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LECTURES ON STRINGS IN FLAT SPACE AND PLANE WAVES FROM $N = 4$ SUPER YANG MILLS

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LECTURES ON STRINGS IN FLAT SPACE

AND PLANE WAVES FROM $\mathcal{N} = 4$ SUPER YANG MILLS

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In these lecture notes we explain how the string spectrum in flat space and plane waves arises from the large N limit of U(N) $\mathcal{N} = 4$ super Yang Mills. We reproduce the spectrum by summing a subset of the planar Feynman diagrams. We also describe some other aspects of string propagation on plane wave backgrounds. These lecture notes are largely based on [1].

1. Introduction

The fact that large N gauge theories have a string theory description was believed for a long time [2]. These strings live in more than four dimensions [3]. One of the surprising aspects of the AdS/CFT correspondence [4,5,6,7] is the fact that for $\mathcal{N} = 4$ super Yang Mills these strings move in ten dimensions and are the usual strings of type IIB string theory. The radius of curvature of the ten dimensional space goes as $R/l_s \sim (g_{YM}^2 N)^{1/4}$. The spectrum of strings on $AdS_5 \times S^5$ corresponds to the spectrum of single trace operators in the Yang Mills theory. The perturbative string spectrum is not known exactly for general values of the 't Hooft coupling, but it is certainly known for large values of the 't Hooft coupling where we have the string spectrum in flat space. In these notes we will explain how to reproduce this spectrum from the gauge theory point of view. In fact we will be able to do slightly better than reproducing the flat space spectrum. We will reproduce the spectrum on a plane wave. These plane waves incorporate, in a precise sense, the first correction to the flat space result for certain states.

The basic idea is the following. We consider chiral primary operators such as $Tr[Z^J]$ with large J. This state corresponds to a graviton with large momentum p^+. Then we consider replacing some of the Zs in this operator by other fields, such as ϕ, one of the other transverse scalars. The position of ϕ inside the operator will matter since we are in the planar limit. When we include interactions ϕ can start shifting position inside the operator. This motion of ϕ among the Zs is described by a field in 1+1 dimensions. We then identify this field with the field corresponding to one of the transverse scalars of a string in light cone gauge. This can be shown by summing a subset of the Yang Mills Feynman diagrams. We will present a heuristic argument for why other diagrams are not important.

Since these results amount to a "derivation" of the string spectrum at large 't Hooft coupling from the gauge theory, it is quite plausible that by thinking along the lines sketched in this paper one could find the string theory for other cases, most interestingly cases where the string dual is not known (such as pure non-supersymmetric Yang Mills).

These notes are organized as follows. In section two we describe various aspects of plane waves. We discuss particle and string propagation on a plane wave as well as their symmetries. In section three we describe how plane waves arise from Penrose limits of various spacetimes, concentrating mostly on $AdS_5 \times S^5$. In section 4 we describe the computation of the spectrum from the $\mathcal{N} = 4$ Yang Mills point of view.

2. Plane waves

A plane wave is a geometry of the general form

$$ds^2 = -4dx^+dx^- + H(x^+, y)(dx^+)^2 + dy^i dy^i \qquad (2.1)$$

where the function H is independent of x^-. We have d_t transverse coordinates y^i. We will sometimes use the coordinates t, x defined by $x^\pm = (t \pm x)/2$. The Ricci tensor has only the following nonzero component

$$R_{++} = -\frac{1}{2}\partial_i\partial_i H \qquad (2.2)$$

where the index i only ranges over the y^i coordinates. If we want (2.1) to be a solution of vacuum Einstein equations then we need to impose that (2.2) vanishes. One possibility would be to set

$$H = \frac{f(x^+)}{|y|^{d_t - 2}} \qquad (2.3)$$

where f is arbitrary. This gives the gravitational field produced by a coherent state of gravitons moving along the direction $-\hat{x}$, with wavefunctions specified by f. If we take $f = \delta(x^+)$ we get the metric outside a point particle that is moving very close to the speed of light. These backgrounds are proper gravitational waves that move in asymptotically flat Minkowski space.

From now on we will consider only plane waves such that H is quadratic in y^i,

$$H = -A_{ij}(x^+)y^i y^j \qquad (2.4)$$

The matrix A can be a function of x^+. From (2.2) we see that $R_{++} = \frac{1}{2}A_{ii}(x^+)$. In these cases, as opposed to (2.3) the background is not asymptotically flat, though it is flat where A vanishes.

We will often consider backgrounds with constant field strengths of the form

$$F = dx^+ \wedge \varphi \qquad (2.5)$$

where φ is a constant form of R^{d_t}, the space spanned by the y^i coordinates. Then we see that Einstein's equations imply that $R_{++} = \frac{1}{2}A_{ii} \sim |\varphi|^2$. In principle φ can depend on x^+, thought we often will consider the case where it is independent of x^+.

2.1. Light font kinematics

If we have a two dimensional metric of the form

$$ds^2 = -4dx^+dx^- = -dt^2 + dx^2 \qquad (2.6)$$

we can define the corresponding momenta $p_\pm$ which generate translations in the $x^\pm$ coordinates. We find that these momenta are related to the energy and P by

$$p_\pm = -E \pm P \qquad (2.7)$$

We see that for massive particles $p_\pm < 0$ and for massless particles one of them can be zero. Massless particles with p_- nonzero and $p_+ = 0$ are moving along the $+\hat{x}$ direction. Their wavefunctions depend only on x^-, so we can form a wave packet localized in the x^- direction. Note that the waves given by (2.3) move in the $-\hat{x}$ direction.

