Heterogeneous Agents and Volatility Persistence in Real Returns

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Abstract

This paper examines the conditions for volatility pesistence in real returns in heterogeneous agent models with borrowing constraints, capital accumulation, production, and idiosyncratic endowment shocks. When agents are "sufficiently heterogeneous", through different endowment shocks volatility persistence in real returns emerges when capital accumulation is introduced, even if adjustment costs are relatively small.

JEL classification: E43, G12

Key words: volatility persistence, learning, heterogeneous agents, adjustment costs

1 Introduction

The significance of heterogeneous agent models for explaining key macroeconomic facts has led to calls for a "requiem" for models based on the single "representative agent". In particular, Carroll (2000) has drawn attention to heterogeneity in agents as a key ingredient for explaining the aggregate marginal propensity to consume as well as skewed wealth distribution. Earlier, Krusell and Smith (1998) found that a "small amount" of heterogenity succeeds in replecating key features of wealth data.

Recent research with heterogeneous agent models has focused on the presence of borrowing constraints to explain another key macroeconomic fact, namely, volatility clustering or persistence in real returns. Den Haan (1997, 1999), den Haan and Spear (1998), and Zhang (2000) all examine the issue of volatility persistence in short-term real interest rates, in an "incomplete markets" framework with a stochastic aggregate endowment process, idiosyncratic income shocks and borrowing constraints. In these models, risk aversion is identical among agents.[1]

However, all three papers leave out capital accumulation, production and growth in their environments with limited borrowing/lending opportunities. Huggett (1997) introduced heterogeneous agents into a Brock-Mirman stochastic growth framework with idiosyncratic endowment shocks and a continuum of agents, but he did not consider question of volatility persistence in real returns.

Rouwenhorst (1995) acknowledged that attempts to explain many asset-pricing phenomena in models with non-trivial production have proved to be less than successful. The reason is simple: fluctuations in consumption can always be reduced by altering production plans, so asset prices need not change due to borrowing or lending pressures.

Jermann (1997) incorporates both habit persistence and adjustment costs of capital into a model with production and a single representative consumer. He finds that both habit persistence and adjustment costs are needed to replicate historical equity premia. His reasoning is direct: with no habit persistence,

[1] Research with these models has also focused on the well-known equity premium "puzzle" first posed by Mehra and Prescott (1985). Heaton and Lucas (1996) add transactions costs as well as short-sales constraints along with borrowing limits into their model with heterogeneous agents.. They find that either large transactions costs or a limited quantity of tradable assets can produce a sizeable equity premium. This approach is in sharp contrast with that of Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999). These authors argue for "habit formation" in individual preferences in addition to the standard time-separable utility function for explaining the equity premium puzzle in models with a single representative agent.

people do not care about sharp fluctuations in consumption, and with no adjustment costs, they can smooth consumption by altering production plans.

This paper examines volatility persistence in heterogeneous agent framework. Analogous to Jermann's findings, without heterogeneity among agents, either through different degrees of risk aversion or different endownment shocks, there would be little reason to trade, but without adjustment costs for productive capital, individual agents would be able to smooth production by altering their production plans. Introducing adjustment costs allows volatility persistence to emerge. However, this result should not be very surprising. After all, the models which produce volatility persistence without capital accumulation may be interpreted as special cases of capital accumulation with very high adjustment costs. There is no need to introduce habit persistence. Perhaps "heterogeneity among agents" and habit perisistence are operationally equivalent, in their ability to deliver similar results for asset returns, when they are combined with adjustment costs.

Once capital accumulation and production play a role in the model, volatility persistence in longer-term returns, rather than short-term rates, takes center stage. Figure 1 pictures the behavior of the real long-term aaa corporate bond yield since 1947.

