It will allow us to replace the actual nuclear Hamiltonian by an ensemble $\mathcal{E}$ of Hamiltonian matrices, quite irrespective of the choice of this ensemble. This step is taken in §3. We then turn to the scattering amplitude for direct reactions. This amplitude is obtained by averaging the actual scattering amplitude over the

compound-nucleus resonances. Such averaging leads to the optical model of elastic scattering (§4) and defines the input parameters of the statistical model. In subsequent sections, the statistical approach is applied to a number of typical cases. First, we use the GOE to calculate compound-nucleus reactions (§5). This can also be done using the maximum-entropy approach (§6). The GOE is then generalised to allow for the breaking of a discrete symmetry or the violation of an invariance (§7), and to allow for the existence of a relaxation time in the compound nucleus (§8).

Following Wigner (see Porter (1965)), we introduce the GOE by the requirement that the ensemble should be invariant under those transformations of Hilbert space that leave the property (1) invariant. These are the orthogonal transformations. The ensemble is accordingly defined in terms of an integration over all independent matrix elements $H_{\mu\nu}$ with $\mu \leq \nu$ and a weight factor P. The integration measure is given by

$$P(H) \prod_{\mu \leq \nu} dH_{\mu\nu}. \tag{2}$$

The product of differentials of matrix elements of H has the required invariance property, and by choosing P to be a function of $\mathrm{tr}(H^m)$ with m integer or of a combination of such terms, overall orthogonal invariance is guaranteed. Averages of observables that depend on H are calculated by integration over matrix space with the weight factor P. The weight factor is needed as a cutoff to make the integrals convergent. It is usually chosen in Gaussian form,

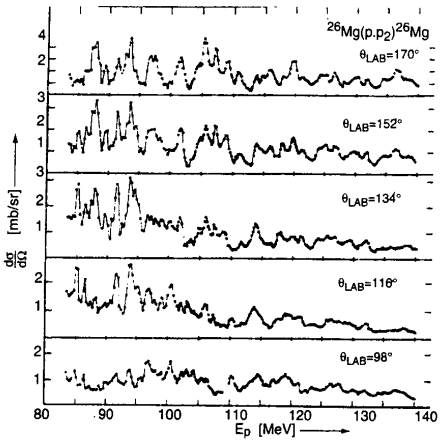
$$P \propto \exp(-(N/\lambda^2)\,\mathrm{tr}(H^2)). \tag{3}$$

Equations (2) and (3) define the GOE. In the limit $N \to \infty$, the average level density of the GOE has the shape of a semicircle of radius 2λ. At the centre of the semicircle, the mean level spacing is given by $d = \pi\lambda/N$. In applications of the GOE to experimental spectra, this relation is used to replace λ in favour of the actual mean level spacing.

Given d, the GOE predicts spectral fluctuation properties in a parameter-free fashion. For comparison with data, these properties are needed in an energy interval of order d. This interval is typically chosen near the centre of the semicircle. For $N \to \infty$, it is infinitely small in comparison with the radius of the semicircle. Hence, the unphysical shape of the overall GOE spectrum plays no role. The Gaussian choice for the weight factor P is obviously arbitrary. It is important to ascertain that other choices of P yield identical fluctuation properties (although the mean level spacings d will have different values). This is indeed the case provided that P is chosen such that the spectrum is again confined to a finite interval of the energy axis. This fact is referred to as universality and justifies the use of the Gaussian ensemble. In tests of the GOE, one typically compares an ensemble average calculated theoretically with an average taken over a single experimental spectrum (referred to as a running average). Equality of ensemble average and running average is the necessary condition for a meaningful application of random-matrix theory. This property is referred to as ergodicity. It is known to hold for a very wide class of observables and in the limit in which the running average is taken over infinitely many resonances (Brody *et al.*, 1981; Guhr *et al.*, 1998). For a finite number K of resonances, the finite-range-of-data error is of order K^{-1}.

The quantities most often used in the analysis of experimental data for energy levels are the nearest-neighbour spacing distribution, and the Δ_3 statistic. In the case of eigenfunctions, it is the Porter–Thomas distribution. There is no room to go into any details; see Porter (1965) and Guhr *et al.* (1998) and references therein. Suffice it to say that the GOE has been thoroughly tested by comparing its predictions with the totality of resonance data (level spacings and, to a lesser extent, resonance widths that involve GOE eigenfunctions) available for a number of nuclei in the form of data shown in Fig. 1, and in the form of similar proton scattering data taken in the vicinity of the Coulomb barrier. The success of the GOE approach to the resonance properties shows that the resonance energies ϵ_μ and resonance wave functions may be considered random variables. By the same token, the cross section shown in Fig. 1 is a random variable. It then makes sense to develop a statistical theory of elastic neutron scattering. This theory would apply in the regime of isolated resonances shown in Fig. 1. In this regime, the average width Γ of the resonances is small compared to the average level spacing d. The scope of the statistical theory is much wider, however. This is demonstrated in Fig. 2. At excitation energies of the compound nucleus higher than that shown in Fig. 1, statistical fluctuations of the cross section ("Ericson fluctuations") appear. The nearly exponential increase of the average nuclear level density with excitation energy goes along with a much weaker but also nearly exponential increase of the average total width Γ of the compound-nucleus resonances, caused by the growing number of channels open for decay into ground and excited states of the residual nuclei. Therefore, a few megaelectronvolts above neutron threshold the inequality $\Gamma \gg d$ holds. The compound-nucleus resonances overlap strongly. From the point of view of the statistical model, each such resonance makes a random contribution to the scattering amplitude, and these coherent contributions may add constructively or destructively, as the case may be. As a result, the cross section shows random fluctuations.

Figure 2 Ericson fluctuations in the differential cross section taken at several lab angles versus proton energy (in MeV) for inelastic proton scattering on ^{26}Mg leading to the second excited state. Taken from Häusser *et al.* (1968).



The coherence width of these fluctuations is expected to be of order Γ. Detailed analyses of many data sets of the type shown in Fig. 2 have definitely established the validity of the statistical point of view in this regime (Ericson and Mayer-Kuckuk, 1966).

Thus, the scope of a statistical theory of nuclear reactions covers the entire range of excitation energies from neutron threshold (the regime of isolated resonances) to several megaelectronvolts above neutron threshold and higher (the regime of strongly overlapping resonances). The statistical approach forfeits any detailed knowledge of the Hamiltonian of the system under study. It is successful if nuclear cross-section data are generic and rather independent of system-specific properties. Indeed, the statistical theory uses a very limited number of input parameters. It predicts fluctuations in terms of mean values. Therefore, input parameters are the values of the mean level spacing d, of the mean level width Γ, of the mean partial widths for the open decay channels and of the average scattering amplitude. These must be determined either phenomenologically or from dynamical models. It is only through these input parameters that system-specific properties enter into the theory. Beyond that, the results of the theory are generic. From a formal point of view, the statistical theory answers the question: What are the characteristic properties of scattering processes in which the compound system is described by an ensemble of random matrices? In this form, the theory has wide applications beyond nuclear physics; see Guhr *et al.* (1998).

Lack of space does not allow me to describe the history of the statistical theory. In subsequent sections, I will briefly mention the main developments. I refer the reader to Guhr *et al.* (1998) and Mahaux and Weidenmüller (1991).

§2. Theory of Resonance Reactions

Any theory of resonance reactions aims at decomposing the scattering matrix into two parts. One part describes nonresonant scattering with a smooth energy dependence while the other is a sum over resonances. This decomposition is generic and applies whenever resonances play a role in scattering processes. The basic idea is very simple: In the case of a single resonance, the scattering matrix is decomposed into a smooth part and a Breit–Wigner term. The decomposition we aim at is similar in form and yields the unitary generalisation to many resonances that may or may not overlap. The resonance parameters are related to properties of the underlying Hamiltonian.

Reaction theory determines the elements $S_{ab}(E)$ of the scattering matrix S as functions of energy E. The channels $a, b, \ldots$ are defined in terms of the intrinsic states of the fragments at asymptotic distance and in terms of the quantum numbers of their relative motion. The angular momenta of relative motion and the intrinsic spins of the fragments are coupled to the overall spin J of the system. Spin J, overall parity π and, if applicable, total isospin T of the system are conserved quantum numbers. We focus attention on the scattering matrix for fixed values of J, π, T. To simplify the notation, we do not carry these indices, however, except for §7 where breaking of parity and isospin symmetry will be considered. We mention here already that in the statistical model it is assumed that S-matrix elements pertaining to different conserved quantum numbers are uncorrelated. A channel a is open (closed) when the threshold energy E_a is smaller (larger) than the energy E of the system. At the energies of interest for the statistical model, only two-body fragmentation is of interest, and only two-body channels are considered in the sequel. When expressed in terms of elements of the scattering matrix, any differential reaction cross section involves angular momentum coupling coefficients, spherical harmonics and kinematical factors. These expressions are part of the standard repertoire of nuclear reaction theory (Lane and Thomas, 1958) and are not given here; see Chapter 3.1.2 by I. J. Thompson. They are not affected by the use of the statistical model.

The first widely used theory of resonance reactions was formulated by Wigner and Eisenbud. It is described in the classic review paper by Lane and Thomas (1958). The configuration space of all nucleons involved in the reaction is divided into two parts separated by a boundary. In one part, all nucleons are close to each other. Using suitable boundary conditions on the boundary, the self-adjoint Hamiltonian

defined on this part yields a set of bound states that turn into resonances when the physical conditions linking both parts of configuration space are used. This "R-matrix theory of nuclear reactions" plays an important role even today in the analysis of experimental resonance reaction data of the type shown in Fig. 1. Aside from early attempts by Moldauer (1976), this theory has, however, not been used as a framework for the implementation of the statistical model and will, therefore, not be considered in the sequel.

A second theory due to Feshbach (1958, 1962, 1992) uses projection operators P and Q, which project Hilbert space onto the space of open and closed channels, respectively. Restricting the Hamiltonian H to the closed-channel part QHQ yields a set of bound states that turn into resonances when the coupling to the open channels is taken into account. This theory has been used for purposes of the statistical model mainly in the context of precompound reactions. We return to it in §8.

A third theory, closely related in spirit to Feshbach's, was formulated in Mahaux and Weidenmüller (1969). It makes explicit use of shell-model ideas but the resulting formulas apply much more widely. This theory will be used here because for a stochastic modelling of the Hamiltonian, it is the most widely used version, with applications to compound and precompound reactions, to the conductance properties of chaotic and/or disordered mesoscopic systems, and to the scattering of light by media with a random index of refraction (Guhr *et al.*, 1998).

Two mechanisms lead to the occurrence of resonances: Barrier penetration causing single-particle resonances and auto-ionising states, a typical many-body phenomenon. For pedagogical reasons, I present the scattering matrices for several simple Hamiltonians that cause resonances through either mechanism in increasing order of complexity. In this way, we arrive at the most general form of the scattering matrix which is used in the sequel.

Single-Particle Resonances

Such resonances occur when a particle is trapped within a potential well with a barrier separating it from the outside world. The particle may tunnel through the barrier. The width of the resonance is proportional to the tunnelling probability. States of nucleons in the nuclear mean-field potential with an angular-momentum barrier and/or the Coulomb barrier serve as examples. The Hamiltonian may generically be modelled in the form

$$H = \sum_k \epsilon(k) a^\dagger(k) a(k) + E_0 d^\dagger d + \sum_k [W_0(k) a^\dagger(k) d + \text{h.c.}]. \quad (4)$$

Here the state $|0\rangle$ within the barrier has energy E_0 and Fermion creation and annihilation operators $d^\dagger$ and d, respectively. The states beyond the barrier form a continuum, with wave number k, energy $\epsilon(k)$ and Fermion creation and annihilation operators $a^\dagger(k)$ and $a(k)$, respectively. The matrix elements $W_0(k)$ are the tunnelling amplitudes connecting the state $|0\rangle$ with the channel states labelled k. It is straightforward to calculate the scattering amplitude corresponding to H, see Eqs. (6) to (8).

