A New Model of Industry Dynamics

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Abstract

This paper discusses externalities experienced by firms in making production decisions in an economy, or a sector of an economy, by specifying the types of conditional probabilities for entries and exists of additional production units, or entries of production units of entirely new goods. Such conditional probability specifications may result from assumptions on firms adjusting their output rates in response to expected excess demand or profit changes in environments where such excess demands or profits are functions of own output rates, as well as those of other firms.

We utilize the notion of holding times in the continuous-time Markov chain models to determine movements of outputs around an equilibrium distributions of sizes of clusters of agents of different types. After a short comment on how entries and exits are modeled in the standard economic literature, we present an alternative approach using continous-time Markov processes. We employ one- and two-parameter versions of the conditional probability specifications, due to Ewens and Pitman respectively, in the context of exchangeable partitons of existing total amounts of production capacities.

In addition to the usual state vector which lists the number of agents by types, we explain the use of partition vector as state vector in models composed of a large number of exchangeable agents of possibly many types. We illustrate the use of this new state vector by describing the equilibrium distribution of agents by types in partition vector form. We suggest that this approach is also useful in agent-based simulations.

Introduction

Economists often face problems of modeling collective behavior of a large number of interacting agents, possibly of several different types. Models are then used to explain such things as equilibrium size distributions of firms, market shares by different types or kinds of goods, and so on, and finally how

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some macroeconomic regularities emerge as the number of agents increases towards infinity.

To date, economists have not been very successful in modeling dynamic or distributional phenomena in economics when markets involve several types of agents. We explicitly assume that there are several types of agents in our models, the number of which may or may not be known in advance, and that models are open, that is, agents may enter or exit the models at any time. In addition, agents may change their minds at any time about the decisions or behavioral rules they use. In other words, agents may change their types any time.[1]

In this paper, we interpret the word "types" broadly. They may refer to the decision rules or behavioral rules adopted by economic agents, or to the kinds of goods they choose to produce. We assume that the number of types are at most countable. We cannot assume in advance that we know all of them because new rules or new goods may be invented in the future. This is the so-called problem of unanticipated knowledge in the sense of Zabell, see Zabell (1992).[2]

We discuss some new concepts not much used in economics in this paper. For example, we introduce the notion of partition vector as state vectors, which is different from the empirical distributions, and use the asssumption of exchangeable partitions induced by agents of different types in the models, in the technical sense of exchangeable random variables in the probability literature. We utilize the notion of holding times from the literature on continous-time Markov chain (also called jump Markov process) to decide randomly which agents act first. We apply the equilibrium distribution discovered by Ewens in the context of population genetics literature into that of cluster sizes of agents by types. We show how to use these concepts help us in modeling dynamics of interacting agents of several types.

The Ewens sampling formula is specified by a single parameter θ, which controls the rate of entries of new types, and correlations among agents of different types. We also describe its two-parameter extension by Pitman (1992), which is specified by two parameters, α and θ discussed later.

We first illustrate the approach of the traditional economists by sketching how the problem of allocating capital stock between two sectors is formulated by Dixit (1989). After briefly mentioning some problems of this traditional approach, we switch to our modeling procedure in terms of continuous-time Markov chains with a large but finite number of interacting agents.

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[1] There is no lock-step behavior by agents.

[2] Zabell describes the problem faced by statisticians in classifying samples of insects collected in unexplored regions, since they may contain new species of insects, say. The naive Bayesian approach is not applicable. See, however, Antoniak (1974) on non-parametric Bayesian approach. He obtained the same distribution as the Ewens sampling formula, Ewens (1972), which was discovered by Ewens in connection with the population gentics work.

A traditional approach to model two-sector economy

In 1989 Dixit has analyzed several economic problems, such as that of how to optimally allocate capital stocks among two sectors, and of assessing the effects of exchange rate changes to induce entries or exits of firms in some export industry.

The reader should note that what Dixit derives are the entry and exit price schedules, that is, the price as a function of the number of firms in one sector, which triggers a move by one more firm from one sector to the other, or move into or out of export business when n firms are already in one sector or in the export business. He is silent about the decision processes of individual firms, that is, he does not say which firms enter or exit next. In other words, the decision problem he solves is that of the economic planner of a centrally controlled economy, and not decision problems of individual firms in a market of a decentralized economy. In spite of random prices his approach is basically deterministic.

