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SMR.1402 - 2

SPRING SCHOOL ON SUPERSTRINGS AND RELATED MATTERS

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COVARIANT QUANTIZATION OF THE SUPERSTRING AND SUPERMEMBRANE

Lectures 1 and 2

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Please note: These are preliminary notes intended for internal distribution only.

COVARIANT QUANTIZATION OF THE SUPERSTRING "AND" SUPERMEMBRANE

Still need to regularize Hamiltonian

Motivation: RNS formalism has various problems

1) Awkward (picture changing, sum over spin structures, susy not manifest)

2) Inadequate (R-R backgrounds, supermembrane)

GS formalism has only been quantized in light-cone gauge (interaction point operators, contact terms, special backgrounds, covariance not manifest)

Basic idea: 1) Add canonical momenta d_α for GS θ^α variables

2) Include pure spinor "ghost" variables λ^α

3) Replace κ-symmetry with BRST invariance with respect to $Q = \int \lambda^\alpha d_\alpha$

Successes: 1) Covariantly quantize the superstring "and" supermembrane (up to regularization of Hamiltonian)

2) Compute superstring N-point tree amplitudes with manifest SO(9,1) super-Poincaré invariance.

3) Quantize the superstring in R-R backgrounds

Related ideas:	Siegel '86 Add d^α variable	Howe '91 Pure spinors related to on-shell SYM and supergravity	Sorokin, Tonin, ... '89-present Twistor and pure spinors are related to κ-symmetry

Outline of Lectures:

I. Covariant quantization of $d=10$ superparticle
   A. Review of GS formalism
   B. On-shell $d=10$ super-Maxwell in superspace
   C. Pure spinor formalism for $D=10$ superparticle
   D. BRST description of $D=10$ super-Maxwell
   E. Comparison with BRST description of $D=3$ Chern-Simons

II. Covariant quantization of superstring
   A. **Review of GS superstring**
   B. Worldsheet action in a flat background
   C. Massless unintegrated and integrated vertex operators
   D. Computation of massless N-point tree amplitudes

III. Quantization of superstring in R-R backgrounds
   A. Worldsheet action in curved supergravity background
   B. Action in $AdS_5 \times S^5$ background and one-loop conf. inv.
   C. Action in R-R plane wave background and all loops conf. inv.

IV. Quantization of $D=11$ superparticle "and" supermembrane
   A. Review of BST formalism
   B. On-shell $d=11$ supergravity in superspace
   C. Pure spinor formalism for $D=11$ superparticle
   D. BRST description of $D=11$ supergravity
   E. Pure spinor formalism for $D=11$ supermembrane
   F. Speculations on connection with M-theory

I. Covariant quantization of d=10 superparticle

A. Review of GS formalism

Brink-Schwarz superparticle: $S = \int d\tau \left(\Pi^m P_m + e P^m P_m\right)$

$\Pi^m = \dot{x}^m - \frac{i}{2}\dot{\theta}^\alpha \gamma^m_{\alpha\beta}\theta^\beta$, $\gamma^m_{\alpha\beta} = \gamma^m_{\beta\alpha}$, $\gamma^{\alpha\beta}_{(m}\gamma_{n)\beta\gamma} = 2\eta_{mn}\delta^\alpha_\gamma$

$f^{(\alpha\beta)} = \gamma^{\alpha\beta}_m f^m + \gamma^{\alpha\beta}_{mnpqr} f^{mnpqr}$, $\Gamma^m = \begin{pmatrix} 0 & \gamma^m_{\alpha\beta} \\ \gamma^{m\,\alpha\beta} & 0 \end{pmatrix}$, $\eta_{mn}\gamma^m_{(\alpha\beta}\gamma^n_{\gamma)\delta} = 0$

