PRE-EQUILIBRIUM NUCLEAR REACTIONS: An introduction to classical and quantum-mechanical models

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1. CLASSICAL MODELS

1. The exciton model
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EXCITON MODEL

The composite nucleus (projectile + target) is treated as a system characterized by basically two quantum numbers, the exciton number $n = p + h$, (particles + holes) and the total energy of the system, E. Furthermore:

- Equiprobability of particle-hole configurations with the same E.
- Two-body interactions only.
- Temporal development in terms of exciton number.
- Free parameter: the average matrix element for transitions to less or more complex stages.
- Correlation of system with initial condition gradually disappears.
- Automatic inclusion of the compound stage.

THE MASTER EQUATION

$$\frac{dq(n,t)}{dt} = \lambda^+(n-2)q(n-2,t) + \lambda^-(n+2)q(n+2,t) - [\lambda^+(n) + \lambda^-(n) + w(n)]q(n,t). \quad (1)$$

- q: occupation probability
- n: exciton number
- $\lambda^+(n)$, $\lambda^-(n)$: internal transition rates from state n with $\Delta n = +2$ and $\Delta n = -2$, respectively.
- $w(n)$: total emission rate from state n summed over all outgoing particles and energies.

EMISSION RATE

Emission rate:

$$w_b(n,\epsilon) = \frac{2s_b+1}{\pi^2\hbar^3}\mu_b\epsilon\sigma_{b,inv}(\epsilon)\frac{\omega(p-p_b,h,U)}{\omega(p,h,E)}Q_b(n)\phi_b, \qquad (2)$$

- ϵ: energy,
- μ_b: relative mass
- s_b: spin
- E: total excitation energy
- U: residual excitation energy
- $\sigma_{b,inv}(\epsilon)$: inverse reaction cross section
- Q: charge-conserving coefficient

PARTICLE-HOLE STATE DENSITY

Williams, Betak and Dobes, Oblozinsky:

$$\omega(p,h,E) = \frac{g^n}{p!h!(n-1)!}\sum_{j=0}^{h}(-1)^j \binom{h}{j} \times (E - A_{p,h} - jF)^{n-1}\Theta(E - E_{PP} - jF). \quad (3)$$

- F: Fermi energy,
- $E_{PP} = [p^2 + h^2 + p - h]/2g$
- Θ unit step function (finite hole depth),
- $g = \frac{6a}{\pi^2}$ single-particle state density
- $A_{p,h} = [p(p-1) + h(h-1)]/4g$: Pauli correction factor.

Transition rates

$$\lambda^{-}(n) = \frac{2\pi}{\hbar}\left\langle M^2\right\rangle \frac{1}{2} gph(n-2)\left[1-\left(\frac{C(p-1,h-1)}{gE-A(p-1,h-1)}\right)^{n-3}\right],$$

$$\lambda^{+}(n) = \frac{2\pi}{\hbar}\left\langle M^2\right\rangle \frac{1}{2}\frac{g}{n+1}\frac{[gE-A(p+1,h+1)]^{n+1}-[C(p+1,h+1)]^{n+1}}{[gE-A(p,h)]^{n-1}}, \quad (4)$$

where $C(p,h) = gh(E-F) - hA(p,h)$

Matrix element:

$$\left\langle M^2\right\rangle = \frac{c}{A^3(E+d)^3} \quad (5)$$

Solution of the master equation

$$\begin{aligned} -q_0(n) &= \lambda^+(n-2)t(n-2) + \lambda^-(n+2)t(n+2) \\ &- \left[\lambda^+(n) + \lambda^-(n) + w(n)\right] t(n-2) \end{aligned} \tag{6}$$

Mean lifetime of an exciton state $t(n)$

$$t(n) = \int_0^\infty q(n,t)dt. \tag{7}$$

Average cross section for emission of particle b with energy ϵ

$$\frac{d\sigma}{d\epsilon}^{PE}(a,b) = \sigma_a \sum_n w_b(n,\epsilon)t(n) \tag{8}$$

METHOD A: Exact solution

Akkermans (1979): Use straightforward properties of tri-diagonal matrices, all possible transitions taken into account.