For simpler explanations, assume that the economy is closed, that is, the total number of firms is fixed at N. All firms are assumed to be indistinguishable, and choose between two alternatives of either producing goods one or goods two.[3] Suppose that there are n firms producing goods two, and $N - n$ firms producing goods one. The prices of the two goods are normalized to be 1 and P, that is, P is the price of goods two in terms of goods one. It depends on n, but this dependence is suppressed for simpler notation.

The economy as a whole receives profit (net revenue) (per unit period) of

$$R(P, n) = PF(n) + G(N - n),$$

where F and G are the production function of sector 2 and 1 respectively, given as functions of the number of firms. If P is sufficiently high, then it is more profitable for firms in sector one to move to sector two. There is an entry cost associated with changing sectors

$$h(P, n) = h_0[G(N - n + 1) - G(N - n)].$$

This assumes that the move takes one period to complete, and that the cost of moving is proportional to the lost production because one firm in sector one shuts down its production to start producing goods two. In a move in the opposite direction, the loss of move is given by

$$l(P, n) = l_0[F(n) - F(n - 1)].$$

Here h_0 and l_0 are some positive constants.

Let $V(P, n)$ be the value function of this economy. It is the discounted expected present value of the stream of profits when the economy (a central planner) optimally allocates the firms between the two sectors. Denoting

[3] This is a problem of birth-death or a binary choice model, see Aoki (1996).

the discount rate by ρ, it is given by

$$\rho V(P,n) = R(P,n) + E(\frac{dV}{dt}).$$

This expression is a typical one in finance. Regarding the expected value of the economy as financial asset, the right-hand side, which is the sum of the revenue and the capital gain (loss) term, must equal to the return from holding the asset on the left-hand side.

Now, one firm moves from sector one to two when P satisfies

$$V(P, n-1) = V(P,n) - h(P,n),$$

and

$$V_P(P, n-1) = V_P(P,n) - h_P(P,n).$$

Denote this P by P_n^+. The first equation states that at this critical or switch-over price, the values with $n-1$ and n minus the cost of moving are the same. The second equation is a technical gradient-matching condition without which the assumption that V is the optimal value is violated. The other switch-over price P_n^- at which a firm moves back to sector one from sector two is similarly expressed. Dixit assumed that P moves as a Brownian process, and solved for V, and then derives these prices. When $P(t)$ cuts the schedule P_n^+ from below, then a move of firm from sector one to two occurs, that is n changes to $n+1$. Similarly for a move from n to $n-1$ which occurs at price P_n^-.

What are some of the objections to this analysis? First, there is no explanation about which of the $N-n$ firms decide to move. On the macroeconomic level, it does not matter when firms are truly indistinguishable, including managerial abilities. As to firm managers' decision problems, how do they decide that it is their turn to move? Does the central planneer of this economy choose one firm, by lining up the firms in the order of marginal productivity or some such measure? Perhaps all firms are lined up linearly, and they enter one by one as P monotonically increase. But, firms are assumed to be indistinguishable. There is apparently no uncertainty as to which firm enters next, or exit next. This may only be possible in a planned economy. Problems of imperfect or incomplete information and externalities among firms (agents) are cleverly hidden or abstracted away in his analysis.

An Alternative Approach: Basic Setup

Aoki and Yoshikawa (2001), and Aoki (2002, Chapt. 8) are examples of some alterntive approaches to that sketched above. Basically, our approach focuses on the random partitions of the set of firms into clusers induced by subsets formed by firms of the same types, and utilizes the conditional probability specifications for new entries and exits to derive equilibrium distributions for cluster sizes. We use the master equation (backward Chapman-Kolomorov equation) as the dynamic equation for the probabilities of state vectors.[4]

Given the total number of agents, N, and the number of possible types, K, both of which are assumed in this paper to be known and finite for ease of explanation, we examine how the N-set, that is, the set $\{1, 2, \ldots, N\}$ is partitioned into K clusters, or subsets. This partition is treated as a random exchangeable partition in the sense of Zabell (1992). We do not discuss here the situation with K tending to infinity or infinite. See Kingman (1993, 1978a,b) who used the order statistics of the fractions of agents by types, and invented what is known as the paint-box process and the resultant Poisson-Dirichlet distribution to solve this problem. In this exposition we mostly keep K finite, but large.