Autoionising States

We consider the mean-field approximation to the nuclear many-body problem (i.e., the shell model). The many-body wave functions are Slater determinants of single-particle states. We assume that the mean field allows for both bound states and scattering states. For simplicity, we consider only Slater determinants where all particles or all particles but one occupy bound states. We label the first set $|\Phi_\mu>$, with Hartree–Fock energies E_μ, and the second set $|\chi_c(E)>$, where the energy E is a continuous variable, and where c labels the channels. These states are taken to be orthonormal, with a delta-function normalisation in energy for the continuum states. To begin with, we consider a single-bound state $|\Phi_0>$ with energy E_0 larger than the threshold energy E_c in channel c, the only open channel. The residual two-body interaction that is not accounted for by the mean field couples $|\Phi_0>$ and $|\chi_c(E)>$. This causes the state $|\Phi_0>$ to become unstable against particle decay into channel c. The state $|\Phi_0>$ turns into a resonance. Such states occur frequently in practice: Consider two particles above the filled Fermi sea, both lacking the energy needed to escape into the continuum. The two-body interaction may transfer energy from one particle to the other that then escapes. In atomic physics, such states are referred to as autoionizing states. In Mahaux and Weidenmüller (1969), the term bound states embedded in the continuum is used. Here we use the term quasi-bound states. The Hamiltonian has the form

$$H = \int_{E_c}^{\infty} dE\, E|\chi_c(E)><\chi_c(E)| + E_0\, |\Phi_0><\Phi_0| + \left[\int_{E_c}^{\infty} dE\, W_0(E)|\chi_c(E)><\Phi_0| + \text{h.c.}\right]. \quad (5)$$

Here W_0 is the matrix element of the residual interaction. We note the close similarity of the Hamiltonians in Eqs. (4) and (5). Because of the normalisation of the continuum wave functions, W_0 has the dimension $E^{1/2}$ in both equations.

The models (4) and (5) yield identical forms of the scattering function (the scattering matrix has dimension one),

$$S(E) = \exp(2i\delta)\left[1 - 2i\pi \frac{W_0^2(E)}{E - E_0 - F(E)}\right], \quad (6)$$

where

$$F(E) = \mathcal{P}\int_{E_c}^{\infty} dE' \, \frac{W_0^2(E')}{E-E'} - i\pi W_0^2(E) \qquad (7)$$

and where $\mathcal{P}$ denotes the principal-value integral. The background phase shift due to potential scattering is denoted by δ. Obviously, unitarity holds, $|S(E)|^2 = 1$. Moreover, Eq. (6) has the form of a Breit–Wigner resonance. In most applications, it is realistic to assume that $W_0(E)$ is practically constant over the width of the resonance. Then, the principal-value integral is negligibly small compared to the width of the resonance, and

$$S(E) = \exp(2i\delta)\left[1 - 2i\pi \frac{W_0^2}{E - E_0 + i\pi W_0^2}\right]. \qquad (8)$$

The formula for the resonance width, $\Gamma = 2\pi W_0^2$, looks like the golden rule but actually is a non-perturbative result. Equation (8) expresses the scattering function in terms of quantities defined by the Hamiltonian: The potential scattering phase shift δ, the energy E_0 of the autoionising state and the coupling matrix element W_0.

For purposes of the statistical model, it is necessary to consider a generalisation to N quasi-bound states $|\Phi_\mu>$, $\mu = 1, \ldots, N$. We also consider Λ open channels. Let $H_{\mu\nu}$ denote the matrix elements of the Hamiltonian between the quasi-bound states. We write the total Hamiltonian in the form

$$H = \sum_{c=1}^{\Lambda} \int_{E_c}^{\infty} dE \, E|\chi_c(E)><\chi_c(E)| + \sum_{\mu\nu}^{N} H_{\mu\nu}\,|\Phi_\mu><\Phi_\nu| + \sum_{c=1}^{\Lambda}\sum_{\mu=1}^{N}\left[\int_{E_c}^{\infty} dE \, W_{c\mu}(E)|\chi_c(E)><\Phi_\mu| + \text{h.c.}\right]. \qquad (9)$$

Here $W_{c\mu}$ is the coupling matrix element between channel c and the quasi-bound state $|\Phi_\mu>$.

The scattering matrix has dimension Λ and the form

$$S_{ab}(E) = \exp(2i\delta_a)\delta_{ab} - 2i\pi \exp(i\delta_a) \times \sum_{\mu\nu} W_{a\mu}\left(D^{-1}\right)_{\mu\nu} W_{\nu b}\exp(i\delta_b). \qquad (10)$$

The quantity D is a matrix in the space of the N quasi-bound states $|\Phi_\mu>$ and is given by

$$D_{\mu\nu}(E) = E\delta_{\mu\nu} - H_{\mu\nu} + i\pi\sum_c W_{\mu c} W_{c\nu}, \qquad (11)$$

with $H_{\mu\nu}$ introduced in Eq. (9). We have again neglected the energy dependence of the matrix elements W. It is straightforward to check that the matrix S is unitary. Equations (10) and (11) constitute the unitary extension of the Breit–Wigner formula to N resonances. These equations apply both for isolated and for overlapping resonances. They expresss the scattering matrix in terms of properties of the Hamiltonian H. We note that as the energy increases and passes a threshold with energy E_c, one or several channels open, and the dimension Λ of the S matrix increases. Equations (10) and (11) have a simple physical interpretation: Aside from elastic nonresonant scattering processes that yield the elastic phase shifts δ_c, scattering proceeds via the N compound-nucleus resonances. The matrix elements $W_{c\mu} = W_{\mu c}$ describe formation and decay of these resonances. The matrix D describes the propagation of the system in the space of the resonance states and contains the projection $H_{\mu\nu}$ of the Hamiltonian onto this space as well as the width matrix with elements $\gamma_{\mu\nu} = 2\pi\sum_c W_{\mu c} W_{c\nu}$. The resonances do (do not) overlap if the $|\gamma_{\mu\nu}|$ are large (small) in comparison with the mean level spacing d of the eigenvalues of the matrix $H_{\mu\nu}$. We distinguish these two cases by the inequalities $\Gamma \gg d$ and $\Gamma \ll d$, respectively. For reasons that will become obvious presently, we refer to the case of Eqs. (10) and (11) as to the "optical-model case". In the definition of this case, we include Eq. (14) introduced below.

This case is not sufficiently general, however. To account for direct reactions between channels, it is necessary to consider a further generalisation of the model. So far it has been assumed that there is no dynamical coupling between the states $|\chi_c(E)>$ pertaining to different channels. This is not realistic: Inelastic scattering processes that do not involve the formation of resonances are important and cause direct reactions to occur. The generalisation consists in allowing for such reactions by adding to the right-hand side of Eq. (9) the term $\sum_{cc'}\int_{E_c}^{\infty} dE \int_{E_{c'}}^{\infty} dE' V_{cc'}(E,E')|\chi_c(E)><\chi_{c'}(E')|$. The S matrix for the resulting Hamiltonian is obtained in two steps. (i) We disregard the matrix elements W and consider the formal solutions $|\Psi_c^{\pm}(E)>$ of the resulting coupled-channels scattering problem. (The actual calculation of these functions would require a numerical approach; see Chapter 3.1.2 by I. J. Thompson.) We denote the resulting scattering matrix by $S_{ab}^{(0)}$. We assume that this matrix depends smoothly on energy. (ii) We take account of the presence of the matrix elements $W_{\mu c}(E) =< \mu|W|\chi_c(E) >$ and define the new elements $W_{\mu c}^{(0)}(E) =< \mu|W|\Psi_c^{+}(E) >$. The scattering matrix of the generalised model is obtained as

$$S_{ab}(E) = S_{ab}^{(0)} - 2i\pi\sum_{\mu\nu} W_{a\mu}^{(0)}\left(D^{-1}\right)_{\mu\nu} W_{\nu b}^{(0)}. \qquad (12)$$

The matrix D has the form

$$D_{\mu\nu}(E) = E\delta_{\mu\nu} - H_{\mu\nu} + i\pi\sum_c W_{\mu c}^{(0)} W_{c\nu}^{(0)}. \qquad (13)$$

The matrix $S^{(0)}$ describes reaction processes without the intermediate formation of compound states. The last term in Eq. (12) is very similar in form to the corresponding term in Eq. (10). The elastic phase shifts have disappeared; they form part of the coupling

matrix elements $W_{\mu c}^{(0)}$. We refer to the case of Eqs. (12) and (13) as to the "direct-reaction case". In the sequel, we distinguish this case with a nondiagonal background S-matrix $S^{(0)}$ from the "optical-model case" of Eqs. (10) and (11). Equations (12) and (13) give the most general form of the scattering matrix.

§3. Implementation of Stochasticity

Equations (10) and (11) or, in the case of a direct coupling between channels, Eqs. (12) and (13) give the scattering matrix in terms of the underlying Hamiltonian. We recall that stochastic features reside in the compound-nucleus resonances, i.e., in the projection $H_{\mu\nu}$ of the actual Hamiltonian of the nuclear system onto the space of quasi-bound states $|\Phi_\mu>$, $\mu = 1, \ldots, N$. To implement stochasticity, we accordingly replace the matrix $H_{\mu\nu}$ by a suitable ensemble of random matrices. In the present context, this ensemble is the GOE, although in later sections different choices will be considered. For pedagogical reasons, we consider first the simpler optical-model case of Eqs. (10) and (11) and return to the direct reactions below. We assume that the matrix elements $W_{c\mu}$ obey the relations

$$\sum_\mu W_{a\mu} W_{\mu b} = N v_a^2 \delta_{ab}. \tag{14}$$

This assumption simplifies the calculation and is removed at the end of this section. We always consider the limit $N \to \infty$.

Use of the GOE implies certain constraints on the properties of the quasi-bound states, which will be lifted later. Indeed, because of the orthogonal invariance of the GOE, all states $|\Phi_\mu>$ are treated on the same footing. This assumption is realistic only if the time scale τ_{eq} for intrinsic mixing of the states $|\Phi_\mu>$ is small compared to the average nuclear decay time $\tau_{dec} = \hbar/\Gamma$ due to coupling to the channels. The value of τ_{eq} is determined by the strength of the residual interaction. It is related to the spreading width $\Gamma^\downarrow$ of the strong interaction, $\tau_{eq} = \hbar/\Gamma^\downarrow$ with $\Gamma^\downarrow \approx$ a few megaelectronvolts. (The spreading width is defined in section 0.1.7). This value changes slowly with excitation energy. In contradistinction, Γ increases very strongly with excitation energy. The inequality $\Gamma \ll \Gamma^\downarrow$ holds at neutron threshold and justifies here the use of the GOE. However, $\Gamma \approx \Gamma^\downarrow$ roughly 10 MeV above neutron threshold. Here the compound system may decay by particle emission before it has reached equilibrium, and precompound reaction processes become important. We address this situation in §8.

The stochastic model is defined by Eqs. (10), (11) and (14) and by substituting for $H_{\mu\nu}$ the GOE. It may appear that this model is ill-defined. Indeed, the parameters of the model are the $W_{\mu a}$'s and λ, which fixes the mean level spacing d. With Λ the number of open channels, the number of parameters is $1 + \Lambda \times N$. This figure diverges as $N \to \infty$. The orthogonal invariance of the GOE saves the situation. Indeed, because of the orthogonal invariance of the GOE, the ensemble average of S_{ab} and of all observables depending on S_{ab} can depend only on orthogonal invariants constructed from these parameters, i.e., on the quantities $\sum_\mu W_{a\mu} W_{\mu b}$ and on λ. Equation (14) reduces this set to λ and $N v_a^2$, $a = 1 \ldots \Lambda$. The S matrix is dimensionless. Hence, only the dimensionless parameters $N v_a^2/\lambda$, $a = 1 \ldots \Lambda$ can play a role. The input for the stochastic model consists in the values of the average S-matrix elements $\overline{S_{ab}}$. In the absence of direct reactions, the average S matrix is diagonal, $\overline{S_{ab}} = \delta_{ab} \overline{S_{aa}}$. We see that the number Λ of parameters of the model equals the number of input variables $\overline{S_{aa}}$: The stochastic model predicts S-matrix fluctuations uniquely in terms of average S-matrix elements. The latter are determined phenomenologically in terms of the optical-model potential; see §4. For the prediction of correlation functions, one further parameter is needed: The energy is scaled in units of the mean level spacing d.