$f^{[\alpha\beta]} = \gamma^{\alpha\beta}_{mnp} f^{mnp}$

Action inv. under $\delta\theta^\alpha = \epsilon^\alpha$, $\delta x^m = \frac{i}{2}\theta\gamma^m\epsilon$, ϵ^α = constant susy parameter

and $\delta\theta^\alpha = (\not{P}\kappa)^\alpha$, $\delta x^m = -\frac{i}{2}\theta\gamma^m\delta\theta$, $\delta e = i\dot{\theta}^\alpha\kappa_\alpha$, κ_α = local K-symmetry parameter

Define $p_\alpha = \frac{\partial L}{\partial\dot{\theta}^\alpha} = -\frac{i}{2}(\not{P}\theta)_\alpha$

Dirac constraints: $d_\alpha = p_\alpha + \frac{i}{2}\not{P}\theta \approx 0$

$\{d_\alpha, d_\beta\} = i P_m \gamma^m_{\alpha\beta} \Rightarrow$ 8 first-class and 8 second-class constraints

No covariant way to separate out second-class constraints.

<u>But</u> in light-cone gauge, can quantize the action:

Gauge $(\gamma^+\theta)_\alpha = 0 \to S = \int d\tau\left(\dot{x}^m P_m + \frac{i}{2}(\dot{\theta}\gamma^-\theta)P^+ + e P^m P_m\right)$

$\boxed{\gamma^\pm = \frac{1}{\sqrt{2}}(\gamma^0 \pm \gamma^9)}$

Assuming $P^+ \neq 0$, define $\xi^a = \sqrt{P^+}(\gamma^-\theta)^a$ for a=1 to 8

$\Rightarrow S = \int d\tau\left(\dot{x}^m P_m + \frac{i}{2}\dot{\xi}^a\xi^a + e P^m P_m\right)$

$\{\xi^a, \xi^b\} = 2\delta^{ab} \Rightarrow \Psi$ is a rep. of "spinor" SO(8) σ-matrix.

$\xi^a \Psi_j = \sigma^j_{a\dot a} \Psi^{\dot a}, \quad \xi^a \Psi^{\dot a} = \sigma^j_{a\dot a} \Psi^j$

$p^m p_m \Psi_j = p^m p_m \Psi^{\dot a} = 0 \Rightarrow \Psi_j(x)$ and $\Psi_{\dot a}(x)$ describe

massless SO(8) vector and spinor $\Rightarrow$ d=10 super-Maxwell theory in light-cone gauge

I.B. Superspace description of on-shell d=10 super-Maxwell

Component fields of super-Maxwell can be covariantly described by spinor gauge superfield $A_\alpha(x,\theta)$ satisfying eq. of motion $(\gamma^{mnpqr})^{\alpha\beta} D_\alpha A_\beta = 0$ with gauge invariance $\delta A_\alpha = D_\alpha \Lambda(x,\theta)$.

where $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \frac{i}{2}(\gamma^m\theta)\frac{\partial}{\partial x^m}$ is supersymmetric derivative.

in gauge $A_\alpha(x,\theta) = \underset{\substack{\uparrow \\ \text{photon}}}{(\gamma^m\theta)_\alpha\, a_m(x)} + (\gamma^m\theta)_\alpha\,(\theta\gamma_m \underset{\substack{\uparrow \\ \text{photino}}}{\chi(x)}) + \ldots$

where $\partial^m \partial_{[m} a_{n]} = (\partial\!\!\!/ \chi)_\alpha = 0$ and ... depends on derivatives of $a_m(x)$ and $\chi^\alpha(x)$.

Note that $\{D_\alpha + A_\alpha, D_\beta + A_\beta\} = i\gamma^m_{\alpha\beta}\left(\frac{\partial}{\partial x^m} + B_m(x,\theta)\right)$

where $B_m(x,\theta) = a_m(x) + \ldots$ is the vector gauge superfield with gauge invariance $\delta B_m = \partial_m \Lambda$.

