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SMR.1402 - 6

SPRING SCHOOL ON SUPERSTRINGS AND RELATED MATTERS

18 - 26 March 2002

COVARIANT QUANTIZATION OF THE SUPERSTRING AND SUPERMEMBRANE

Lecture 3

N. BERKOVITS
Instituto de Fisica Teorica
Universidade Estadual Paulista
Sao Paulo
BRAZIL

Please note: These are preliminary notes intended for internal distribution only.

III. Quantization in R-R Backgrounds

III A. Open superstring in super-Maxwell background

For bosonic string in Maxwell background,

$$S = \frac{1}{\alpha'}\int d^2z \, \tfrac{1}{2}\partial X^m \bar\partial X_m + \int d\tau \, A_m(x)\partial X^m$$

can be used to obtain string corrections to Maxwell eq (Born-Infeld + α' corrections). Conf. inv $\Rightarrow$ eqns of motion for $A_m(x)$.

For GS superstring in super-Maxwell background,

$$S_{GS} = \frac{1}{\alpha'}\int dz\, d\bar z \left(\tfrac{1}{2}\Pi^m \bar\Pi_m + B^{flat}_{MN}\partial Y^M \bar\partial Y^N\right) + \int d\tau \left(\partial\theta^\alpha A_\alpha(x,\theta) + \Pi^m B_m(x,\theta)\right)$$

Under $\delta\theta^\alpha = \xi^\alpha$, $\delta\bar\theta^{\bar\alpha} = \bar\xi^{\bar\alpha}$, $\delta x^m = -\frac{1}{2}(\theta\gamma^m\delta\theta + \bar\theta\gamma^m\delta\bar\theta)$

$$\delta S = \frac{i}{\alpha'}\int dz\, d\bar z\left[\xi \not\Pi \bar\partial\theta + \bar\xi \bar{\not\Pi}\partial\bar\theta\right] + i\int d\tau \left[\xi \not\Pi W(x,\theta) + \xi^\alpha \partial\theta^\beta\left(D_\alpha A_\beta + D_\beta A_\alpha - \gamma^m_{\alpha\beta}B_m\right)\right]$$

$\Rightarrow \delta S = 0$ if $\xi^\alpha = (\not\Pi \kappa)^\alpha$, $\bar\xi^{\bar\alpha} = (\bar{\not\Pi}\bar\kappa)^{\bar\alpha}$, $D_\alpha A_\beta + D_\beta A_\alpha - \gamma^m_{\alpha\beta}B_m = 0$

$\Rightarrow$ Inv. under κ-symmetry if background is on-shell.

How to compute string corrections (supersymmetric Born-Infeld)?

Use pure spinor version of action:

$$S_{pure} = \frac{1}{\alpha'}\int dz\, d\bar z\left(\tfrac{1}{2}\partial x^m \bar\partial x_m + p_\alpha \bar\partial\theta^\alpha + \bar p_{\bar\alpha}\partial\bar\theta^{\bar\alpha} + \text{``}w_\alpha\bar\partial\lambda^\alpha\text{''} + \text{``}\bar w_{\bar\alpha}\partial\bar\lambda^{\bar\alpha}\text{''}\right)$$
$$+ \int d\tau\left(\partial\theta^\alpha A_\alpha + \Pi^m B_m + d_\alpha W^\alpha + N^{mn}F_{mn}\right)$$

BRST invariance implies eqns. of motion for $(A_\alpha, B_m, W^\alpha, F_{mn})$.

Quantum computations $\Rightarrow$ superstring corrections

Can relate GS action and pure spinor action using BRST method of Oda and Tonin:

Under $Q+\bar{Q}=\int dz\,\lambda^\alpha d_\alpha+\int d\bar{z}\,\bar{\lambda}^{\bar\alpha}\bar{d}_{\bar\alpha}$: $\delta\theta^\alpha=\lambda^\alpha$, $\delta\bar\theta^{\bar\alpha}=\bar\lambda^{\bar\alpha}$,

$\delta x^m=-\frac{i}{2}(\theta\gamma^m\delta\theta+\bar\theta\gamma^m\delta\bar\theta)$, $\delta d_\alpha=-i(\not\Pi\lambda)_\alpha$, $\delta\bar d_{\bar\alpha}=-i(\bar{\not\Pi}\bar\lambda)_{\bar\alpha}$,

$\delta\omega_\alpha=d_\alpha$, $\delta\bar\omega_{\bar\alpha}=\bar d_{\bar\alpha}$.

