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SPRING SCHOOL ON SUPERSTRINGS AND RELATED MATTERS

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D-BRANES AND HOLOGRAPHY

Lecture 3

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Please note: These are preliminary notes intended for internal distribution only.

3 'Gluing CFTs' (or the holographic view of AdS branes)

Consider type IIB theory on $T^4 \times \mathbb{R}^{1,5}$.

↳ This has a variety of strings

$$F1,\ D1,\ \underbrace{NS5,\ D5}_{\text{wrapping } T^4},\ \underbrace{D3's}_{\text{wrapping 2 cycles in } T^4}$$

which span a Narain lattice $\Gamma_{5,5}$.

The U-duality group is $O(5,5;\mathbb{Z})$. It transforms the general 'charge vector'

$$\underset{\sim}{q} = (\vec{q}_L, \vec{q}_R)$$

$$5 + 5$$

leaving $\underset{\sim}{q}^2 \equiv \vec{q}_L^{\,2} - \vec{q}_R^{\,2}$ invariant.

Note that $\underset{\sim}{q}$ depends on (integer) #s of branes, and on the moduli of the compactification. Let us use $\vec{n}$ for the vector of 10 integer charges

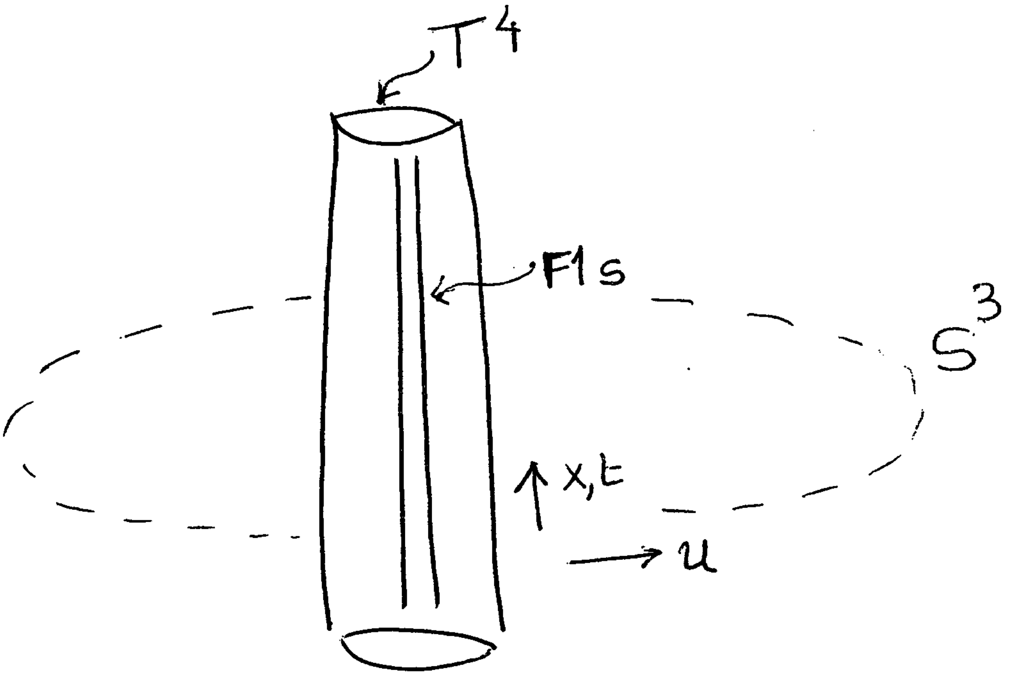
$\left(\text{cf } (n,m) \to \left(\frac{n}{2r} + mr,\ \frac{n}{2r} - mr\right)\right)$

↳ Any string with $\tilde{q}^2 \neq 0$ can be mapped to a configuration of

N_1 F1s , N_5 NS5-branes

and $\tilde{q}^2 = 2N_1N_5$. For large enough N_1, N_5 we can write a supergravity solution whose near-horizon geometry is:

$$AdS_3 \times S_3 \times T^4 \qquad \text{(WZW models)}$$



The dual CFT at the boundary has central charge $c = 6N = 6 N_1 N_5$

↳ the moduli space of the compactification is:

$$O(5,5;\mathbb{Z})\backslash O(5,5)/O(5)\times O(5)$$

Picking a particular vector $\vec{n}$ fixes some scalars in the nhg through the 'attractor mechanism', such that

$$\vec{q}_L = 0 \qquad (\text{'BPS' condition})$$

Thus the dual spacetime CFT has a smaller moduli space:

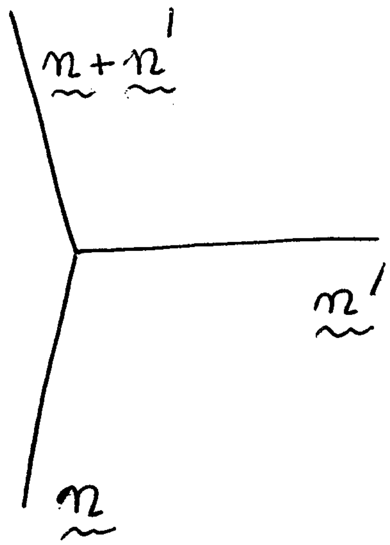
$$O(4,5;\mathbb{Z})\backslash O(4,5)/O(4)\times O(5)$$

↳ T-dualities of dual CFT

In fact its full moduli space contains several such fundamental domains, corresponding to different factorizations of $N_1 N_5$.