Also, $[D_\alpha + A_\alpha, \partial_m + B_m] = \gamma_{m\alpha\beta} W^\beta(x,\theta)$

where $W^\beta(x,\theta) = \chi^\beta(x) + \ldots$ is the spinor superfield strength

and $\{D_\alpha + A_\alpha, W^\beta\} = (\gamma^{mn})_\alpha{}^\beta F_{mn}(x,\theta)$

where $F_{mn}(x,\theta) = \partial_{[m} a_{n]} + \ldots$ is the vector superfield strength,

I.C. Pure spinor formalism for D=10 superparticle

Treat d_α as an independent variable:

$$S = \int d\tau\, (\Pi^m P_m + \dot\theta^\alpha d_\alpha) + S_{ghost}$$
$$= \int d\tau\, (\dot x^m P_m + \dot\theta^\alpha p_\alpha) + S_{ghost} \quad \text{where} \quad p_\alpha = d_\alpha - \tfrac{i}{2}(\not{P}\theta)_\alpha$$

To find S_{ghost}, consider nilpotent BRST operator

$$\tilde Q = \tilde\lambda^\alpha d_\alpha + \frac{\xi^a}{\sqrt{P^+}}(\gamma^+ \not{P}\tilde\lambda)^a + c P^m P_m + \frac{b}{P^+}(\tilde\lambda\gamma^+\tilde\lambda)$$

where $\tilde\lambda^\alpha$ is unconstrained bosonic spinor and $\{\xi^a, \xi^b\} = 2\delta^{ab}$.

Light-cone gauge obtained by using $\tilde\lambda^\alpha d_\alpha$ to gauge $(\tilde\lambda^\alpha, d_\alpha, \tilde\omega_\alpha, \theta^\alpha)$ to zero $\Rightarrow$ $Q = cP^m P_m$.

But can instead use $\frac{b}{P^+}(\tilde\lambda\gamma^+\tilde\lambda)$ to gauge $(\tilde\lambda\gamma^+\tilde\lambda, b, c, \omega_+) = 0$

$\Rightarrow (\gamma^+\tilde\lambda)^{\dot a}$ is a null $SO(8)$ spinor which breaks $SO(8) \to U(4)$

$\Rightarrow$ $\tilde\lambda\gamma^{\bar A}\tilde\lambda = 0$ where $(A, \bar A) = 1$ to 4 is $SU(4)$ index and $v^{\dot a} \to (v^A, v^{\bar A})$.

Now use $\frac{\xi^a}{\sqrt{P^+}}(\gamma^+\not{P}\tilde\lambda)^a$ to gauge $(\tilde\lambda\gamma^A\tilde\lambda, \xi^a, \omega_A) = 0$.

In this gauge, $\boxed{Q = \lambda^\alpha d_\alpha}$ where $\boxed{\lambda\gamma^m\lambda = 0 \text{ for } m=0 \text{ to } 9}$

λ^α is a "pure spinor" with eleven independent components.

After Wick rotation, λ^α parameterizes $\frac{SO(10)}{U(5)} \times \mathbb{C}$

$$S_{ghost} = \int d\tau \text{ ``}\dot\lambda^\alpha \omega_\alpha\text{''}$$ where only 11 of 16 components are included.

Although $(\lambda^\alpha, \omega_\alpha)$ are complex, their complex conjugates never appear in formalism and will be ignored. Although solving pure spinor constraint breaks manifest Lorentz covariance, one never will need to solve constraint to do computations.

I.D. BRST description of super-Maxwell

Physical states of superparticle defined as states in cohomology of $Q = \lambda^\alpha d_\alpha$ at ghost number one.

Ghost-number one $\Rightarrow$ $\Psi(\lambda, x, \theta) = \lambda^\alpha A_\alpha(x,\theta)$

$$Q\Psi = 0 \Rightarrow \lambda^\alpha d_\alpha \lambda^\beta A_\beta = \lambda^\alpha \lambda^\beta D_\alpha A_\beta = 0$$

$$\Rightarrow (\lambda\gamma^{mnpqr}\lambda)\gamma_{mnpqr}^{\alpha\beta} D_\alpha A_\beta = 0 \Rightarrow \gamma_{mnpqr}^{\alpha\beta} D_\alpha A_\beta = 0$$

$$\delta\Psi = Q\Lambda \Rightarrow \delta(\lambda^\alpha A_\alpha) = \lambda^\alpha D_\alpha \Lambda \Rightarrow \delta A_\alpha = D_\alpha \Lambda$$

$\Rightarrow$ Cohomology at ghost-number one is on-shell super-Maxwell.