$Q^2=\bar Q^2=0$ except on ω^α, $\bar\omega^{\bar\alpha}$: $Q^2\omega_\alpha=-i(\not\Pi\lambda)_\alpha$, $\bar Q^2\bar\omega_{\bar\alpha}=-i(\bar{\not\Pi}\bar\lambda)_{\bar\alpha}$
which is OK since it is a gauge transf. $\delta\omega_\alpha=(\gamma^m\lambda)_\alpha\Lambda_m$ and $\delta\bar\omega_{\bar\alpha}=(\gamma^m\bar\lambda)_{\bar\alpha}\bar\Lambda_m$

From K-symmetry, $(Q+\bar Q)\,S_{GS}=\frac{i}{\alpha'}\int dz\,d\bar z\,[\lambda\not\Pi\bar\partial\theta+\bar\lambda\bar{\not\Pi}\partial\bar\theta]+i\int dz\,\lambda\not\Pi W$

$\Rightarrow S=S_{GS}+(Q+\bar Q)\left[\frac{1}{\alpha'}\int dz\,d\bar z\,[w\bar\partial\theta+\bar w\partial\bar\theta]+\int dz\,w_\alpha W^\alpha\right]$

is BRST-invariant

$\Rightarrow S=S_{GS}+\frac{1}{\alpha'}\int dz\,d\bar z\,[d_\alpha\bar\partial\theta^\alpha+w_\alpha\bar\partial\lambda^\alpha+\bar d_{\bar\alpha}\partial\bar\theta^{\bar\alpha}+w_{\bar\alpha}\partial\bar\lambda^{\bar\alpha}]$

$+\int dz\,(d_\alpha W^\alpha+\frac{1}{2}(w\gamma^{mn}\lambda)F_{mn})$

$=S_{pure}$

III.B. Closed superstring in supergravity background

$$S_{GS}=\frac{1}{\alpha'}\int dz\,d\bar z\left(\frac{1}{2}\eta_{ab}\Pi^a\bar\Pi^b+B_{MN}\,\partial Y^M\bar\partial Y^N\right)$$

$Y^M=(x^m,\theta^\mu,\bar\theta^{\bar\mu})$, $\Pi^a=E^a{}_M\,\partial Y^M$, $\bar\Pi^a=E^a{}_M\,\bar\partial Y^M$, $\partial_{[M}B_{NP]}=H_{MNP}$

$(E^a{}_M(x,\theta,\bar\theta),\ E^\alpha{}_M(x,\theta,\bar\theta),\ E^{\bar\alpha}{}_M(x,\theta,\bar\theta))$ is $N=2$ $D=10$ super-vierbein

$M=(m,\mu,\bar\mu)$ are curved indices, $A=(a,\alpha,\bar\alpha)$ are tangent-space indices

On-shell, E^A_M and H_{MNP} are related. Dilaton φ and R-R fields appear in (E^A_M, H_{MNP}) with derivatives.

Classical action inv. under κ-symmetry when background on-shell.
∴ How to compute α' corrections to eqns of motion?
∴ Where is Fradkin-Tseytlin term $\int d^2z\, \varphi(x)\, r$?

To compute S_{pure}, use Oda-Tonin method:
In curved background, $\delta\theta^\alpha = \lambda^\alpha$, $\delta\bar\theta^{\bar\alpha} = \bar\lambda^{\bar\alpha}$, $\delta x^m = -\frac{i}{2}(\theta\gamma^m\delta\theta + \bar\theta\gamma^m\delta\bar\theta)$
becomes $\delta Y^M = E^M{}_\alpha \lambda^\alpha + E^M{}_{\bar\alpha}\bar\lambda^{\bar\alpha}$.
Also, $\delta d_\alpha = -i(\not\Pi\lambda)_\alpha$, $\delta\bar d_{\bar\alpha} = -i(\bar{\not\Pi}\bar\lambda)_{\bar\alpha}$, $\delta w_\alpha = d_\alpha$, $\delta\bar w_{\bar\alpha} = \bar d_{\bar\alpha}$

$$\Rightarrow (Q+\bar Q) S_{GS} = \frac{i}{\alpha'}\int dz d\bar z \left[(\lambda\not\Pi)_\alpha E^\alpha_M \bar\partial Y^M + (\bar\lambda\bar{\not\Pi})_{\bar\alpha} E^{\bar\alpha}_M \partial Y^M\right]$$

$$\Rightarrow \quad S = S_{GS} + \frac{1}{\alpha'}\int dz d\bar z \Big[Q\,(w_\alpha E^\alpha_M \bar\partial Y^M) + \bar Q\,(\bar w_{\bar\alpha} E^{\bar\alpha}_M \partial Y^M)$$
$$- Q\bar Q\,(w_\alpha \bar w_{\bar\beta} F^{\alpha\bar\beta})\Big]$$