Mean lifetime:

$$t(n) = \tau_n h_n \left(\prod_{m=n_0,\Delta m=2}^{n-2} \lambda^+(m)\tau_m h_m \right) \times \left(1 + \sum_{s=n+2,\Delta s=2} \left[\prod_{k=n,\Delta k=2}^{s-2} \lambda^+(k)\tau_k h_k \lambda^-(k+2)\tau_{k+2} h_{k+2} \right] \right) \quad (9)$$

$$\tau_i = [\lambda^+(i) + \lambda^-(i) + w(i)]^{-1}$$
$$h_{i+1} = [1 - h_i \lambda^+(i)\tau_i \lambda^-(i+1)\tau_{i+1}]^{-1}, \; h_1 = 1 \quad (10)$$

METHOD B: Never-come-back approximation

- Works good for high energies ($E > 20$ MeV)
- $\lambda^{-}(n) = 0$
- More consistent with complementary use of Hauser-Feshbach model

Pre-equilibrium cross section:

$$\frac{d\sigma}{d\epsilon}^{PE}(a,b) = \sigma_a[\frac{w_b(n,\epsilon)}{w(n)+\lambda^{+}(n)} + \sum_{n=n_0+2,\Delta n=2}^{\overline{n}} \left(\prod_{j=n_0,\Delta j=2}^{n-2} \frac{\lambda^{+}(j+2)}{w(j)+\lambda^{+}(j+2)} \right) \frac{w_b(n,\epsilon)}{w(n)+\lambda^{+}(n+2)}] \quad (11)$$

Kalbach systematics

Kalbach (1988) derived a powerful expression for continuum angular distributions:

$$\frac{d^2\sigma}{d\Omega d\epsilon_b} = \frac{1}{4\pi}\frac{d\sigma}{d\epsilon_b}\frac{a}{\sinh(a)}[\cosh(a\cos\Theta) + f_{PE}\sinh(a\cos\Theta)] \quad (12)$$

with,

$$\begin{aligned} a(e'_b, e'_a) &= 0.04\frac{E_1 e'_b}{e'_a} + 1.8\times10^{-6}(\frac{E_1 e'_b}{e'_a})^3 + 6.7\times10^{-7} M_a m_b (\frac{E_3 e'_b}{e'_a})^4, \\ E_1 &= \min(e'_a, 130 \pm 10\text{MeV}) \\ E_3 &= \min(e'_a, 41 \pm 5\text{MeV}) \\ e'_b &= \epsilon_b + S_b \\ e'_a &= \epsilon_a + S_a. \end{aligned} \quad (13)$$

ϵ_a, ϵ_b: incident and outgoing energy

Other extensions

Possible generalizations:

- Angle Ω: generalized exciton model (Mantzouranis 1975)
- distinction between neutrons and protons (Betak and Dobes 1976)
- leading-particle energy ϵ (Iwamoto 1984)
- Spin J and Parity Π (Plyuiko 1978, Shi 1987)
- Continuum pickup/stripping and knockout model (Kalbach 2002)

"Mother of all master equations" in terms of the probability $q(n_\pi, n_\nu, J, \Pi, \Omega, \epsilon, t)$.

Two-component exciton model revisited (Koning and Duijvestijn, 2002)

Two-component exciton model

Cross section:

$$\frac{d\sigma_k^{EM}}{dE_k} = \sigma^{CF} \sum_{p_\pi=p_\pi^0}^{p_\pi^{eq}} \sum_{p_\nu=p_\nu^0}^{p_\nu^{eq}} w_k(p_\pi, h_\pi, p_\nu, h_\nu, E_k) S_{pre}(p_\pi, h_\pi, p_\nu, h_\nu) \qquad (14)$$

- Physically more intuitive (a nucleus consists of neutron and protons)
- More flexibility with particle-hole state densities
- More flexibility with internal transition rates

