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SMR.1402 - 11

SPRING SCHOOL ON SUPERSTRINGS AND RELATED MATTERS

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D-BRANES AND HOLOGRAPHY

Lectures 1 and 2

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Please note: These are preliminary notes intended for internal distribution only.

⓪ Introduction

In these lectures I will focus to a large extent on D(irichlet) branes in AdS3 geometry. This is a very specific problem, but with diverse motivations & ramifications.

Here are a few:

* D-branes are privileged probes of quantum geometry. Only in last couple of years have we started to systematically explore their properties in curved space, beyond the semiclassical approximation. AdS3 is perhaps the simplest example where the time coordinate enters non-trivially.

* AdS_3, its orbifolds & cosets, arise in the near-horizon geometry of many basic black holes of string theory.

They are the obvious place for understanding holography beyond the supergravity approximation.

* Branes in AdS3 are toy models for the study of warped string compactifications

* They lead, as I will show, naturally to an extension (specialization) of boundary CFT that is of great interest in statistical mechanics (applications in string field theory ?)

Much of what I will describe I learned during enjoyful collaborations with: Mike Douglas, Christoph Schweigert, Marios Petropoulos, Nicolas Couchoud, Paul Windey, Jan de Boer, Robbert Dijkgraaf & Hirosi Ooguri. Other refs. as I go on.

My plan :

1. Brief review of BCFT
2. D-branes in AdS3
3. Gluing together CFTs.

(1) Boudary CFT (a quick review)

We consider a bulk 2d CFT.

Its data includes the following:

* Some chiral algebra $\mathcal{A}$ with generators A_n, such that Virasoro $\subset$ Envelop($\mathcal{A}$)

example current algebra:

$$[J_n^a, J_m^b] = i f^{ab}{}_c J^c_{n+m} + \frac{k}{2} n \delta^{ab} \delta_{n,-m}$$

Sugawara construction:

$$L_n = \sum_{\substack{m \\ a}} \frac{:J^a_{n+m} J^a_{-m}:}{k+2}$$

In general the left- & right-moving sectors have different chiral algebras, but for our purposes here we consider $\mathcal{A}_L = \mathcal{A}_R$.

* A set of primary fields and associated highest-weight vectors

$$\varphi^{\alpha} \longleftrightarrow |h_\alpha, \alpha\rangle$$

↑ conformal weight

$\mathcal{H}_\alpha$: rep. of $\mathcal{A}$ built on top of $|h_\alpha, \alpha\rangle$

The characters are:

$$\chi_\alpha(q) = \mathrm{tr}_{\mathcal{H}_\alpha}\left(q^{L_0 - c/24}\right)$$

$$q = e^{2\pi i \tau}$$

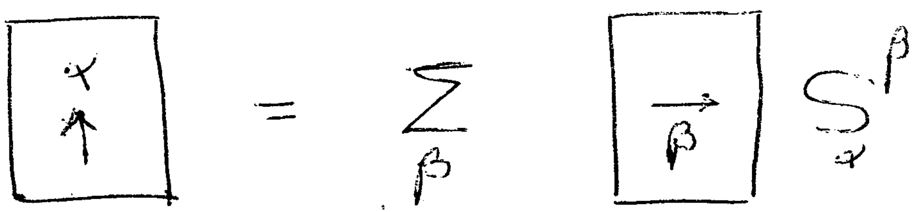
They have modular-transfo. properties:

$\underline{\tau \to \tau + 1}$ $\qquad \chi_\alpha \to T_\alpha{}^\beta \chi_\beta$

$$T = \mathrm{diag}\left(e^{2\pi i (h - c/24)}\right)$$

$\underline{\tau \to -1/\tau}$ $\qquad \chi_\alpha \to S_\alpha{}^\beta \chi_\beta$

'channel duality'



the matrix S obeys the Verlinde formula:

$$N_{\alpha\beta}{}^{\gamma} = \sum_{\delta} \frac{S_{\alpha}{}^{\delta} S_{\beta}{}^{\delta} S_{\delta}{}^{\gamma}}{S_{0}{}^{\delta}}$$

$\uparrow$ identity rep.

(indices raised/lowered with conjugation matrix $C_{\alpha\beta} = \delta_{\alpha\beta^*}$)

where

$N_{\alpha\beta}{}^{\gamma}$ are non-negative integer <u>fusion</u> coef<u>fici</u>ents

they give the # of times the irrep. γ appears in the decomposition of the tensor product of α & β.

All this is chiral data. Next we need to put left and right sectors together to make a consistent theory.

* Multiplicities $N_{\alpha\bar{\alpha}}$ of pairs of left & right irreps corresponding to bulk fields. The partition function of the theory is

$$Z = \sum N_{\alpha\bar{\alpha}} \; \chi_{\alpha}(q) \; \chi_{\bar{\alpha}}(\bar{q})$$