We turn to the case of direct reactions, Eqs. (12) and (13), and do not assume the validity of Eq. (14). This case can be reduced to that considered in the previous paragraph (Nishioka and Weidenmüller, 1985): There exists a unitary transformation U_{ab} in the space of open channels that reduces the scattering matrix of Eqs. (12) and (13) to the form given in Eqs. (10) and (11). At the same time, U_{ab} can be chosen in such a way that the W's obey Eq. (14). The matrix U is obtained through the following steps. (i) Find the orthogonal transformation $O^{(0)}$ in channel space that brings the symmetric matrix $S^{(0)}$ in Eq. (12) to diagonal form, $O^{(0)} S^{(0)} [O^{(0)}]^T = \exp(2i\delta^{(0)})$. Here T denotes the transpose and $\delta^{(0)}$ is a diagonal matrix, the elements of which are the (real) eigenphase shifts of $S^{(0)}$. Define $W^{(1)} = O^{(0)} W^{(0)}$. (ii) Find a second orthogonal transformation $O^{(1)}$ in channel space that diagonalises the symmetric bilinear form $\sum_\mu W_{a\mu}^{(1)} W_{\mu b}^{(1)}$ so that $\sum_{ab} O_{ca}^{(1)} \sum_\mu W_{a\mu}^{(1)} W_{\mu b}^{(1)} O_{bd}^{(1)} = N v_c^2 \delta_{cd}$. Define $W = O^{(1)} W^{(1)}$. Then, the unitary matrix U is defined as

$$U = O^{(1)} \exp(-i\delta^{(0)}) O^{(0)}, \tag{15}$$

and we put

$$S^{(1)} = U S U^T. \tag{16}$$

The matrix $S^{(1)}$ has the form of Eqs. (10) and (11) with all δ_a's put equal to 0; the W's obey Eq. (14). Stochasticity is now implemented into the matrix $S^{(1)}$ in exactly the same way as described in the previous

paragraph. Formally, the matrix U in Eq. (16) plays the same role as the diagonal phase matrix $\delta_{ab}\exp[i\delta_a]$ in Eq. (10).

We remark in parentheses that whenever Eqs. (10) and (11) do apply but the matrix elements W do not obey the condition (14), step (ii) of the construction just outlined can be used to satisfy this condition. Hence, the condition (14) can always be imposed without loss of generality.

In summary, we have presented formal arguments that show that in the very general case of Eqs. (12) and (13), knowledge of the average scattering matrix and of the mean level spacing d of the compound-nucleus resonances is sufficient to determine completely the fluctuation properties of S and, thus, averages of observables containing any number of S-matrix elements (cross sections, correlation functions, polarisations, etc.). These formal arguments must now be supplemented by answers to the following concrete questions: How are the average S-matrix elements related to phenomenology (optical-model and coupled-channels calculations)? And given a phenomenological model or a fit procedure to experimental data that do allow us to determine the average S matrix, how do we proceed to actually calculate the fluctuation properties of S and, thereby, averages over observables? These questions are dealt with in the following two sections.

§ 4. Optical Model and Direct Reactions

We wish to determine the values of the ensemble averages of the S matrices for the case of the optical model (Eqs. (10), (11) and (14)) and for the case of direct reactions (Eqs. (12), (13)) from empirical input. This will fix the input parameters of the statistical model and will allow us to predict the fluctuations in terms of the mean values. The introduction of the optical model into nuclear physics (Feshbach *et al.*, 1953) was a pivotal step: The earlier black-box theory was replaced by a model that could be related to nuclear forces. Without this step, the development of the statistical theory described here would not have been possible.

We denote the actual nuclear scattering matrix by S^{nuc}, the energy average by angular brackets, and the ensemble average by an overbar. We require that the ensemble averages of the scattering matrices defined in Eqs. (10),(11) and (14) (and in Eqs. (12),(13)) be equal to the energy average of S^{nuc},

$$\overline{S_{ab}} = \langle S_{ab}^{\mathrm{nuc}} \rangle. \tag{17}$$

In the case of the optical model where $\overline{S_{ab}}$ is diagonal, this postulate is meaningful only if $\langle S_{ab}^{\mathrm{nuc}} \rangle$ is (nearly) diagonal and direct reactions are negligible. In postulate (17), we have implicitly used ergodicity, i.e., the equality of running average and ensemble average.

The implementation of Eq. (17) makes use of the optical model of elastic scattering and of coupled-channels calculations with complex optical-model potentials. We first focus on the optical-model case (Eqs. (10), (11) and (14)) and turn to the case of direct reactions below. Theoretically, the optical model at low energy is defined (Feshbach, 1992; Mahaux and Weidenmüller, 1969; Austern, 1970) by the requirement that the elastic scattering amplitude S_{aa}^{opt} calculated from the single-particle Schrödinger equation with a complex optical-model potential coincides with the energy-averaged elastic element of the actual nuclear scattering matrix S^{nuc},

$$S_{aa}^{\mathrm{opt}} = \langle S_{aa}^{\mathrm{nuc}} \rangle. \tag{18}$$

In practice this equation is useful only for channels where both reaction partners are in their ground states; it is commonly assumed that the optical-model potential determined in this way applies likewise to channels involving the same reaction partners in excited states. In the case of nucleon scattering, the optical-model potential is the sum of the mean-field potential and an imaginary part that accounts for absorption of the incident particle. There are two contributions to the imaginary part: Inelastic scattering processes and compound-nucleus formation. We have excluded direct reactions and, therefore, consider the case where only compound-nucleus formation occurs. Formation of the compound nucleus is not an absorptive process, of course. Absorption does occur, however, upon averaging the scattering amplitude over an energy interval containing a sufficiently large number of compound-nucleus resonances. We see this in a time-dependent picture: We consider a wave packet incident on a target nucleus. By Fourier transformation, it is easily seen that the average of the scattering amplitude taken over an energy interval of length I describes the asymptotic contribution of that part of the incident wave packet that is emitted from the interaction region after short delay times, i.e., times $\leq \hbar/I$. This is the "fast" part of the wave packet. The part of the wave packet that forms compound-nucleus resonances is reemitted from the interaction region at times of order $\hbar/\Gamma$. For $I \gg \Gamma$, this part will not be counted in the fast part. Therefore, the average scattering amplitude cannot be unitary. This causes the optical model to acquire an imaginary part even in the absence of direct inelastic processes. This is the case, for instance, at low energies where only the elastic channel is open; see Fig. 1.

Rather than using directly Eq. (17) to determine the input parameters of the statistical model, we combine Eqs. (17) and (18) to obtain

$$\overline{S_{aa}} = S_{aa}^{\text{opt}}. \tag{19}$$

We take it for granted that an optical-model potential has been found and that S_{aa}^{opt} is known for all open channels a. In order to use Eq. (19) for the determination of the input parameters of the statistical model, we must calculate the ensemble average of S_{aa} defined in Eqs. (10), (11) and (14). We again invoke ergodicity and calculate the energy average instead. It is not difficult to show the equality of both averages. To perform the energy average, we use a Lorentzian weight factor with width I centred at energy E_0. We transform the integration over the real axis into a contour integral by adding to the path of integration the semicircle with radius $R \to \infty$ in the upper half of the complex energy plane. We use the fact that $S_{ab}(E)$ has no poles in this part of the plane. Then $\langle S_{ab} \rangle = S_{ab}(E_0 + iI)$: Averaging over energy amounts to replacing the energy argument of S_{ab} by $E_0 + iI$. A straightforward calculation of $S_{ab}(E_0 + iI)$ yields (Brown, 1959)

$$\overline{S_{ab}} = S_{ab}(E_0 + iI) = \delta_{ab} \exp(2i\delta_a) \frac{1 - x_a}{1 + x_a}. \tag{20}$$

Here $x_a = \pi N v_a^2/\lambda = \pi^2 v_a^2/d$ with v_a^2 defined in Eq. (14). Equations (19) and (20) determine the x_a, $a = 1, \ldots, \Lambda$ and, thus, the parameters of the statistical model.

It turns out that except for overall phase factors, the higher moments of S do not depend on the variables S_{aa}^{opt} but only on the "sticking probabilities" or "transmission coefficients" T_a defined by

$$T_a = 1 - |S_{aa}^{\text{opt}}|^2 = 1 - |\overline{S_{aa}}|^2. \tag{21}$$

Obviously, we have $0 \leq T_a \leq 1$ for all channels a. The relation (21) has a simple physical interpretation: The coefficients T_a measure the unitarity deficit of $\overline{S}$, i.e., of the "fast part" of S. In other words, the T_a values measure that part of the incident flux that is not scattered instantaneously back into the channels but populates the long-lived resonant states. Fluctuation properties and correlation functions of S depend only on this part. Combining the definition (21) with Eq. (20), we obtain the explicit relation $T_a = 4x_a/(1 + x_a)^2$. Taken as a function of the x's, each T_a depends only on the parameter x_a and vanishes for both $x_a \to 0$ and $x_a \to \infty$. Thus, T_a vanishes both for very weak and, perhaps surprisingly, also for very strong coupling of the channel a to the levels μ. We return to this point in §5.

We turn to the implementation of the postulate (17) in the case of direct reactions, Eqs. (12) and (13). The argument runs parallel to the case of the optical model: We assume that a coupled-channels approach with complex optical-model potentials yields a scattering matrix S^{dir} that satisfies the equation.

$$S_{ab}^{\text{dir}} = \langle S_{ab}^{\text{nuc}} \rangle. \tag{22}$$

The task consists in determining the parameters of the statistical model from the postulate $\overline{S_{ab}} = S_{ab}^{\text{dir}}$. For $\overline{S_{ab}}$, we use Eq. (16) and the fact that the matrix U is a smooth function of energy. Then, $(U\bar{S}U^{\text{T}})_{ab} = \overline{S_{ab}^{(1)}}$. As shown in the previous paragraphs, the matrix $\overline{S^{(1)}}$ is diagonal and can be written as $\overline{S_{ab}^{(1)}} = \delta_{ab}(1 - x_a)/(1 + x_a)$. We proceed similarly for S_{ab}^{dir}: There exists (Engelbrecht and Weidenmüller, 1973) a unitary transformation U^{dir} that "diagonalises" S_{ab}^{dir} in the sense that $(U^{\text{dir}} S_{ab}^{\text{dir}} [U^{\text{dir}}]^{\text{T}})_{ab} = \delta_{ab}(\pm\sqrt{1 - p_a})$. Without loss of generality, we have assumed that the "eigenvalues" are real and that $0 \leq p_a \leq 1$. In the presence of direct reactions, the coefficients p_a play the same role as the transmission coefficients T_a in the case of the optical model: These coefficients are the eigenvalues of a matrix with elements $\delta_{ab} - (S^{\text{dir}} S^{\text{dir}\dagger})_{ab}$ and measure the unitarity deficit of the matrix S^{dir}. We compare the resulting forms for $\overline{S}$ and S^{dir} and equate U with U^{dir} and $(1 - x_a)/(1 + x_a)$ with $\pm\sqrt{1 - p_a}$. This determines the parameters of the statistical model in the case of direct reactions, with $x_a = \pi N v_a^2/\lambda$ as defined earlier.

In applications of this formalism to compound-nucleus reactions, the standard procedure consists in using only the optical model, and to disregard direct reactions. This is because direct reactions tend to populate low-lying states of the reaction products while particle emission from the compound-nucleus favours high-lying states where we know very little about direct reaction processes. For this reason, we will use the parlance that the input of the statistical model is determined by the optical model. The situation is different in case of the multistep-direct reaction; see §8.

