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*WATER BALANCE*

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## WATER BALANCE

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### 1. Introduction

The water cycling in a watershed or in a cropped field can be characterized and quantified by a water balance, which is the computation of all water fluxes at the boundaries of the system under consideration. It is an itemized statement of all gains, losses and changes of water storage within a specified volume element of soil. Its knowledge is of extreme importance for the correct water management in natural and agro-systems. It gives an indication of the strength of each component, which is important for their control and to ensure the utmost productivity with a minimum interference in the environment.

### 2. Volume Element and Components

Considering the whole physical environment of a field crop, we define a volume element of soil to establish the balance, having a unit surface area ( $1 \text{ m}^2$ ), and a height ranging from the soil surface ( $z = 0$ ) to the bottom of the root zone ( $z = L$ ), where  $z$  (m) is the vertical position coordinate. Water fluxes are considered only in the  $z$  direction, with exception to the runoff. It is, therefore, an unidirectional approach, which is a simplification that is best valid when the soil is fairly homogeneous.

Water fluxes are actually water flux densities, which corresponds to amounts of water that flow per unit of cross-sectional area and per unit of time. One convenient unit is liters of water

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per square meter per day, which corresponds to  $\text{mm.day}^{-1}$ . They are vectors, assumed positive when entering the volume element (gain), and negative when leaving (loss).

At the upper boundary, the soil surface ( $z = 0$ ), rainfall ( $p$ ) and irrigation ( $i$ ) are considered gains; evaporation ( $e$ ), transpiration ( $t$ ), or evapotranspiration ( $et$ ), are losses, and the runoff ( $r$ ) can be either a loss or a gain, depending on the water flow being into or out of the area considered for the balance.

At the lower boundary, the bottom of the root zone ( $z = L$ ), soil water fluxes ( $q_L$ ) can be gains (upward) or losses (downward).

Figure 1 is a schematic view of the volume element and of the fluxes that compose the balance.

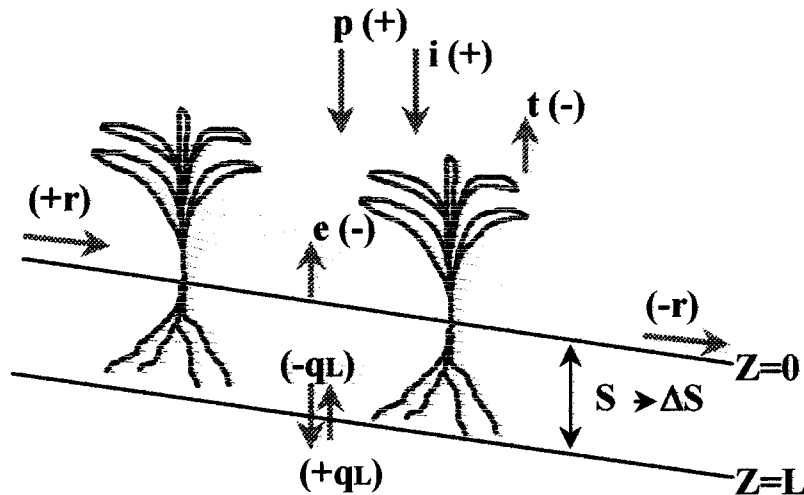


Figure 1: Schematic view of the volume element and of the fluxes that compose the water balance

### 3.The Balance

The balance is an expression of the mass conservation law, which can be written for the elemental volume as follows:

$$\sum f = \int_0^L \frac{\partial \theta}{\partial t} dz \quad (1)$$

where  $\theta$  is the soil water content ( $\text{m}^3 \cdot \text{m}^{-3}$ ),  $t$  the time (day) and  $f$  stands for the flux densities  $p$ ,  $i$ ,  $t$ ,  $e$  (or  $et$ ),  $r$  and  $q$ . The entrance or leave of the fluxes  $f$  in the elemental volume give rise to changes in soil water contents  $\partial\theta/\partial t$ , which integrated over the depth interval,  $z = 0$  and  $z = L$ , represent changes in soil water storage  $S$ . Therefore, equation (1) can be rewritten as:

$$p + i - et \pm r \pm q_L = \frac{\partial S}{\partial t} \quad (1a)$$

where  $S$  is defined by

$$S = \int_0^L \theta dz \quad (2)$$

Equation (1a) is an instantaneous view of the balance. When integrated over a time interval  $\Delta t = t_f - t_i$ , in days, yields amounts of water (mm):

$$\int_{t_i}^{t_f} (p + i - et \pm r \pm q_L) dt = \int_{t_i}^{t_f} \int_0^L \frac{\partial \theta}{\partial t} dz dt \quad (3)$$

or

$$P + I - ET \pm R \pm Q_L = \Delta S \quad (3a)$$

When all but one of the above components are known, the unknown is easily calculated algebraically. Five short examples are given below.