Two-component exciton model

Emission rate:

$$w_k(p_\pi, h_\pi, E_k) = \frac{2s_k + 1}{\pi^2 \overline{h}^3} \mu_k E_k \sigma_{k,inv}(E_k) \frac{\omega(p_\pi - Z_k, h_\pi, p_\nu - N_k, h_\nu, E_x)}{\omega(p_\pi, h_\pi, p_\nu, h_\nu, E^{tot})}, \tag{15}$$

Time-integrated strength

$$S_{pre}(p_\pi, h_\pi, p_\nu, h_\nu) = P_2(p_\pi, h_\pi, p_\nu, h_\nu) t(p_\pi, h_\pi, p_\nu, h_\nu) \tag{16}$$

Lifetime:

$$\begin{aligned} t(p_\pi, h_\pi, p_\nu, h_\nu) &= [\lambda_{\pi+}(p_\pi, h_\pi, p_\nu, h_\nu) + \lambda_{\nu+}(p_\pi, h_\pi, p_\nu, h_\nu) + \lambda_{\pi\nu}(p_\pi, h_\pi, p_\nu, h_\nu) \\ &+ \lambda_{\nu\pi}(p_\pi, h_\pi, p_\nu, h_\nu) + w(p_\pi, h_\pi, p_\nu, h_\nu)]^{-1} \end{aligned} \tag{17}$$

Two-component exciton model

Internal transition rate

$$\lambda_{\pi+}(p_\pi, h_\pi, p_\nu, h_\nu) = \frac{2\pi}{\hbar}\frac{g_\pi^2}{2n(n+1)}\frac{\left[E^{tot} - A(p_\pi+1, h_\pi+1, p_\nu, h_\nu)\right]^{n+1}}{\left[E^{tot} - A(p_\pi, h_\pi, p_\nu, h_\nu)\right]^{n-1}} \times (n_\pi g_\pi M_{\pi\pi}^2 + 2n_\nu g_\nu M_{\pi\nu}^2)\, f(p+1, h+1, E^{tot}, V) \quad (18)$$

f: finite well function

$$B_{\pi\nu}(p_\pi, h_\pi, p_\nu, h_\nu) = max[A(p_\pi, h_\pi, p_\nu, h_\nu), A(p_\pi - 1, h_\pi - 1, p_\nu + 1, h_\nu + 1)] \quad (19)$$

Two-component exciton model

Strength:

$$
\begin{aligned}
P_2(p_\pi, h_\pi, p_\nu, h_\nu) &= P_2(p_\pi - 1, h_\pi - 1, p_\nu, h_\nu)\Gamma_{\pi+}(p_\pi - 1, h_\pi - 1, p_\nu, h_\nu) \\
&+ P_2(p_\pi, h_\pi, p_\nu - 1, h_\nu - 1)\Gamma_{\nu+}(p_\pi, h_\pi, p_\nu - 1, h_\nu - 1) \\
&+ ((P_2(p_\pi - 2, h_\pi - 2, p_\nu + 1, h_\nu + 1)\Gamma_{\pi+}(p_\pi - 2, h_\pi - 2, p_\nu + 1, h_\nu + 1) \\
&+ P_2(p_\pi - 1, h_\pi - 1, p_\nu, h_\nu)\Gamma_{\nu+}(p_\pi - 1, h_\pi - 1, p_\nu, h_\nu)) \\
&\times \Gamma_{\nu\pi}(p_\pi - 1, h_\pi - 1, p_\nu + 1, h_\nu + 1) \\
&+ (P_2(p_\pi, h_\pi, p_\nu - 1, h_\nu - 1)\Gamma_{\pi+}(p_\pi, h_\pi, p_\nu - 1, h_\nu - 1) \\
&+ P_2(p_\pi + 1, h_\pi + 1, p_\nu - 2, h_\nu - 2)\Gamma_{\nu+}(p_\pi + 1, h_\pi + 1, p_\nu - 2, h_\nu - 2)) \\
&\times \Gamma_{\pi\nu}(p_\pi + 1, h_\pi + 1, p_\nu - 1, h_\nu - 1)) \\
&\times \frac{t'(p_\pi, h_\pi, p_\nu, h_\nu)}{t(p_\pi, h_\pi, p_\nu, h_\nu)}
\end{aligned}
\tag{20}
$$

$$
\Gamma_{\pi+}(p_\pi, h_\pi, p_\nu, h_\nu) = \lambda_{\pi+}(p_\pi, h_\pi, p_\nu, h_\nu)t(p_\pi, h_\pi, p_\nu, h_\nu) \tag{21}
$$