2.2. Particle propagation on plane waves

In order to understand the physics of these backgrounds let us consider particle propagation on these plane waves. Starting with the particle action

$$S = \frac{1}{2}\int d\tau(e^{-1}\dot{X}^2 - em^2) \qquad (2.8)$$

where X is contracted with the spacetime metric. We now set $x^+ = \tau$ and we insert the plane wave metric in (2.8)[1]. Since the metric is independent of x^- we can eliminate $\dot{x}^-$ using the fact that the corresponding momentum, which is $p_- = -2e^{-1}$ is conserved[2]. In this way we find an action where the einbein e is eliminated and which depends only on the y^i coordinates

$$S = \int dt \frac{|p_-|}{4}(\dot{y}^2 - A_{ij}(x^+)y^iy^j) - m^2/|p_-| \tag{2.9}$$

The equations of motion for y^i are

$$\frac{d^2y^i}{dx^{+2}} + A_{ij}y^j = 0 \tag{2.10}$$

The light cone hamiltonian which implements translations in x^+ is equal to $-p_+$

$$-p_+ = H = \frac{p_i^2}{|p_-|} + \frac{|p_-|}{4}A_{ij}y^iy^j + \frac{m^2}{|p_-|} \tag{2.11}$$

We see that for $A = 0$ we recover the usual dispersion relation. If A is independent of x^+ we get a set of harmonic oscillators for the y^i coordinates, some of which could have negative w^2 if A has negative eigenvalues. If A depends on time we essentially get harmonic oscillators with time dependent frequencies. Note that $y = 0$ is a solution which corresponds to a particular geodesic.

Let us analyze a couple of cases. Let us first assume that the metric is a solution of the vacuum Einstein equations. Then we see that A has some negative eigenvalues and some positive eigenvalues. This is just the familiar fact that tidal forces for geodesics near the $y = 0$ geodesic will point towards the initial geodesic or away from it depending on the direction of the displacement, i.e. they are "focusing" in some directions and "defocusing" in others.

If we have constant field strengths we can consider As with only positive eigenvalues. In that case tidal forces are focusing and particles with nonzero p_- feel in a gravitational potential well from which they cannot escape. The

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[1] We call this light cone gauge since p_- is lightlike, though in general x^+ is not lightlike.

[2] Note that p_- is then naturally negative since e non negative.

frequency of oscillations in this well is independent of p_-, thought the amplitude of oscillations, for fixed p_+ is proportional to $1/\sqrt{|p_-|}$. We see that as we take $p_- \to 0$ we obtain that particles move to $y = \infty$ and back in finite x^+ time. Note that in the case that all eigenvalues of A are positive the x^+ direction is timelike except at $y = 0$.

If we have $A_{ij} = \mu^2\delta_{ij}$. Then we have a set of harmonic oscillators of frequency μ and their energies are

$$-p_+ = \sum_{i=1}^{d_t} \mu(n_i + \frac{1}{2}) \tag{2.12}$$

where n_i is the occupation number of the oscillator i.

Another way to treat the problem would be to solve the wave equation for a scalar field in the plane wave background. Fourier expanding the wave equation in x^- and x^+ (if A is x^+ independent) we find the harmonic oscillator equation for the y^i coordinates.

2.3. Symmetries of plane waves

Let us start by considering a plane wave of the form

$$ds^2 = -4dx^+dx^- - A_{ij}y^iy^j(dx^+)^2 + dy^idy^i \tag{2.13}$$

Let us first consider the bosonic symmetries. We start with general x^+ dependent A. There is an obvious symmetry generated by the killing vector

$$p_- = -i\frac{\partial}{\partial x^-} \tag{2.14}$$

Though it is not apparent from (2.13) a generic plane wave has also, in addition, $2d_t$ killing vectors. To understand the origin of these let us first take the case where we have only one transverse dimension so that A is just a single function and not a matrix. Let us choose $y_\alpha(x^+)$, $\alpha = 1, 2$, two linearly independent solutions of the equation (2.10). Then we can form two Killing vectors of the form

$$\xi^\alpha = y_\alpha(x^+)\frac{\partial}{\partial y} + \frac{1}{2}\dot{y}_\alpha(x^+)y\frac{\partial}{\partial x^-} \tag{2.15}$$

It can be checked that these are killing vectors for (2.13). We also find that their commutator is

$$[\xi^\alpha, \xi^\beta] = \frac{1}{2}(y_\alpha \dot{y}_\beta - \dot{y}_\alpha y_\beta)\frac{\partial}{\partial x^-} \qquad (2.16)$$

Note that due to the equation of motion (2.10) the prefactor of p_- in (2.16) is a constant independent of x^+. We see that once we diagonalize p_- these two Killing vectors obey a Heisenberg algebra. This algebra is realized on the light cone gauge Hilbert space as the Heisenberg algebra of position and momentum for the coordinate y. We see that the simplicity of the particle action in light cone gauge is intimately related to the existence of these Killing vectors. These two Killing vectors are realized on the particle light cone Hilbert space as creation and annihilation operators for the corresponding harmonic oscillator modes. More explicitly, if $A = \mu$, independent of x^+, we can choose $y_\pm \sim e^{\pm i\mu x^+}$ then (2.16) becomes the commutation relation of harmonic oscillators. In the case that A is a matrix we can similarly find $2d_t$ Killing vectors by solving (2.10) and choosing $2d_t$ linearly independent solutions, etc.

If A is x^+ independent there is another Killing vector, p_+, corresponding to translations in x^+. This becomes the energy in light cone gauge. Note that in this case $[-p_+, y_\pm] = \mp y_\pm$. So we see that the energies of the harmonic oscillators on the light cone lagrangian are determined by the spacetime symmetry algebra.

If (2.13) is a solution of a supersymmetric lagrangian we can ask whether we preserve any supersymmetries. In general we preserve half of the supersymmetries, the half given the equation

$$\Gamma_- \epsilon = 0 \qquad (2.17)$$

on the supersymmetry generating spinor parameter ϵ. We call these supersymmetries Q_{-a}. The a index depends on the theory we consider. These supersymmetries obey usual anti commutation relations of the form

$$\{Q_{-a}, Q_{-b}\} \sim -p_- \delta_{ab} \qquad (2.18)$$

If we have a constant field strength of the form (2.5) we find that the x^+ dependence of the susy generating spinor parameter is of the form

$$\epsilon = e^{i \int^{x^+} dx'^+ \varphi_{i_1 \cdots i_n}(x'^+)\Gamma^{i_1 \cdots i_n}} \epsilon_0 \qquad (2.19)$$

where ϵ_0 is constant, independent of x^+, and obeys (2.17). In the case of type IIB string theory we can choose

$$F = dx^+ \varphi , \qquad \varphi = \mu(dy^1 dy^2 dy^3 dy^4 + dy^5 dy^6 dy^7 dy^8) \qquad (2.20)$$

which would give a self dual F_5 [3]. In this case we see that

$$\epsilon = e^{i\mu I}\epsilon_0 , \qquad \Gamma_- \epsilon_0 = 0 , \qquad I = \Gamma^{1234}. \qquad (2.21)$$

where we used the self duality condition as well as the positive chirality of ϵ in ten dimensions.