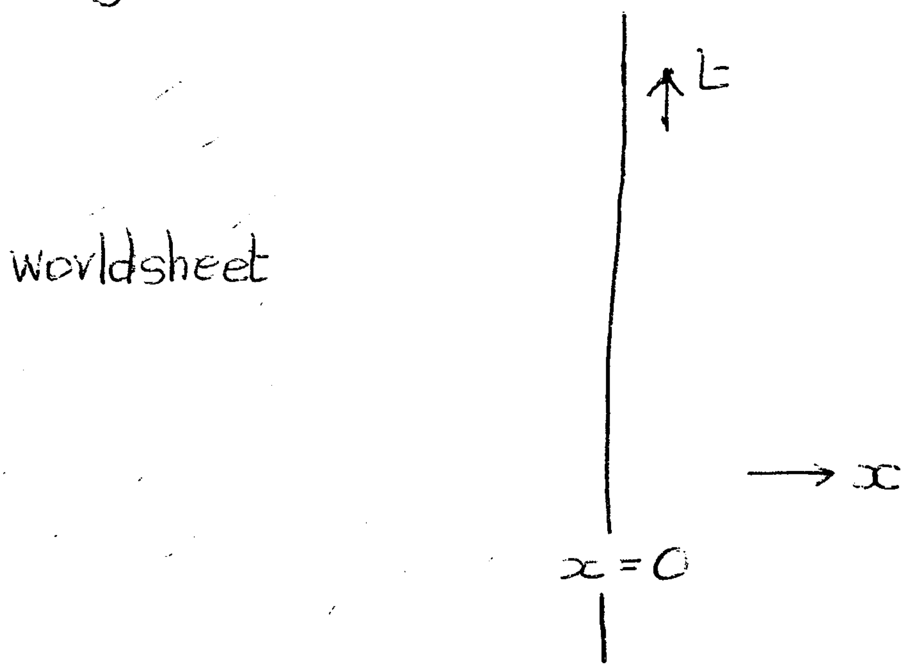
Modular invariance $\Longrightarrow$

$$\begin{cases} S_{\alpha}{}^{\beta} S_{\bar{\alpha}}{}^{\bar{\beta}} N_{\beta\bar{\beta}} = N_{\alpha\bar{\alpha}} \\ h(\alpha) - h(\bar{\alpha}) \in \mathbb{Z} \end{cases}$$

All this is necessary but not sufficient
'Algebraic approach' axioms: consistent factorizn of 4-point function on plane, and 0-point, 1-point functions on torus
(Moore Seiberg ; Sonoda)

All this automatic given local action principle.

We want now to introduce a boundary of space



No <u>net</u> flow of 2D energy to the boundary

$$\Updownarrow$$

$$\boxed{0 = T_{xt} = T_{++} - T_{--}} \quad \text{at } x = 0$$

$$x^{\pm} = x \pm t$$

after Wick rotation:

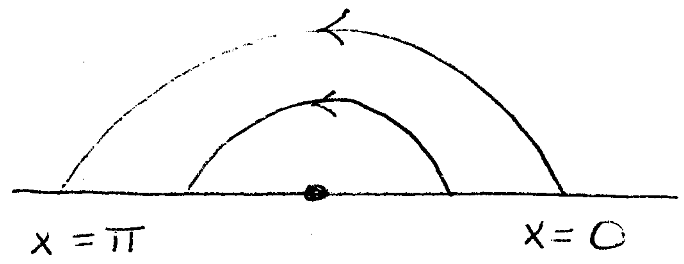
$$T_{zz} = \overline{T}_{\bar{z}\bar{z}} \qquad \text{at } \mathrm{Re}\, z = 0$$

Let us translate this in open-
& closed-string language:

* <u>Open strings</u>

$$w = e^{t-ix} = e^{-iz}$$

(radial-quantiznt variables)
for <u>strip</u>



$$T_{ww} = \left(\frac{\partial z}{\partial w}\right)^2 T_{zz} + \frac{c}{12}\{z, w\}$$

Schwarzian derivative

$$= \frac{\partial^3 z/\partial w^3}{\partial z/\partial w} - \frac{3}{2}\left(\frac{\partial^2 z/\partial w^2}{\partial z/\partial w}\right)^2$$

$$\Rightarrow T_{ww} = -\frac{1}{w^2}\left(T_{zz} - \frac{c}{24}\right)$$

$$\Rightarrow T_{ww} = \overline{T}_{\bar{w}\bar{w}} \quad \text{at} \quad w = \bar{w}$$

So can define Virasoro algebra of conserved charges:

$$\mathcal{L}_n = \oint dw\, w^{n+1} T_{ww} + \oint d\bar{w}\, \bar{w}^{n+1} \bar{T}_{\bar{w}\bar{w}}$$

↳ half contours

{can be translated in time since boundary contributions cancel out.

preservation of conformal sym.

$\Updownarrow$

solution of classical open-string equations of motion.

↳ Generalize to arbitrary $\mathcal{A}$:

$$A(z) = \Omega \bar{A}(\bar{z})$$

↳ automorphism of algebra

symmetry-preserving boundary states

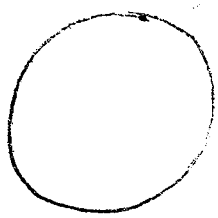
A priori only Vir need be preserved.

* Closed-string (boundary states)

compactify $t = t + 2\pi$ to obtain cylinder

$$\begin{cases} w = e^{z} = e^{x+it} \\ \text{boundary at } |w| = 1 \end{cases}$$

worldsheet



as before

$$w^2 T_{ww} = \bar{w}^2 \bar{T}_{\bar{w}\bar{w}} \Big|_{\text{bound}}$$

$$\Rightarrow \oint \frac{dw}{w} w^{n+2} T_{ww} = -\oint \frac{d\bar{w}}{\bar{w}} \bar{w}^{-n+2} \bar{T}_{\bar{w}\bar{w}}$$

which implies formal equality:

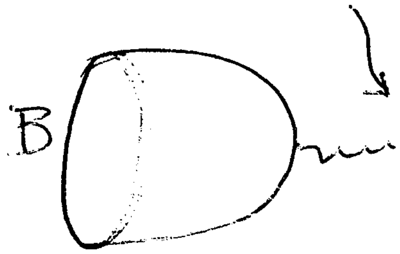
$$\left(L_n - \bar{L}_{-n} \right) |B\rangle\!\rangle = 0$$

closed-string boundary 'state' (not correctly normalizable)

The 'state' $|B\rangle\rangle$ defines (equivalently) a map between the holomorphic & antiholomorphic sectors:

$$B_{\alpha\bar\alpha} = \langle\langle B | \left(|\alpha\rangle \otimes |\bar\alpha\rangle \right)$$

closed-string

$=$ 

This commutes with the action of the Virasoro algebra (or more generally $\mathcal{A}$)

$$\begin{tikzcd}
\mathcal{H} \arrow[r, "B"] \arrow[d, "Vir"'] & \overline{\mathcal{H}} \arrow[d, "\overline{Vir}"] \\
\mathcal{H} \arrow[r, "B"'] & \overline{\mathcal{H}}
\end{tikzcd}$$

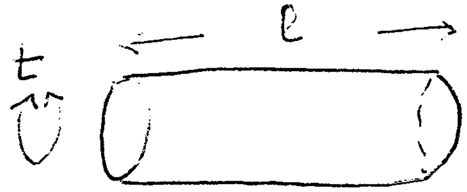
We conclude that within irreducible reps. B must act as identity map (isomorphism):

$$|B\rangle\rangle = \sum_{\alpha} B_{\alpha}\, |\alpha\rangle\rangle_{\text{Ishibashi}}$$

$$|\alpha\rangle\rangle_{\text{Ishibashi}} \to \sum_{\{n\}} |\alpha, n\rangle \otimes |\bar{\alpha}, n\rangle$$

The problem then is to determine the coefficients B_α.