At ghost number (0, 1, 2, 3), cohomology reproduces (ghosts, fields, antifields, antighosts) of super-Maxwell theory
No cohomology at ghost number greater than 3.

$$\Psi(\lambda,x,\theta) = \omega + (\lambda\gamma^m\theta)a_m + (\lambda\gamma^m\theta)(\theta\gamma_m)_\alpha \chi^\alpha$$
$$+ (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\theta\gamma_{mn})^\alpha \chi^*_\alpha + (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\theta\gamma_{mnp}\theta)\, a^{*p}$$
$$+ (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\,\omega^*$$

Can define Batalin-Vilkovisky action for super-Maxwell theory by

$$\mathcal{S} = \int d^{10}x \, \langle \Psi(\lambda,x,\theta)\, Q\Psi(\lambda,x,\theta) \rangle$$

where one defines $\langle(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle = 1$.
Definition is gauge invariant since $(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta) \neq Q\Omega$.

$$\frac{16\ \theta^\alpha \text{ integrations}}{11\ \lambda^\alpha \text{ integrations}} \Rightarrow 5\ \theta^\alpha \text{ integrations}$$

I. E. Comparison with BRST description of D=3 Chern-Simons

First-quantized version of D=3 Chern-Simons (Witten '92)

$$S = \int d\tau \, (\dot{x}^j P_j + e^j P_j) \rightarrow S = \int d\tau \, (\dot{x}^j P_j + \dot{c}^j b_j) \quad j = 1 \text{ to } 3$$

States defined by cohomology of $Q = c^j P_j$.

$$\Psi(c^j, x^j) = \omega + c^j A_j + \epsilon_{jkl} c^j c^k A^{*l} + \epsilon_{jkl} c^j c^k c^l \omega^*$$

$$Q\Psi = 0 \Rightarrow \partial_{[j} A_{k]} = 0, \quad \delta\Psi = Q\Lambda \Rightarrow \delta A_j = \partial_j \Lambda$$

$$\mathcal{S}_{BV} = \int d^3x \, \langle \Psi Q \Psi \rangle \quad \text{where } \langle c^j c^k c^l \rangle = \epsilon^{jkl}.$$

Note that $\langle \Psi \rangle = \omega^*$ as in super-Maxwell.

II. Covariant quantization of superstring

IIA. Review of GS superstring:

$$S=\int dz\,d\bar z\,\left(\Pi^m\bar\Pi_m + B_{MN}(\partial Y^M\bar\partial Y^N - \bar\partial Y^M\partial Y^N)\right)$$

$\Pi^m = \partial x^m + \frac{i}{2}(\theta\gamma^m\partial\theta+\bar\theta\gamma^m\partial\bar\theta)$, $\bar\Pi^m=\bar\partial x^m+\frac{i}{2}(\theta\gamma^m\bar\partial\theta+\bar\theta\gamma^m\bar\partial\bar\theta)$

$Y^M=(x^m,\theta^\alpha,\bar\theta^{\bar\alpha})$ $\quad$ IIA/IIB $\Rightarrow (\alpha,\bar\alpha)$ have $\frac{\text{different}}{\text{same}}$ chirality

$d(B_{MN}\,dY^M dY^N) = H_{MNP}\,dY^M dY^N dY^P$ where

$H_{m\alpha\beta}=\gamma_{m\alpha\beta}$, $H_{m\bar\alpha\bar\beta}=-\gamma_{m\bar\alpha\bar\beta}$