(cf Larsen + Martinec)

The strings of the theory can form (supersymmetric ?) junctions:



↳ Will restrict discussion to situation where:

$\underset{\sim}{n}$ has NS5s & F1s : heavy background

$\underset{\sim}{n}'$ has D1s & F1s : light probe

Since $\frac{1}{g_6^2} = \frac{N_1}{N_5}$ (one of the fixed scalars)

and $L^2 = N_5 \alpha'$

we need $N_1 \gg N_5 \gg 1$ to stay in supergravity limit.

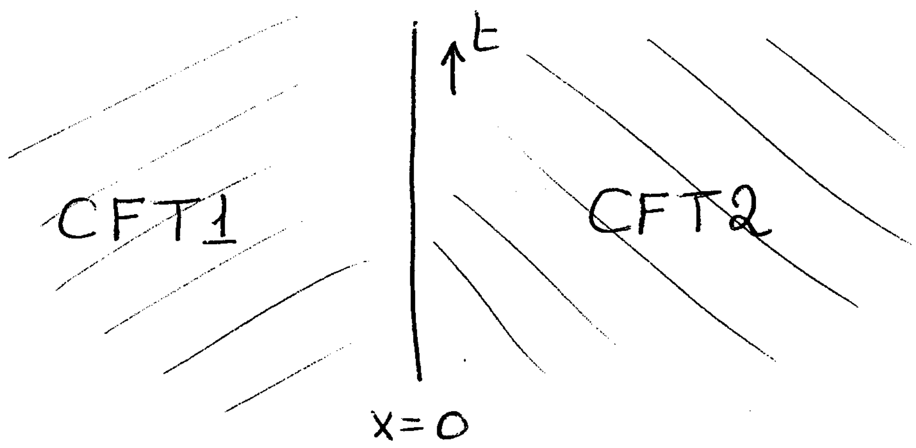
In the nhg of the background, the probe looks like one of our $AdS_2 \times S_2$ branes.

On either side of the junction one has a different (in general) CFT, with different c, and/or moduli.

$\therefore$ We need to understand the gluing of two CFTs. Since our branes are AdS_2, they preserve one (super) conformal invariance.

we proved it.

and so should the interfaces.



No net flow of energy to the boundary

$\Rightarrow$ T_{xt} continuous across defect

$$\Rightarrow \quad T^{(1)}_{++} - T^{(1)}_{--} = T^{(2)}_{++} - T^{(2)}_{--} \Big|_{x=0}$$

Let us first think about a free scalar with a discontinuous jump in radius:

$$S = 2r_1^2 \int_{x<0} (\partial_\mu \tilde{\varphi})^2 + 2r_2^2 \int (\partial_\mu \tilde{\varphi})^2$$

$$\tilde{\varphi} = \tilde{\varphi} + 2\pi$$

boundary conds:

$$r_1^2 \partial_x \tilde{\varphi} \Big|_{x=0_-} = r_2^2 \partial_x \tilde{\varphi} \Big|_{x=0_+}$$

Redefine:

$$\varphi = \begin{cases} r_1 \tilde{\varphi} & x<0 \\ r_2 \tilde{\varphi} & x>0 \end{cases}$$

to normalize the energy-momentum tensor

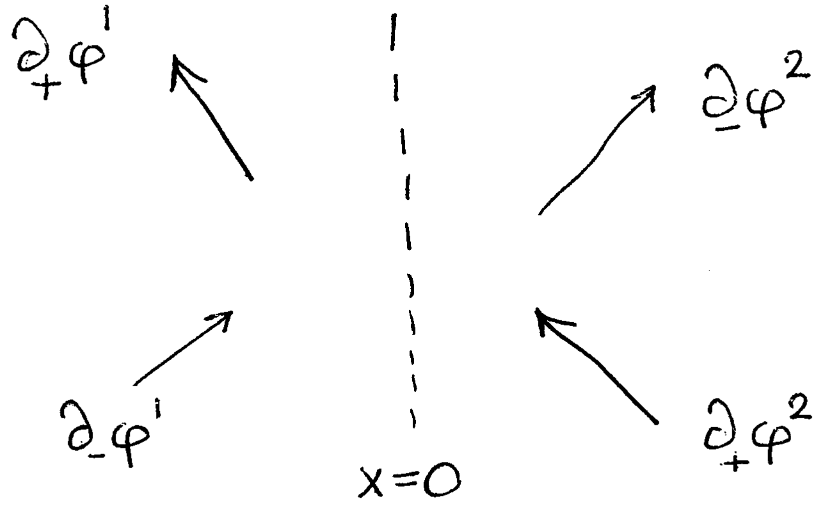
$$\Rightarrow \quad \begin{pmatrix} \partial_x \varphi \\ \partial_t \varphi \end{pmatrix}_{x=0_-} = \begin{pmatrix} \tan\theta & 0 \\ 0 & \cot\theta \end{pmatrix} \begin{pmatrix} \partial_x \varphi \\ \partial_t \varphi \end{pmatrix}_{x=0_+}$$