In conclusion, we have shown that the parameters of the statistical model can be determined from a standard optical-model or coupled-channels approach. More specifically, when we write the scattering matrix S as the sum of the ensemble average and the fluctuating part S^{fluc},

$$S_{ab} = \overline{S_{ab}} + S_{ab}^{\text{fluc}}, \tag{23}$$

where $\overline{S_{ab}^{\text{fluc}}} = 0$ by definition, then the average part is determined by the optical model. As pointed out in § 3, all moments and correlation functions of S_{ab}^{fluc} are then predicted uniquely and in a parameter-free fashion by the statistical model.

§ 5. The Compound Nucleus

The concept of the compound nucleus has a long history and dates back well past the beginnings of a proper statistical theory of nuclear reactions; see (Blatt and Weisskopf, 1952; Mahaux and

Weidenmüller, 1979). The concept was introduced by Bohr (1936). It was based on the idea that nucleons interact very strongly in the nuclear interior and that, therefore, energy exchange and equilibration are extremely rapid processes. This picture led to the expectation that formation and decay of the compound nucleus are independent processes. Aside from kinematical and geometrical factors, we identify the compound-nucleus cross section with $\overline{|S_{ab}^{\rm fluc}|^2}$. This expression should factorise. One factor should depend only on the entrance channel a and be given by the probability of formation of the compound system. According to the arguments following Eq. (21), this factor is T_a. The second factor should give the normalised probability of decay of the compound system into a given final channel b. Using detailed balance, we find that the second factor has the form $T_b/\sum_c T_c$. In summary, the Bohr picture implies the "Hauser–Feshbach formula"

$$\overline{|S_{ab}^{\rm fluc}|^2} = (1+\delta_{ab})\frac{T_a T_b}{\sum_c T_c}. \tag{24}$$

The factor $(1+\delta_{ab})$ goes beyond the Bohr argument. It embodies time-reversal symmetry and is referred to as "elastic enhancement factor". In the original black-box model, all T_a values were put equal to 1, and the emission probability was, therefore, simply given by the density of states of the fragments ("evaporation model").

Does the statistical model yield the result (24)? And if so, what are the limits of validity of this formula? What is the value of the variance of the cross section, of the cross-section autocorrelation function, and of other observables that depend on S_{ab}? How do the predictions of the statistical model compare with experimental data? These are the questions we address in the present section. The relevant energy interval extends from neutron threshold where $\Gamma \ll d$ to about 20 MeV excitation energy where $\Gamma \gg d$ and where precompound processes must also be included. We recall that the statistical model refers to S-matrix elements pertaining to fixed values of the overall conserved quantum numbers like spin and assumes that S-matrix elements pertaining to different values of the quantum numbers are uncorrelated. Thus, Eq. (24) and all similar equations below always involve S-matrix elements with the same overall quantum numbers. This fact automatically implies that the average compound-nucleus cross section is symmetric about 90° c.m., irrespective of the precise form of the second moment of S.

To calculate $\overline{|S^{\rm fluc}|^2}$ from a statistical input, several authors (see Lane and Lynn (1957) and Moldauer (1976)) used a perturbative expansion valid for $\Gamma \ll d$, while Agassi *et al.* (1975) used an asymptotic expansion valid for $\Gamma \gg d$; see Mahaux and Weidenmüller (1979). In either case, the Hamiltonian matrix $H_{\mu\nu}$ in Eq. (11) or (13) is diagonalised with the help of an orthogonal matrix $O_{\mu\nu}$. We denote the eigenvalues by E_μ. The transformed matrix $ODO^{\rm T}$ has the form $[(E-E_\mu)\delta_{\mu\nu} + (i/2)(O\gamma O^{\rm T})_{\mu\nu}]$. We recall that $\gamma_{\mu\nu}$ stands for the elements $2\pi\sum_a W_{\mu a}W_{a\nu}$ of the width matrix. The matrix $ODO^{\rm T}$ is either expanded in powers of the nondiagonal matrix elements of the transformed width matrix $O\gamma O^{\rm T}$ ($\Gamma \ll d$) and averaged or expanded in powers of $O\gamma O^{\rm T}$ and averaged, and the leading terms in powers of d/Γ are resummed ($\Gamma \gg d$). The full regime (any value of Γ/d) remained inaccessible for a long time. The problem was solved (Verbaarschot *et al.*, 1985) by adapting a novel technique due to Efetov (the supersymmetry technique) from condensed matter theory. The correlation function $\overline{S^{\rm fluc}(E_1)S^{\rm fluc*}(E_2)}$ and, by implication, the intensity $\overline{|S^{\rm fluc}|^2}$, could be calculated from the statistical input. This is not the place to describe the technical aspects of this method; see Efetov (1997) and Verbaarschot *et al.* (1985). We confine ourselves to a number of results that have been obtained with this method.

The average of a product of fluctuating S-matrix elements vanishes unless the product contains at least one factor $S^{\rm fluc}$ and one factor $S^{\rm fluc*}$. Indeed, if this condition is not met, we replace the ensemble average by the energy average, use the fact that the poles of all factors in the product lie on the same side of the real energy axis, close the contour in the other half-plane (see §4) and find zero as a result. Interest thus concentrates on the simplest non trivial case: The correlation function of a pair of complex conjugate fluctuating S-matrix elements with energy arguments E_1 and E_2 and carrying different channel indices (ab) and (cd), respectively. This function is given by a threefold integral,

$$\begin{aligned}
\overline{S_{ab}^{\rm fluc}(E_1)S_{cd}^{\rm fluc*}(E_2)} &= \frac{1}{8}\int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda \\
&\times \frac{(1-\lambda)\lambda|\lambda_1-\lambda_2|}{((1+\lambda_1)\lambda_1(1+\lambda_2)\lambda_2)^{1/2}(\lambda+\lambda_1)^2(\lambda+\lambda_2)^2} \\
&\times \exp\left[-\frac{i\pi\varepsilon}{d}(\lambda_1+\lambda_2+2\lambda)\right] \prod_e \frac{(1-T_e\lambda)}{(1+T_e\lambda_1)^{1/2}(1+T_e\lambda_2)^{1/2}} \\
&\times \left(\delta_{ab}\delta_{cd}\overline{S_{aa}}\,\overline{S_{cc}}T_aT_c\left(\frac{\lambda_1}{1+T_a\lambda_1}+\frac{\lambda_2}{1+T_a\lambda_2}+\frac{2\lambda}{1-T_a\lambda}\right)\right. \\
&\times \left(\frac{\lambda_1}{1+T_c\lambda_1}+\frac{\lambda_2}{1+T_c\lambda_2}+\frac{2\lambda}{1-T_c\lambda}\right) \\
&+(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})T_aT_b\left(\frac{\lambda_1(1+\lambda_1)}{(1+T_a\lambda_1)(1+T_b\lambda_1)}\right. \\
&\left.\left.+\frac{\lambda_2(1+\lambda_2)}{(1+T_a\lambda_2)(1+T_b\lambda_2)}+\frac{2\lambda(1-\lambda)}{(1-T_a\lambda)(1-T_b\lambda)}\right)\right).
\end{aligned} \tag{25}$$

Here, $\varepsilon = E_2 - E_1$. Aside from overall phases contained in the factors $\overline{S_{aa}}$ and $\overline{S_{cc}}$, the correlation

function is indeed completely determined by the values of the transmission coefficients T_c, $c = 1, \ldots, \Lambda$. Moreover, this function depends only on the dimensionless energy difference ε/d. The Kronecker deltas reflect the absence of direct reactions. Analogous statements hold for all higher moments of S^{fluc}. We emphasise that Eq. (25) holds for any number of channels and for all values of the transmission coefficients. Thus, it can be used to calculate the compound-nucleus cross section considered in Eq. (24) for any value of the ratio Γ/d. Equation (25) is used for this purpose in numerical codes that apply statistical nuclear reaction theory to the calculation of cross sections. These calculations are needed for reactor safety, radiation shielding and similar purposes in the case of short-lived or rare target isotopes and are successfully tested against empirical data for stable and abundant target nuclei; see Qaim (1991) and Reffo (1997). In such applications, Eq. (25) is referred to as the "threefold integration formula".

The result (25) cannot be simplified further. Aside from a numerical evaluation, the implications of Eq. (25) can, therefore, only be displayed in limiting cases. We first consider the case of strongly overlapping resonances ($\Gamma \gg d$), i.e., of Ericson fluctuations. Ericson (Mayer-Kuchut (1966)) predicted the existence of these fluctuations and their properties. His arguments were intuitive. His results were later shown to follow from the statistical model. For $\Gamma \gg d$, it can be shown (Mahaux and Weidenmüller, 1979) that the complex elements of S^{fluc} have a Gaussian distribution. Thus, it suffices to know the second moments. We know that $\overline{S_{ab}^{\text{fluc}}(E_1) S_{cd}^{\text{fluc}}(E_2)} = 0$. For $\overline{S_{ab}^{\text{fluc}}(E_1) S_{cd}^{\text{fluc}*}(E_2)}$, on the other hand, the leading term in an asymptotic expansion in powers of d/Γ has the form

$$\overline{S_{ab}^{\text{fluc}}(E_1) S_{cd}^{\text{fluc}*}(E_2)} = (\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) \frac{T_a T_c}{\sum_e T_e + 2i\pi\varepsilon/d}, \quad (26)$$

with ε as defined above. This follows from Eq. (25). Thus, to leading order in d/Γ, the distribution of S^{fluc} is completely known. We note that Eq. (26) implies the Hauser–Feshbach formula (24). It is straightforward to work out the autocorrelation function of the cross section in the same limit. We find that

$$\frac{\overline{|S_{ab}^{\text{fluc}}(E_1)|^2 |S_{ab}^{\text{fluc}*}(E_2)|^2} - \overline{|S_{ab}^{\text{fluc}}(E_1)|^2}\ \overline{|S_{ab}^{\text{fluc}*}(E_2)|^2}}{\overline{|S_{ab}^{\text{fluc}}(E_1)|^2}\ \overline{|S_{ab}^{\text{fluc}*}(E_2)|^2}} = \frac{1}{1 + (\varepsilon/\Gamma)^2}. \quad (27)$$

Here, the average total width Γ of the compound-nucleus resonances is given by

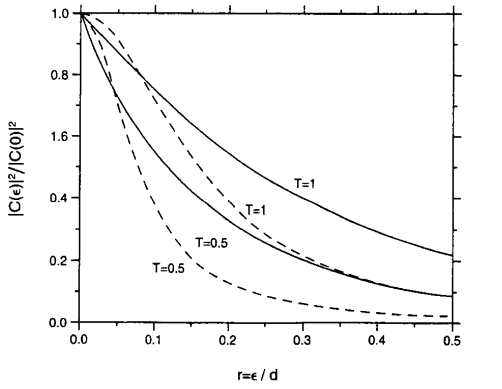
$$\Gamma = \frac{d}{2\pi} \sum_a T_a. \quad (28)$$

Equation (27) shows that Γ is the coherence width of the cross-section autocorrelation function. On the other hand, we can read Eq. (28) as giving the average total width Γ in terms of a sum of average partial widths $\Gamma_a = \frac{d}{2\pi} T_a$ for decay into channel a. And the form of Γ_a possesses a simple semiclassical interpretation (Blatt and Weisskopt, 1952): $d/2\pi\hbar$ estimates the period with which a wave packet evolving in time in the space of quasi-bound states returns to a given point (the "opening" of channel a) in that space, and T_a is the probability with which the wave packet escapes into that channel.

In summary: In the regime $\Gamma \gg d$, the statistical theory implies a Gaussian distribution of the elements of S^{fluc} as its central result. This regime is attained whenever the condition $\sum_c T_c \gg 1$ is met. The distribution of the values of the transmission coefficients T_c over the individual channels is irrelevant. The nonvanishing second moments of S_{ab}^{fluc} are given in Eq. (26). Equation (28) identifies the correlation width with the average decay width Γ of the compound-nucleus resonances. These results suffice to derive all assumptions used in the review of Ericson and Mayer-Kuckuk (1966) to describe average cross sections, energy and angular correlation functions, and the role of direct reactions in Ericson fluctuations. Lack of space forces me to refer to this review for details.