<u>Cardy</u> consistency conditions:



$$\langle\langle B_1 |\, \tilde{q}^{\frac{1}{2}\left(L_0 + \bar{L}_0 - \frac{c}{12}\right)} \,| B_2 \rangle\rangle$$

$$\begin{bmatrix} \tilde{q} = e^{2\pi i (2il)} \\ q = e^{-4\pi/t} \end{bmatrix}$$

$$= \sum_{\alpha} B_{1,\alpha} B_{2,\alpha}\, \chi_\alpha(\tilde{q})$$

$$= \sum_{\alpha,\beta} \underbrace{B_{1,\alpha} B_{2,\alpha}\, S_\alpha^{\ \beta}}_{= \, n_{12}^{\ \beta}} \, \chi_\beta(q)$$

The $n_{12}{}^{\beta}$ must be non-negative integers since they give the multiplicity of open strings in the rep $\mathcal{H}_\beta$.

↳ Cardy's solution for diagonal RCFT:

$$|\alpha\rangle\rangle_{\text{Cardy}} = \sum_{\beta} \frac{S_\alpha{}^{\beta}}{\sqrt{S_0{}^{\beta}}} |\beta\rangle\rangle_{\text{Ishibashi}}$$

↳ for each primary

Verlinde's formula $\Rightarrow$ $n_{\alpha\beta}{}^{j} = N_{\alpha\beta}{}^{j}$

↳ Cardy conditions necessary but not sufficient.

Counterexample (Gaberdiel) in $c = 1/2$ model:

$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & \sqrt{2} \\ 1 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 0 \end{pmatrix}$$

Cardy states:

$$|\alpha_0\rangle\rangle = \frac{1}{\sqrt{2}}|0\rangle\rangle + \frac{1}{\sqrt{2}}|\tfrac{1}{2}\rangle\rangle + \frac{1}{\sqrt[4]{2}}|\tfrac{1}{16}\rangle\rangle$$

$$|\alpha_{1/2}\rangle\rangle = \frac{1}{\sqrt{2}}|0\rangle\rangle + \frac{1}{\sqrt{2}}|\tfrac{1}{2}\rangle\rangle - \frac{1}{\sqrt[4]{2}}|\tfrac{1}{16}\rangle\rangle$$

$$|\alpha_{1/16}\rangle\rangle = |0\rangle\rangle - |\tfrac{1}{2}\rangle\rangle$$

'Alternative set':

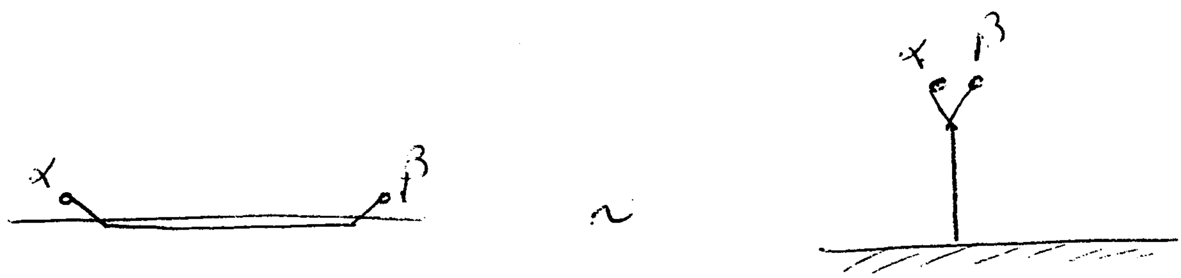
$$|1\rangle\rangle = |0\rangle\rangle + |\tfrac{1}{2}\rangle\rangle + \sqrt[4]{2}\,|\tfrac{1}{16}\rangle\rangle$$
$$|2\rangle\rangle = |0\rangle\rangle + |\tfrac{1}{2}\rangle\rangle$$
$$|3\rangle\rangle = \sqrt{2}\,|0\rangle\rangle - \sqrt{2}\,|\tfrac{1}{2}\rangle\rangle$$

believed to be inconsistent (?)
identity appears twice on $|1\rangle\rangle$ & $|3\rangle\rangle$

↳ Sewing conditions
classifying algebra

Lewellen, Cardy
Pradisi, Sagnotti, Stanev
⋮

factorize bulk 2-point
function in two diff. ways



$$\mathcal{B}_\alpha \mathcal{B}_\beta = \sum_\gamma C_{\alpha\beta}{}^{\gamma} \, F_{\gamma 0}\begin{bmatrix}\beta & \beta \\ \alpha & \alpha\end{bmatrix} \mathcal{B}_\gamma$$

3-point structure constants

fusing matrices (relate chiral four-point blocks)

Difficult to check explicitly in most cases (need to know explicitly C, F)

'Axiomatization' of BCFT still to be done

further reading:

Behrend et al., hep-th/9908036
Gaberdiel, hep-th/0201113
Felder et al, hep-th/9909140
Sagnotti + Stanev, hep-th/9605042
etc etc

Much has been learned in recent years for specific backgrounds:

↳ Minimal models

↳ free fields
(subtler than first sight) (*)

↳ WZW models (*)

↳ general GKO models
Gepner models

↳ Non-compact CFTs (**)

I will say a few words here about (*), then focus in following lecture on (**).