These supersymmetries are very analogous to the bosonic symmetries we had above. When we act with Q_- on the light cone Hilbert space we raise or lower the energy by $\pm \mu$.

Finally in type IIB supergravity, if $A_{ij} = \mu^2 \delta_{ij}$ we have sixteen extra supersymmetries. Their explicit form can be found in [8]. What we want to note here is that they obey an algebra of the form

$$\{Q_{+a}, Q_{+b}\} = -p_+ \delta_{ab} + \Gamma^{ij} J^{ij} \qquad (2.22)$$

where the currents J appearing on the right hand side are the $SO(4) \times SO(4)$ generators which rotate the first four or the second four coordinates. The field strength (2.20) breaks the $SO(8)$ symmetry of the metric to $SO(4) \times SO(4)$. These supersymmetries are fairly standard supersymmetries in light cone gauge. They anticommute to the Hamiltonian plus the currents. Similarly if we commute or anticommute Q_+ with Q_- or the killing vectors (2.15) we obtain some combination of (2.15) and Q_- respectively, see [8]. These plane waves are maximally supersymmetric backgrounds that preserve 32 supersymmetries.

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[3] Note that the self duality condition relates the + component of F to the + component of F due to the off diagonal term in the metric (2.13).

Let us consider first bosonic strings. We start with the metric (2.13) and we choose light cone gauge $x^+ = \tau$ where τ is the worldsheet time. We can then follow all the steps performed in the light cone treatment of the Polyakov action (see for example [9]). As we found above for the particle case we can see that the action reduces to a quadratic action for the transverse coordinates

$$S = \frac{1}{2\pi\alpha'} \int dt \int_0^{\pi\alpha'|p_-|} d\sigma \frac{1}{2} \left[\dot{y}^2 - y'^2 - A_{ij}(\tau) y^i y^j \right] \qquad (2.23)$$

In addition we need to impose the constraint that the total momentum in the sigma direction of the string worldsheet is zero. We see that the length of the string in the σ direction is $L = \pi\alpha'|p_-|$. We can then decompose the fields into Fourier modes in σ as $y^i = \sum_n y^i_n(\tau) e^{2\pi i n\sigma/L}$. Then the action decouples into a sum of harmonic oscillators with time dependent frequency. The fact that the frequency of these oscillators can depend on time gives rise to particle creation on the worldsheet. Note that in spacetime there is no particle, or string, creation since the background has a lightlike Killing vector which could be used to define positive and negative frequencies. The particle creation on the string worldsheet means that a string that crosses the plane wave will get excited, for example we could send in a graviton and get a highly excited string state on the other side. The string number does not change but the mass of the string can change. These effects due to x^+ dependence of A were studied in various papers [10].

If A is x^+ independent we do not have particle creation on the string worldsheet. In fact from now on we will consider only the type IIB case where $A_{ij} = \mu\delta_{ij}$ and F_5 is (2.20), so that we get the metric

$$ds^2 = -4dx^+ dx^- - \mu^2 y^i y^j (dx^+)^2 + dy^i dy^i \qquad (2.24)$$

In the superstring case we can start with the Green Schwarz action and choose light cone gauge as described in [11] by choosing $x^+ = \tau$ and $\Gamma_-\theta^\alpha = 0$, $\alpha = 1, 2$. Then, as it was shown in [12,13] we obtain the action

$$S = \frac{1}{2\pi\alpha'} \int dt \int_0^{\pi\alpha'|p_-|} d\sigma \left[\frac{1}{2}\dot{z}^2 - \frac{1}{2}z'^2 - \frac{1}{2}\mu^2 z^2 + i\bar{S}(\not{\partial} + \mu I)S \right] \qquad (2.25)$$

where $I = \Gamma^{1234}$ and S is a Majorana spinor on the worldsheet and a positive chirality $SO(8)$ spinor under rotations in the eight transverse directions. We quantize this action by expanding all fields in Fourier modes on the circle labeled by σ. For each Fourier mode we get a harmonic oscillator (bosonic or fermionic depending on the field). Then the light cone Hamiltonian is

$$2p^- = -p_+ = H_{lc} = \sum_{n=-\infty}^{+\infty} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha'|p_-|/2)^2}} \qquad (2.26)$$

Here n is the label of the fourier mode, $n > 0$ label left movers and $n < 0$ right movers. N_n denotes the total occupation number of that mode, including bosons and fermions. Note that the ground state energy of bosonic oscillators is canceled by that of the fermionic oscillators.

The constraint on the momentum in the sigma direction reads

$$P = \sum_{n=-\infty}^{\infty} n N_n = 0 \qquad (2.27)$$

It is useful to understand how the symmetries we discussed above act on the string Hilbert space. The spacetime symmetries generated by Q_{-a} and the Killing vectors (2.15), are realized by the zero momentum modes on the string. The rest of the symmetries, the symmetries that act linearly on the fields of the light cone gauge lagrangian, are the $SO(4) \times SO(4)$ rotations and the Q_+ supersymmetry. If we first find multiplets of Q_+ supersymmetries with no zero momentum oscillators, then we can find the full spacetime multiplets by adding the zero momentum oscillators in all possible ways.

When only the $n = 0$ modes are excited we reproduce the spectrum of massless supergravity modes propagating on the plane wave geometry. Since the zero momentum modes are related to symmetries of the background we see that all gravity modes with the same p_- are in the same multiplet.

In the limit that μ is very small, or in other words if

$$\mu\alpha'|p_-| \ll 1 \qquad (2.28)$$

we recover the flat space spectrum. Indeed we see from (2.24) that the metric reduces to the flat space metric if we set μ to zero. The quantity $\mu|p_-|$ measures the size of the tidal forces that particles (or strings) propagating along geodesics close to the $y = 0$ geodesic feel. The limit (2.28) corresponds to the case that the tension is much larger than these tidal forces. In this limit the length of the string $L \sim \alpha'|p_-| \ll \mu^{-1}$.