is BRST invariant where the superspace torsion satisfies
$$T_{c\alpha}{}^{\bar\beta} = \gamma_{c\alpha\delta} F^{\delta\bar\beta} \quad \text{and} \quad T_{c\bar\alpha}{}^{\beta} = -\gamma_{c\bar\alpha\bar\delta} F^{\beta\bar\delta}.$$
So $\bar Q\,(E^\alpha_M \bar\partial Y^M) = \bar\lambda^{\bar\beta} T_{c\bar\beta}{}^\alpha \bar\Pi^c = -(\bar\lambda\bar{\not\Pi})_{\bar\beta} F^{\alpha\bar\beta}$
and $Q\,(E^{\bar\alpha}_M \partial Y^M) = \lambda^\beta T_{c\beta}{}^{\bar\alpha} \Pi^c = (\lambda\not\Pi)_\alpha F^{\alpha\bar\beta}$.

$\theta = \bar\theta = 0$ component of $F^{\alpha\bar\beta}$ is $e^\varphi f^{\alpha\bar\beta}$ where $f^{\alpha\bar\beta}$ is R-R field strength.

$$\Rightarrow S_{pure} = \frac{1}{\alpha'}\int dz d\bar z \Big[\tfrac{1}{2}\eta_{ab}\Pi^a\bar\Pi^b + B_{MN}\,\partial Y^M \bar\partial Y^N + d_\alpha E^\alpha_M \bar\partial Y^M + \bar d_{\bar\alpha} E^{\bar\alpha}_M \partial Y^M$$
$$+ w_\alpha \bar\nabla\lambda^\alpha + \bar w_{\bar\alpha}\nabla\bar\lambda^{\bar\alpha} + d_\alpha \bar d_{\bar\beta} F^{\alpha\bar\beta} + d_\alpha \bar N_{ab} C^{\alpha ab} + \bar d_{\bar\alpha} N_{ab} \bar C^{\bar\alpha ab}$$
$$+ N_{ab}\bar N_{cd} R^{abcd}\Big]$$

where $\bar\nabla\lambda^\alpha = \bar\partial\lambda^\alpha + \Omega_M{}^{cd}(\gamma_{cd}\lambda)^\alpha \bar\partial Y^M$
$\nabla\bar\lambda^{\bar\alpha} = \partial\bar\lambda^{\bar\alpha} + \bar\Omega_M{}^{cd}(\gamma_{cd}\bar\lambda)^{\bar\alpha}\partial Y^M$

$C^{\alpha ab} = (\gamma^{ab})^{\bar\gamma}{}_{\bar\beta}\nabla_{\bar\gamma} F^{\alpha\bar\beta}$
$\bar C^{\bar\alpha ab} = (\gamma^{ab})^{\gamma}{}_{\beta}\nabla_\gamma F^{\beta\bar\alpha}$

After including the F-T term $\int d\bar{z}dz\ \Phi\, r$, can check that $\lambda^\alpha d_\alpha$ is nilpotent and holomorphic when background superfields are on-shell. Fradkin-Tseytlin term is needed at one-loop order (hep-th/0112160 with Paul Howe)

III.C. Superstring in $AdS_5\times S^5$ background

In $AdS_5\times S^5$ background, $E^A{}_M$ can be constructed using Metsaev-Tseytlin currents.

Define $g(x,\theta,\bar\theta) = \exp\left(x^c P_c + \theta^\alpha Q_\alpha + \bar\theta^{\bar\alpha} Q_{\bar\alpha}\right)$

where $\{Q_\alpha, Q_\beta\} = \gamma^c_{\alpha\beta} P_c$, $\{Q_{\bar\alpha}, Q_{\bar\beta}\} = \gamma^c_{\bar\alpha\bar\beta} P_c$

$[P^c, P^d] = R^{cdef} J_{ef}$, $[Q_\alpha, P_c] = \gamma_{c\alpha\beta} F^{\beta\bar\gamma} Q_{\bar\gamma}$

$[Q_{\bar\alpha}, P_c] = \gamma_{c\bar\alpha\bar\beta} F^{\gamma\bar\beta} Q_\gamma$, $\{Q_\alpha, Q_{\bar\gamma}\} = J_{cd}\, \gamma^c_{\alpha\beta} F^{\beta\bar\delta} \gamma^d_{\bar\delta\bar\gamma}$

and J_{ab} generates $SO(4,1)\times SO(5)$ Lorentz algebra.