Free boson

$$\varphi = \varphi + 2\pi r$$

$\mathcal{A} = U(1)$ current algebra

$$\{\alpha\} = \{ P_L = \frac{n}{2r} + mr \mid n, m \in \mathbb{Z} \}$$

$$\{\bar\alpha\} = \{ P_R = \frac{n}{2r} - mr \mid n, m \in \mathbb{Z} \}$$

↳ symmetric states imply gluing condition:

$$\left(J_t + \Omega(\bar J_{-t}) \right) |B\rangle\rangle = 0$$

$$\Omega(\bar J_{-t}) = \pm \bar J_{-t} \quad \begin{array}{l} \nearrow \text{Neumann} \\ \searrow \text{Dirichlet} \end{array}$$

* $$|N, w\rangle\rangle = \sqrt{r} \sum_{m \in \mathbb{Z}} e^{iwmr} \, |mr\rangle\rangle^{N}_{\text{Ishibashi}}$$

is a D1-brane with Wilson-line w

* $$|D, x\rangle\rangle = \frac{1}{\sqrt{2r}} \sum_{n \in \mathbb{Z}} e^{in x/r} \, \left| \frac{n}{2r} \right\rangle\rangle^{D}_{\text{Ishibashi}}$$

is a D0-brane at position x on the circle where

$$|mr\rangle\rangle^{N}_{\text{Ishibashi}} = \exp\left(\sum_{\ell=1}^{\infty} -\frac{1}{\ell}\, a_{-\ell}\bar{a}_{-\ell}\right)|n=0, m\rangle$$

$$\left|\frac{n}{2r}\right\rangle\rangle^{D}_{\text{Ishibashi}} = \exp\left(\sum_{\ell=1}^{\infty} \frac{1}{\ell}\, a_{-\ell}\bar{a}_{-\ell}\right)|n, m=0\rangle$$

Is this all ?

No — Friedan; Gaberdiel, Recknagel & refs. therein

at $r_* = \frac{1}{\sqrt{2}}$ (self-dual point)

there is $SU(2)_1$ current algebra

$\Rightarrow$ continuous moduli space $\approx SU(2)$

(N & D branes special points)

Will see this later.

T-duality = gauge symmetry.

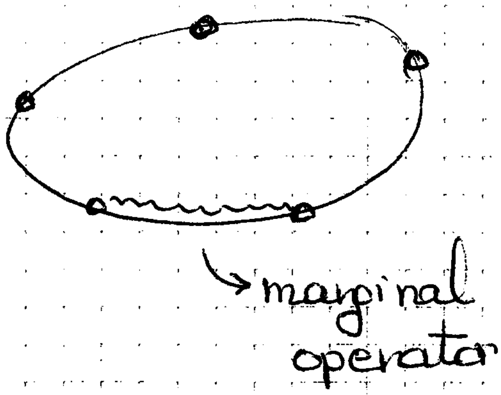
More <u>striking</u>

at rational multiples of r_* :

$$r = \frac{N}{M} r_*$$

$\exists$ continuous deformations of (collections) of D0 or D1 branes that are 'fundamental branes' breaking U(1) sym.

<u>ex</u> for $M=1$, any N



regular array of N D0's

$\downarrow$

unstable D1 brane

To be sure, we find no new stable branes, but illustrates rich structure of classical open-string-theory solutions.

$SU(2)_k$ WZW model

(serve to illustrate connection with geometry)

↳ primaries: $|j\rangle \qquad j = 0, \tfrac{1}{2}, 1, \ldots, \tfrac{k}{2}$

$$S_i^{\ j} = \sqrt{\frac{2}{k+2}}\, \sin\left[\frac{(2j+1)(2i+1)\pi}{k+2}\right]$$

Cardy states:

$$\left| i = \tfrac{n-1}{2} \right\rangle\!\!\rangle_{\text{Cardy}} = \sum_j \frac{S_i^{\ j}}{\sqrt{S_0^{\ j}}}\, |j\rangle\!\rangle_{\text{Ishibashi}}$$

satisfy symmetric gluing conditions:

$$\left(J^a_n + \omega\left(\bar{J}^a_{-n}\right)\right) |B\rangle\!\rangle = 0$$

↳ inner automorphism of $SU(2)$.

Open-string spectrum on ith brane:

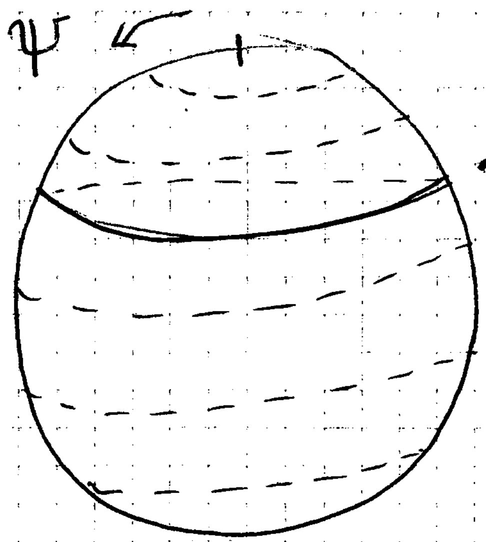
$$j = 0, 1, \ldots \min(2i, k-2i), \qquad h = \frac{j(j+1)}{k+2}$$

↑ integer spin

↳ Geometrically these correspond to spherical D2-branes carrying

$$0 < n < k+2$$

units of magnetic flux, which stabilizes them at some finite radius



$\psi_n = \frac{\pi n}{k+2}$ location of nth brane, whose center of mass is at North pole.