Conclusions (Exciton Model)

- The basic Exciton Model is simple, but very powerful.
- Applicable up to at least 200 MeV.
- Most significant extension seems to be the two-component approach.
- Inferior to quantum-mechanical approach from physical point of view.

2. QUANTUM-MECHANICAL MODELS

1. History
2. Two-component multi-step direct model: Theory
3. Calculation method
4. Complementary reactions
   - Collective effects
   - P-Q transitions and multi-step compound reactions
   - Compound and multiple emission
   - Multiple pre-equilibrium emission
5. Conclusions

History

1966-1974:

- Direct: DWBA, coupled channels: Quantum-mechanical
- Compound: Hauser-Feshbach, etc. : Quantum-mechanical
- Pre-equilibrium: Exciton, Hybrid model, etc.: Classical

Pre-equilibrium models: why quantum-mechanical?

- Derived from first principles
- Prediction of continuum angular distributions
- Continuum analyzing power

History

1975-1984:

- Development of quantum-mechanical pre-equilibrium theories: Agassi-Weidenmüller-Mantzouranis, Tamura-Udagawa-Lenske, Feshbach-Kerman-Koonin
- Implementation in computer codes: Tamura, Bonetti

1985-present:

- Development of new MSD/MSC models: Nishioka-Weidenmüller-Yoshida-Verbaarschot
- More computer codes (healthy situation!)
- More emphasis on *ingredients*: Optical model, level densities, nucleon-nucleon interaction, RPA strength functions, single-particle level schemes

Differences between MSD Models

Multi-step direct:

- Extrapolation of direct reactions to the continuum
- Distorted wave theory + statistical assumptions:
- $\longrightarrow$ Kill quantum-mechanical interference terms
- $\longrightarrow$ Manageable formulae

Residual-system statistics vs. leading-particle statistics
$\longrightarrow$ Unifying scheme for MSD models

Hot theoretical item: Causality problems in the Feshbach-Kerman-Koonin model ($\chi - \hat{\chi}$ problem).

The one-step cross section

DWBA to the continuum:

$$\frac{d^2\sigma^{(1)}_{j\leftarrow i}(E,\Omega\leftarrow E_0,\Omega_0)}{d\Omega dE} = \frac{m^2}{(2\pi\hbar^2)^2}\frac{k}{k_0}\sum_{\mu}\hat{\rho}_{\mu}(p_\pi, h_\pi, p_\nu, h_\nu, E_x)$$

$$\times\ |\langle\chi^{(-)}_j(E,\Omega)|\langle\mu(p_\pi, h_\pi, p_\nu, h_\nu)|\mathcal{V}|0\rangle|\chi^{(+)}_i(E_0,\Omega_0)\rangle|^2 \qquad (22)$$

μ: $1p1h$ state

p_π: proton particle, h_π: proton hole, p_ν: neutron particle, h_ν: neutron hole

χ: distorted wave

E_x: excitation energy

$\mathcal{V}$: effective nucleon-nucleon interaction

The one-step cross section

$1p1h$-distribution:

$$\hat{\rho}_\mu(p_\pi, h_\pi, p_\nu, h_\nu, E_x) = \sum_f \overline{|a_\mu^f|^2 \delta(E_f - E_x)} \tag{23}$$