$F^{\alpha\bar\beta} = \frac{1}{ng}\left(\gamma^{01234}\right)^{\alpha\bar\beta}$ self-dual R-R field strength

$R^{cdef} = \pm\eta^{c[e}\eta^{f]d}$ when $(c,d,e,f) \to [0,\dots,4]$ / $\to [5,\dots,9]$

Then $$g^{-1}\partial g = P_c E^c_M \partial Y^M + Q_\alpha E^\alpha_M \partial Y^M + Q_{\bar\alpha} E^{\bar\alpha}_M \partial Y^M + J_{cd}\, \omega_M{}^{cd} \partial Y^M$$

$$S_{GS} = \int d^2z\left(\tfrac{1}{2}\eta_{cd} E^c \bar E^d + 2F^{-1}_{\alpha\bar\beta}\left(E^\alpha \bar E^{\bar\beta} - \bar E^\alpha E^{\bar\beta}\right)\right)$$

$$\left(E^\alpha \bar E^{\bar\beta} - \bar E^\alpha E^{\bar\beta}\right) \;\text{"}\; B_{MN}\left(\partial Y^M \bar\partial Y^N - \bar\partial Y^M \partial Y^N\right)$$

$$S_{pure} = S_{GS} + \frac{1}{\alpha'}\int d^2z \Big[d_\alpha \bar{E}^\alpha + \bar{d}_{\bar{\alpha}} E^{\bar{\alpha}} + \omega_\alpha \bar{\nabla}\lambda^\alpha + \bar{\omega}_{\bar{\alpha}} \nabla \bar{\lambda}^{\bar{\alpha}} + d_\alpha \bar{d}_{\bar{\beta}} F^{\alpha\bar{\beta}} + N_{ab}\bar{N}_{cd} R^{abcd} \Big]$$

($AdS_3 \times S^3$ w/ Vafa, Witten ; $AdS_2 \times S^2$ w/ Bershadsky, Hauer, Zukov, Zwiebach ; $AdS_5 \times S^5$ w/ Chandía

Integrate out d_α and $\bar{d}_{\bar{\beta}}$:

$$S_{pure} = \frac{1}{\alpha'}\int d^2z \left(\frac{1}{2}\eta_{cd} E^c \bar{E}^d + F^{-1}_{\alpha\bar{\beta}} (3E^\alpha \bar{E}^{\bar{\beta}} - \bar{E}^\alpha E^{\bar{\beta}}) \right) + S_{ghost}$$

K-symmetry is replaced with BRST invariance.
Quantization is straightforward using normal coordinate expansion.
One-loop conformal invariance proven (not yet for S_{ghost})
by Feynmann diagram computation in $AdS_2 \times S^2$ paper.

III. D. Superstring in R-R plane wave background (to appear soon)

Can take Penrose limit of $AdS_5 \times S^5$ background $\Rightarrow F^{\alpha\bar{\beta}} = \mu(\gamma^{+1234})^{\alpha\bar{\beta}}$
$\Rightarrow F^{\alpha\bar{\beta}}$ is not invertible. Break $SO(9,1) \to SO(8)$
$\alpha \to (a, \dot{a})$ and $\bar{\alpha} \to (\bar{a}, \dot{\bar{a}})$, $F^{a\dot{\bar{b}}} = F^{\dot{a}\bar{b}} = F^{\dot{a}\dot{\bar{b}}} = 0$

To prove conf. inv. of S_{pure}, first consider S_{pure} at $\theta^{\dot{a}} = \bar{\theta}^{\dot{\bar{a}}} = 0$

$$S_{pure}\big|_{\theta^{\dot a} = \bar\theta^{\dot a} = 0} = S_{GS}\big|_{\theta^{\dot a} = \bar\theta^{\dot a} = 0} + \frac{1}{\alpha'}\int d^2z \Big[d_a \bar{E}^a + \bar{d}_{\bar{a}} E^{\bar{a}} + d_a \bar{d}_{\bar{b}} F^{a\bar{b}} + \omega_\alpha \bar{\nabla}\lambda^\alpha + \bar{\omega}_{\bar{\alpha}} \nabla \bar{\lambda}^{\bar{\alpha}} + N_{+b}\bar{N}_{+c}\eta^{bc} \Big)$$

$$= \int d^2z \left(\frac{1}{2}\eta_{cd} E^c \bar{E}^d + F^{-1}_{a\bar{b}} (3E^a \bar{E}^{\bar{b}} - \bar{E}^a E^{\bar{b}}) \right) + S_{ghost}$$

$\Rightarrow$ one-loop conformal invariant. But can easily show
there are no multiloop Feynmann diagrams $\Rightarrow$ inv. at all loops.

But isometries imply S_{pure} is conf. inv. if $S_{pure}\big|_{\theta^{\dot a} = \bar\theta^{\dot{\bar a}} = 0}$ is conf. inv.