This follows from straightforward minimaization of effective D2-brane action:

$$S = T_2 \int d^3\sigma \sqrt{-\det(\hat{G}_{ab} + \hat{B}_{ab} + 2\pi\alpha' F_{ab})}\, e^{-\phi}$$

↳ Dirac-Born-Infeld

$$+ T_2 \int (\hat{C}^{(3)} + \hat{C}^{(1)} \wedge F)$$

↳ Wess-Zummo

where $\hat{G}, \hat{B}$ induced metric & NS 2-form field

$F = dA$ worldvolume $U(1)$ gauge field

$C^{(n)}$ n-index antisym. RR potential (vanishing for particular backg.)

↳ One can compute semiclassically :

- $S_i{}^j \propto$ couplings of i brane to hyperspherical harmonic $f_{j\, \underbrace{l=m=0}_{\text{diagonal } SU(2)}}$ on S^3

- $h = \dfrac{j(j+1)}{k+2}$: spectrum of open-string excitations

- moduli ~ rigid translations of center of mass

all in perfect agreement with CFT data

Remarks

* nth brane has n units of D0 charge (induced by magnetic flux)
  → blows up in background B field
  (dielectric, Myers effect)

* In condensed matter flow from n D0's at north pole $\to |n\rangle\rangle_{\text{Cardy}}$ is screening of magnetic impurity by electron cloud (Kondo effect)

* Cardy states in 1-to-1 correspondence with charges in twisted version of K-theory (cf. Moore). Extra (stable??) non-symmetric branes can be constructed (Maldacena, Moore, Seiberg)

* Extensions to orbifolds, orientifolds by several groups

* Some words on supersymmetry

In type II A or B backgrounds we need at least $N=(1,1)$ superconformal symmetry, with associated supercurrents

G_r, $\overline{G}_r$ ($r \in \mathbb{Z}$ (R) or $r \in \mathbb{Z}+\frac{1}{2}$ (NS))

Boundary states respecting the symmetry must obey:

$$\left(G_r + i\eta\, \overline{G}_{-r}\right)|B,\eta\rangle\rangle = 0$$

$$(\eta = \pm 1)$$

Since

$$(-)^F \left(G_r + i\eta \overline{G}_{-r}\right) = \left(-G_r + i\eta \overline{G}_{-r}\right)(-)^F$$

both values of η required to implement the GSO projections in closed sector.

Backgrounds with at least $N=(2,2)$ superconformal symmetry can have target-space supersymmetric branes

Two distinct possibilities, according to gluing of $U(1)$ currents in $N=2$ superalgebra

(A) $$(H_\ell - \bar{H}_{-\ell})|B\rangle\rangle = 0$$

preserves axial $U(1)$ symmetry

{ brane = middle-dimensional Lagrangian submanifold of sympl. manifold

(B) $$(H_\ell + \bar{H}_{-\ell})|B\rangle\rangle = 0$$

preserves vector $U(1)$ symmetry

{ brane = holomorphic submanifold of complex manifold

(cf Hori)

② D-branes in AdS3

AdS_3 is an important solution in closed-string theory for several reasons:

* it, and its orbifolds, arise in the near-horizon geometry of the basic string-theory black holes

* string theory is exactly soluble in this background (no RR fields) and its close cousins ($SL(2,R)/U(1)$, Liouville): only known examples with non-trivial time coordinate (except pp waves? cf Maldacena)

* it is toy model for studying, warped string 'compactifications'. (cf also Karch + Randall)

AdS3 is universal cover of the $SL(2,\mathbb{R})$ group manifold, which can be parametrized as follows:

$$g = \frac{1}{L}\begin{pmatrix} X^0+X^1 & X^2+X^3 \\ X^2-X^3 & X^0-X^1 \end{pmatrix}$$

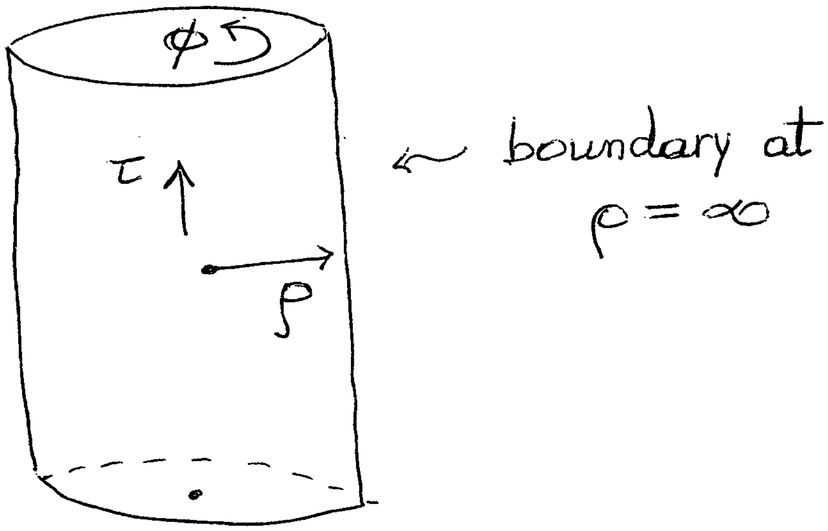
with $\quad (X^0)^2+(X^3)^2-(X^1)^2-(X^2)^2 = L^2$

Some useful coordinate systems are:

↳ Cylindrical coordinates (global)

$$\begin{cases} X^0+iX^3 = L\cosh\rho\, e^{i\tau} \\ X^1+iX^2 = L\sinh\rho\, e^{i\phi} \end{cases}$$

$$ds^2 = L^2\left(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\phi^2\right)$$



↳ Poincaré coordinates

$$\begin{cases} X^0 + X^1 = L u \\ X^0 - X^1 = L\left(\frac{1}{u} + u\,\omega^+\omega^-\right) \\ X^2 \pm X^3 = L u\, \omega^{\pm} \end{cases}$$

$$ds^2 = L^2 \left(\frac{du^2}{u^2} + u^2 \underbrace{d\omega^+ d\omega^-}_{\mathbb{R}^{1,1} \text{ slice}} \right)$$

