Industrial innovations, R&D spillovers and knowledge accumulation[1]

Gian-Italo Bischi[a,*] , *Fabio Lamantia*[b]

[a] *Istituto di Scienze Economiche, Università di Urbino, Italy*

[b] *O.A.A.P. Department, Facoltà di Economia, Università della Calabria, Arcavacata di Rende, Italy.*

ABSTRACT. In this paper we propose a dynamic model of competition among firms, with optimal resource allocation in *R&D*, based on a game-theoretic approach. The effects of knowledge accumulated via *R&D* activities of the past and knowledge spillovers are considered. In the case of competition between two firms, we study the existence and stability of a Nash Equilibrium, as well as the effects of the main parameters on its basin of attraction.

Keywords: Rent seeking contest; Knowledge spillover; R&D; Stability; Basins of attraction.

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*Corresponding author.
Istituto di Scienze Economiche,Università di Urbino
via Saffi, 42
I-61029 URBINO (Italy)
tel.(+39) 0722 305553, fax (+39) 0722 305550
E-mail: *bischi@econ.uniurb.it*

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1 Introduction

It is now well known that competition among firms producing high tech goods is strongly influenced by their *Research and Development* ($R\&D$) activities. Indeed, a firm's optimal resource allocation in $R\&D$ is often necessary for introducing innovations before rivals or obtaining patents, with consequent competitive advantages and greater than rivals market shares. This kind of competition has been denoted as "$R\&D$ with rivalry" (see Reinganum, 1981, 1982). However, many authors stress that the effectiveness of $R\&D$ efforts is also related to the "knowledge capital stock", that is, the gradually accumulated knowledge via $R\&D$ activities of the past (see e.g. Nelson, 1982). Moreover, $R\&D$ activities often create knowledge which may spill from one firm to its competitors, due to the difficulty of protecting intellectual properties (see e.g. Spence, 1984, D'Aspremont and Jaquemin, 1988, Kultti and Takalo, 1998, Bischi and Lamantia, 2002, just to cite a few). The relations between "knowledge capital stock" and $R\&D$ activity, as well as the problem of public and private nature of the knowledge, have been clearly outlined by Nelson (1982).

In this paper, we propose a dynamic representation of competition with optimal decisions on $R\&D$ efforts occurring over time, together with an associated process of knowledge accumulation. The dynamic model is based on a game theoretic approach, in the spirit of rent seeking contests (Tullock, 1980; see also Pérez-Castrillo and Verdier, 1993, Okuguchi and Szidarovszky, 1999, Bischi et al., 2001), with competitive firms engaging $R\&D$ activities repeatedly over time and deciding at each time period optimal $R\&D$ efforts in order to maximize their expected profits, that also depend on their accumulated amount of knowledge capital stock.

2 A dynamic representation of accumulated knowledge with spillovers

Assume that there are n firms, indexed by $i = 1, ..., n$. Let $x_i(t)$ be a measure of $R\&D$ activity of firm i at time period t. The knowledge gain for firm i during time period t is assumed to be

$$X_i(t) = x_i(t) + \sum_{j \neq i} \vartheta_{ij} x_j(t) \tag{1}$$

where the parameters $\vartheta_{ij} \in [0, 1]$ are intended to capture knowledge spillovers from firm j to firm i. If $\vartheta_{ij} = 1$ then knowledge gained by firm j, via its $R\&D$ efforts, is fully shared by firm i (the case of $\vartheta_{ij} = 1$ for each $i \neq j$ corresponds to the assumption of public knowledge). At the other limiting situation, $\vartheta_{ij} = 0$, no knowledge from firm j is transferred to firm i (the case of $\vartheta_{ij} = 0$ for each i, j corresponds to the assumption of private companies that have proprietary rights to avoid any knowledge diffusion among competitors). The knowledge accumulated, up to period t, is given by

$$z_i(t) = \sum_{k=0}^{t} \rho^{t-k} X_i(k) \tag{2}$$

where the parameter $\rho \in [0, 1]$ represents a discount factor which "exponentially discounts older information". This captures the idea that firm's i accumulated knowledge, $z_i(t)$, is augmented by present $R\&D$ expenditures of effort, and the stock of knowledge may depreciate as well (Nelson, 1982). The value of ρ gives a measure of how rapidly information becomes obsolete: values close to 1 represent a system where even the results of very old $R\&D$ efforts are important at time t, whereas values close to 0 imply that only very recent $R\&D$ efforts give significant contributions to the present knowledge.

The knowledge capital stock can be obtained recursively as:

$$z_i(t) = X_i(t) + \rho \sum_{k=0}^{t-1} \rho^{t-1-k} X(k) = X_i(t) + \rho z_i(t-1) \tag{3}$$

i.e. the accumulated knowledge at time t is the sum of the knowledge $X_i(t)$, acquired during last round, and a discounted fraction of the knowledge gained in the past.