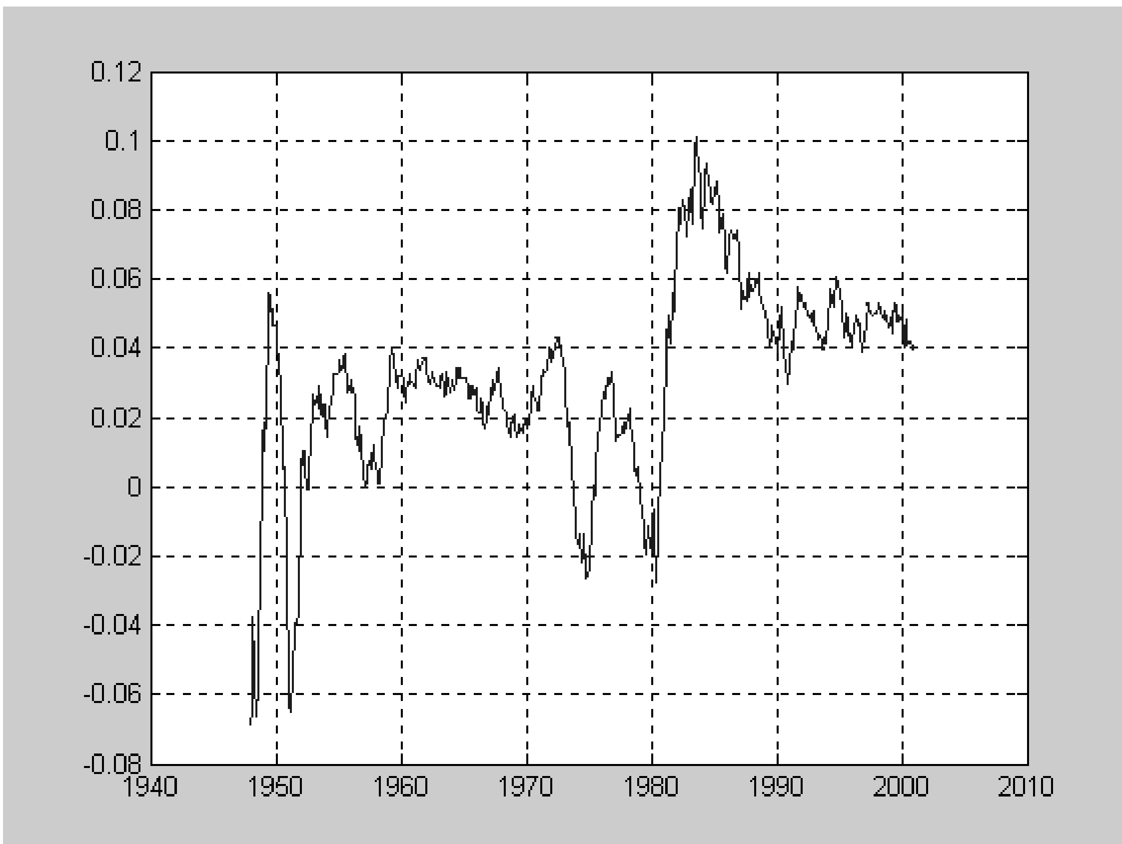


Figure 1: Real Corporate Bond Yield

Table I lists the relevant statistical information as well as the GARCH propoerties of the long term bond yeild pictured in Figure 1.

Statistical Properties of Real Returns, 1948-2000

Mean	.0311
Standard Deviation	.0283
Skewness	-.6330
Kurtosis	4.4363

GARCH Model for First-Differenced Real Bond Returns

$$\mathrm{r}_t = r_{t-1} + \epsilon_t$$
$$\epsilon_t = N(0, \sigma_t^2)$$
$$\sigma_t^2 = \beta_0^2 + \beta_1^2 \widehat{\epsilon}_{t-1}^2 + \beta_2^2 \sigma_{t-1}^2$$

	Esimates	/T-Statistics
β_0	.0008	1.724
β_1	.3491	3.117
β_2	.9065	13.664

Table 1

Negative skewness and excess kurtosis are well-documented phemonenona in empirical asset-pricing studies. The GARCH behavior is statistically significant for the first-differenced real returns.[2]

The next section describes the model to be used for this analysis as well as the solution algorithm. The third section assesses the results and the final section concludes.

2 The Model

The framework of this paper first follows the incomplete markets framework of den Haan and Spear (1998), and then adds capital accumulation. Then adjustment costs and learning enter the framework.

2.0.1 den Haan and Spear Framework

The usual constant relative risk aversion (CRRA) utility function characterizes the preferences of each agent or household:

$$U(c_t^i) = \frac{(c_t^i)^{1-\sigma_i}}{1-\sigma_i} \tag{1}$$

where σ_i is the coefficient of relative risk aversion for agent i..

Each maximizes the following intertemporal discounted utility function over an infinite horizon:

$$E\left[\sum_{t=0}^{\infty} \beta^t U(c_t^i)\right] \tag{2}$$

with $0 < \beta < 1$.

Each agent faces the following budget constraint:

$$c_t^i + b_t^i = e_t^i + (1 + r_{t-1}^l) b_{t-1}^i \text{ when } b_{t-1}^i \geq 0 \tag{3}$$
$$c_t^i + b_t^i = e_t^i + (1 + r_{t-1}^b) b_{t-1}^i \text{ when } b_{t-1}^i < 0 \tag{4}$$
$$e_t^i = s_t^i \cdot e_t \tag{5}$$
$$\sum_{i=1}^{N} b_t^i = 0 \tag{6}$$

[2] Evans (1998) has called attention to the role of instability in underlying long-run fundamental processes for explaining much of the volatility in asset prices. In particular, Evans argues that instability in the dividend and discount-rate process contribute "significantly" to the predictability of long-horizon asset returns. He conjectures that such instability may be due to systematic forecasting errors by agents.

where s_t^i is the share of agent i at time t of aggregate endowment e_t, b_t^i is a risk-free one-period bond held by agent i at time t, and r_{t-1}^l, r_{t-1}^b are the lending and borrowing rates at time t-1. Aggregate lending or borrowing sums to zero.