It is also interesting to consider the opposite limit, where

$$\mu\alpha'p^+ \gg 1 \qquad (2.29)$$

This limit corresponds to strong tidal forces on the strings. It corresponds to strong curvatures. In this limit all the low lying string oscillator modes have almost the same energy. This limit corresponds to a highly curved background with RR fields. In fact we will later see that the appearance of a large number of light modes is expected from the Yang-Mills theory. In this limit the size of the string $L \gg \mu^{-1}$, so the typical excitation will correspond to well separated massive particles that are not moving very fast along the string. Since these "particles" along the string correspond to oscillations of the string in the target space we can say that in this limit different pieces of the string are oscillating independently.

It is interesting to note that in the plane wave (2.24) we can also have giant gravitons as we have in $AdS_5 \times S^5$. These giants are D3 branes classically sitting at fixed x^- and wrapping the S^3 of the first four directions or the S^3 of the second four directions with a size

$$r^2 = \pi g|p_-|\mu\alpha'^2 \qquad (2.30)$$

where p_- is the momentum carried by the giant graviton. This result follows in a straightforward fashion from the results in [14]. Its p_+ eigenvalue is zero. We see that the description of these states in terms of D-branes is correct when their size is much bigger than the string scale. If we are at weak string coupling this size is substringy and the description of the states in terms of fundamental strings is good.

It is well known that in conformal gauge the equation of motion for the background is conformal invariance of the two dimensional worldsheet theory. It would be nice to understand what the equation of motion for the background is in these more general massive cases, where we have chosen the light cone gauge. In flat space conditions like $D = 26$ appear, in light cone gauge, from the proper realization of the non-linearly realized Lorentz generators. These plane wave backgrounds generically break those Lorentz generators.

3. Plane waves as Penrose limits.

Penrose showed that plane waves can be obtained as limits of various backgrounds [15]. Here we first consider a specific case and then we will say something about the general case.

3.1. Type IIB plane wave from $AdS_5 \times S^5$

In this subsection we obtain the maximally supersymmetric plane wave of type IIB string theory as a limit of $AdS_5 \times S^5$.

The idea is to consider the trajectory of a particle that is moving very fast along the S^5 and to focus on the geometry that this particle sees. We start with the $AdS_5 \times S^5$ metric written as

$$ds^2 = R^2 \left[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho \, d\Omega_3^2 + d\psi^2 \cos^2 \theta + d\theta^2 + \sin^2 \theta \, d\Omega_3'^2 \right]$$
$$(3.1)$$

We want to consider a particle moving along the ψ direction and sitting at $\rho = 0$ and $\theta = 0$. We will focus on the geometry near this trajectory. We can do this systematically by introducing coordinates $\tilde{x}^\pm = \frac{t \pm \psi}{2}$ and then performing the rescaling

$$x^+ = \tilde{x}^+ \,, \qquad x^- = R^2 \tilde{x}^- \,, \qquad \rho = \frac{r}{R} \,, \qquad \theta = \frac{y}{R} \,, \qquad R \to \infty \qquad (3.2)$$

In this limit the metric (3.1) becomes

$$ds^2 = -4dx^+dx^- - (\vec{r}^{\,2} + \vec{y}^{\,2})(dx^+)^2 + d\vec{y}^{\,2} + d\vec{r}^{\,2} \qquad (3.3)$$

where $\vec{y}$ and $\vec{r}$ parametrize points on R^4. We can also see that only the components of F with a plus index survive the limit. The mass parameter μ can be introduced by rescaling (3.2) $x^- \to x^-/\mu$ and $x^+ \to \mu x^+$. These solutions where studied in [8].

It will be convenient for us to understand how the energy and angular momentum along ψ scale in the limit (3.2). The energy in global coordinates in AdS is given by $E = i\partial_t$ and the angular momentum by $J = -i\partial_\psi$. This angular momentum generator can be thought of as the generator that rotates the 56 plane of R^6. In terms of the dual CFT these are the energy and R-charge of a state of the field theory on $S^3 \times R$ where the S^3 has unit radius. Alternatively, we can say that $E = \Delta$ is the conformal dimension of an operator on R^4. We find that

$$\begin{aligned}
2p^- &= -p_+ = i\partial_{x^+} = i\partial_{\tilde{x}^+} = i(\partial_t + \partial_\psi) = \Delta - J \\
2p^+ &= -p_- = -\frac{\tilde{p}_-}{R^2} = \frac{1}{R^2}i\partial_{\tilde{x}^-} = \frac{1}{R^2}i(\partial_t - \partial_\psi) = \frac{\Delta + J}{R^2}
\end{aligned} \qquad (3.4)$$

Configurations with fixed non zero p_- in the limit (3.2) correspond to states in AdS with large angular momentum $J \sim R^2 \sim (gN)^{1/2}$. It is useful also to rewrite (2.26) in terms of the Yang Mills parameters. Then we find that the contribution of each oscillator to $\Delta - J$ is

$$(\Delta - J)_n = w_n = \sqrt{1 + \frac{4\pi g N n^2}{J^2}} \qquad (3.5)$$

Notice that gN/J^2 remains fixed in the $gN \to \infty$ limit that we are taking.

When we perform the rescalings (3.2) we can perform the limit in two ways. If we want to get the plane wave with finite string coupling then we take the $N \to \infty$ limit keeping the string coupling g fixed and we focus on operators with $J \sim N^{1/2}$ and $\Delta - J$ fixed.

On the other hand we could first take the 't Hooft limit $g \to 0$, gN =fixed, and then after taking this limit, we take the limit of large 't Hooft coupling keeping $J/\sqrt{gN}$ fixed and $\Delta - J$ fixed. Taking the limit in this fashion gives us a plane wave background with zero string coupling. Since we will be interested in these notes in the free string spectrum of the theory it will be more convenient for us to take this second limit.