IV. Covariant Quantization of D=11 Superparticle "and" Supermembrane

IVA. On-shell D=11 Supergravity in superspace

(x^r, θ^μ) $\quad r=1$ to 11 (x^{10} = time), $\mu = 1$ to 32

$\Gamma^r_{\mu\nu} = \Gamma^r_{\nu\mu}$, Can use $C^{\mu\nu} = -C^{\nu\mu}$ and $C_{\mu\nu} = -C_{\nu\mu}$ to raise and lower D=11 spinor indices.

$$f^{(\mu\nu)} = f^r \Gamma_r^{\mu\nu} + f^{rs}\Gamma_{rs}^{\mu\nu} + f^{rstuv}\Gamma_{rstuv}^{\mu\nu}$$

$$f^{[\mu\nu]} = f\, C^{\mu\nu} + f^{rst}\Gamma_{rst}^{\mu\nu} + f^{rstu}\Gamma_{rstu}^{\mu\nu}$$

$$\eta_{rs}\Gamma^r_{(\mu\nu}\Gamma^{st}_{\rho\sigma)} = 0$$

D=11 supergravity fields g_{rs}, b_{rst}, χ_r^μ are described on-shell by superfields

$E_M^A(x,\theta)$, $B_{MNP}(x,\theta)$ where $M = (m,\mu)$

Just as $A_\alpha(x,\theta)$ contains all D=10 super-Maxwell fields,

$B_{\mu\nu\rho}(x,\theta)$ contains all D=11 supergravity fields.

Eq. of motion: $D_{(\alpha}A_{\beta)} = \gamma^m_{\alpha\beta}A_m \;\rightarrow\; D_{(\sigma}B_{\mu\nu\rho)} = \Gamma^r_{(\sigma\mu}B_{\nu\rho)r}$ (Cederwall, et al '01)

Gauge inv.: $\delta A_\alpha = D_\alpha \Lambda \;\rightarrow\; \delta B_{\mu\nu\rho} = D_{(\mu}\Lambda_{\nu\rho)}$

$\Rightarrow$ Can gauge $B_{\mu\nu\rho}(x,\theta) = (\Gamma^r\theta)_\mu(\Gamma^s\theta)_\nu(\Gamma^t\theta)_\rho\, b_{rst}(x) + (\Gamma^r\theta)_{(\mu}(\Gamma^{st}\theta)_\nu(\Gamma_t\theta)_{\rho)}g_{rs}(x)$

$+\left[(\Gamma^r\theta)_{(\mu}(\Gamma^s\theta)_\nu(\Gamma^t\theta)_{\rho)}(\Gamma_{st}\theta)_\sigma - (\Gamma^r\theta)_{(\mu}(\Gamma^{st}\theta)_\nu(\Gamma_s\theta)_{\rho)}(\Gamma_t\theta)_\sigma\right]\chi_r^\sigma(x)$

$+ \ldots$ where $\ldots$ is determined by $(b_{rst}, g_{rs}, \chi_r^\sigma)$

Also, these component fields are all on-shell.

IV B. D=11 Superparticle

Brink-Schwarz: $S = \int d\tau \, (\Pi^r P_r + e P^r P_r)$, $\Pi^r = \dot{x}^r - \frac{i}{2}\dot{\theta}\Gamma^r\theta$

Inv. under κ-symmetry, $p_\mu = -\frac{i}{2}(\not{P}\theta)_\mu \Rightarrow d_\mu = p_\mu + \frac{i}{2}(\not{P}\theta)_\mu = 0$

$\{d_\mu, d_\nu\} = i P_r \Gamma^r_{\mu\nu} \Rightarrow$ 16 first-class and 16 second-class

Can quantize in light-cone gauge and get linearized D=11 supergravity

[Note error in lecture I]

Pure spinor action: $S = \int d\tau \, (\dot{x}^r P_r - \underline{\frac{1}{2} P^r P_r} + \dot{\theta}^\mu p_\mu + \text{"}\dot{\lambda}^\mu \omega_\mu\text{"})$

$= \int d\tau \, (\frac{1}{2}\dot{x}^r \dot{x}_r + \dot{\theta}^\mu p_\mu + \text{"}\dot{\lambda}^\mu \omega_\mu\text{"})$

BRST operator: $Q = \lambda^\mu d_\mu$, $\lambda\Gamma^r\lambda = 0$ for $r = 1$ to 11.