Jump Markov Process Models

Here we follow Aoki (1996, 1998, 2000a,b,c, 2002) and sketch the basic ingredients for our modeling procedure without too much detail. The reader is asked to consult the cited references for detail.

Excluding pathological phenomenon of an infinitely many jumps in a small time interval, continuous-time Markov chains, also known as jump Markov processes, are specified by transition rates.

Define a state vector X_t which takes on the value $\mathbf{n} := (n_1, n_2, \ldots, n_K)$, called frequency or occupancy vector, where n_i is the number of agents of type i, $i = 1, 2, \ldots, K$, $N = n_1 + n_2 + \cdots + n_K$.

In our model we need to specify entry rates, exit rates and rates of type changes. Over a small time interval Δ, rates are multiplied by the length of interval to approximate the conditional probabilities up to $O(\Delta)$. Entry rates by an agent of type j is given by

$$w(\mathbf{n}, \mathbf{n} + \mathbf{e_j}) = \phi_j(n_j, \mathbf{n}),$$

where $\mathbf{e}_j$ is a vector with the only nonzero element of one at component j.[5] Exit rates of an agent of type k specified by

$$w(\mathbf{n}, \mathbf{n} - \mathbf{e_k}) = \psi_k(n_k, \mathbf{n})$$

and transition rates of type i agent changing into type j agent by

$$w(\mathbf{n}, \mathbf{n} - \mathbf{e_i} + \mathbf{e_j}) = \lambda_{i,j}\nu(n_i, n_j, \mathbf{n}).$$

With transition rates between states specified, the dynamics for the probability is given by the following equation, where $\mathbf{s}$, $\mathbf{s}'$, and $\mathbf{s}''$ refer to some states

$$dP(\mathbf{s}, t)/dt = \sum_{\mathbf{s}'} w(\mathbf{s}', \mathbf{s}) P(\mathbf{s}', t)$$

[4] In the above two-sector model the scalar variable of the number of firms in sector one, say, serves as the state variable. In an open model with K sectors, a K-dimensional vector is used.

[5] For example, $w(\mathbf{n}, \mathbf{n} + \mathbf{e_j})\Delta \approx Pr(X_{t+\Delta} = \mathbf{n} + \mathbf{e_j})$.

$$-\sum_{\mathbf{s}''} w(\mathbf{s}, \mathbf{s}'')P(\mathbf{s}, t).$$

This is called the master equation in phyics, ecology and chemistry, and we follow their usage of the name.

A special example of interest has the transition rates:

$$w(\mathbf{n}, \mathbf{n} + \mathbf{e}_k) = c_k(n_k + h_k),$$

for $n_k \geq 0$,

$$w(\mathbf{n}, \mathbf{n} - \mathbf{e}_j) = d_j n_j,$$

$n_j \geq 1$, and

$$w(\mathbf{n}, \mathbf{n} - \mathbf{e}_j + \mathbf{e}_k) = \lambda_{jk} d_j n_j c_k (n_k + h_k),$$

with $\lambda_{jk} = \lambda_{kj}$, and where $j, k = 1, 2, \dots K$. We assume that $d_j \geq c_j > 0$, and $h_j > 0$, and $\lambda_{jk} = \lambda_{kj}$ for all j, k pairs.

The first transition rate specifies entry rate of type k agents, and the second that of the exit or departure rate by type j agents and the last specifies the transition intensity of changing types by agents from type j to type k. In the entry transition rate specification $c_k n_k$ stands for attractiveness of larger group, such as network externality which makes it easier for others to join the cluster or group, and $c_k h_k$ stands for the innovation effects which is independent of the group size. These transition rates for type changes are in Kelly (1979). We need interactions or correlations among agents. It turns out that parameter θ, to be introduced in connection with (2) below, plays this role. See Aoki (2000a, 2002b). The jump Markov process thus specified has the steady state or stationary distribution

$$\pi(\mathbf{n}) = \prod_{j=1}^{K} \pi_j(n_j),$$

where

$$\pi_j(n_j) = (1 - g_j)^{-h_j} \begin{pmatrix} -h_j \\ n_j \end{pmatrix} (-g_j)^{n_j}$$

where $g_j = c_j / d_j$.