From this point of view it is clear that the full supersymmetry algebra of the metric (3.1) is a contraction of that of $AdS_5 \times S^5$ [8]. This algebra implies that $p^\pm \geq 0$.

In other $AdS_d \times S^p$ geometries we can take similar limits, see [16]. The only minor difference as compared to the above computation is that in general the radius of AdS_d and the sphere are not the same. Performing the limit for $AdS_7 \times S^4$ or $AdS_4 \times S^7$ we get the same geometry, the maximally supersymmetric plane wave metric discussed in [17,18]. For the $AdS_3 \times S^3$ geometries that arise in the D1-D5 system the two radii are equal and the computation is identical to the one we did above for $AdS_5 \times S^5$.

In general the geometry could depend on other parameters besides the radius parameter R. It is clear that in such cases we could also define other interesting limits by rescaling these other parameters as well. For example one could consider the geometry that arises by considering D3 branes on A_{k-1} singularities [19]. These correspond to geometries of the form $AdS_5 \times S^5/Z_k$ [20]. The Z_k quotient leaves an S^1 fixed in the S^5 if we parametrize this S^1 by the ψ direction and we perform the above scaling limit we find the same geometry that we had above except that now $\vec{y}$ in (3.3) parametrizes an A_{k-1} singularity. It seems possible to deform a bit the singularity and scale the deformation parameter with R in such a way to retain a finite deformation in the limit. We will not study these limits in detail below but they are of clear physical interest.

3.2. Penrose limits in general

Penrose showed that the neighborhood of any light like geodesic in any manifold looks like a plane wave [15].

Penrose showed that if we take a null geodesic in any spacetime then the metric near it can be written as

$$ds^2 = \epsilon^2 \left[-4dx^+dx^- - A_{ij}(x^+)y^iy^j + dy^idy^i + o(\epsilon) \right] \qquad (3.6)$$

where ϵ is a small parameter, i.e. by taking $x^\pm, y^i$ fixed we are looking at a small neighborhood of the geodesic $x^- = y^i = 0$. This shows that the geometry close to this geodesic is that of a plane wave.

If we can rescale the overall factor in the metric to get rid of the factor of ϵ then we would obtain a plane wave in the limit. Einstein's equations are invariant under such rescaling, so that the rescaled metric will also be a solution. If there are other fields present in the initial solution, then one can show [21], that only the + component is left in the limit. Similarly, all supergravity theories normally encountered in string theory are invariant under an overall scaling of the metric and the fieldstrengths [21], [22]. Stringy corrections to the metric are not invariant under such a rescaling, but fortunately the rescaling goes in the direction of making these corrections less and less relevant for the asymptotic regions of the plane waves.

4. Strings from $\mathcal{N} = 4$ Super Yang Mills

After taking the 't Hooft limit, we are interested in the limit of large 't Hooft coupling $gN \to \infty$. We want to consider states which carry parametrically large R charge $J \sim \sqrt{gN}$.[4] This R charge generator, J, is the SO(2) generator rotating two of the six scalar fields. We want to find the spectrum of states with $\Delta - J$ finite in this limit. We are interested in single trace states of the Yang Mills theory on $S^3 \times R$, or equivalently, the spectrum of dimensions of single trace operators of the euclidean theory on R^4. We will often go back and forth between the states and the corresponding operators.

Let us first start by understanding the operator with lowest value of $\Delta - J = 0$. There is a unique single trace operator with $\Delta - J = 0$, namely $Tr[Z^J]$, where

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[4] Since we first took the 't Hooft limit then giant gravitons are not important.

$Z \equiv \phi^5 + i\phi^6$ and the trace is over the N color indices. We are taking J to be the SO(2) generator rotating the plane 56. At weak coupling the dimension of this operator is J since each Z field has dimension one. This operator is a chiral primary and hence its dimension is protected by supersymmetry. It is associated to the vacuum state in light cone gauge, which is the unique state with zero light cone hamiltonian. In other words we have the correspondence

$$\frac{1}{\sqrt{J}N^{J/2}}Tr[Z^J] \longleftrightarrow |0, p_+\rangle_{l.c.} \qquad (4.1)$$

We have normalized the operator as follows. When we compute $\langle Tr[\bar{Z}^J](x)Tr[Z^J](0)\rangle$ we have J possibilities for the contraction of the first $\bar{Z}$ but then planarity implies that we contract the second $\bar{Z}$ with a Z that is next to the first one we contracted and so on. Each of these contraction gives a factor of N. Normalizing this two point function to one we get the normalization factor in (4.1).[5]

Now we can consider other operators that we can build in the free theory. We can add other fields, or we can add derivatives of fields like $\partial_{(i_1} \cdots \partial_{i_n)}\phi^r$, where we only take the traceless combinations since the traces can be eliminated via the equations of motion. The order in which these operators are inserted in the trace is important. All operators are all "words" constructed by these fields up to the cyclic symmetry, these were discussed and counted in [3]. We will find it convenient to divide all fields, and derivatives of fields, that appear in the free theory according to their $\Delta - J$ eigenvalue. There is only one mode that has $\Delta - J = 0$, which is the mode used in (4.1). There are eight bosonic and eight fermionic modes with $\Delta - J = 1$. They arise as follows. First we have the four scalars in the directions not rotated by J, i.e. ϕ^i, $i = 1, 2, 3, 4$. Then we have derivatives of the field Z, $D_iZ = \partial_iZ + [A_i, Z]$, where $i = 1, 2, 3, 4$ are four directions in R^4. Finally there are eight fermionic operators $\chi^a_{J=\frac{1}{2}}$ which are the eight components with $J = \frac{1}{2}$ of the sixteen component gaugino χ (the other eight components have $J = -\frac{1}{2}$). These eight components transform in

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[5] In general in the free theory any contraction of a single trace operator with its complex conjugate one will give us a factor of N^n, where n is the number of fields appearing in the operator.

the positive chirality spinor representation of $SO(4) \times SO(4)$ [6]. We will focus first on operators built out of these fields and then we will discuss what happens when we include other fields, with $\Delta - J > 1$, such as $\bar{Z}$.