Decompose in SO(9,1) components

$\lambda\Gamma^r\lambda = 0 \nearrow \lambda^\alpha \gamma^m_{\alpha\beta}\lambda^\beta + \bar{\lambda}_\alpha \gamma^{m\alpha\beta}\bar{\lambda}_\beta = 0$
$\searrow \lambda^\alpha \bar{\lambda}_\alpha = 0$

$\lambda^\mu \nearrow \lambda^\alpha$, $\searrow \bar{\lambda}_\alpha$

$\Rightarrow \bar{\lambda}_\alpha \gamma^{m\alpha\beta}\bar{\lambda}_\beta = -\lambda^\alpha \gamma^m_{\alpha\beta}\lambda^\beta \Rightarrow$ 9 constraints on $\bar{\lambda}_\alpha$

$\eta_{rs} \Gamma^r_{(\mu\rho} \Gamma^{st}_{\sigma\nu)} = 0 \Rightarrow \eta_{rs}(\lambda\Gamma^r\lambda)(\lambda\Gamma^{st}\lambda) = 0$

So $\lambda\Gamma^m\lambda = 0 \Rightarrow (\lambda\Gamma^{11}\lambda)(\lambda\Gamma^{11\,t}\lambda) = -(\lambda\Gamma^m\lambda)(\lambda\Gamma^{mt}\lambda) = 0$

$\Rightarrow$ Either $\lambda\Gamma^{11}\lambda = 0$ or $\lambda\Gamma^{11\,t}\lambda = 0$ for $t = 1$ to 10

Generically, get $\lambda\Gamma^{11}\lambda = 0 \Rightarrow \lambda^\mu$ has $\underline{23}$ independent components

Light-cone = $\underset{\theta^\mu}{32} + \underset{p_\mu}{32} - \underset{\lambda^\mu}{23} - \underset{\omega_\mu}{23}$ fermions = 18 fermions = 16 light-cone + (b, c)

Can dim. reduce at $P_{11} = 0$ to N=2 D=10 superparticle

$S = \int d\tau \, (\dot{x}^m P_m - \frac{1}{2}P^m P_m + \dot{\theta}^\alpha p_\alpha + \dot{\bar{\theta}}_\alpha \bar{p}^\alpha + \dot{\lambda}^\alpha w_\alpha + \dot{\bar{\lambda}}^\alpha \bar{w}_\alpha)$

$Q = \lambda^\alpha d_\alpha + \bar{\lambda}_\alpha \bar{d}^\alpha$

But this version of N=2 d=10 superparticle is different from naive infinite tension limit of Type IIB superstring since $\lambda\gamma^m\lambda - \bar\lambda\gamma^m\bar\lambda = \lambda\Gamma^{m''}\lambda \neq 0$. Have chosen "other" branch of $\lambda\Gamma^m\lambda=0$ constraint where $\lambda\Gamma^{''}\lambda = \lambda^\alpha\bar\lambda_\alpha = 0$.

In naive limit, light cone $\Rightarrow$ $64 - 2(22) = 20$ fermion $= 16$ light-cone $+ (b, c) + (\bar b, \bar c)$.

Still need to impose $b - \bar b = 0$ constraint to get zero mom. dilaton and R-R gauge fields

IVC. BRST quantization of linearized D=11 Supergravity

Define physical states = cohom. of $Q = \lambda^\mu d_\mu$ at ghost-number three (d=3 worldvol. of supermembrane)

$$\Psi(\lambda, x, \theta) = \lambda^\mu\lambda^\nu\lambda^\rho B_{\mu\nu\rho}(x,\theta)$$

$$Q\Psi = 0 \Rightarrow \lambda^\mu\lambda^\nu\lambda^\rho\lambda^\sigma D_\sigma B_{\mu\nu\rho} = 0 \Rightarrow D_{(\sigma} B_{\mu\nu\rho)} = \Gamma^r_{(\sigma\mu} B_{\nu\rho)r}$$

for some $B_{\nu\rho r}$

$$\delta\Psi = Q(\lambda^\mu\lambda^\nu\Lambda_{\mu\nu}) = \lambda^\mu\lambda^\nu\lambda^\rho D_\rho\Lambda_{\mu\nu} \Rightarrow \delta B_{\mu\nu\rho} = D_{(\mu}\Lambda_{\nu\rho)}$$

$\Rightarrow$ $B_{\mu\nu\rho}(x,\theta)$ describes (linearized) D=11 supergravity

At other ghost numbers, cohom. of Q gives (ghosts, fields, antifields, antighost

At zero momentum,
$$\Psi(\lambda,\theta) = \omega'' + (\lambda\theta)^r\omega'_r + (\lambda^2\theta^2)^{[rs]}\omega_{rs} + (\lambda^2\theta^2)^r\rho_r$$
$$+ (\lambda^2\theta^3)^\mu\xi_\mu + (\lambda^3\theta^3)^{[rst]}b_{rst} + (\lambda^3\theta^3)^{(rs)}g_{rs} + (\lambda^3\theta^4)^{r\alpha}\chi_{r\alpha}$$
$$+ (\lambda^4\theta^5)^{r\alpha}\chi^*_{r\alpha} + (\lambda^4\theta^6)^{(rs)}g^*_{rs} + (\lambda^4\theta^6)^{[rst]}b^*_{rst}$$
$$+ (\lambda^5\theta^6)^\mu\xi^*_\mu + (\lambda^5\theta^7)^r\rho^*_r + (\lambda^5\theta^7)^{[rs]}\omega^*_{rs} + (\lambda^6\theta^8)^r\omega^{*\prime}_r + (\lambda^7\theta^9)\omega^{*\prime\prime}$$