These expressions are derived straightforwardly by applying the detailed balance conditions to the transition rates. See Kelly (1979, Chapt.1) for example.

To provide simpler explanation, suppose that $g_j = g$ for all j. Then, noting that $\prod_j (1-g)^{-h_j} = (1-g)^{-\sum_j h_j}$, the joint probability distribution is expressible as

$$\pi(\mathbf{n}) = \begin{pmatrix} -\sum h_k \\ n \end{pmatrix}^{-1} \prod_{j=1}^{K} \begin{pmatrix} -h_j \\ n_j \end{pmatrix}. \tag{1}$$

Another class of interesting transition rates arise by applying what is called the Johnson's sufficientness hypothesis[6] in the statistical literature.

[6] Johnson's sufficientness postulte stipulates that the conditional probability that the next agent which enters is of type i, given the current state vector, is $f(n_i, n)$, that is, a function of the existing number of agents of type i and that of the total number of agents in the model. See Zabell (1982).

In modeling industrial sector with n_i being the number of agents of type i, the word type may refer to the kinds of goods being produced by firm i or n_i may refer to the size of the "production line", that is, a measure of capacity utilization by firm producing typ i good. Zabell (1982) proved that under the assumption of exchangeable partitions the functional form of f is specified by

$$f(n_i, n) = \frac{n_i}{n+\theta}, \tag{2}$$

with some positive scalar parameter θ. Therefore, the entry rate of a new type is given by $\theta/(n+\theta)$. More generally, they are of the form

$$w(\mathbf{n}, \mathbf{n}+\mathbf{e}_k) = \frac{\alpha + n_k}{K\alpha + n},$$

which reduces to (2) in the limit of α going to zero, and K to infinity while their product approaches θ, and

$$w(\mathbf{n}, \mathbf{n}-\mathbf{e}_j) = \frac{n_j}{n}.$$

See Costantini (1979, 2000), and Zabell (1982) for circumstances under which these transition rates arise. See Aoki and Yoshikawa (2001) and Aoki (2002, Sec.8.6) for an application of this type of transition rates in models of economy or sectors of economy.

Partition Vectors and Ewens Distribution

Now, we introduce the partition vector $\mathbf{a} = (a_1, a_2, \ldots, a_n)$, so called by Zabell,[7] where a_k is the number of types or clusters with exctly k agents. Consequently we have an inequality

$$\sum_i a_i = K_n \leq K,$$

where K_n is the number of groups or clusters formed by n agents, and

$$\sum_i i a_i = n,$$

which is an accounting identity.

In the occupancy problems we count the number of different ways for placing distinguishable or indistinguishable balls into boxes, where boxes are assumed to be distinguishable or indistinguishable, that is, unlabelled. See Feller (1957, p. 36). As we mentioned earlier, partitions are induced by the clusters which contain agents of the same types. When boxes are unlabelled, the partition vector describes their state.

To further simplify our presentation, let us suppose that $h_j = h$ for all j.[8] Then

$$\pi(\mathbf{n}) = \binom{-Kh}{n}^{-1} \prod_{j=1}^{K} \binom{-h}{j}^{a_j}.$$

[7] This vector is called by different names by Kingman (1980), and Sachkov (1996).

[8] This assumption is not necessary. All we need is that the product Kh_j has the same positive limit as K goes to infinity and h_j to zero.

This is so because there are a_j of the ns which equal j.