Each agent's effective labor endowment depends on two processes: the overall labor endowment process $\{e_t\}$, as well as the share of each agent, $\{s_t^i\}$ in the overall endowment:

$$\begin{aligned} s_t^i &= \left[\frac{1}{N}\right](1-\rho_s) + \rho_s s_{t-1}^i + \epsilon_t^i, i = 1..N-1 &(7)\\ s_t^N &= 1 - \sum_{i=1}^{N-1} s_t^i &(8)\\ \epsilon_t^i &\sim N(0, \sigma_s^2) &(9)\\ \Delta \ln(e_t) &= c_e + \rho_e \Delta \ln(e_{t-1}) + \epsilon_t^e &(10)\\ \epsilon_t^e &\sim N(0, \sigma_e^2) &(11)\end{aligned}$$

where N is the number of agents.

The amount of debt of any agent is assumed to be limited to the present value of their endowment:

$$b_t^i \geq -\frac{\overline{b} e_t^i}{1+r^b} \tag{12}$$

The spread between the borrowing and lending rate is a function of the level of debt and past interest, the current income of the agent, and the rate of growth of aggregate endowment:

$$r_t^b - r_t^l = \omega_0 \left(\frac{(1+r_{t-1}^b)|b_t^i|}{e_t}\right)^{\omega_1} \left(\overline{s}_t^i\right)^{-\omega_2} \left(\overline{e}_t^i\right)^{-\omega_3} \tag{13}$$

where the variables $\overline{s}_t^i$,$\overline{e}_t^i$ are monotone transformations of s_t^i, e_t^i for obtaining values between zero and one.

2.1 Adding Capital and Production

Extending the model for capital and production opportunities requires the budget constraints to be amended in the following way:

$$\begin{aligned} c_t^i + b_t^i + k_t^i &= w_t e_t^i + r_t k_{t-1}^i + (1-\delta)k_{t-1}^i + (1+r_{t-1}^l)b_{t-1}^i \text{ when } b_{t-1}^i \geq 0 &(14)\\ c_t^i + b_t^i + k_t^i &= w_t e_t^i + r_t k_{t-1}^i + (1-\delta)k_{t-1}^i + (1+r_{t-1}^b)b_{t-1}^i \text{ when } b_{t-1}^i < 0 &(15)\end{aligned}$$

The variables w_t and r_t represent the real wages and real returns on productive capital at time t. There is a single firm that operates the technology, with marginal productivity conditions for wages and capital returns based on aggregate capital and labor, k_t and e_t:

$$\begin{aligned} f(k,e) &= Ak^\alpha e^{1-\alpha} &(16)\\ w &= f_e = (1-\alpha)Ak^\alpha e^{-\alpha}\\ r &= f_k = \alpha A k^{\alpha-1} e^{1-\alpha}\\ k &= \sum_{i=1}^N k^i\\ e &= \sum_{i=1}^N e^i\end{aligned}$$

2.2 Adding Capital and Production with Adjustment Costs

When adjustment costs are added for capital accumulation for each agent, the budget constraints are amended as follows:

$$
\begin{aligned}
c_t^i + b_t^i + k_t^i &= w_t e_t^i + r_t k_{t-1}^i + (1-\delta) k_{t-1}^i + \frac{\phi_K}{2}\left(k_t^i - k_{t-1}^i\right)^2 + (1+r_{t-1}^l) b_{t-1}^i && (17)\\
\text{when } b_{t-1}^i &\geq 0 && (18)\\
c_t^i + b_t^i + k_t^i &= w_t e_t^i + r_t k_{t-1}^i + (1-\delta) k_{t-1}^i + + \frac{\phi_K}{2}\left(k_t^i - k_{t-1}^i\right)^2 (1+r_{t-1}^b) b_{t-1}^i && (19)\\
\text{when } b_{t-1}^i &< 0 && (20)
\end{aligned}
$$

where adjustment costs are captured by the quadratic relation, $\frac{\phi_K}{2}\left(k_t^i - k_{t-1}^i\right)^2$,with a fixed adjustment coefficient ϕ_K.

3 Solution Methods

The solution method we use is the parameterized expectations algorithms (PEA), presented in den Haan. and Marcet (1990a), Marcet (1988, 1993), and Marcet and Lorenzoni (1998). This method is first used in the case of full rationality and then in the case of "bounded rationality" with last squares learning.