For each final nuclear state:

$$|f\rangle = \sum_{m\mu} a_{m\mu}^f |m\mu\rangle \tag{24}$$

$a_{m\mu}$: distribution amplitudes

The one-step cross section

(p,n) reaction:

$$\frac{d^2\sigma^{(1)}_{\nu\leftarrow\pi}(E,\Omega\leftarrow E_0,\Omega_0)}{d\Omega dE} = \frac{m^2}{(2\pi\hbar^2)^2}\frac{k}{k_0}\sum_\mu \hat{\rho}_\mu(1,0,0,1,E_x)$$
$$\times\ |\langle\chi^{(-)}_\nu(E,\Omega)|\langle\mu(1,0,0,1)|\mathcal{V}_{\pi\nu}|0\rangle|\chi^{(+)}_\pi(E_0,\Omega_0)\rangle|^2 \qquad (25)$$

(p,p') reaction:

$$\frac{d^2\sigma^{(1)}_{\pi\leftarrow\pi}(E,\Omega\leftarrow E_0,\Omega_0)}{d\Omega dE} = \frac{m^2}{(2\pi\hbar^2)^2}\frac{k}{k_0}\sum_\mu$$
$$\times\ [\hat{\rho}_\mu(1,1,0,0,E_x)|\langle\chi^{(-)}_\pi(E,\Omega)|\langle\mu(1,1,0,0)|\mathcal{V}_{\pi\pi}|0\rangle|\chi^{(+)}_\pi(E_0,\Omega_0)\rangle|^2$$
$$+\ \hat{\rho}_\mu(0,0,1,1,E_x)|\langle\chi^{(-)}_\pi(E,\Omega)|\langle\mu(0,0,1,1)|\mathcal{V}_{\pi\nu}|0\rangle|\chi^{(+)}_\pi(E_0,\Omega_0)\rangle|^2] \qquad (26)$$

State densities

(p,n) reaction:

$$\frac{d^2\sigma^{(1)}_{\nu\leftarrow\pi}(E,\,\Omega\leftarrow E_0,\,\Omega_0)}{d\Omega dE} = \sum_J \rho(1,\,0,\,0,\,1,\,J,\,E_x)\left\langle\frac{d\sigma_{\nu\leftarrow\pi}(E,\,\Omega\leftarrow E_0,\,\Omega_0)}{d\Omega}\right\rangle_J^{\mathcal{V}_{\pi\nu}} \quad (27)$$

(p,p') reaction:

$$\begin{aligned}\frac{d^2\sigma^{(1)}_{\pi\leftarrow\pi}(E,\,\Omega\leftarrow E_0,\,\Omega_0)}{d\Omega dE} &= \sum_J \rho(1,\,1,\,0,\,0,\,J,\,E_x)\left\langle\frac{d\sigma_{\pi\leftarrow\pi}(E,\,\Omega\leftarrow E_0,\,\Omega_0)}{d\Omega}\right\rangle_J^{\mathcal{V}_{\pi\pi}} \\ &+ \sum_J \rho(0,\,0,\,1,\,1,\,J,\,E_x)\left\langle\frac{d\sigma_{\pi\leftarrow\pi}(E,\,\Omega\leftarrow E_0,\,\Omega_0)}{d\Omega}\right\rangle_J^{\mathcal{V}_{\pi\nu}} \quad (28)\end{aligned}$$

State densities

Assume J, E_x dependence can be decoupled:

$$\rho(p_\pi, h_\pi, p_\nu, h_\nu, J, E_x) = (2J+1) R_n(J) \omega(p_\pi, h_\pi, p_\nu, h_\nu, E_x) \tag{29}$$

Betak and Dobes (for two components):

$$\begin{aligned} \omega(p_\pi, h_\pi, p_\nu, h_\nu, E_x) &= \frac{g_\pi^{p_\pi+h_\pi} g_\nu^{p_\nu+h_\nu}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} \sum_{k=0}^{h_\pi} \sum_{l=0}^{h_\nu} (-1)^{k+l} \binom{h_\pi}{k} \binom{h_\nu}{l} \\ &\times (E_x - \Delta - A_{\text{Pauli}} - (k+l)E_F)^{n-1} \\ &\times \Theta(E_x - \Delta - E_{PP} - (k+l)E_F) \end{aligned} \tag{30}$$

$$R_n(J) = \frac{2J+1}{\pi^{1/2} n^{3/2} \sigma^3} \exp\left[-\frac{(J+\frac{1}{2})^2}{n\sigma^2} \right] \tag{31}$$