The state (4.1) describes a particular mode of ten dimensional supergravity in a particular wavefunction [6]. Let us now discuss how to generate all other massless supergravity modes. On the string theory side we construct all these states by applying the zero momentum oscillators a_0^i, $i = 1, \ldots, 8$ and S_0^b, $b = 1, \ldots 8$ on the light cone vacuum $|0, p_+\rangle_{l.c.}$. Since the modes on the string are massive all these zero momentum oscillators are harmonic oscillators, they all have the same light cone energy. So the total light cone energy is equal to the total number of oscillators that are acting on the light cone ground state. We know that in $AdS_5 \times S^5$ all gravity modes are in the same supermultiplet as the state of the form (4.1)[23]. The same is clearly true in the limit that we are considering. More precisely, the action of all supersymmetries and bosonic symmetries of the plane wave background (which are intimately related to the $AdS_5 \times S^5$ symmetries) generate all other ten dimensional massless modes with given p_-. For example, by acting by some of the rotations of S^5 that do not commute with the $SO(2)$ symmetry that we singled out we create states of the form

$$\frac{1}{\sqrt{J}} \sum_l \frac{1}{\sqrt{J} N^{J/2+1/2}} Tr[Z^l \phi^r Z^{J-l}] = \frac{1}{N^{J/2+1/2}} Tr[\phi^r Z^J] \qquad (4.2)$$

where ϕ^r, $r = 1, 2, 3, 4$ is one of the scalars neutral under J. In (4.2) we used the cyclicity of the trace. Note that we have normalized the states appropriately in the planar limit. We can act any number of times by these generators and we get operators roughly of the form $\sum Tr[\cdots z\phi^r z \cdots z\phi^k]$. where the sum is over

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    [6]   The first SO(4) corresponds to rotations in R^4, the space where the Yang Mills theory is defined, the second $SO(4) \subset SO(6)$ corresponds to rotations of the first four scalar fields, this is the subgroup of $SO(6)$ that commutes with the $SO(2)$, generated by J, that we singled out to perform the analysis. By positive chirality in $SO(4) \times SO(4)$ we mean that it has positive chirality under both $SO(4)$s or negative under both $SO(4)$. Combining the spinor indices into $SO(8)$, $SO(4) \times SO(4) \subset SO(8)$ it has positive chirality under $SO(8)$. Note that $SO(8)$ is not a symmetry of the background.

all the possible orderings of the ϕs. We can repeat this discussion with the other $\Delta - J = 1$ fields. Each time we insert a new operator we sum over all possible locations where we can insert it. Here we are neglecting possible extra terms that we need when two $\Delta - J = 1$ fields are at the same position, these are subleading in a $1/J$ expansion and can be neglected in the large J limit that we are considering. We are also ignoring the fact that J typically decreases when we act with these operators. In other words, when we act with the symmetries that do not leave Z invariant we will change one of the Zs in (4.1) to a field with $\Delta - J = 1$, when we act again with one of the symmetries we can change one of the Zs that was left unchanged in the first step or we can act on the field that was already changed in the first step. This second possibility is of lower order in a $1/J$ expansion and we neglect it. We will always work in a "dilute gas" approximation where most of the fields in the operator are Zs and there are a few other fields sprinkled in the operator.

For example, a state with two excitations will be of the form

$$\sim \frac{1}{N^{J/2+1}} \frac{1}{\sqrt{J}} \sum_{l=1}^{J} Tr[\phi^r Z^l \psi^b_{J=\frac{1}{2}} Z^{J-l}] \qquad (4.3)$$

where we used the cyclicity of the trace to put the ϕ^r operator at the beginning of the expression. We associate (4.3) to the string state $a_0^{\dagger k} S_0^{\dagger\,b} |0, p_+\rangle_{l.c.}$. Note that for planar diagrams it is very important to keep track of the position of the operators. For example, two operators of the form $Tr[\phi^1 Z^l \phi^2 Z^{J-l}]$ with different values of l are orthogonal to each other in the planar limit (in the free theory).

The conclusion is that there is a precise correspondence between the supergravity modes and the operators. This is of course well known [5,6,7]. Indeed, we see from (2.26) that their $\Delta - J = -p_+$ is indeed what we compute at weak coupling, as we expect from the BPS argument.

In order to understand non-supergravity modes in the bulk it is clear that what we need to understand the Yang Mills description of the states obtained by the action of the string oscillators which have $n \neq 0$. Let us consider first one of the string oscillators which creates a bosonic mode along one of the

four directions that came from the S^5, let's say $a_n^{\dagger\,8}$. We already understood that the action of $a_0^{\dagger\,8}$ corresponds to insertions of an operator ϕ^4 on all possible positions along the "string of Z's". By a "string of Zs" we just mean a sequence of Z fields one next to the other such as we have in (4.1). We propose that $a_n^{\dagger 8}$ corresponds to the insertion of the same field ϕ^4 but now with a position dependent phase

$$\frac{1}{\sqrt{J}}\sum_{l=1}^{J}\frac{1}{\sqrt{J}N^{J/2+1/2}}Tr[Z^l\phi^4 Z^{J-l}]e^{\frac{2\pi inl}{J}} \qquad (4.4)$$

In fact the state (4.4) vanishes by cyclicity of the trace. This corresponds to the fact that we have the constraint that the total momentum along the string should vanish (2.27), so that we cannot insert only one $a_n^{\dagger\,i}$ oscillator. So we should insert more than one oscillator so that the total momentum is zero. For example we can consider the string state obtained by acting with the $a_n^{\dagger\,8}$ and $a_{-n}^{\dagger\,7}$, which has zero total momentum along the string. We propose that this state should be identified with

$$a_n^{\dagger\,8}a_{-n}^{\dagger\,7}|0,p_+\rangle_{l.c.} \longleftrightarrow \frac{1}{\sqrt{J}}\sum_{l=1}^{J}\frac{1}{N^{J/2+1}}Tr[\phi^3 Z^l\phi^4 Z^{J-l}]e^{\frac{2\pi inl}{J}} \qquad (4.5)$$

where we used the cyclicity of the trace to simplify the expression. The general rule is pretty clear, for each oscillator mode along the string we associate one of the $\Delta - J = 1$ fields of the Yang-Mills theory and we sum over the insertion of this field at all possible positions with a phase proportional to the momentum. States whose total momentum is not zero along the string lead to operators that are automatically zero by cyclicity of the trace. In this way we enforce the $L_0 - \bar{L}_0 = 0$ constraint (2.27) on the string spectrum.