Now suppose that K becomes very large and h very small, while the product Kh approaches a positive constant θ. We note that the negative binomial expression

$$\begin{pmatrix} -h \\ j \end{pmatrix}^{a_j}$$

approaches $(h/j)^{a_j}(-1)^{ja_j}$ as h becomes smaller. Suppose $K_n = k \leq K$. Then, there are

$$\frac{K!}{a_1!a_2!\cdots a_n!(K-k)!}$$

many ways of realizing **a** vector. Hence

$$\pi(\mathbf{a}) = \begin{pmatrix} -\theta \\ n \end{pmatrix} (-1)^n \frac{K!}{a_1!a_2!\cdots a_n!(K-k)!} \prod_j (\frac{h}{j})^{a_j}. \tag{3}$$

Noting that $K!/(K-k)! \times h^k$ approaches θ^k in the limit of K becoming infinite and h approaching 0 while keeping Kh at θ, we arrive, in the limit, at the probability distribution, known as the Ewens distribution, or Ewens sampling formula very well known in the genetics literature, Ewens (1972), and Kingman (1987).

$$\pi_n(\mathbf{a}) = \frac{n!}{\theta^{[n]}} \prod_{j=1}^{n} (\frac{\theta}{j})^{a_j} \frac{1}{a_j!},$$

where $\theta^{[n]} := \theta(\theta+1)\cdots(\theta+n-1)$. This distribution has been investigated in several ways. See Arratia and Tavaré (1992), or Hoppe (1987). Kingman (1980) states that this distribution arise in many applications. There are other ways of deriving this distribution. We next examine some of its properties.

The number of clusters and value of θ

Ewens sampling formula has a single parameter θ. Its value influences the number of clusters formed by the agents. Smaller values of θ tends to produce a few large clusters, while larger values produce a large number of smaller clusters.

To obtain some quick feels for the influences of the value of θ, take $n = 2$ and $a_2 = 1$. All other as are zero. Then

$$\pi_2(a_1 = 0, a_2 = 1) = \frac{1}{1+\theta}.$$

This shows that two randomly chosen agents are of the same type with high probability when θ is small, and with small probability when θ is large. In fact, θ controls correlation between agents' types or classification. Furthermore, the next two expreme situations may convey the relation between the value of θ and the number of clusters. We note that the probability of n agents forming a single cluster is given by

$$\pi_n(a_j = 0, 1 \leq j \leq (n-1), a_n = 1) = \frac{(n-1)!}{(\theta+1)(\theta+2)\cdots(\theta+n-1)}$$

while the probability that n agents form n singleton is given by

$$\pi_n(a_1 = n, a_j = 0, j \neq 1) = \frac{\theta^{n-1}}{(\theta+1)(\theta+2)\cdots(\theta+n-1)}.$$

With θ much smaller than one, the former probability is approximately equal to 1, while the latter is approximately equal to zero. When θ is much larger than n the opposite is approximately true.

We can show that

$$P_n(K_n = k) = \frac{1}{\theta^{[n]}} c(n,k)\theta^k,$$

where $c(n,k)$ is known as the signless Stirling numbers of the first kind, and is defined by

$$\theta^{[n]} = \sum_1^n c(n,k)\theta^k.$$

See Hoppe (1987) for the derivation. Stirling numbers are discussed in van Lint and Wilson (1992, p.104) for example.

Two-parameter generalization of the Ewens distribution

Pitman (1992) generalized the Ewens' distribution by useing the transition rates

$$w(\mathbf{n}, \mathbf{n} + \mathbf{e_j}) = \frac{n_j - \alpha}{n + \theta},$$

where $\theta + \alpha > 0$.

With this, the conditional probability that a new type enters in the next Δ time interval is approximately given by $\frac{K_n\alpha+\theta}{n+\theta}\Delta$. Pitman also derived the equilibrium distribution for this two-parameter version.

There are interpretations in terms of what is called size-biased sampling of these, but we will not stop here to explain them. See Kingman (1992), and Pitman (1992, 1995)

Clusters in partition vector

We cite some examples from Kelly (1979)as being suggestive of other applications to economic modeling.[9] There are N basic units partitioned into distinct clusters or collections, with a_i being the number of groups consisiting of i units. Recall that we mean by units some basic building blocks from which objects that cluster are made up.

The transition rate $w(\mathbf{a}, \mathbf{a} + e_i) = \alpha$ represents the process in which a basic unit or singleton (called isolate in Kelly (1979, chapt.8)) joins a group of size i at rate α, $i = 1, 2, \ldots$. The transition rate $w(\mathbf{a}, \mathbf{a} - e_i) = \beta$ refers to

[9] This subsection is based in part on Kelly (1979, Chap. 8). We use a_i rather than m_i used in Kelly.

the rate at which a singleton (one basic unit) leaves that group to become an isolate. Call a cluster of size i i-cluster.