3 The rent-seeking contest

Many situations of competition among economic agents can be modelled as rent-seeking contests, in which the contestants expend effort to win a prize. For example, firms compete to obtain a procurement contract or to win a patent by investing in *R&D* (see e.g. Nitzan, 1994, for a survey of various applications). In the basic rent-seeking contest n players are confronted with the opportunity to win prize, usually fixed at a constant level R. If we let e_i denote the effort of player i to win the rent, then the probability that player i wins the rent is $p_i(e_1, ..., e_n)$, where $\sum_{i=1}^{n} p_i = 1$ and p_i is nondecreasing in x_i and nonincreasing in x_j, $j \neq i$. A commonly used form of the probability functions in rent-seeking games is the logit type (see Baik, 1994), which specifies that player i's probability for winning the rent is

$$p_i = \frac{f_i(e_i)}{\sum f_j(e_j)},$$

where $f_i(e_i), i = 1, ...n$ may be interpreted as the likelihood that player i wins the contest when she expends effort e_i. If all the contestants do not expend any effort, we just simply define $p_i = 1/n \; \forall i$. The players choose their efforts e_i trying to maximize their expected utility. In our model of "*R&D* with rivalry", we consider a rent seeking game where the probability of winning the game for the $t+1$ period is proportional to the *R&D* efforts $x_i(t+1)$, i.e. for firm i

$$p_i(t+1) = \frac{a_i x_i(t+1)}{a_i x_i(t+1) + \sum_{j \neq i} a_j x_j(t+1)}. \tag{4}$$

However, differently from standard rent seeking games, where the cost is just assumed to be proportional to x_i, here we assume that the cost is a decreasing function of the accumulated knowledge

$$C_i(t+1) = \frac{x_i(t+1)}{1 + \lambda_i z_i(t)} \tag{5}$$

That is, the cost of *R&D* at time $(t+1)$ is assumed to be proportional to the amount of *R&D* activities at time $(t+1)$ and inversely proportional to the knowledge stock (i.e. the knowledge accumulated up to period t). This assumption captures the idea that *R&D* efforts are a kind of investment, with an immediate effect, the associated cost for innovating, and the delayed beneficial consequence of a stronger knowledge base, which enhances the ability to research and makes technology advance. Hence, obtaining *R&D* results more efficiently lowers unitary costs. The parameters $\lambda_i \geq 0$, $i = 1, .., n$ can be interpreted as a measure of *efficiency* of the use of the acquired knowledge capital stock.

If the prize R is normalized to 1, then agent's i expected profit is

$$\pi_i^e(t+1) = \frac{a_i x_i(t+1)}{a_i x_i(t+1) + \sum_{j \neq i} a_j x_j^e(t+1)} - \frac{x_i(t+1)}{1 + \lambda_i z_i(t)} \tag{6}$$

where $x_j^e(t+1)$ represents the expected *R&D* effort at time $(t+1)$ of the competitor j. Notice that, according to (1) and (2), the accumulated knowledge z_i of firm i depends both on *R&D* efforts of firm i itself and of the competing firms through the spillover parameters. So, besides the interdependence caused by (4), as in standard rent seeking games, another source of interdependence is present, due to the positive cost externalities related to the role of accumulated knowledge with spillover effects. Of course, if $\lambda_i = 0$, $i = 1, ..., n$, a standard rent seeking game is obtained.

3.1 The case of two firms

In order to keep things as simple as possible, let us consider the case of two firms, i.e. $n = 2$. In the spirit of rent-seeking games, we assume that at every time t, in order to decide *R&D* effort for the next time period $t+1$, each player solves the optimization problem $\max\limits_{x_i(t+1)} \pi_i^e(t+1)$, $i = 1, 2$. So, firm i solves its maximization problem calculating

$$\frac{\partial \pi_i^e(t+1)}{\partial x_i(t+1)} = \frac{a_1 a_2 x_j^e(t+1)}{(a_i x_i(t+1) + a_j x_j^e(t+1))^2} - \frac{1}{1+\lambda_i z_i(t)} = 0 \quad i,j = 1,2; \;\; j \neq i$$

and solving for x_i we get the reaction function

$$x_i = r_i(x_j^e, z_i) = \frac{1}{a_i}\left(-a_j x_j^e + \sqrt{a_1 a_2 x_j^e\left(1+\lambda_i z_i\right)}\right), \quad i,j = 1,2; \;\; j \neq i \tag{7}$$