The multi-step cross section

Feshbach-Kerman-Koonin:

- Extra energy average over intermediate channels
- Rotation of S-matrix poles from 4th into 1st quadrant
- $\overline{\hat{\chi}^{(+)}} \longrightarrow \overline{\chi}^{(-)}$
- Leading-particle statistics
- Two-step cross section = Product of two one-step cross sections
- Bonetti, Koning, Akkermans and Hodgson (Phys. Rep. 247, no. 1, 1 (1994))

The multi-step cross section

Two-component extension:

$$\frac{d^2\sigma^{(2)}_{j\leftarrow i}(E,\Omega\leftarrow E_0,\Omega_0)}{d\Omega dE} = \frac{2m^5}{(2\pi)^8\hbar^{10}}\frac{k}{k_0}2\pi^2\sum_{k=\pi,\nu}\sum_{\mu}\sum_{\mu'}\int \mathrm{d}\Omega_1\int \mathrm{d}E_1 E_1$$
$$\times\ \hat{\rho}_{\mu'}(p_\pi,h_\pi,p_\nu,h_\nu,E''_x)\hat{\rho}_{\mu}(p_\pi,h_\pi,p_\nu,h_\nu,E'_x)$$
$$\times\ |\langle\bar{\chi}^{(-)}_j(E,\Omega)|\langle\mu'(p_\pi,h_\pi,p_\nu,h_\nu)|\mathcal{V}|0\rangle|\bar{\chi}^{(+)}_k(E_1,\Omega_1)\rangle|^2$$
$$\times\ |\langle\bar{\chi}^{(-)}_k(E_1,\Omega_1)|\langle\mu(p_\pi,h_\pi,p_\nu,h_\nu)|\mathcal{V}|0\rangle|\bar{\chi}^{(+)}_i(E_0,\Omega_0)\rangle|^2 \quad (32)$$

Recursive expression:

$$\frac{d^2\sigma^{(n)}_{j\leftarrow i}(E,\Omega\leftarrow E_0,\Omega_0)}{d\Omega dE} = \frac{m}{4\pi^2\hbar^2}\sum_{k=\pi,\nu}\int \mathrm{d}\Omega_{n-1}\int \mathrm{d}E_{n-1}E_{n-1}$$
$$\times\ \frac{d^2\sigma^{(1)}_{j\leftarrow k}(E,\Omega\leftarrow E_{n-1},\Omega_{n-1})}{d\Omega dE}\frac{d^2\sigma^{(n-1)}_{k\leftarrow i}(E_{n-1},\Omega_{n-1}\leftarrow E_0,\Omega_0)}{d\Omega_1 dE_1} \quad (33)$$

The multi-step cross section

(p,n) reaction:

$$\frac{d^2\sigma^{(2)}_{\nu\leftarrow\pi}(E,\Omega\leftarrow E_0,\Omega_0)}{d\Omega dE} = \frac{2m^5}{(2\pi)^8\hbar^{10}}\frac{k}{k_0}2\pi^2\sum_{\mu}\sum_{\mu'}\int \mathrm{d}\Omega_1\int \mathrm{d}E_1 E_1$$