1. A soil profile stores 280 mm of water and receives 10 mm of rain and 30 mm of irrigation. It loses 40 mm by evapotranspiration. Neglecting runoff and soil water fluxes below the root zone, what is its new storage?

2. A soybean crop loses 35 mm by evapotranspiration in a period without rainfall and irrigation. It loses also 8 mm through deep drainage. What is its change in storage?
3. During a rainy period, a plot receives 56 mm of rain, of which 14 mm are lost by runoff. Deep drainage amounts to 5 mm. Neglecting evapotranspiration, what is the storage change?
4. Calculate the daily evapotranspiration of a bean crop which, in a period of 10 days, received 15 mm of rainfall and two irrigations of 10 mm each. In the same period, the deep drainage was 2 mm and the change in storage  $-5$  mm.
5. How much water was given to a crop through irrigation, knowing that in a dry period its evapotranspiration was 42 mm and the change in storage was  $-12$  mm? Soil was at field capacity and no runoff occurred during irrigation.

### SOLUTIONS

$n^{\circ}$	P	+	I	-	ET	$\pm R$	$\pm Q_L$	=	$\Delta S_L$	Answer
1	10		30		-40	0	0		0	280 mm
2	0		0		-35	0	-8		-43	-43 mm
3	56		0		0	-14	-5		+37	+37 mm
4	15		20		-38	0	-2		-5	$-3.8 \text{ mm.day}^{-1}$
5	0		30		-42	0	0		-12	+30 mm

The time interval  $\Delta t$  used to integrate equation (3) is the time interval over which the water balance is made. Its choice depends on the objectives of the balance. Intervals shorter than one day are seldomly used. Periods of 3, 7, 10, 15 days are very common, and larger ones are used in environmental studies.

## **4. Discussion of the Components**

### **4.1. Rainfall**

Rainfall is easily measured with simple rain gauges which consist of containers of a cross sectional area  $A$  ( $m^2$ ), which collect a volume  $V$  (liters) of rain, corresponding to a rainfall depth  $h$  (mm) equal to  $h = V/A$ . The problem in its measurement lies in the variability of the rain in space and time. In the case of whole watersheds, rain gauges have to be well distributed, following a scheme based on rainfall variability data. For the case of small experimental fields, attention must be given to the distance of the gauge in relation to the water balance plots. Reichardt et al. (1995) is an example of a rainfall variability study, carried out in a tropical zone, where localized thunder-storms play an important role.

### **4.2. Irrigation**

The measurement of the irrigation depth that effectively infiltrated into a given soil at a given area is not an easy task. Different methods of irrigation (sprinkler, furrow, drip, flooding, etc....) present great space variability which have to be taken into account.

### **4.3. Evapotranspiration (ET)**

Evapotranspiration can be measured independently or estimated from the balance, if all other components are known. In the first case, a great number of reports are found in the literature, covering classical methods like those proposed by Thornthwaite, Braney-Criddle and Penmann, which are based on atmospheric parameters such as air temperature and humidity, wind, solar radiation, etc. These methods have all their own shortcomings, mainly because they do not take into account plant and soil factors. Several models, however, include aspects of plant and soil, and yield much better results.

The main problem of estimating ET from the balance lies in the separation of the contribution of the components ET and  $Q_L$ , since both lead to changes in soil water storage  $\Delta S$ . One important thing is that the depth  $L$  has to be such that it includes the whole root system. If there are roots below  $z = L$ , ET is under estimated. If  $L$  covers the whole root system and  $Q_L$  is well estimated, which is difficult as will be seen below, ET can be estimated from the balance. Villagra et al. (1995) discuss these problems in detail.