In summary, each string oscillator corresponds to the insertion of a $\Delta - J = 1$ field, summing over all positions with an n dependent phase, according to the rule

$$a^{\dagger i} \longrightarrow D_i Z \quad \text{for } i = 1, \cdots, 4$$
$$a^{\dagger j} \longrightarrow \phi^{j-4} \quad \text{for } j = 5, \cdots, 8 \qquad (4.6)$$
$$S^a \longrightarrow \chi^a_{J=\frac{1}{2}}$$

In order to show that this identification makes sense we want to compute the conformal dimension, or more precisely $\Delta - J$, of these operators at large 't Hooft coupling and show that it matches (2.26). First note that if we set $\frac{gN}{J^2} \sim 0$ in (3.5) we find that all modes, independently of n have the same energy, namely one. This is what we find at weak 't Hooft coupling where all operators of the form (4.5) have the same energy, independently of n. Expanding the string theory result (3.5) we find that the first correction is of the form

$$(\Delta - J)_n = w_n = 1 + \frac{2\pi gNn^2}{J^2} + \cdots \qquad (4.7)$$

This looks like a first order correction in the 't Hooft coupling and we can wonder if we can reproduce it by a a simple perturbative computation.

In order to compute the corrections it is useful to view the $\mathcal{N} = 4$ theory as an $\mathcal{N} = 1$ theory. As an $\mathcal{N} = 1$ theory we have a Yang Mills theory plus three chiral multiplets in the adjoint representation. We denote these multiplets as W^i, where $i = 1, 2, 3$. We will often set $Z = W^3$ and $W = W^1$. The theory also has a superpotential
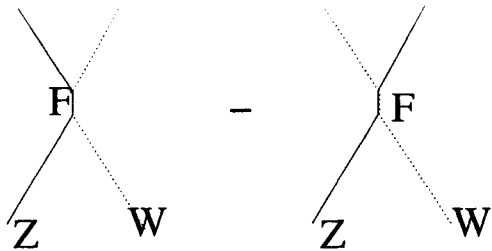
$$\mathcal{W} \sim g_{YM}Tr(W^i W^j W^k)\epsilon_{ijk} \qquad (4.8)$$

The potential for the Yang Mills theory is the sum of two terms, $V = V_F + V_D$, one coming from F terms and the other from D-terms. The one coming from F terms arises from the superpotential and has the form

$$V_F \sim \sum_{ij}Tr\left([W^i,W^j][\bar{W}^i,\bar{W}^j]\right) \qquad (4.9)$$

On the other hand the one coming from D terms has the form

$$V_D \sim \sum_{ij}Tr([W^i,\bar{W}^i][W^j,\bar{W}^j]) \qquad (4.10)$$

Fig. 1: Diagrams that come from F terms. The two diagrams have a relative minus sign. The F terms propagator is a delta function so that we could replace the three point vertex by a four point vertex coming from (4.9). If there are no phases in the operator these contributions vanish.

We will concentrate in computing the contribution to the conformal dimension of an operator which contains a W insertion along the string of Zs. There are various types of diagrams. There are diagrams that come from D terms, as well as from photons or self energy corrections. There are also diagrams that come from F terms. The diagrams that come from F terms can exchange the W with the Z. The F term contributions cancel in the case that there are no phases, see fig. 1. This means that all other diagrams should also cancel, since in the case that there are no phases we have a BPS object which recieves no corrections. All other one loop diagrams that do not come from F terms do not exchange the position of W, this means that they vanish also in the case that there are phases since they will be insensitive to the presence of phases. In the presence of phases the only diagrams that will not cancel are then the diagrams that come from the F terms. These are the only diagrams that give a momentum, n, dependent contribution.

In the free theory, once a W operator is inserted at one position along the string it will stay there, states with W's at different positions are orthogonal to each other in the planar limit (up to the cyclicity of the trace). We can think of the string of Zs in (4.1) as defining a lattice, when we insert an operator W at different positions along the string of Zs we are exciting an oscillator $b_l^\dagger$ at the site l on the lattice, $l = 1, \cdots J$. The interaction term (4.9) can take an excitation from one site in the lattice to the neighboring site. So we see that the

effects of (4.9) will be sensitive to the momentum n. In fact one can precisely reproduce (4.7) from (4.9) including the precise numerical coefficient. Below we give some more details on the computation.

We will write the square of the Yang-Mills coupling in terms of what in AdS is the string coupling that transforms as $g \to 1/g$ under S-duality. The trace is just the usual trace of an $N \times N$ matrix.

We define $Z = \frac{1}{\sqrt{2}}(\phi^5 + i\phi^6)$ and similarly for W. Then the propagator is normalized as

$$\langle Z_i{}^j(x)\bar{Z}_k{}^l(0)\rangle = \delta_i^l\delta_k^j \frac{2\pi g}{4\pi^2}\frac{1}{|x|^2} \tag{4.11}$$

In (4.9) there is an interaction term of the form the form $\frac{1}{\pi g}\int d^4 x Tr([Z,W][\bar{Z},\bar{W}])$, where W is one of the (complex) transverse scalars, let's say $W = W^1$. The contribution from the F terms shown in (4.9) give

$$< O(x)O^*(0) > = \frac{\mathcal{N}}{|x|^{2\Delta}}\left[1 + N(4\pi g)(-2 + 2\cos\frac{2\pi n}{J})I(x)\right] \tag{4.12}$$

where $\mathcal{N}$ is a normalization factor and $I(x)$ is the integral

$$I(x) = \frac{|x|^4}{(4\pi^2)^2}\int d^4 y \frac{1}{y^4(x-y)^4} \sim \frac{1}{4\pi^2}\log|x|\Lambda + \text{finite} \tag{4.13}$$

We extracted the log divergent piece of the integral since it is the one that reflects the change in the conformal dimension of the operator.