with $\boxed{\tan\theta = \frac{r_2}{r_1}}$

The reader can check easily continuity of $\partial_x \varphi \partial_t \varphi = T_{xt}$ across the wall.

Alternatively, define a S' matrix

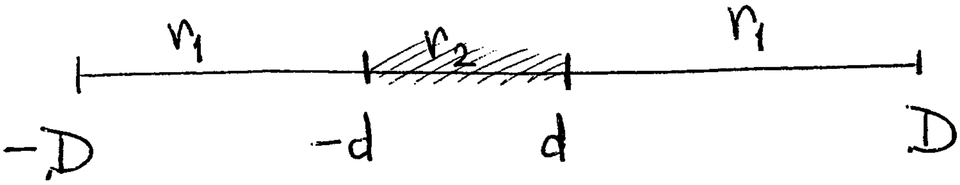
↳ do not confuse with modular transfo.



$$\begin{pmatrix} \partial_-\varphi^1 \\ \partial_+\varphi^2 \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \partial_+\varphi^1 \\ \partial_-\varphi^2 \end{pmatrix}$$

Ex: show this.

One simple observable is the Casimir energy associated with a bubble of radius r_2 in a space of radius r_1:



Plane-wave solns:

$$\varphi = e^{i\omega t} A_j \sin(\omega x + \delta_j)$$

$j = 1,2,3 \rightarrow$ three intervals

Impose b.cs $\Rightarrow$ transcendental eqn:

$$\tan[\omega(d-D)] = \tan^2\theta \cdot \tan[\omega d + O(\mathrm{mod}\ \pi/2)]$$

Solve in $D \to \infty$ limit, and find after regularizing the sum:

$$\mathcal{E} = \sum_n \tfrac{1}{2}\omega_n = -\frac{1}{8\pi d} \mathrm{Li}_2(\cos^2 2\theta)$$

where $\mathrm{Li}_2(z) = \sum_1^\infty \frac{z^m}{m^2} = \int_0^z \frac{\log(1-w)}{w} dw$

Check that:

$\underline{\theta = \pi/4}$ perfect transmission (no defect)
$\Rightarrow \mathcal{E} = 0$ ✓

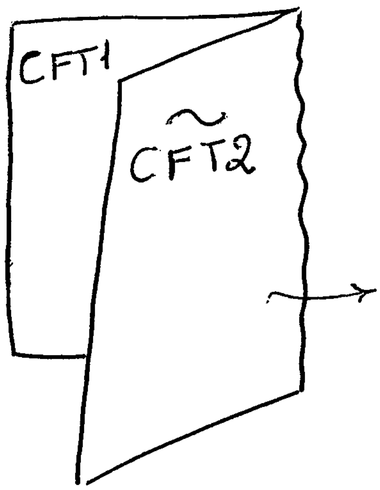
$\underline{\theta = 0, \pi/2}$ perfect reflection $(v_1 \gtrless v_2)$

$$\Rightarrow \mathcal{E} = -\frac{1}{8\pi d} \mathrm{Li}_2(1) = -\frac{\pi}{48 d}$$

standard Casimir energy of scalar confined

Folding trick

To describe defect lines using standard BCFT, we may fold the plane as follows:



exchange left & right movers

We now have a regular conformal boundary state in a tensor-product theory:

$$\text{CFT1} \otimes \widetilde{\text{CFT2}}$$

When this state can be written in terms of Ishibashi states of the individual theories, it satisfies

$$\left(L_n^{(1)} - \bar{L}_{-n}^{(1)}\right)|\text{state}\rangle\rangle = \left(L_n^{(2)} - \bar{L}_{-n}^{(2)}\right)|\text{state}\rangle\rangle = 0$$

$\Longleftrightarrow$ perfectly-reflecting boundary conditions.

Non-trivial defects are boundary states that cannot be written in terms of Ishibashi's of the factor theories.

Rest of this lecture is entirely based on:

hep-th/0111210