Indeed, the profit function reaches a maximum when calculated in the points of the reaction function (7), as it follows from a simple check on the second derivative. Under the assumptions of naive expectations, i.e. $x_j^e(t+1) = x_j(t)$, and that $a_1 = a_2 = a$, usually made in rent-seeking games, we get a dynamical system in the four dynamic variables $(x_1, x_2, z_1, z_2) \in \mathbb{R}^4$

$$\begin{cases} x_1(t+1) = -x_2(t) + \sqrt{x_2(t)\left(1+\lambda_1 z_1(t)\right)} \\ x_2(t+1) = -x_1(t) + \sqrt{x_1(t)\left(1+\lambda_2 z_2(t)\right)} \\ z_1(t+1) = -x_2(t) + \sqrt{x_2(t)\left(1+\lambda_1 z_1(t)\right)} + \vartheta_{12}\left(-x_1(t) + \sqrt{x_1(t)\left(1+\lambda_2 z_2(t)\right)}\right) + \rho z_1(t) \\ z_2(t+1) = -x_1(t) + \sqrt{x_1(t)\left(1+\lambda_2 z_2(t)\right)} + \vartheta_{21}\left(-x_2(t) + \sqrt{x_2(t)\left(1+\lambda_1 z_1(t)\right)}\right) + \rho z_2(t) \end{cases} \tag{8}$$

with initial conditions that, according to the definition (2) of the variables z_i, are given by

$$x_1(0);\;\; x_2(0); \quad z_1(0) = x_1(0) + \vartheta_{12} x_2(0); \quad z_2(0) = x_2(0) + \vartheta_{21} x_1(0) \tag{9}$$

i.e. the initial conditions of the four-dimensional dynamical system (8) must be taken in a two-dimensional submanifold (a plane) of $\mathbb{R}^4$ in order to obtain trajectories of the process we are modelling.

As usual in the study of nonlinear dynamical systems, the first step is the localization of the fixed points (or steady states) and the analysis of their stability. The fixed points of (8) are the solutions of the algebraic system obtained from (8) with $x_i(t+1) = x_i(t)$ and $z_i(t+1) = z_i(t)$, $i = 1, 2$. Of course, we are only interested in the fixed points with positive coordinates, which represent the Nash equilibria of the game. After some simple algebraic manipulations, the equations which give the equilibrium values of the two *R&D* efforts become

$$\begin{cases} x_1^2 + \left(1 - \frac{\lambda_1 \vartheta_{12}}{1-\rho}\right) x_2^2 + \left(2 - \frac{\lambda_1}{1-\rho}\right) x_1 x_2 - x_2 = 0 \\ \left(1 - \frac{\lambda_2 \vartheta_{21}}{1-\rho}\right) x_1^2 + x_2^2 + \left(2 - \frac{\lambda_2}{1-\rho}\right) x_1 x_2 - x_1 = 0 \end{cases} \tag{10}$$

with equilibrium values for the corresponding knowledge capital stocks given by $z_1 = (x_1 + \vartheta_{12} x_2)/(1-\rho)$ and $z_2 = (x_2 + \vartheta_{21} x_1)/(1-\rho)$. The equations (10) represent two conic-sections (ellipses, parabolas or hyperboles, according to the parameters) and their numerical solutions suggest that they have at most one positive intersection in the positive orthant of the (x_1, x_2) plane. Numerical simulations of the discrete dynamical system (8) can be easily performed in order to explore the influence of the parameters on existence and stability of equilibria, in particular the role of heterogeneities in *knowledge spillovers*, i.e. $\vartheta_{12} \neq \vartheta_{21}$, as well as the influence of the *memory parameter* ρ.

However, in order to obtain some analytical results we now assume that the two firms are identical (or homogenous) in the sense that are characterized by identical values of the parameters

$$\lambda_1 = \lambda_2 = \lambda; \vartheta_{12} = \vartheta_{21} = \vartheta. \tag{11}$$

The homogeneous case can be considered as a benchmark, since the analytical results that we obtain under assumption (11) can lead us to perform numerical explorations of the more general model (8) with heterogeneous players.

3.2 Existence and stability of the Nash Equilibrium with homogeneous firms

Under the assumption (11) the system (10) is symmetric, and the analysis can be carried out analytically. In fact it straightforward to see that a unique Nash equilibrium E exists, of the form

$$E = (x, x, z, z) \quad \text{with} \quad x = \frac{\rho - 1}{\lambda(1+\theta) + 4(\rho - 1)} \quad \text{and} \quad z = -\frac{1+\theta}{\lambda(1+\theta) + 4(\rho - 1)}$$

which is positive if the inequality

$$\lambda(1+\theta) + 4\rho - 4 < 0 \tag{12}$$

is satisfied. Condition (12) states that a Nash equilibrium exists provided that the parameters λ, θ and ρ are sufficiently small, i.e. knowledge does not accumulate too much or it does not decrease too much unitary cost. Notice that if $\rho \to 1$, then condition (12) is not satisfied. This is quite obvious, because the process we are describing can have a steady state, characterized by constant values of x_i and z_i, only if the rate at which new knowledge is gained via $R\&D$ equals the rate at which old knowledge is dissipated, so that stationariety is impossible without sufficiently high knowledge dissipation.