↳ AdS coordinates

$$X^0 + iX^3 = L \cosh\psi \cosh\omega\, e^{i\tau}$$

$$X^1 = L \cosh\psi \sinh\omega$$

$$X^2 = L \sinh\psi$$

$$ds^2 = L^2 \left[d\psi^2 + \cosh^2\psi \underbrace{\left(-\cosh^2\omega\, d\tau^2 + d\omega^2 \right)}_{\text{AdS2 slice of varying radius}} \right]$$

To generalize to $n+2$ dimensions replace $\mathbb{R}^{1,1} \longrightarrow \mathbb{R}^{1,n}$ in Poincaré,

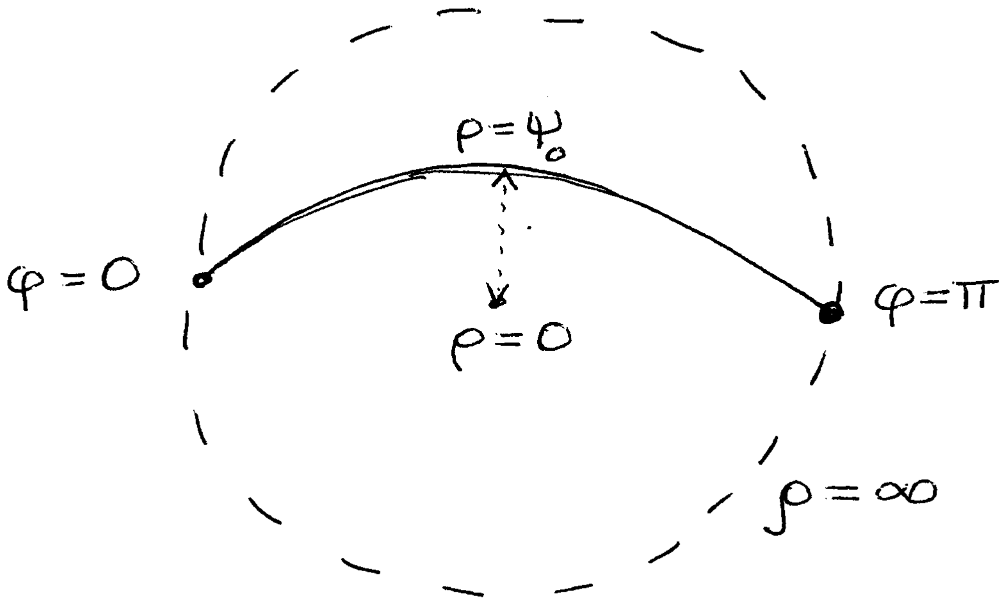
or AdS2 $\longrightarrow$ AdS$(n+1)$ by adding

$$+ \sinh^2\omega\, d\Omega_{n-1}^2$$

↳ To visualize the AdS slices, note that:

$$\sinh\psi = \sinh\rho \sin\varphi$$

so in cylindrical coordinates fixed $\psi = \psi_0$ looks as follows:

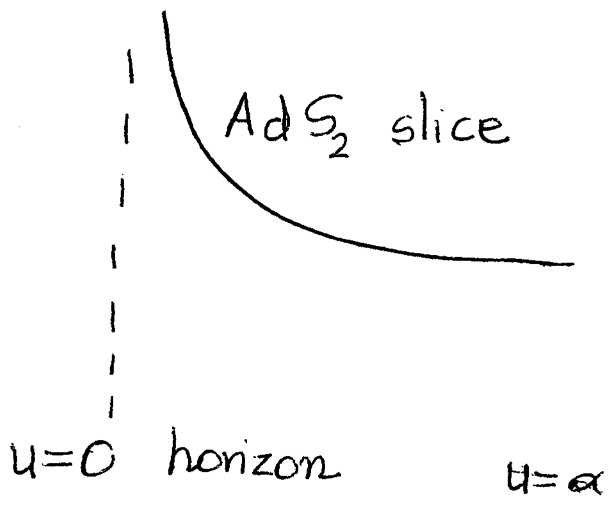


As $\psi_0 \to \infty$ the slice is pushed towards the boundary of AdS_3, and its radius of curvature grows: $\underline{\text{radius} = L\cosh\psi_0}$

↳ In Poincaré coordinates:

$$\sinh\psi = u x$$

$$\Rightarrow \quad u = \frac{\sinh\psi_0}{x}$$



These AdS_2 slices are the 'physical', symmetric D-branes.

Indeed, consider the gluing conditions for the $SL(2,R)$ currents (in the open picture):

$$J = k(\partial_+ g)g^{-1} \quad ; \quad \bar{J} = k g^{-1}\partial_- g$$

$$\boxed{J = -\omega \bar{J} \omega^{-1} \Big|_{x=0}}$$

automorphism

$\Rightarrow$ Dirichlet condition confining string endpoints on (twisted) conjugacy class

$$\boxed{tr(\omega g) = 2C_1}$$

For $\omega = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (outer automorphism)

$$\Rightarrow tr(\omega g) = \frac{2X^2}{L} = 2\sinh\rho \sin\varphi = 2C_1$$

$$\begin{cases} AdS_2 \text{ brane with} \\ C_1 = \sinh\psi_0 \end{cases}$$

Let's check that this solves the Born-Infeld equations. In Poincaré coordinates:

$$ds^2 = L^2\left[\frac{du^2}{u^2} + u^2(dx^2 - dt^2)\right] \qquad \underline{\text{metric}}$$

$$H = dB = 2L^2 u \, dx \wedge dt \wedge du$$

$$\Rightarrow \quad B = L^2 u^2 \, dx \wedge dt$$

NS <u>background</u>

∴ for a D-string with embedding $u(x,t)$ and carrying a world-volume electric field

$$f \equiv 2\pi\alpha' F_{xt}/L^2$$

we find:

$$S_{BI} = -\int T_D \sqrt{-\det(\hat{G} + \hat{B} + 2\pi\alpha' F)} \, dx \, dt$$

$$= -T_D L^2 \int dx \, dt \sqrt{u^4 + u'^2 - \dot{u}^2 - (u^2+f)^2}$$

with $\quad \dot{u} = \dfrac{\partial u}{\partial t}$

$\quad u' = \dfrac{\partial u}{\partial x}$

* Eqn. for gauge field is Gauss constraint (continuity of electric flux):

$$\frac{2\pi\alpha' T_D \, (\hat{B} + 2\pi\alpha' F)_{xt}}{\sqrt{-\det(\quad)}} = -q \in \mathbb{Z}$$