How large are the corrections to these results, i.e., the terms of order $(\sum_c T_c)^{-2}$? It turns out that the right-hand side of Eq. (26) gives a surprisingly good semiquantitative account of the actual autocorrelation function even in the case of few open channels. The extreme case is that of a single open channel with transmission coefficient T. Figure 3 shows the square $|C(\varepsilon)|^2$ of the autocorrelation function $\overline{S^{\text{fluc}}(E) S^{\text{fluc}*}(E+\varepsilon)}$ of the scattering function versus

Figure 3 The normalised S-matrix autocorrelation function for a single open channel versus the dimensionless energy variable r; see text. Taken from Lewenkopf and Weidenmüller (1991).



$r = \varepsilon/d$ for two cases defined by $T = 0.5$ and 1. The dashed lines give the Lorentzians expected from Eq. (26) and the full lines the actual behaviour. The difference is quantitatively significant. In particular, the non-Lorentzian behaviour of the full lines implies that the decay of the compound nucleus is not exponential in time but follows a power law; see the next paragraph. This behaviour is also found in the case of several open channels. It is a direct consequence of the Porter–Thomas distribution of the partial widths. This is most easily seen for well-isolated resonances. Here, each S-matrix element S_{ab} is a sum of resonance terms of the form $\gamma_{a\mu}(E-E_\mu+(1/2)i\Gamma_\mu)^{-1}\gamma_{\mu b}$. Averaging this sum over the partial width amplitudes (which also contribute to Γ_μ) changes the energy dependence and leads to the non-Lorentzian dependence of the autocorrelation function on E.

The autocorrelation function of the scattering matrix is related to the decay in time of the compound system (Harney *et al.*, 1992). An incident narrow wave packet populates the compound nucleus at time $t = 0$. We consider the time evolution of the projection $\rho(t)$ of the density matrix onto the space of quasi-bound states $|\Phi_\mu\rangle$ with $\mu = 1, \ldots, N$. We define the occupation probability of the compound system as $p(t) = \text{trace}[\rho(t)]$. This function is closely related (Harney *et al.*, 1992) to the Fourier transform with respect to ε of the S-matrix autocorrelation function $\overline{S^{\text{fluc}}(E)S^{\text{fluc}*}(E+\varepsilon)}$. In the Ericson limit $\Gamma \gg d$ of Eq. (26), $p(t)$ decreases exponentially in time. The deviations from the Lorentzian form of the S-matrix autocorrelation function addressed in the previous paragraph imply an asymptotic power-law dependence of $p(t) \sim t^{-1-\Lambda/2}$ for large t and Λ open channels (Harney *et al.*, 1992).

In the case of isolated resonances ($\Gamma \ll d$), poles of the scattering matrix in the complex E plane are in a one-to-one correspondence with resonances. It is tempting to generalise this correspondence and to ask for the distribution of poles of the S matrix in the general case. Here a caveat is needed, however. The S matrix depends on the energy E and, with m_c the reduced mass of the fragments in channel c, on the wave numbers $k_c = \sqrt{2m_c(E-E_c)}/\hbar$ in all Λ channels. The dependence on k_c arises because the states $|\chi_c(E)>$ in Eq. (9) depend on the k_c's, and so do the matrix elements $W_{\mu c}$. As a function of the complex variable E, the S matrix, therefore, has branch points on the real E axis located at the energies E_c. Each pair of nearest but different threshold energies defines a section of the real E axis. Different sections separated by one or more branch points connect to different Riemann sheets. Poles of S are given by zeros of $\det(D)$. Causality requires these poles to occur below the real physical E axis, but these poles have different locations on different sheets. The canonical simplification used in all applications of statistical nuclear reaction theory consists in disregarding the branch points and, thereby, the channels with threshold energies in the energy interval of interest. This simplification is legitimate in two cases: (i) All omitted channels are weakly coupled to the system. Because of the angular momentum and Coulomb barriers, this may not be an unrealistic assumption. (ii) The mean spacing d of resonances is very small in comparison with the spacing d_{thresh} of neighbouring thresholds. Then, the relevant energy interval may not comprise any thresholds. This is the case near neutron threshold in heavy nuclei where with $d \approx 10$ eV, we have $d_{\text{thresh}} \approx 100$ keV.

With this simplification, the S matrix has N poles in the lower E plane. For the model (10), (11), S takes the form (I omit the background phase shifts)

$$S_{ab} = \delta_{ab} - 2i\pi \sum_{\mu}^{N} \frac{g_{a\mu}g_{\mu b}}{E-\mathcal{E}_\mu}, \tag{29}$$

where $\text{Im}(\mathcal{E}_\mu) \leq 0$. It is tempting to use the form (29) as the starting point for further analytical work since this form displays explicitly all N resonances. This, however, is not easy to do because of the constraints imposed by unitarity on the resonance parameters $g_{a\mu}$ and $\mathcal{E}_\mu$. For isolated resonances (resonance spacing large compared to resonance width) unitarity yields only the relation $-2\,\text{Im}\,(\mathcal{E}_\mu) = \Gamma_\mu = \sum_a |g_{\mu a}|^2$: The total width equals the sum of the partial widths over all open channels, but whenever the resonances overlap, the unitarity relation for S leads to a set of $\Lambda(\Lambda-1)/2$ equations that connect all partial width amplitudes $g_{\mu a}$ with all resonance energies $\mathcal{E}_\nu$. This is why it is preferable in general to use the expressions (10), (11) as the starting point for the statistical theory since these expressions obey unitarity automatically.

These statements have a straightforward physical interpretation. An isolated pole does signify an isolated resonance, visible in the local energy dependence of the cross section. Poles with spacings smaller than their distance from the real axis describe overlapping resonances. Such resonances jointly contribute to a perhaps very complicated dependence of the cross section on energy. In this case it is not possible to establish a one-to-one correspondence between a specific feature of the cross section and one of the poles of S. Therefore, it is not possible to attach physical meaning to any one of these poles individually, and positions and other values of the resonance parameters of individual poles of the S matrix are devoid of physical interest. Despite these facts, the global distribution of the poles of the S matrix is of interest even for strongly overlapping resonances, especially when $1 \ll \Lambda \ll N$. This distribution can be worked

out analytically with the help of the supersymmetry method (Lehmann *et al.*, 1995).

Qualitative features of the distribution of poles can be inferred from the form of Eq. (11). We consider the case where the coupling to the channels described by the matrix elements W becomes very large. Then it is convenient to bring the symmetric width matrix $2\pi\sum_c W_{\mu c}W_{c\nu}$ appearing in Eq. (11) to diagonal form. This can be accomplished by an orthogonal transformation $\overline{O}$ in the space of quasi-bound states. We denote the eigenvalues by w_μ^2 with $\mu = 1 \ldots N$. The transformed matrix $\overline{O}D\overline{O}^{\mathrm{T}}$ has the form $E\delta_{\mu\nu} + (1/2)iw_\mu^2\delta_{\mu\nu} - [\overline{O}H\overline{O}^{\mathrm{T}}]_{\mu\nu}$. The width matrix $2\pi\sum_c W_{\mu c}W_{c\nu}$ is a sum of Λ separable terms. This implies that only Λ of the eigenvalues w_μ^2 differ from 0. These eigenvalues obey $w_\mu^2 \geq 0$. Typically, the number Λ of open channels is much smaller than the number N of levels. If the nonvanishing eigenvalues w_μ^2 are much larger than the nondiagonal elements of $[\overline{O}H\overline{O}^{\mathrm{T}}]_{\mu\nu}$, then Λ poles of the S matrix have a distance from the real axis that is much larger than that of the remaining $(N-\Lambda)$ poles. These far-away poles only change the overall phase of the scattering matrix and eventually merge into the background contained in $\bar{S}$ while only the $(N-\Lambda)$ close-lying poles contribute to S^{fluc}. The distance of the far-away poles increases monotonically with increasing coupling to the channels. By the same token, the distance of the remaining $N-\Lambda$ poles from the real axis actually shrinks and approaches zero as $w_\mu^2 \to \infty$. This can easily be seen using perturbation theory. As this happens, absorption is reduced. These observations explain why the transmission coefficients $T_a = 4x_a/(1+x_a)^2$ first increase but eventually decrease with increasing x_a. Indeed, the nonvanishing eigenvalues w_μ^2 of the width matrix are monotonically increasing functions of the x_a's. Increasing all the x_a's causes the motion of poles just described. Absorption attains its maximum value, $\overline{S_{aa}} = 0$ or $T_a = 1$, at $x_a = 1$. This is why the eigenvalues w_μ^2 are not suitable measures for the physical strength of the coupling between channels and resonances. Rather, this role is played by the transmission coefficients T_a.

These qualitative arguments are fully supported by the quantitative analysis. In Lehmann *et al.* (1995), the limit $N \to \infty$ was considered with $m = \Lambda/N$ held fixed. The ratio m denotes the fraction of resonances that move toward $-i\infty$ in the complex E plane as all $x_a \to \infty$. For fixed and small values of the x_a, the distribution of poles in the complex E plane shows a cloud located below the real axis. As the x_a increase, this cloud deforms and eventually splits into two. The upper cloud contracts toward the real axis. It was numerically observed by Moldauer (1975) and later confirmed analytically by Gaspard and Rice (1989) that all poles keep a finite distance from the real axis. This distance is related to but different from the average decay width Γ.

The statistical theory of compound-nucleus reactions presented in this section uses the optical model of elastic scattering and/or a coupled-channels approach as input. The parameters determining these models are assumed to be known and do not form part of the statistical theory. Nevertheless, it is only fair to point out that optical-model parameters must be known for all channels, not only those with both fragments in their ground states. The essential step then consists in replacing the projection $H_{\mu\nu}$ of the Hamiltonian matrix onto the space of quasi-bound states by an ensemble of random matrices, the GOE. In this way, averages over all observables depending on S^{fluc} are uniquely determined in terms of the optical-model or coupled-channels input. It is the strength of this statistical approach that it is capable of predicting not only variances but also correlation functions, both versus energy (Eq. (25)) and, as will be shown in §7, also with respect to a parameter that measures symmetry breaking. This is possible because in Eqs. (11) and (13) the energy E and the projected Hamiltonian both appear explicitly. The exact calculation of the correlation functions is not easy, however. Even in the simplest nontrivial case, that of the S-matrix autocorrelation function of Eq. (25), the calculation involves the use of the supersymmetry technique. And calculating the average of a product of four elements of S^{fluc} (two with and two without complex conjugate sign) has been possible only in very special cases. So far, this has not been a serious drawback because cross-section correlation functions outside the Ericson regime (where the inequality $\Gamma \gg d$ allows for an asymptotic expansion) are very difficult to measure, and have not been of particular experimental interest. Observables involving more than four S-matrix elements in a nontrivial way are beyond the present reach of the supersymmetry method.

§6. Maximum Entropy Approach

A second approach to the statistical theory of compound-nucleus scattering does not take recourse to Eqs. (11) and (13). Rather, the scattering matrix itself is considered a member of a suitable ensemble of random matrices, without the detour of implementing stochasticity into the projection of the Hamiltonian onto the space of quasi-bound states. Dyson's circular ensembles may be considered an early precursor of this approach. Dyson (see Porter (1965)) introduced

the circular orthogonal ensemble (COE) of symmetric unitary matrices S of dimension Λ: The matrix S is written as $S = U\,U^{\mathrm{T}}$, and the ensemble is defined by integration over the Haar measure for the unitary matrices U in Λ dimensions. Since S is unitary and symmetric, we may consider the COE as an ensemble of scattering matrices. For this ensemble, $\bar{S} = 0$. It has been shown (Lewenkopf and Weidenmüller, 1991; Brouwer, 1995) that the COE is identical to the ensemble of S matrices defined for compound-nucleus scattering, see §5, with $\bar{S} = 0$ or, equivalently, with $T_a = 1$ for all a.