In order to study the stability properties of the equilibrium E, we must evaluate the eigenvalues of the Jacobian matrix of the map (8) computed at the equilibrium. Under assumption (11) of homogeneous players, the computation of the eigenvalues gives $\eta_1 = 0$; $\eta_2 = 0$; $\eta_3 = 0.25\,(1+\theta)\,\lambda + \rho$; $\eta_4 = 0.25\,(1-\theta)\,\lambda + \rho$. It is straightforward to see that if E exists, i.e. the condition (12) holds, then $0 \leq \eta_i < 1$ for each $i = 1, ..., 4$, and the following proposition can be stated:

Proposition. *If* (11) *holds and* $\lambda(1+\theta) + 4\rho - 4 < 0$ *then a unique Nash equilibrium exists and is asymptotically stable.*

Condition (12) clearly shows that the existence of E is favored by decreasing values of λ, θ and ρ. Moreover, smaller values of λ and ρ increase the speed of convergence to the stable equilibrium, since both η_3 and η_4 decrease if λ and ρ are decreased. Notice also that the convergence is nonoscillatory around E, because the eigenvalues are real and non negative.

3.3 Basin of attraction of the Nash equilibrium

From the discussion of the previous section one may argue that decreasing values of the parameters λ and ρ have not only the effect of helping the existence of E, but also to reinforce its stability. However, if we compare stability in terms of the extension of the basin of attraction, i.e. if we consider more robust an equilibrium when it has a larger basin of attraction, then different conclusions may follow. We have performed this analysis through a series of numerical studies of the basin of E. For example, in fig. 1a, obtained with parameters $\lambda = 0.3$, $\theta = 0.5$ and $\rho = 0.5$, the white region represents the basin of attraction of the equilibrium E, i.e. the set of the initial conditions (9) starting from which trajectories converging to E are generated, whereas the grey region represent the set of initial conditions which generate diverging trajectories (i.e. the process does not settle to any equilibrium and $R\&D$ efforts grow up indefinitely). In the situation shown in fig. 2a, obtained with the same parameters θ and ρ, but with $\lambda = 0.1$, a smaller basin of E is obtained. This is confirmed by many other numerical explorations, from which we can state that even if decreasing values of λ favors the existence of E and also the speed of convergence, an opposite effect is obtained on its robustness measured in terms of the extension of the basin. A similar effect is observed for the parameter θ, whereas changes in the parameter ρ seem to have no remarkable effects on the basin.

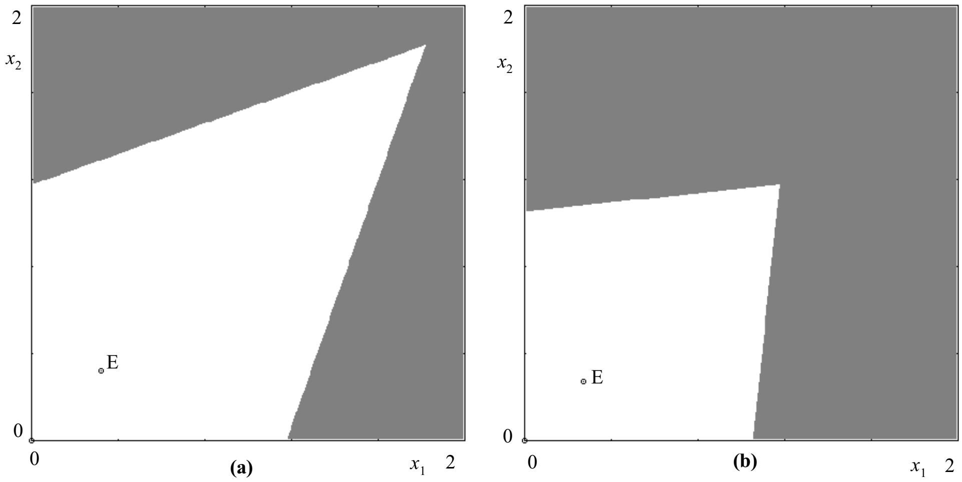

Figure 1

4 Further studies

Numerical explorations of the model (8) can be made even in the case of heterogenous players, through a computation of the equilibria, by solving the algebraic system (10), followed by a computation of the eigenvalues of the Jacobian matrix and a numerical estimate of the basin of E. By properly tuning the *spillover parameters* ϑ_{ij} we can study the effects of different balances between public and private knowledge, including cases of asymmetric spillovers, and, by tuning the *memory parameter* ρ, we can simulate different degrees of knowledge depreciation.

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