#### 4.4.Runoff (R)

Runoff is difficult to be estimated since its magnitude depends on the slope of the land, the length of the slope, soil type, soil cover, etc. For very small slopes, runoff is in general neglected. If the soil is managed correctly, using contour lines, even with significant slopes runoff can be neglected. In cases it can not be neglected, runoff is measured using ramps, about 20 m long and 2 m wide, covering an area of 40 to 50 m<sup>2</sup>, with a water collector at the lower end. Again, the runoff depth  $h$  (mm) is the volume  $V$  (liters) of the collected water, divided by the area  $A$  (m<sup>2</sup>) of the ramp. Several reports in the literature cover the measurement of  $R$ , and its extrapolation to different situations of soil, slope, cover, etc. This is a subject very well considered in other opportunities of this College.

#### 4.5.Soil Water Fluxes at $z = L$ , $Q_L$

The estimation of soil water fluxes at the lower boundary  $z = L$ , can be estimated using Darcy-Buckingham's equation, integrated over the time:

$$Q_L = \int_{t_i}^{t_r} [K(\theta) \partial H / \partial z] dt \quad (4)$$



where  $K(\theta)$ , ( $\text{mm.day}^{-1}$ ), is the hydraulic conductivity estimated at the depth  $z = L$ , and  $\partial H/\partial z$  ( $\text{m.m}^{-1}$ ) the hydraulic potential head gradient,  $H$  (m) being assumed to be the sum of the gravitational potential head  $z$  (m), and the matric potential head  $h$  (m). Therefore it is necessary to measure  $K(\theta)$  at  $z = L$  and the most common procedures used are those presented by Hillel et al. (1972), Libardi et al. (1980), and Sisson et al. (1980). These methods present several problems, discussed in detail in Reichardt et al. (1998). The use of these  $K(\theta)$  relations involves two main constraints: (i.) the strong dependence of  $K$  upon  $\theta$ , which leads to exponential or power models, and (ii.) soil spatial variability.

Two commonly used  $K(\theta)$  relations are:

$$K = K_o \exp[\beta(\theta - \theta_o)] \quad (5)$$

and

$$K = a\theta^b \quad (6)$$

in which  $\beta$ ,  $a$  and  $b$  are fitting parameters,  $K_o$  the saturated hydraulic conductivity, and  $\theta_o$  the soil water content saturation. Reichardt et al. (1993) used model (5), and for 25 observation points of a transect on a homogeneous dark red latosol, obtained an average equation with  $K_{o\text{average}} = 144.38 \pm 35.33 \text{ mm.day}^{-1}$ , and  $\beta_{\text{average}} = 111.88 \pm 33.16$ . Assuming  $\theta_o = 0.442 \text{ m}^3.\text{m}^{-3}$ , the value of  $K$  is  $1.04 \text{ mm. day}^{-1}$  for  $\theta = 0.4 \text{ m}^3.\text{m}^{-3}$ . If this value of  $\theta$  has an error of 2%, which is very small for field conditions, we could have  $\theta$  ranging from 0.392 to  $0.408 \text{ m}^3.\text{m}^{-3}$ , and the corresponding values of  $K$  are: 0.43 and  $2.55 \text{ mm.day}^{-1}$ , with a difference of almost 500%. This example shows in a simple manner the effect of the exponential character of the  $K(\theta)$  relations. The standard deviations of  $K_o$  and  $\beta$ , shown above, reflect the problem of spatial variability. Added to this is the spatial variability of  $\theta$  itself.

#### 4.6.Changes in Soil Water Storage $\Delta S$

Soil water storage  $S$ , defined by equation (2) are, in general, estimated either by: (i.) direct auger sampling; (ii.) tensiometry, using soil water characteristic curves; and (iii.) using neutron probes. The direct sampling is the most disadvantageous due to soil perforations left behind after each sampling event. Tensiometry embeds the problem of the establishment of soil water characteristic curves, and neutron probes have calibration problems.

Once  $\theta$  versus  $z$  data at fixed times are available,  $S$  is estimated by numerical integration, the trapezoidal rule being an excellent approach, and in this case, equation (2) becomes:

$$S = \int_0^L \theta dz \cong \sum \theta \Delta z = \bar{\theta} L \quad (2a)$$

The changes  $\Delta S$  are simply the difference of  $S$  values obtained at different times.

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