$$\begin{aligned}
\times\ [\quad (\quad & \hat{\rho}_{\mu'}(0,0,1,1,E''_x)|\langle\bar{\chi}^{(-)}_{\nu}(E,\Omega)|\langle\mu'(0,0,1,1)|\mathcal{V}_{\nu\nu}|0\rangle|\bar{\chi}^{(+)}_{\nu}(E_1,\Omega_1)\rangle|^2 \\
+\quad & \hat{\rho}_{\mu'}(1,1,0,0,E''_x)|\langle\bar{\chi}^{(-)}_{\nu}(E,\Omega)|\langle\mu'(1,1,0,0)|\mathcal{V}_{\nu\pi}|0\rangle|\bar{\chi}^{(+)}_{\nu}(E_1,\Omega_1)\rangle|^2 \quad) \\
\times\quad & \hat{\rho}_{\mu}(1,0,0,1,E'_x)|\langle\bar{\chi}^{(-)}_{\nu}(E_1,\Omega_1)|\langle\mu(1,0,0,1)|\mathcal{V}_{\pi\nu}|0\rangle|\bar{\chi}^{(+)}_{\pi}(E_0,\Omega_0)\rangle|^2 \\
+\quad & \hat{\rho}_{\mu'}(1,0,0,1,E''_x)|\langle\bar{\chi}^{(-)}_{\nu}(E,\Omega)|\langle\mu'(1,0,0,1)|\mathcal{V}_{\pi\nu}|0\rangle|\bar{\chi}^{(+)}_{\pi}(E_1,\Omega_1)\rangle|^2 \\
\times\quad (\quad & \hat{\rho}_{\mu}(0,0,1,1,E'_x)|\langle\bar{\chi}^{(-)}_{\pi}(E_1,\Omega_1)|\langle\mu(0,0,1,1)|\mathcal{V}_{\pi\nu}|0\rangle|\bar{\chi}^{(+)}_{\pi}(E_0,\Omega_0)\rangle|^2 \\
+\quad & \hat{\rho}_{\mu}(1,1,0,0,E'_x)|\langle\bar{\chi}^{(-)}_{\pi}(E_1,\Omega_1)|\langle\mu(1,1,0,0)|\mathcal{V}_{\pi\pi}|0\rangle|\bar{\chi}^{(+)}_{\pi}(E_0,\Omega_0)\rangle|^2 \quad) \quad (34)
\end{aligned}$$

Calculation method

DWBA cross sections:

- Single-particle states generated with spherical Nilsson model
- Seeger and Howard coefficients
- Only normal-parity states, spin transfer reactions excluded
- Yukawa interaction, range 1 fm.

Calculate DWBA cross sections with ECIS97 for ALL 1p1h-states

Typical case: 80 MeV (p, xp) and (p, xn) on ^{90}Zr:

- 62 000 DWBA cross sections
- 26 Mb input file for ECIS97
- 1 hour on 1 GhZ Linux/Pentium machine

Complementary reaction mechanisms

Direct, collective reactions: Include all discrete levels with known deformation lengths (and transform into β_L for each component of the optical potential)

Multi-step compound:

- FKK model
- Gradual absorption (P-Q transitions) following Marcinkowski et al.

Multiple emission:

- Continuum Hauser-Feshbach
- Energy dependent level density parameters
- Multiple evaporation until all channels are closed
- Multiple MSD emission: Chadwick method

Conclusions (MSD Model)

- Two-component MSD model: distinction between protons and neutrons at all stages of the reaction for
  - The excited particle-hole pair
  - The leading particle
- Inconsistency for higher steps in MSD model removed
- Strength of the Yukawa interaction:
  - $E < 40$ MeV: $V_{\nu\nu} \approx 23 - 27$ MeV
  - $E > 40$ MeV: $V_{\pi\pi} = V_{\pi\nu} \approx 12(160\text{MeV}) - 23(45\text{MeV})$

Conclusions (MSD Model)

- Austin (1979) finds for $20 < E < 50$ MeV: $V_{\pi\nu} = 3V_{\nu\nu}$. What's missing?
- Still too many parameters:
  - Use optical models dedicated per nucleus
  - Use realistic microscopic level scheme (Hartree-Fock-(Bogolyubov); Gogny force) or associated microscopic level densities.
  - $V_{\pi\nu}$, $V_{\pi\pi}$ can always "repair" too simple physics.
- Include non-normal parity states and more up-to-date prescription for the nucleon-nucleon interaction (use code DWBA98)
- Microscopic calculation of full second and higher order matrix elements (circumvents the controversy around Feshbach-Kerman-Koonin model) This is arguably the best approach.