This raises the question whether an ensemble of S matrices can be found also for the general case (arbitrary values for the average S-matrix elements $\bar{S}$) in a direct fashion, i.e., without implementing stochasticity into the projection of the Hamiltonian onto the space of quasi-bound states. This is indeed the case (Mello *et al.*, 1985; Brouwer, 1995). The ensemble is constrained by the following conditions. (i) The members must be unitary and symmetric. This constraint imposes probability conservation and time-reversal invariance. (ii) Any function $f(S)$ that can be expanded in a series of nonnegative powers of S must obey $\overline{f(S)} = f(\bar{S})$, and similarly for $f(S^*)$. This constraint was mentioned as an obvious property above Eq. (25) but must now be imposed. (iii) The ensemble average of S is given by the optical S matrix, $\bar{S} = S^{\mathrm{opt}}$, or by the coupled-channels S matrix, $\bar{S} = S^{\mathrm{dir}}$, as the case may be. This is the constraint that defines the statistical model. The measure of the ensemble is written as

$$dp(S) = F(S)d\mu(S). \tag{30}$$

The measure of integration $d\mu(S)$ is found by writing $S = U\,U^{\mathrm{T}}$ and varying S so that $S \to S + dS = U(I + i\delta M)U^{\mathrm{T}}$ with I the Λ-dimensional unit matrix and δM real and symmetric. This yields $d\mu(S) = 2^{\Lambda(\Lambda-1)/4}\prod_{j\le k} dM_{jk}$. The weight factor $F(S)$ is found by maximising the information entropy S defined by

$$S = -\int d\mu(S)F(S)\ln F(S) \tag{31}$$

with respect to $F(S)$, subject to the three constraints mentioned above. Variation yields for $F(S)$ the result (Mello *et al.*, 1985)

$$F(S) = \frac{1}{V}\frac{[\det(I-\bar{S}\,\overline{S^*})]^{(\Lambda+1)/2}}{[\det(I-\bar{S}\,S^*)]^{(\Lambda+1)}} \tag{32}$$

with V some normalisation factor. For $\bar{S} = 0$, we retrieve the COE. It was mentioned above that the COE is completely equivalent to the statistical model of §5 provided we put all $T_a = 1$ in the latter. A similar equivalence has not been established yet for the general case where $\bar{S} \neq 0$, i.e., for the ensemble defined by Eqs. (30) and (32). Numerical studies have shown perfect agreement for the case of two channels, and the Hauser–Feshbach formula (24) has been derived from Eqs. (30) and (32) for the case $\Lambda \gg 1$. A comparison in other cases is difficult because of the unwieldy form of the integral kernel (32).

The strength of the maximum entropy approach lies in the fact that it deals directly with the quantity of interest, i.e., the scattering matrix, and avoids introducing the Hamiltonian. It yields Eq. (32) for $F(S)$ from which in principle all moments of S can be obtained. The approach has not been much used in nuclear physics but has found important applications to properties of quasi one-dimensional disordered conductors. The main drawback of the approach is its inability to predict correlation functions, except for select cases. This is because it is not clear how a dependence on energy and/or on a symmetry-breaking field can be incorporated into this approach in a physically correct fashion. This is precisely because the approach avoids using the Hamiltonian explicitly.

§7. Symmetry Breaking

The strong interaction conserves isospin and parity and is time-reversal invariant. In nuclei, isospin is violated by the electromagnetic interaction and parity, by the weak interaction. Violation of both quantum numbers has been observed in compound-nucleus reactions. To account for these observations and to deduce the strength of the symmetry-breaking interaction from the data, it is necessary to modify the approach of §5. The same holds true for a possible violation of time-reversal invariance for which an upper bound has been established in compound-nucleus reactions. A violation of symmetry and/or invariance may affect the values of $S^{(0)}$, of the $W^{(0)}$'s and of $H_{\mu\nu}$ in Eqs. (12) and (13) for the S matrix. Because of the long lifetime of the quasi-bound states, the effect is expected to be strongest when it occurs in $H_{\mu\nu}$. We follow common practice and focus attention on this term only which for brevity we denote by $\mathcal{H}$. In compound-nucleus reactions, isospin violation (parity violation) has mainly been investigated for strongly overlapping (for isolated) resonances, respectively. Both cases require a separate treatment. The case of time-reversal symmetry is different altogether because the operator of time-reversal symmetry is anti-Hermitean. Lack of space forces me to omit this topic.

Isospin Violation

Violation of isospin symmetry in compound-nucleus reactions for strongly overlapping resonances has been reviewed in Harney *et al.* (1986) and Reiter and Harney (1990). By assumption, isospin violation occurs only in $\mathcal{H}$, i.e., in the compound nucleus. Therefore, among the labels characterising the channels $(a, b, \ldots)$ there is also the isospin quantum

number T. The physical channels are characterised by the isospin projection t of the light reaction partner. A simple orthogonal transformation involving a Clebsch–Gordan coefficient connects the two representations. To model isospin violation in $\mathcal{H}$, we confine ourselves to the simplest case: Only two different values, $T = T_1$ and $T = T_2$, of the isospin quantum number are considered. This suffices for most applications. A generalisation is easily possible but would require more complex notation. We label the quasi-bound states by the isospin quantum number T and write the matrix $\mathcal{H}$ in block form,

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_1 & V \\ V^{\mathrm{T}} & \mathcal{H}_2 \end{pmatrix}. \tag{33}$$

The first (second) diagonal block refers to quasi-bound states with $T = T_1$ ($T = T_2$, respectively). The matrix V violates isospin. In the absence of isospin violation ($V = 0$), we would proceed as in §5: We would model $\mathcal{H}_1$ and $\mathcal{H}_2$ as two independent GOEs of dimensions N_1 and N_2, with semicircle radii $2\lambda_1$ and $2\lambda_2$, and mean level spacings $d_1 = \pi\lambda_1/N_1$ and $d_2 = \pi\lambda_2/N_2$, respectively. We consider the regime of strongly overlapping resonances, $\Gamma_k \gg d_k$, for both isospin values $k = 1, 2$. The Γ_k values are defined as in Eq. (28). In the presence of isospin-symmetry breaking ($V \neq 0$), Coulomb effects and other isospin-violating interactions are contained not only in V. Such effects contribute also to $\mathcal{H}_1$ and to $\mathcal{H}_2$ but are here neglected because such effects are small in comparison with the matrix elements of the nuclear force. In the presence of V, we accordingly retain the modelling of $\mathcal{H}_1$ and $\mathcal{H}_2$ in terms of two independent GOEs. In the spirit of the statistical model, we assume that the elements of V are uncorrelated Gaussian distributed random variables with mean value zero and a second moment v^2. The dependence of v^2 on N_1, N_2 is determined by the observation that the dimensionless ratio $v^2/[d_1 d_2]$ must be independent of N_1 and N_2: The matrix elements of the isospin-violating interaction scale with the N_k values as do the mean level spacings. We accordingly write $v^2 = \overline{\mathcal{H}_{12}^2}/[N_1 N_2]$. Then $\overline{\mathcal{H}_{12}^2}$ is the average of the square of the isospin-violating element in the nucleus. The analysis of data on compound-nucleus scattering with isospin violation aims at determining $\overline{\mathcal{H}_{12}^2}$ or another suitable measure of the strength of isospin-symmetry breaking related to this quantity. We mention in passing that the representation used in Harney *et al.* (1986) and Reiter and Harney (1990) differs from that adopted here: In the former, it is assumed that the matrices $\mathcal{H}_1$ and $\mathcal{H}_2$ have been diagonalised, the limits $N_k \to \infty$ have been taken, the eigenvalues of both matrices have been rescaled to attain finite mean level spacings d_1 and d_2 and the matrix elements of $\mathcal{H}_{12}$ have been introduced. The technique used in Harney *et al.* (1986) and Reiter and Harney (1990) to calculate averages of observables was later shown to give the same results in the Ericson regime as the supersymmetry technique.

We observe that the average S matrix of the model conserves isospin, i.e., is diagonal in T. This is because isospin-symmetry violation occurs only in the compound nucleus and is entirely due to V. The isospin-violating elements of S are necessarily odd in V and, thus, vanish on average. As in §5, we assume that $\bar{S}$ is given in terms of an optical-model or coupled-channels calculation, which, for consistency, must conserve isospin. The statistical model for isospin-symmetry breaking differs in one fundamental aspect from the model for compound-nucleus scattering in §5: The model contains an additional parameter, the strength $\overline{\mathcal{H}_{12}^2}$ of the isospin-symmetry breaking interaction V. This parameter is not determined by the values of the average S-matrix elements. Rather, it is determined by a fit to data on isospin-symmetry breaking. The validity of the model can only be established by showing that it consistently describes a large body of such experimental data; see Fig. 4. We also observe that the model of Eq. (33) violates the orthogonal invariance in Hilbert space originally postulated in the defining equations of the GOE, Eqs. (2) and (3): The model keeps this invariance with respect to states with fixed isospin but not in the entire Hilbert space. This is the unavoidable consequence of a weakly broken symmetry.

In the regime of strongly overlapping resonances, the distribution of the fluctuating part S^{fluc} of the S matrix has many properties in common with that described in §5 for compound-nucleus scattering: The distribution is Gaussian, and the only nonvanishing second-order correlation function has the form $\overline{S^{\mathrm{fluc}}(E_1) S^{\mathrm{fluc}*}(E_2)}$. It is given by

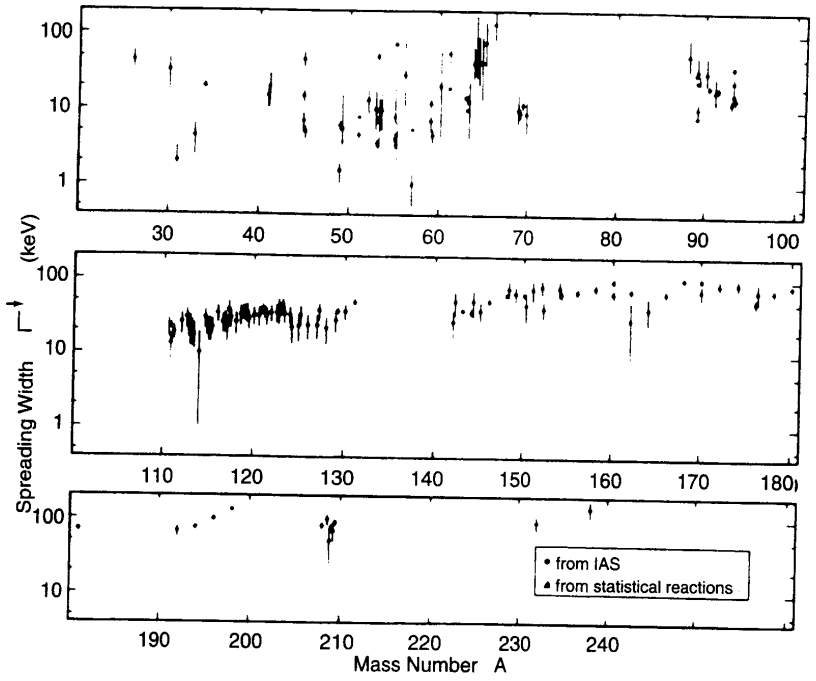
$$\overline{S^{\mathrm{fluc}}_{at_1 bt_2}(E_1) S^{\mathrm{fluc}*}_{ct_3 dt_4}(E_2)} = \delta_{ac}\delta_{bd} \sum_{mn} \tau_{am}^{t_1 t_3} \Pi_{mn} \tau_{bn}^{t_2 t_4} + \delta_{ad}\delta_{bc} \sum_{mn} \tau_{am}^{t_1 t_4} \Pi_{mn} \tau_{bn}^{t_2 t_3}. \tag{34}$$

The indices (at) refer to the physical channels with t denoting the isospin projection of the light reaction partner. The matrix Π_{mn} in the space of isospin classes has dimension two. The inverse of Π has the form

$$\Pi^{-1} = \begin{pmatrix} \sum_{ct} \tau_{c1}^{tt} + z + 2i\pi\varepsilon/d_1 & -z \\ -z & \sum_{ct} \tau_{c2}^{tt} + z + 2i\pi\varepsilon/d_2 \end{pmatrix}. \tag{35}$$

Here, $\varepsilon = E_2 - E_1$. The dimensionless parameter z measures isospin violation. Except for a correction that usually is unimportant, z is given by $z = 4\pi^2 \overline{\mathcal{H}_{12}^2}/[d_1 d_2]$. The definition of the transmission coefficients $\tau_{am}^{t_1 t_2}$ involves charge-dependent barrier penetration factors and is not given here; see Harney

Figure 4 The spreading width $\Gamma^{\downarrow}$ of Eq. (36) in kiloelectronvolts versus mass number A. Open triangles (full dots) indicate values determined from the statistical theory (from isobaric analogue states, respectively). Data available at different excitation energies cause the occurrence of different data points for the same nucleus. Taken from Harney *et al.* (1986) and Reiter and Harney (1990)).



et al. (1986) and Reiter and Harney (1990). Equations (34) and (35) give the leading terms of an asymptotic expansion in inverse powers of $\sum_{ct}\tau_{c1}^{tt}$ and of $\sum_{ct}\tau_{c2}^{tt}$. We observe that for $z = 0$, the correlation function (34) reduces to two independent Lorentzians, one for each isospin value. This is consistent with Eq. (26) except for the fact that the channels are now also explicitly labelled by the (conserved) isospin quantum number. Investigation of the converse limit $z \to \infty$ shows that the correlation function is governed by a single Lorentzian: Isospin mixing between the two classes is so strong as to make any distinction between them meaningless. In this limit, the orthogonal invariance of the matrix ensemble (33) in the combined Hilbert space of the states with isospins T_1 and T_2, is restored. These limits show that Eqs. (34) and (35) constitute the simplest nontrivial extension of Eq. (26).