$(\omega_{rs}, \rho_r, \xi_\mu)$ ghosts for $\delta b_{rst} = \partial_{[r}\Lambda_{st]}$, $\delta g_{rs} = \partial_{(r}\Lambda_{s)}$, $\delta\chi_{r\alpha} = \partial_r\xi_\alpha$

ω'_r ghost-for-ghost $\delta\Lambda_{st} = \partial_{[s}\Lambda'_{t]}$, ω'' ghost-for-ghost-for-ghost $\delta\Lambda'_t = \partial_t\Lambda$

Can define BV action for linearized D=11 supergravity

$$\mathcal{S} = \int d^{11}x \, \langle \Psi(\lambda,x,\theta) Q \Psi(\lambda,x,\theta) \rangle \quad \text{where } \langle \lambda^7 \theta^9 \rangle = 1.$$

IV.D. BST action for supermembrane

$$S_{BST} = \int d\tau d^2\sigma \left[\tfrac{1}{2} \hat{\Pi}_0^r \hat{\Pi}_{0r} + B^{flat}_{RST} \partial_0 Y^R \partial_1 Y^S \partial_2 Y^T \right] \qquad Y^R = (x^r, \theta^\mu)$$

with Hamiltonian $\hat{\Pi}_0^r \hat{\Pi}_{0r} = \det(\Pi_I^r \Pi_{Jr})$ $\quad I,J = 1,2$

$$\boxed{\hat{\Pi}_0^r = \Pi_0^r + e^J \Pi_J^r} \qquad \Pi^r = \partial x^r + \tfrac{i}{2} \theta \Gamma^r \partial \theta$$

Have gauge fixed $e^0 = 1$, but left e^J unfixed (spatial reparam's).

Under $\delta\theta^\mu = \xi^\mu$, $\delta x^r = -\frac{i}{2}(\theta \Gamma^r \delta\theta)$, $\delta e^J = i \xi_\mu (\partial_I \theta^\mu) \epsilon^{IJ}$,

$$\delta S_{BST} = i \int d\tau d^2\sigma \, \xi^\mu \left(\Gamma^r_{\mu\nu} \hat{\Pi}_{0r} - \tfrac{1}{2} \Gamma^{rs}_{\mu\nu} \Pi_{Ir} \Pi_{Js} \epsilon^{IJ} \right) \nabla\theta^\mu$$

where $\nabla\theta^\mu = \hat{\partial}_0 \theta^\mu + \Gamma_r^{\mu\nu} (\partial_I \theta_\nu) \Pi_J^r \epsilon^{IJ}$

$\Rightarrow \delta S_{BST} = 0$ if $\xi_\mu = (\Gamma^r_{\mu\nu} \hat{\Pi}_{0r} + \frac{1}{2} \Gamma^{rs}_{\mu\nu} \Pi_{Ir} \Pi_{Js} \epsilon^{IJ}) \kappa^\nu$

up to reparam. constraints.

$$d_\mu = p_\mu - \frac{\partial L}{\partial \dot{\theta}^\mu} \approx 0, \quad \{d_\mu, d_\nu\} = -\hat{\Pi}_{0r} \Gamma^r_{\mu\nu} + \tfrac{1}{2} \epsilon^{IJ} \Pi_{Ir} \Pi_{Js} \Gamma^{rs}_{\mu\nu}$$

$\Rightarrow$ 16 first-class and 16 second-class constraints.

Two new features: 1) $\delta e^J \neq 0$; 2) $\{d_\lambda, d_\nu\}$ has term with $\Gamma^{rs}_{\mu\nu}$.