\# of bound fundamental strings

---

* Second eqn. can be integrated by using continuity of 2d energy-momentum $\Theta^\alpha{}_\beta$. For a static string it implies:

$$\Theta^x{}_x = L^2 \left(\frac{T_{(1,q)}\, u^4}{\sqrt{u^4 + u'^2}} - q T_F u^2 \right) = $$

$$= \text{constant}$$

where T_F = fundamental string tension

$T_{(1,q)} = \sqrt{T_D^2 + q^2 T_F^2}$ = tension of $(1,q)$ string

Free boundary condition at $\infty \Rightarrow$ no momentum flowing out at $x=0 \Rightarrow \Theta^x{}_x = 0$

implies $\boxed{u = \dfrac{C}{x - x_0}}$ AdS_2 brane

Notice that: $C = \pm \frac{q T_F}{T_D} = \sinh \psi_0$

so the radius of AdS_2 is:

$$\ell_{AdS_2} = L\sqrt{1+C^2} = L\,\frac{T_{(1,q)}}{T_D} \geq L$$

The parameter q (electric field) controls the tension and NS charge of our brane.

In general this falls towards the interior of AdS_3, attracted by the background NS5/F1 configuration (see later).

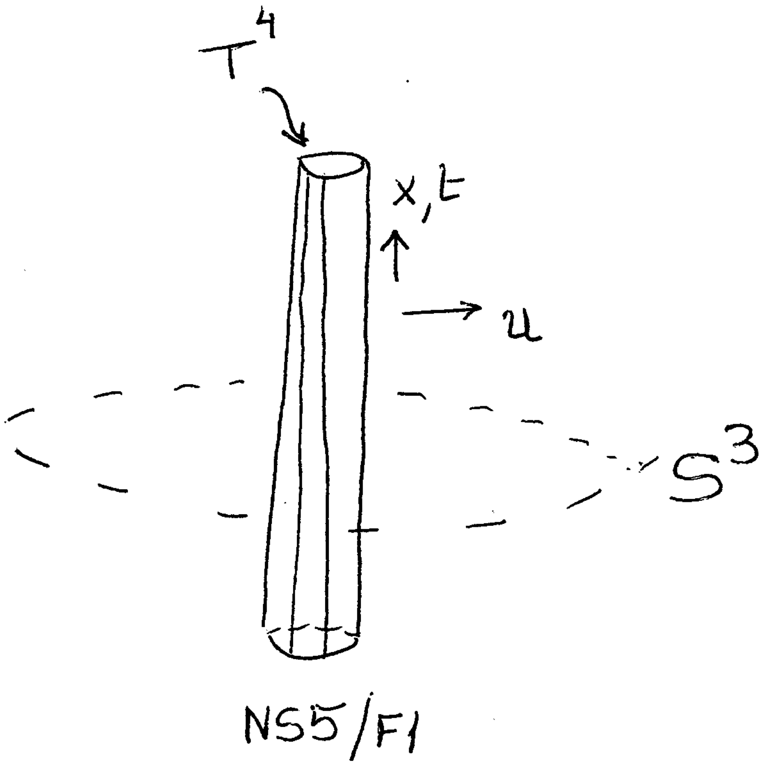
But as $q \to \infty$ we approach a BPS limit, and the probe brane stays (almost) flat.

This is similar to the behavior of the Randall-Sundrum brane if one does not fine tune Λ_5 and the brane tension.

Supersymmetry

To embedd these branes in a susy background (and ensure stability) consider the geometry near a configuration of NS5 branes & F1 strings, wrapping the fivebranes around T^4 (or K3). The full geometry is:

$$\underbrace{AdS3}_{SL(2,R)} \times \underbrace{S3}_{SU(2)} \times \underbrace{T^4}_{U(1)^4} \quad \text{WZW models}$$



The relation between the $AdS3 \times S3$ radius & # of fivebranes is:

$$L^2 = Q_5 \alpha' = (k+2)\alpha'$$

↳ level of current algebra

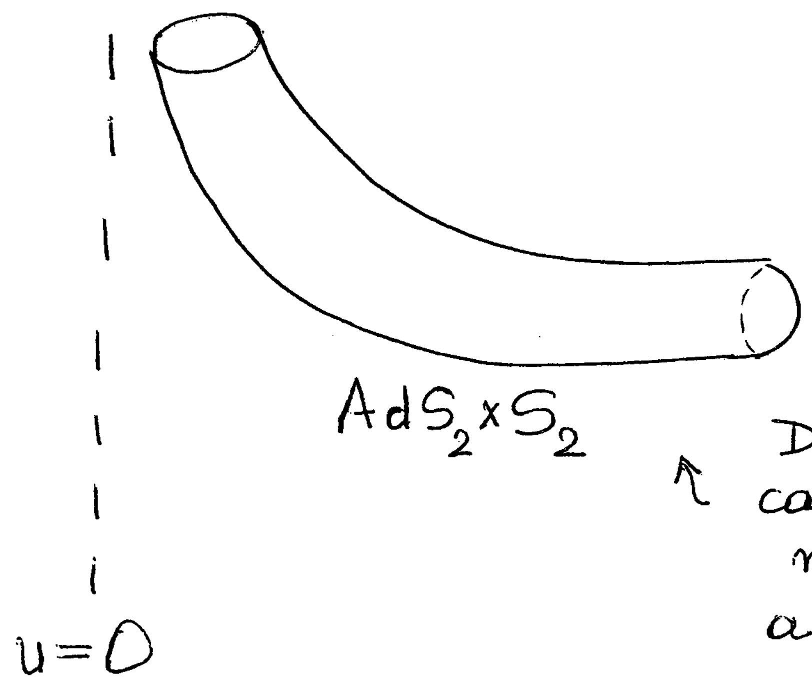
The supersymmetric branes are:

$$AdS2 \times S2$$

and carry p units of magnetic, q units of electric } flux

The constant C & radius of AdS_2 are modified as follows:

$$C = \pm \frac{q T_F}{p T_D} \, , \quad \ell_{AdS_2} = L \frac{T(p,q)}{p T_D}$$



D3 brane but carries quantum numbers of a (p, q) string

This is solution because:

* DBI action can be factorized
* boundary states can be tensored (modulo GSO projection).