Inv. under $\delta\theta^\alpha=(\not\Pi\kappa)^\alpha$, $\delta\bar\theta^{\bar\alpha}=(\bar{\not\Pi}\bar\kappa)^{\bar\alpha}$, $\delta x^m=-\frac{i}{2}(\theta\gamma^m\delta\theta+\bar\theta\gamma^m\delta\bar\theta)$

up to Virasoro constraints $\Pi^m\Pi_m=\bar\Pi^m\bar\Pi_m=0$ $\Rightarrow$ κ-symmetry

$\{d_\alpha,d_\beta\}=i\,\Pi_m\gamma^m_{\alpha\beta}$, $\{\bar d_{\bar\alpha},\bar d_{\bar\beta}\}=i\,\bar\Pi_m\gamma^m_{\bar\alpha\bar\beta}$, $\{d_\alpha,\bar d_{\bar\beta}\}=0$

where $\quad d_\alpha = p_\alpha+\frac{i}{2}(\not\Pi\theta)_\alpha+\frac14(\theta\gamma^m\partial\theta)(\gamma_m\theta)$

$\bar d_{\bar\alpha}=\bar p_{\bar\alpha}+\frac{i}{2}(\bar{\not\Pi}\bar\theta)_{\bar\alpha}+\frac14(\bar\theta\gamma^m\bar\partial\bar\theta)(\gamma_m\bar\theta)$

No covariant way to separate out second-class constraints

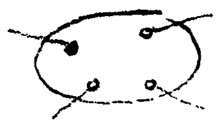
In light-cone gauge, $(\gamma^+\theta)_\alpha=(\gamma^+\bar\theta)_{\bar\alpha}=0$, $\partial x^+=\bar\partial x^+=p^+$

$\Rightarrow S=\int dz\,d\bar z\,\left(\partial x^m\bar\partial x_m+\partial x^+(\theta\gamma^-\bar\partial\theta)+\bar\partial x^+(\bar\theta\gamma^-\partial\bar\theta)\right)$

$=\int dz\,d\bar z\,\left(\partial x^m\bar\partial x_m+\xi^a\bar\partial\xi^a+\bar\xi^{\bar a}\partial\bar\xi^{\bar a}\right)$

$\Rightarrow$ quadratic action (but interactions are complicated)

In pure spinor description, action is quadratic and interactions are simple



II.B. Worldsheet action in flat background

$$S = \int d^2z \left(\tfrac{1}{2}\partial x^m \bar\partial x_m + p_\alpha \bar\partial \theta^\alpha + \bar p_{\bar\alpha} \partial \bar\theta^{\bar\alpha} + \text{"}w_\alpha \bar\partial \lambda^\alpha + w_{\bar\alpha} \partial \bar\lambda^{\bar\alpha}\text{"} \right)$$

$d_\alpha = p_\alpha + \frac{i}{2}(\not\partial x\,\theta)_\alpha + \frac{1}{4}(\theta\gamma\partial\theta)(\gamma_m\theta)_\alpha$, $\Pi^m = \partial x^m + \frac{i}{2}\theta\gamma^m\partial\theta$

satisfy OPE's $\quad d_\alpha(y)\, d_\beta(z) \to \gamma^m_{\alpha\beta} \dfrac{\Pi^m}{y-z}$ (Siegel '86)

$$d_\alpha(y)\, \Pi^m(z) \to \gamma^m_{\alpha\beta} \frac{\partial\theta^\beta}{y-z}$$

$\Rightarrow \quad Q = \int dz\, \lambda^\alpha d_\alpha$, $\bar Q = \int d\bar z\, \bar\lambda^{\bar\alpha} \bar d_{\bar\alpha}$ are nilpotent

if $\lambda\gamma^m\lambda = \bar\lambda\gamma^m\bar\lambda = 0$

Physical state defined as ghost-number one states in cohomology of Q (open superstring).
Proven to give correct physical spectrum (hep-th/0006003, 0105149 w/ Osvaldo Chandía)

$$T = \tfrac{1}{2}\partial x^m \partial x_m + p_\alpha \partial\theta^\alpha + \text{"}w_\alpha \partial\lambda^\alpha\text{"}$$