To get pure spinor action, use Oda-Tonin method and compute BRST transf. of S_{BST} under $Q = \int \lambda^\mu d_\mu$

$\Rightarrow \delta\theta^\mu = \lambda^\mu$, $\delta x^r = -\frac{i}{2}(\theta \Gamma^r \delta\theta)$, $\delta d_\mu = -i(\hat{\Pi} \lambda)_\mu + \frac{i}{2} \epsilon^{IJ} \Pi_{Ir} \Pi_{Js} (\Gamma^{rs} \lambda)_\mu$, $\delta w_\mu = d_\mu$

IV.E. Pure spinor version of supermembrane action

Two new features ⇒ 1) Define $\delta e^J = i\xi_\mu(\partial_I\theta^\mu)\epsilon^{IJ}$

2) $\lambda\Gamma^r\lambda=0$ is not enough to guarantee nilpotence of Q

since $Q^2 = \int \lambda^\mu\lambda^\nu\{d_\mu,d_\nu\} = \int\lambda^\mu\lambda^\nu\left(-\hat{\Pi}_{0r}\Gamma^r_{\mu\nu}+\frac{1}{2}\Pi_{Ir}\Pi_{Js}\Gamma^{rs}_{\mu\nu}\epsilon^{IJ}\right)$

and $Q^2 e^J \neq 0$ ⇒ Need to impose $(\lambda\Gamma^{rs}\lambda)\Pi_{Js}=0$ and $\lambda^\mu\partial_J\lambda_\mu=0$

Not visible for superparticle where $\Pi_{Js}=\partial_J\lambda^\mu=0$

$$\Rightarrow\quad S_{pure} = S_{BST} + \int d\tau\, d^2\sigma\; Q(\omega_\mu\nabla\theta^\mu)$$

$$= \int d\tau\, d^2\sigma\,\Big(\hat{\Pi}_0^r\hat{\Pi}_{0r} + B^{flat}_{RST}\,\partial_0 Y^R\partial_1 Y^S\partial_2 Y^T$$

$$+ d_\mu\nabla\theta^\mu + \omega_\mu\nabla\lambda^\mu$$

$$+ i\epsilon^{IJ}\left[(\omega\Gamma^r\partial_I\theta)(\lambda\Gamma_r\partial_J\theta) - (\omega\partial_I\theta)(\lambda\partial_J\theta)\right]\Big)$$

Can be generalized to a curved background by replacing $\nabla\theta^\mu$ and $\partial_J\theta^\mu$ with $E^\alpha_M\nabla Y^M$ and $E^\alpha_M\partial_J Y^M$, $\Pi^r\to E^c_M\partial Y^M$, $B^{flat}_{RST}\to B_{RST}$.

Upon double-dimensional reduction, $S^{flat}_{pure}\to S^{flat\ IIA}_{pure}$

$\partial_2 X^{11}=1,\ \partial_2\theta^\mu=\partial_2 x^m=0$

But $S^{curved}_{pure} \not\to S^{curved\ IIA}_{pure}$. No Fradkin-Tseytlin term in $D=11$ and no $d_\alpha\bar{d}_{\bar\beta}F^{\alpha\bar\beta}$ term.

But $S^{curved\ IIA}_{pure}$ only well-defined for small coupling constant.

No reason to agree with $S^{membrane}_{pure}$ non-perturbatively.

Can S_{pure} be used to compute non-perturbative IIA amplitudes?

VI F. Speculations on connection with M-theory

To compute amplitudes, need vertex operators. Massless integrated vertex operator $\int d\tau d^2\sigma\, V$ is linear contribution to curved action. BRST invariance implies

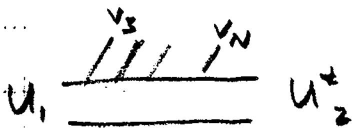
$$Q V = \partial_A W^A \quad \text{for} \quad A = \tau, \sigma_1, \sigma_2 ;$$

$$Q^2 V = 0 \Rightarrow Q W^A = \epsilon^{ABC} \partial_B X_C \; ; \; Q^2 W^A = 0 \Rightarrow Q X_C = \partial_C U$$

where $U = \lambda^\mu \lambda^\nu \lambda^\rho B_{\mu\nu\rho}$. So $d=3$ worldvolume implies U has ghost-number three for physical fields.

$$\langle \lambda^7 \theta^9 \rangle = 1 \Rightarrow +7 \text{ ghost number anomaly}$$

Can define non-vanishing "tree" amplitudes as $\langle U_1 U_2^* \int V_3 \dots \int V_N \rangle$ where U^* carries g.n. $= 4$ and depends on antifields.



In string theory, gives correct answer up to infinite $SL(2,R)$ factor.

Why just tree amplitudes? Supermembrane action needs to be regularized (cutoff, discretization, M(atrix) theory). Since supermembranes allow "spikes", cutoff destroys topology and cannot distinguish trees from loops. No dimensionless parameter in $D=11$ supergravity.

Using pure spinors, should be possible to covariantize light-cone quantization methods such as M(atrix) theory or light cone superparticle methods. Maybe can do perturbative expansion in $\frac{1}{\text{tension}}$ and hope that results are cutoff independent.