↳ To check susy note that WZW model has 10 Majorana fermions:

$$\psi^A,\ \bar{\psi}^A \qquad A = 1 \ldots 10$$

↑ adjoint index of $SL(2,R) \times SU(2) \times U(1)^4$

Unbroken background susys obey:

$$\Big(\prod_{\text{all } A} \Gamma^A\Big) Q = Q \quad ; \quad \Big(\prod_{\text{all } A} \Gamma^A\Big) \bar{Q} = \pm \bar{Q}$$

zero modes of ψ^A in Ramond sector

<u>and</u>

$$\Big(\prod_{A \notin U(1)^4} \Gamma^A\Big) Q = Q \quad \text{\& same for } \bar{Q}$$

Last constraint follows from:

$$T_F \sim \psi^A J^A + f^{ABC} \psi^A \psi^B \psi^C$$

which implies extra constraint on Ramond vacuum

Now current-gluing conditions for our D-branes are :

$$J^A = -\omega(\bar{J}^A) \Rightarrow \psi^A = -\omega(\bar{\psi}^A)$$

to preserve world-sheet susy

ω is automorphism of Γ-matrix algebra
$\Rightarrow$ has action Ω on $SO(1,9)$ spinors

explicitly:

$$\left.\begin{array}{l} \omega(j^3) = -j^3 \\ \omega(j^\pm) = j^\mp \end{array}\right\} \Rightarrow \Omega = \Gamma^3(\Gamma^+ + \Gamma^-)/2$$

has even # of Γ-matrices
$\Rightarrow$ it is consistent with the chiral projections

so unbroken supercharges

$Q + \Omega\bar{Q}$ can be defined

chiral $SO(1,5) \times SO(4)$ spinor

Susy of branes raises a puzzle:

the radii of $AdS_2 \times S_2$ for generic (p,q) are unequal

$$\ell_{AdS_2} = L \frac{T_{(p,q)}}{p T_D} \geq L \geq \ell_{S_2} = L \sin\left(\frac{\pi p \alpha'}{L^2}\right)$$

Existence of a covariantly-constant spinor on the worldvolume requires, however, equal radii (??)

The radii are indeed equal when measured in the effective open string metric. This can be seen in many ways:

* Supersymmetry
* Spectrum of covariant box operator

$$h = \frac{j(j+1)}{k+2} \quad \text{for } SU(2)$$

(no p-dependence, maximum radius)

* From explicit formula:

$$G^{(open)}_{\alpha\beta} = \hat{G}^{(closed)}_{\alpha\beta} - \tilde{\mathcal{F}}_{\alpha\gamma} \hat{G}^{(closed)\,\gamma\delta} \tilde{\mathcal{F}}_{\delta\beta}$$

$\hookrightarrow \hat{B}_{\alpha\gamma} + 2\pi\alpha' F_{\alpha\gamma}$

exercise: check this explicitly for the AdS_2 branes.

This fact could be significant for the R-S scenario in string theory.

Stabilization of brane & bulk requires in general RR fluxes. We don't know how exactly these modify the effective metric on the brane.

But to describe our world, need to ensure that this latter metric (not its closed-string counterpart) can be tuned to be flat.

Question deserves to be analyzed further (note: purely stringy phenomenon)

Few words on the construction of corresponding boundary states

cf Ponsot, Schomerus, Teschner / 0112198

also: Giveon, Kutasov, Schwimmer / 0106005
Parnachev, Sahakyan / 0109150
Rajaraman, Rozali / 0108001

Technically hard because:

* CFT not rational ⇒ problems of regularization

* Euclidean rotation (of target and worldsheet) non-trivial

Many of the problems encountered & clarified in bulk theory

(Maldacena, Ooguri, Son)

Spectrum is left-right symmetric and contains the following chiral reps:

* highest-weight reps of the current algebra built on the unitary $SL(2,R)$ reps:

$$D_j^+ \quad \text{with} \quad \tfrac{1}{2} < j < \frac{k-1}{2}$$

$$C_{\frac{1}{2}+is}^{\alpha} \quad \text{with} \quad 0 \leq \alpha < 1$$

* Spectral flow of these reps, obtained through substitution:

$$\begin{cases} J_n^3 \to J_n^3 + \frac{k}{2}\omega\,\delta_{n,0} \\ J_n^{\pm} \to J_{n\mp\omega}^{\pm} \\ T_{++} \to T_{++} - \omega J^3 - \frac{k}{4}\omega^2 \end{cases}$$

NB – Unflowed continuous rep. has no physical states other than the tachyon

– Spectral flow of these reps. gives long strings that can expand out to the boundary of AdS

After (double) Euclidean rotation, normalizable primary reps. are $C^{a}_{\frac{1}{2}+is=j}$ of $SL(2,\mathbb{C})$. Ponsot et al solve the 'classifying algebra' constraints for the couplings $\mathcal{B}(j)$ of the boundary state to these closed strings.

They confirm that these reduce, in the large-k limit, to the couplings to a D-brane at fixed $\psi = \psi_0$.

↳ This allows a calculation of the exact annulus diagram $\Rightarrow$ density of open-string states (which includes long & short strings).