In conclusion we find that for large J and N the first correction to the correlator is

$$< O(x)O^*(0) > = \frac{\mathcal{N}}{|x|^{2\Delta}}\left[1 - \frac{4\pi g N n^2}{J^2}\log(|x|\Lambda)\right] \tag{4.14}$$

which implies that the contribution of the operator W inserted in the "string of Zs" with momentum n gives a contribution to the anomalous dimension

$$(\Delta - J)_n = 1 + \frac{2\pi g N n^2}{J^2} \tag{4.15}$$

which agrees precisely with the first order term computed from (4.7).

There are similar computations we could do for insertions of $D_i Z$, $\bar{W}$ or the fermions $\chi^a_{J=1/2}$. In the case of the fermions the important interaction term will be a Yukawa coupling of the form $\bar{\chi}\Gamma_z[Z\chi] + \bar{\chi}\Gamma_{\bar{z}}[\bar{Z}, \chi]$. We have not done these computations explicitly since the 16 supersymmetries preserved by the state (4.1) relate them to the computation we did above for the insertion of a W operator.

Encouraged by the success of this comparison we want to reproduce the full square root[7] in (3.5). At first sight this seems a daunting computation since it involves an infinite number of corrections.

In order to reproduce the full square root we essentially need to iterate these diagrams. We can define creation operators $b_l^\dagger$ which create a ϕ^i operator at the position l along the string of Zs and annihilation operators b_l operator that remove a ϕ^i from position l or gives zero if there is no ϕ^i at position l. Then we see that we can write an effective Hamiltonian of the form

$$H = \sum_j b_j^\dagger b_j + \frac{gN}{2\pi}(b_j + b_j^\dagger - b_{j+1} - b_{j+1}^\dagger)^2 \qquad (4.16)$$

An important feature of this Hamiltonian is that it includes terms with two creation or two annihilation operators. These come from the same vertices that we show in fig. 1 but with W and $\bar{W}$ lines both in the past or both in the future. Again we focus on the terms which depend on phases which are of this form.

The operators b_i do not strictly obey usual harmonic oscillator commutation rules, but if we form the operators

$$b_n^\dagger \equiv \frac{1}{\sqrt{J}} \sum_{l=1}^{J} e^{\frac{i2\pi l n}{J}} b_l^\dagger \qquad (4.17)$$

then the b_n oscillators obey the standard commutation relations up to terms of order $1/J$ which we neglect in the large J limit. For this reason the large J limit of (4.16) will give the same as the continuum hamiltonian

$$H = \int_0^L d\sigma \frac{1}{2}\left[\dot{\phi}^2 + \phi'^2 + \phi^2\right], \qquad L = J\sqrt{\frac{\pi}{gN}} \sim |p_-| \qquad (4.18)$$

In this formulation we see that $\epsilon = (gN)^{-1/2}$ plays the role of a short distance cutoff on the worldsheet.

In summary, the "string of Zs" becomes the physical string and each Z carries one unit of J which is one unit of $-p_-$. Locality along the worldsheet of the string comes from the fact that planar diagrams allow only contractions of neighboring operators. So the Yang Mills theory gives a string bit model (see [24]) where each bit is a Z operator. Each bit carries one unit of J which through (4.18) is one unit of $-p_-$.

The reader might, correctly, be thinking that all this seems too good to be true. In fact, we have neglected many other diagrams and many other operators which, at weak 't Hooft coupling also have small $\Delta - J$. In particular, we considered operators which arise by inserting the fields with $\Delta - J = 1$ but we did not consider the possibility of inserting fields corresponding to $\Delta - J = 2, 3, \ldots$, such as $\bar{Z}$, $\partial_k \phi^r$, $\partial_{(l}\partial_{k)}Z$, etc.. The diagrams of the type we considered above would give rise to other 1+1 dimensional fields for each of these modes. These are present at weak 't Hooft coupling but they should not be present at strong coupling, since we do not see them in the string spectrum. We believe that what happens is that these fields get a large mass in the $N \to \infty$ limit. In other words, the operators get a large conformal dimension. One can compute the first correction to the energy (the conformal weight) of the of the state that results from inserting $\bar{Z}$ with some "momentum" n. In contrast to our previous computation for $\Delta - J = 1$ fields we find that besides an effective kinetic term as in (4.7) there is an n independent contribution that goes as gN with no extra powers of $1/J^2$ [1]. This is an indication that these excitations become very massive in the large gN limit. In addition, we can compute the decay amplitude of $\bar{Z}$ into a pair of ϕ insertions. This is also very large, of order gN.

Though we have not done a similar computation for other fields with $\Delta - J > 1$, we believe that the same will be true for the other fields. In general we expect to find many terms in the effective Lagrangian with coefficients that are of order gN with no inverse powers of J to suppress them. In other words, the lagrangian of Yang-Mills on S^3 acting on a state which contains a large number of Zs gives a lagrangian on a discretized spatial circle with an infinite number

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[7] Square roots of the 't Hooft coupling are ubiquitous in the AdS computations.

of KK modes. The coefficients of this effective lagrangian are factors of gN, so all fields will generically get very large masses.

The only fields that will not get a large mass are those whose mass is protected for some reason. The fields with $\Delta - J = 1$ correspond to Goldstone bosons and fermions of the symmetries broken by the state (4.1). Note that despite the fact that they morally are Goldstone bosons and fermions, their mass is non-zero, due to the fact that the symmetries that are broken do not commute with p_+, the light cone Hamiltonian. The point is that their masses are determined, and hence protected, by the (super)symmetry algebra.

Having described how the single string Hilbert space arises it is natural to ask whether we can incorporate properly the string interactions. Clearly string interactions come when we include non-planar diagrams [2].

Some of the arguments used in this section look very reminiscent of the DLCQ description of matrix strings [25][26]. It would be interesting to see if one can establish a connection between them. Notice that the DLCQ description of ten dimensional IIB theory is in terms of the M2 brane field theory. Since here we are extracting also a light cone description of IIB string theory we expect that there should be a direct connection.

It would also be nice to see if using any of these ideas we can get a better handle on other large N Yang Mills theories, particularly non-supersymmetric ones. The mechanism by which strings appear in this paper is somewhat reminiscent of [27].

Acknowledgements

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