The parameter typically deduced from experimental data is the spreading width $\Gamma_2^{\downarrow}$ for isospin mixing defined by

$$\Gamma_2^{\downarrow} = \frac{zd_1}{2\pi} = 2\pi\frac{\overline{\mathcal{H}_{12}^2}}{d_2}. \tag{36}$$

This quantity has a simple physical interpretation, and it possesses an important property. Let $|1\alpha\rangle$ and $|\beta\rangle$ denote eigenstates of $\mathcal{H}_1$ and of $\mathcal{H}$, respectively. We order the states $|\beta\rangle$ so that the associated eigenvalues grow monotonically. The probability $|\langle\beta|1\alpha\rangle|^2$ for finding a fixed state 1α in the states β has on average a Lorentzian form with width $\Gamma_2^{\downarrow}$. Put differently, $\hbar/\Gamma_2^{\downarrow}$ is the average mixing time of the states that have isospin T_1 with those that have isospin T_2. The spreading width $\Gamma_2^{\downarrow}$ is a much more useful measure (Brody *et al.*, 1981) of isospin mixing than the mean square mixing matrix element $\overline{\mathcal{H}_{12}^2}$. This is because the latter involves the overlap between states with isospin values T_1 and T_2. The overlap decreases rapidly with increasing complexity of the states involved, and so does $\overline{\mathcal{H}_{12}^2}$. This strong decrease is almost completely compensated by the factor d_2^{-1} in the definition (36). This was shown theoretically (Brody *et al.*, 1986) and is supported by the data. Indeed, values of $\overline{\mathcal{H}_{12}^2}$ determined from experimental data (Harney *et al.*, 1986; Reiter and Harney, 1990) vary over seven orders of magnitude while the corresponding values of $\Gamma_2^{\downarrow}$ lie in a narrow band; see Fig. 4. We mention in passing that a definition completely analogous to Eq. (36) applies to the spreading width of the strong interaction introduced in §3. Here the state 2α is identified with an eigenstate of the shell model,

while the states β are the eigenstates of the full nuclear Hamiltonian.

Isospin-symmetry breaking in the compound nucleus has been studied with the help of several reaction processes: Through the comparison of average cross sections leading to isospin-allowed and to isospin-forbidden reaction channels, through the intensity of isospin-forbidden electric dipole radiation induced by light or by heavy ions, or the intensity of the isospin-forbidden neutron decay of the giant dipole resonance, through ratios of evaporation spectra that should be unity if isospin were completely broken or through the comparison of cross-section fluctuations in isobaric mirror channels. All these cases involve a fairly straightforward application of the formalism of Eqs. (33) to (36). The results are summarised in Fig. 4. We note that $\Gamma_2^{\downarrow}$ is nearly independent of A and excitation energy. This result is not obvious, especially since the primary experimental observable is z. To obtain $\Gamma_2^{\downarrow}$ from Eq. (36), z must be multiplied with d_1. The average level density d_1^{-1} of states with isospin T_1 depends nearly exponentially on energy and is not always well known. We also note that independent and different sources of information (the statistical theory and isobaric analogue states) lead to almost identical values for $\Gamma_2^{\downarrow}$. The results summarised in Fig. 4 constitute one of the best vindications of the statistical theory of nuclear reactions.

Parity Violation

The weak interaction induces a parity-violating term also in the effective nucleon–nucleon interaction. The scattering of epithermal polarised neutrons on medium-weight and heavy nuclei provides a measure for parity violation in the compound nucleus (Mitchell *et al.*, 1999): The total neutron cross section depends on the helicity of the incident neutron. It is customary to present the data in terms of the "asymmetry"

$$P = \frac{\sigma_+^p - \sigma_-^p}{\sigma_+^p + \sigma_-^p}, \tag{37}$$

where $\sigma_\pm^p$ is the total p-wave cross section for neutrons with helicities ±. An effect is observed when the neutron populates an isolated p-wave compound nucleus resonance. We focus on the typical case where the target nucleus has spin zero. Then the p-wave resonances have spin values 1/2 or 3/2. Parity mixing of the former (the latter) involves s-wave (d-wave) resonances. The angular momentum barrier prevents the d-wave resonances from contributing, and only p-wave resonances with spin 1/2 need be considered. Because of two enhancement factors explained below, values of P in the percent region are measured even though the strength of the weak interaction is only one part in 10^6 or so of the strong one.

The statistical model is that of Eq. (34) with $\mathcal{H}_1$ ($\mathcal{H}_2$) representing quasi-bound states with spin and parity $1/2^+$ ($1/2^-$, respectively), and V modelling the effective weak interaction. The goal of the theory consists in deducing the strength of the effective weak interacion or, more appropriately, the associated spreading width $\Gamma_W^{\downarrow}$ from data given in the form of Eq. (37). To this end, we proceed as in the case of isospin violation and write the second moment of V in the form $\overline{V^2} = \overline{\mathcal{H}_W^2}/[N_1 N_2]$. Then the spreading width is defined in analogy to Eq. (36) as $\Gamma_W^{\downarrow} = 2\pi\overline{\mathcal{H}_W^2}/d_s$. Here d_s is the mean level spacing of the s-wave resonances with spin 1/2.

We use the model of Eq. (34) and Eqs. (10) and (11) for isolated resonances, and first-order perturbation theory in V to evaluate P. We label the p-wave (s-wave) resonances consecutively by an index μ (ν, respectively). Then

$$P_\mu = 2\sum_\nu \frac{\langle \nu|\mathcal{H}_W|\mu\rangle}{E_\nu - E_\mu}\frac{g_\nu g_\mu}{\Gamma_\mu^n}. \tag{38}$$

Here g_μ and g_ν are the neutron decay amplitudes of levels μ and ν, with $\Gamma_\mu^n = g_\mu^2$ the partial width for p-wave neutron decay. The neutron bombarding energy has been taken in the centre of the p-wave resonance, and the p-wave background and the resonance widths have been neglected. In the calculation, the eigenvalues E_μ and E_ν have been rescaled to attain finite mean level spacing.

Equation (38) displays the central aspects of parity violation in the regime of isolated resonances. (i) It shows the origin of two enhancement factors. The first is associated with the ratio $\langle \nu|\mathcal{H}_W|\mu\rangle/[E_\nu - E_\mu]$. The small expected size of the weak interaction matrix element $\langle \nu|\mathcal{H}_W|\mu\rangle$ is partially compensated by the small level spacing $[E_\nu - E_\mu]$. Hence, parity violation is a relatively larger effect near neutron threshold than at lower excitation energies where level spacings are much larger. Quantitatively, we recall the fact that the spreading width is roughly independent of excitation energy and conclude that $|\langle \nu|\mathcal{H}_W|\mu\rangle/[E_\nu - E_\mu]|$ scales with excitation energy as $d_s^{-1/2}$. This implies that in comparison with the ground-state region where d_s is typically 100 keV, the ratio $|\langle \nu|\mathcal{H}_W|\mu\rangle/[E_\nu - E_\mu]|$ is enhanced by a factor $(100\text{ keV} / 10\text{ eV})^{1/2} = 10^2$. The second enhancement factor stems from the partial width amplitudes in Eq. (38): In the case of parity mixing, the p-wave resonance decays by s-wave emission. Without parity mixing, the decay would be back into the p-wave channel and would be hindered by the barrier penetration factor for p waves. The resulting enhancement amounts to another factor $10^2 - 10^3$ near neutron threshold. The combination of both factors explains qualitatively why P_μ values on the order of 10^{-2} are found. (ii) Equation (38) also demonstrates the need of a statistical approach.

Indeed, experimental knowledge of P_μ and of the quantities E_μ, E_ν, g_μ and g_ν will in general not allow us to determine the matrix elements $\langle \nu|\mathcal{H}_W|\mu\rangle$. And even when this is possible (because only a single s-wave resonance lies sufficiently close to E_μ, and the sum in Eq. (38) reduces to a single term), the complexity of the states $|\mu\rangle, |\nu\rangle$ prevents us from using a single matrix element $\langle \nu|\mathcal{H}_W|\mu\rangle$ to deduce properties of the effective weak interaction in nuclei. Here the statistical theory comes to the rescue: It states that the matrix elements $\langle \nu|\mathcal{H}_W|\mu\rangle$ are uncorrelated random variables with a Gaussian distribution centered at 0 and with a common second moment $\overline{\mathcal{H}_W^2}$. We determine the value of $\overline{\mathcal{H}_W^2}$ by writing Eq. (38) as $P_\mu = \sum_\nu A_{\mu\nu}\langle \nu|\mathcal{H}_W|\mu\rangle$ and observe that as sums of random variables, the P_μ are also Gaussian-distributed random variables with mean value zero. The variance $\mathrm{var}(P_\mu)$ of P_μ with respect to both μ and the ensemble is given by $A^2\overline{\mathcal{H}_W^2}$, where $A^2 = (1/N_p)\sum_{\mu\nu} A_{\mu\nu}^2$ and N_p the number of p-wave resonances. Thus,

$$\overline{\mathcal{H}_W^2} = \frac{\mathrm{var}(P_\mu)}{A^2}. \tag{39}$$

Values of the weak spreading width $\Gamma_W^\downarrow$ determined via Eq. (39) are known (Mitchell *et al.*, 1999) for about 15 nuclei and are typically a few times 10^{-7} eV, in keeping with the estimate of a few megaelectronvolts for the spreading width of the strong interaction, and with a factor 10^{-6} relating the strengths of the strong and the weak interactions. These facts, and tests of the distribution of the P_μ values, constitute another successful application of the statistical theory.

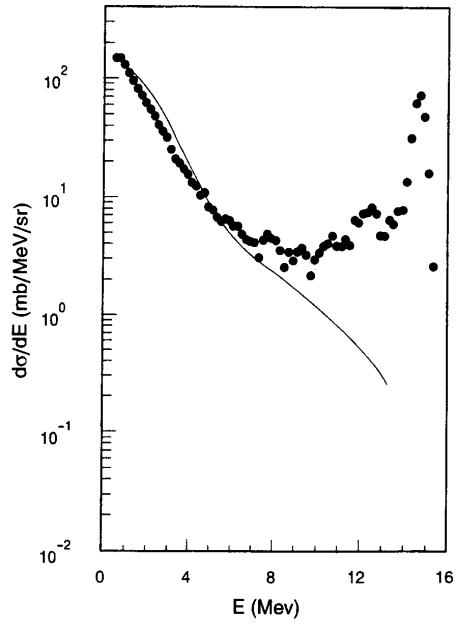
§8. Precompound Reactions

As mentioned in §5, the compound-nucleus picture breaks down at excitation energies of about 15 to 20 MeV. Here the internal equilibration time $\hbar/\Gamma^\downarrow$ becomes comparable with the compound-nucleus decay time $\hbar/\Gamma$, and "precompound decay" sets in: The system decays before it has reached internal equilibrium. The emitted particles have, on average, energies higher than would correspond to compound-nucleus decay. Moreover, the angular distribution of these fragments is forward-peaked and "remembers" the direction of the incident beam, while compound-nucleus decay is symmetric about 90° c.m.