$c=10 \qquad c=-32 \qquad c=22$

No (b,c) ghosts. BRST invariance "implies" Virasoro constraints

II.C. Massless unintegrated and integrated vertex operators

Ghost number one massless unintegrated vertex operator is $U = \lambda^\alpha A_\alpha(x,\theta)$. A_α on-shell $\Rightarrow QU = 0$

Massless integrated vertex operator $\int dz\, V$ should satisfy $QV = \frac{\partial}{\partial z} U$. Find

$$V = \partial\theta^\alpha A_\alpha(x,\theta) + \Pi^m B_m(x,\theta) + d_\alpha W^\alpha(x,\theta) + \underbrace{N_{mn}}_{=\,\frac{1}{2}(\lambda\gamma_{mn}w)} F^{mn}(x,\theta)$$

$$QV = \frac{\partial}{\partial\tau}(\lambda^\alpha A_\alpha) + \lambda^\alpha \partial\theta^\beta (D_\beta A_\alpha + D_\alpha A_\beta - \gamma^m_{\alpha\beta} B_m)$$
$$+ \lambda^\alpha \Pi^m (D_\alpha B_m - \partial_m A_\alpha + \gamma_{m\alpha\beta} W^\beta) + \lambda^\alpha d_\beta (D_\alpha W^\beta - (\gamma_{mn})_\alpha{}^\beta F^{mn})$$
$$+ \lambda^\alpha N^{mn} (D_\alpha F_{mn})$$ ← vanishes since $(\lambda\gamma^m\gamma^n\omega)(\gamma_m\lambda)_\alpha = 0$

In components, $V = \partial x^m a_m(x) + (\frac{1}{2} p\gamma^{mn}\theta + \frac{1}{2}w\gamma^{mn}\lambda)\partial_m a_n(x) + \dots$

$\frac{1}{2} p\gamma^{mn}\theta + \frac{1}{2}\omega\gamma^{mn}\lambda$ is Lorentz current which replaces $\psi^m\psi^n$ in RNS gluon vertex operator

Can show that $N^{mn}(y) N^{pq}(z) \to \frac{\eta^{m[q} N^{p]n}}{y-z} - 3\frac{\eta^{m[q}\eta^{n]p}}{(y-z)^2}$

$$\tfrac{1}{2} p\gamma^{mn}\theta(y)\ \tfrac{1}{2} p\gamma^{mn}\theta(z) \to \tfrac{1}{2}\frac{\eta^{m[q}(p\gamma^{p]n}\theta)}{y-z} + \frac{4\eta^{m[q}\eta^{n]p}}{(y-z)^2}$$

which reproduces $\psi^m\psi^n(y)\ \psi^p\psi^q(z) \to \frac{\eta^{m[q}\psi^{p]}\psi^n}{y-z} + \frac{\eta^{m[q}\eta^{n]p}}{(y-z)^2}$

II. D. Massless N-point tree amplitudes

$$\mathcal{A} = \int dz_4 \dots dz_N \langle U_1(z_1) U_2(z_2) U_3(z_3) V_4(z_4) \dots V_N(z_N) \rangle$$

where (z_1, z_2, z_3) are fixed using $SL(2,R)$ invariance.

Use free-field OPE's to compute contribution of non-zero modes of worldsheet fields

Normalization for zero modes is <u>defined</u> by

$$\langle (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle = 1.$$

Analog of $\langle c\,\partial c\,\partial^2 c\rangle = 1$ for bosonic string

Prescription is supersymmetric and gauge invariant since there are no states f in cohomology of Q which satisfy $\delta f = (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)$.

"On-shell harmonic superspace" integration where eleven λ's cancel eleven of sixteen θ integrations.

Proven w/ Brenno Carlini Vallilo to coincide with RNS tree amplitudes with up to four external fermions. For first time, have N-point manifestly super Poincaré covariant amplitudes.

Still unknown how to compute loop amplitudes.