We assume $\mathbf{a}$ is a Markov process in which the transition rate $w(\mathbf{a}, \mathbf{a} - e_1 - e_i + e_{i+1}) = \alpha a_1 a_i$, $i \geq 2$. This refers to the rate at which an isolate joins an i-cluster, hence forming one more $i+1$-cluster. When two isolates form a new group of size 2, the transition rate is $w(\mathbf{a}, \mathbf{a} - 2e_1 + e_2) = \alpha a_1(a_1 - 1)$. The rate at which an i-cluster breaks up into an isolate and a cluster of size $i-1$ is represented by $w(\mathbf{a}, \mathbf{a} + e_1 + e_{i-1} - e_i) = i\beta a_i$, $i \geq 2$. The transition rate $w(\mathbf{a}, \mathbf{a} + 2e_1 - e_2) = 2\beta a_2$ refers to one cluster of size 2 divides into 2 isolates. In a more general setting, suppose that the transition rate of one r-cluster and one s-cluster form one u-cluster. It is written as

$$w(\mathbf{a}, \mathbf{a} - e_r - e_s + e_u) = \lambda_{rsu} a_r a_s,$$

when $r \neq s$. With $r = s$, we specify the transition rate by

$$w(\mathbf{a}, \mathbf{a} - 2e_r + e_u) = \lambda_{rru} a_r (a_r - 1),$$

and

$$w(\mathbf{a}, \mathbf{a} - e_u + e_r + e_s) = \mu_{rsu} a_u$$

is the transition rate of one u-cluster breaking up into one r-cluster and one s-clustter. In the simple example described above, we have

$$\lambda_{1,i,i+1} = \alpha,$$

and

$$\mu_{1,i-1,i} = i\beta,$$

for $i \geq 2$.

Assume that $\lambda_{rsu} = \lambda_{sru}$ and $\mu_{rsu} = \mu_{sru}$. We check the detailed balance conditions and verify that the equilibrium distribution is of the form

$$\pi(\mathbf{a}) = B \prod_r \frac{c_r^{a_r}}{a_r!},$$

provided there are positive numbers $c_1, c_2, \ldots,$ such that

$$c_r c_s \lambda_{rsu} = c_u \mu_{rsu}.$$

We can easily verify that the detailed balance conditions are satisfied. In the simple example, we note tht

$$c_r = \frac{\beta}{\alpha r!}$$

satisfies the detailed balance conditions.

For a closed model with N fixed, the equilibrium distribution

$$\pi(\mathbf{a}) = B \prod_{i=1}^{N} \frac{1}{a_i!} (\frac{\beta}{\alpha i!})^{a_i}$$

is an example of assemblies analyzed by Arratia and Tavaré. Here $m_i = (\beta/\alpha)$ serves as the "number" of labelled structures on a set of size i, that is the number of the labelled structure in this example is independent of the size of the block.

In the more general development that follow the example, if we set

$$c_r = \frac{m_r}{r!}$$

then m_r is the number of the labelled structures on a set of size r.

Concluding Remarks

This paper proposes a finitary approach to economic modeling, that is to start with a finite number of agents with discrete choice sets, and with explicit transition rates. It discusses several entry and exit transition rates in economic models. In particular, it presented Ewens and related distributions as candidates for distributions of cluster sizes formed by a large number of economic agents who interact in a market. This distribution seems to be very useful in economic modelings, although we have only a few examples so far. However, see Arratia, Barbour and Tavaré (1992), and Kingman (1980). These and other investigations strongly suggest that the Ewens' and related distributions are robust and ubiquitous.

Although no application is described in this paper, Aoki (2002a, 2002b) has one simple application in which stocks of a holding company is traded by a large number of agents. With $\theta = .3$, two largest groups are shown to capture nearly 80 per cent of the market shares and hence dominate the market excess demands for the shares, which in turn determine the stationary distributions of price. In this way it is also possible to relate the tail distribution of the market clearing prices with entry and exit assumptions.

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