Figure 5 shows a typical data set. Neutrons at 13.5 Mev bombarding energy strike a ^{96}Nb target. The full dots show the measured spectrum of neutrons produced at an angle of 143° c.m. in a logarithmic plot. The peak at the highest energy corresponds to direct reactions populating low-lying states. The contribution that decays nearly exponentially with energy is partly due to compound-nucleus decay; see below. The contributions between 7 and 14 MeV are largely due to precompound decay processes.

The statistical modelling must deviate from the GOE to allow for the existence of an equilibration time. The underlying physical picture is based on the shell model with a residual two-body interaction and is explained most simply if the incident particle is a nucleon and the target nucleus is doubly magic. The extension to composite projectiles is straightforward. In a sequence of two-body collisions, the incident nucleon generates a series of two-particle one-hole, three-particle two-hole, etc., states. The n-particle $(n-1)$-hole states form a class with index n. The states in each class are treated in a statistical fashion and can decay by emitting particles. The two-body interaction couples only classes that differ in n by not more than one unit (this is Feshbach's "chaining hypothesis"; see Feshbach *et al.* (1980)). At a given excitation energy E, the partial level density $\rho_n(E)$ for states in class n grows strongly with n and, after reaching a peak value at $n_{\max}$, falls off strongly with n. The equilibration time is the time needed to reach class $n_{\max}$ from the incident channel. This picture is the starting point for a number of theoretical approaches (Feshbach *et al.*, 1980; Tamura *et al.*, 1982; Nishioka *et al.*, 1986, 1988). Common to all

Figure 5 Differential cross section for the reaction ^{93}Nb(n,xn) at E_n = 13.47 MeV and 143° c.m. and the results of a multistep-compound calculation (solid line). Taken from (Herman *et al.*, 1992).



is the distinction between the "multistep-compound" and the "multistep-direct" process. In the multistep-compound process, the states in all classes but the first are quasi-bound states, while in the multistep-direct process, these states are continuum states. These two processes are not independent, however, but states in either are coupled to those in the other by the residual two-body interaction. The relative importance of the multistep-direct process increases with increasing excitation energy. Indeed, in a given class n of the multistep-compound process, the available energy E is shared by $(2n+1)$ "excitons" (particles or holes). Particles cannot absorb more than their binding energy, and the creation of deep-lying hole states is limited by the extremely short lifetimes of such states. Therefore, the number of states available for multistep-compound processes in each class decreases as the excitation energy increases above some fixed class-dependent value. No such restriction exists for the multistep-direct process.

Multistep-Compound Process

Theoretical approaches to the multistep-compound process are fairly similar and essentially use a modification of the procedure described in §7 for the mixing of isospin or parity. The Hamiltonian $\mathcal{H}$ is modelled as in Eq. (33) except that the number of blocks is larger than 2 and given by the number of classes under study. The states in each class n define a block of dimension N_n and are modelled in terms of a GOE, with $N_n \to \infty$. The GOEs for different classes are uncorrelated. Classes differing in class label n by one unit are connected by a block of two-body matrix elements that are taken to be Gaussian-distributed random variables with mean value zero. The second moments $v^2_{nn+1} = \overline{|\mathcal{H}_{nn+1}|^2}/[N_n N_{n+1}]$ together with the level densities $\rho_n(E)$ determine the equilibration time. The model breaks the overall orthogonal invariance of the GOE model for the compound nucleus used in §5. The invariance is restored, and the compound-nucleus limit is attained, if the spreading widths $2\pi\overline{|\mathcal{H}_{nn+1}|^2}\rho_{n+1}(E)$ for the mixing of states in class n with those in class $(n+1)$ become sufficiently large. In practice, it suffices that these quantities are much larger than the decay widths Γ_n of the states in class n. This is the situation realised in the regime of compound-nucleus scattering.

The formula for the average cross section is an obvious generalisation of the expressions (34) and (35). For $a \neq b$, it reads

$$\overline{|S^{\text{fluc}}_{ab}|^2} = \sum_{mn} T_{am}\Pi_{mn}T_{bn}. \tag{40}$$

Here T_{am} is the transmission coefficient for populating class m from channel a and obeys the sum rule $\sum_m T_{am} = T_a$ with T_a as defined in §4. The quantity Π is a matrix with class indices (m,n). Lack of space permits me to give only the central features. The inverse of Π is given by

$$(\Pi^{-1})_{mn} = \delta_{mn}2\pi\rho_m(\Gamma^{\downarrow}_m + \Gamma_m) - T_{mn}. \tag{41}$$

Here $T_{mn} = 2\pi\rho_m\overline{|\mathcal{H}_{mn}|^2}\rho_n$ mixes classes m and n provided that $m = n \pm 1$. The interpretation of Eqs. (40) and (41) follows that of Eqs. (34) and (35). The expressions (40) and (41) can be further simplified if one uses the "never come back" approximation (Feshbach *et al.*, 1980): The level densities ρ_n increase so strongly with n that the decay back from class n into class $(n-1)$ is negligible.

To evaluate Eqs. (40) and (41), one uses level densities of the particle-hole model and calculates the transmission and mixing coefficients and the spreading widths with the help of the optical model (Herman *et al.*, 1992): The imaginary part of the optical model defines the lifetime for decay of each particle or hole into more complicated states. Thus, combining the particle-hole and the optical model makes it possible to evaluate Eqs. (40) and (41) without additional fit parameters. The solid line in Fig. 5 shows the result of such a calculation. Single (multiple) neutron decay is calculated from the multistep-compound process (compound nucleus theory, respectively). The difference between the calculated and the measured results is attributed to the multistep-direct process.

Multistep-Direct Process

The multistep-compound process introduces a statistical model for the quasi-bound states, i.e., for the matrix $H_{\mu\nu}$ in Eq. (13). The model differs from the GOE model used to describe compound-nucleus reactions. In contradistinction, theories of the multistep-direct process address the calculation of the nonresonant scattering matrix $S^{(0)}$ in Eq. (12). As mentioned in §4, this matrix is, in principle, determined by a coupled-channels calculation. Such a calculation becomes impractical, however, whenever the final channel involves excited states in either fragment with excitation energies of several megaelectronvolts. Indeed, the level density in the fragment(s) is then so high that the number of channels needed in the coupled-channels calculation exceeds the possibilities of numerical calculation. Equally important is the fact that very little is known about the spectroscopic properties of the excited states involved in the calculation. Such properties are needed as input for the coupled-channels approach. Hence, one again takes recourse to a statistical modelling. Such modelling has less secure foundations, however, than the GOE model used for compound-nucleus scattering. Indeed, the

experimental evidence presented in §1 is direct proof for the applicability of the GOE to the compound-nucleus regime. In the case of excited states of either fragment, there must be a transition from the regular behaviour of low-lying states that can be described by the shell model and/or by one of the collective models, to the GOE regime that prevails at neutron threshold. Unfortunately, experimental data on this transition are practically nonexistent, and much room is, therefore, left to theoretical modelling.

In all theories (Feshbach *et al.* 1980; Tamura *et al.* (1982); Nishioka *et al.* 1986, 1988) of the multistep-direct process, the coupled-channels problem is solved perturbatively (Born series). The transition from the initial channel to the final channel is then described as a sequence of steps. Each step involves a matrix element that has the form of a direct-reaction amplitude, hence the name "multistep-direct" process. In order to arrive at expressions that are amenable to calculation, the sums over intermediate states are simplified. First, these states are modelled in a schematic fashion, similar to the modelling of the quasi-bound states in the multistep-compound process: It is assumed that the multistep-direct process populates a sequence of n-particle n-hole states in either fragnment. Second, statistical assumptions on these n-particle n-hole states are used to simplify the sums over intermediate states. More precisely, the statistical average of $|S_{ab}^{(0)}|^2$ is calculated and in this average, only incoherent sums over intermediate states appear.

The three approaches (Feshbach *et al.* 1980; Tamura *et al.*, 1982; Nishioka *et al.* 1986, 1988) use different statistical assumptions. It was emphasised above that a statistical assumption usually implies an assumption about a time scale: The states that are treated as statistically equivalent must mix on a time scale that is short in comparison with all other time scales of the problem. Moreover, the three approaches use different simplifications to arrive at workable formulas for practical calculations. The three approaches are compared in Konning and Akkermans (1991). Problems in using these theories are discussed in Chadwick *et al.* (1999). The work of Feshbach, *et al.* (1980) has probably found the widest application. Applications of the formalism described here are reviewed in Gadioli and Hodgson (1992).

§9. Outlook

Starting at neutron threshold, the regime of compound-nucleus scattering extends over an energy interval of about 10 MeV. In this regime, the statistical theory is highly successful. It is based on a well-defined input (the average scattering matrix). In the absence of symmetry breaking, the theory makes parameter-free predictions that are in very good agreement with experimental data and that are used in applications where such data are not available. In cases of symmetry breaking, the theory involves one parameter, the spreading width. This parameter has been consistently determined in many cases of isospin or parity violation.

Precompound reactions are important at excitation energies above the regime of compound-nucleus scattering. The multistep-compound process seems well understood. Under the assumption that a sequence of quasi-bound particle-hole states of increasing complexity is populated in the process, the additional characteristic parameter of the model (the equilibration time) can be determined from the optical model and from particle-hole level densities. For the multistep-direct process, the situation is less satisfactory. The handling of the intermediate states including statistical aspects is currently under debate. As the excitation energy increases, the relative importance of the multistep-direct process increases. It should eventually become equivalent to a semiclassical theory.

Statistical concepts play a large role in other areas of nuclear reaction theory. An outstanding example is deeply inelastic heavy ion scattering. The transfer of large amounts of energy and angular momentum of relative motion into intrinsic excitation of both fragments is phenomenologically described in terms of models with dissipation and requires, on a microscopic scale, statistical concepts. The same is probably true of the multitude of gamma rays emitted in the deexcitation of compound nuclei formed by the fusion of two medium-weight nuclei.

The statistical theory described in this chapter has much in common with the theory of chaotic scattering, and with theories of transport of electrons through disordered and/or chaotic devices and of photons through a medium with an irregular index of refraction. Much work has been done over the past few years on these topics (Guhr *et al.*, 1998).

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Chapter 3.1.5

POLARISATION IN NUCLEAR REACTIONS

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PART 3 SCATTERING IN NUCLEAR PHYSICS

Topic 3.1 Nuclear Physics

Contents

§ 1. Introduction

Scope of the Chapter

A basic fact of nuclear physics is the strong spin dependence of the force between nucleons. This shows up at the most elementary level. Two neutrons do not bind, though a neutron and proton do form a deuteron, indicating a significant difference between the spin-singlet and spin-triplet nucleon–nucleon potentials. Nucleon–nucleon scattering experiments and detailed studies of the deuteron have shown that the nuclear force has the full spin-dependence that is consistent with general symmetries, including spin–orbit and tensor forces. A crucial contribution to these results has come from high precision data obtained with polarised beams and targets. Here we use the term "polarised" to denote any spin state of a spin-I system that differs from a mixed state having a uniform distribution over the $2I+1$ spin projections along some direction in space. Polarisation measurements also play an important role in studies of the 3-nucleon system. (see Chapter 3.1.1).

In this chapter we have chosen to discuss the special phenomena that give rise to spin dependence in the interaction of heavy nuclei and that are revealed by polarisation experiments. Some of the most interesting of these spin-dependent effects have very little