3.1 Parameterized Expectations with Fully Rational Heterogeneous Agents

The Euler equation for the den Haan and Spear pure endowment economy, or for bonds in the production economy, takes the following form for the "lending" agent i:

$$\left(c_t^i\right)^{-\sigma_i} = \beta \mathbf{E}\left[\left(c_{t+1}^i\right)^{-\sigma_i}(1+r_t^l)\right] \tag{21}$$

In the extended production economy, without adjustment costs, the Euler equation for capital has the following form:

$$\left(c_t^i\right)^{-\sigma_i} = \beta \mathbf{E}\left[\left(c_{t+1}^i\right)^{-\sigma_i}(1+r_t-\delta)\right] \tag{22}$$

By arbitrage, the lending rate is equal to the net return on capital:

$$r_t^l = r_t - \delta \tag{23}$$

Finally, in the case of a production economy with adjustment costs, the Euler equation has the form:

$$\left(c_t^i\right)^{-\sigma_i}\left[1+\phi_K\left(k_t^i - k_{t-1}^i\right)\right] = \beta \mathbf{E}\left[\left(c_{t+1}^i\right)^{-\sigma_i}\left\{1+r_t-\delta-\phi_K\left(k_{t+1}^i - k_t^i\right)\right\}\right] \tag{24}$$

By arbitrage, the gross lending rate is equal to the net return on capital, less adjustment costs:

$$(1+r_t^l) = \frac{1+r_t-\delta-\phi_K\left(k_{t+1}-k_t\right)}{1+\phi_K(\phi_K\left(k_t-k_{t-1}\right)} \tag{25}$$

To solve the Euler equation for the optimal decision rule for each agent, we make use of parameterized expectations, extensively analyzed by Marcet (1988, 1993), and den Haan and Marcet (1990).

The Euler equation is parameterized for each agent in the following way:

$$\left(c_t^i\right)^{-\sigma} = \beta \Psi^i(x_t^i; \gamma^i) \tag{26}$$

where the functional form Ψ^i is a neural network, with instrument set x_t^i,and parameters γ^i.

Each agent forms expectations on the basis of observing personal consumption and labor endowments, as well as aggregate endowment in the den Haan/Spear framework. Hence, $\mathrm{x}_t^i = [c_t^i, e_t^i, e_t]$. In the extended model with production, each agent forms expectations on the basis of observing personal captial, personal labor endowment, as well as aggregate capital and aggregate labor endowment. Thus $x_t^i = [k_t^i, e_t^i, k_t, e_t]$.

The neural network specification of the expectations function $\Psi^i(k_t^i, e_t^i, K_t; \gamma^i)$ has the following form:

$$
\begin{aligned}
n_{l,t}^i &= \sum_{j=1}^{J^*} b_j^i x_{j,t}^i \qquad (27)\\
N_{l,t}^i &= \frac{1}{1+e^{-n_{l\circ,t}^i}}\\
\widehat{\Psi_t^i} &= \sum_{l=1}^{L^*} \kappa_l^i N_{l,t}^i
\end{aligned}
$$

where J* is the number of exogenous or input variables, K* is the number of neurons, n_t^i is a linear combination of the input variables, N_t^i is a logsigmoid or logistic transformation of n_t^i, and $\widehat{\Psi_t^i}$ is the neural network prediction at time t of $E\left[\left(c_{t+1}^i\right)^{-\sigma_i}(1+r_{t+1}-\delta\right]$ for agent i, summarized by the function $\Psi^i(k_t^i, e_t^i, K_t; \gamma^i)$, with the parameter set $\{\gamma^i\} = \{b_j^i, \kappa_k^i\}, j = 1, ..., J^*, l = 1, ..., L^*$.

As seen in this equation, the only difference from ordinary non-linear estimation relating "regressors" to a "regressand" is the use of the hidden nodes or neurons, N. One forms a neuron by taking a linear combination of the regressors and then transforming this variable by the logistic or logsigmoid function. One then proceeds to thus one or more of these neurons in a linear way to forecast the dependent variable $\hat{\psi}_t$.

Sargent (1997) has shown that the neural network specification does a better job of "approximating" any non-linear function than polynomial approximations, in that sense that a neural network achieves the same degree of in-sample predictive accuracy with fewer parameters than a polynomial approximation, or achieves greater accuracy than a polynomial one, using the same number of parameters.

The main choices that one has to make for a neural network is L*, the number of hidden neurons, for predicting a given variable Ψ^i Generally, a neural network with only one hidden neuron closely approximates a simple linear model, whereas larger numbers of neurons approximate more complex non-linear relationships. Obviously, with a larger number of neurons in the hidden layer of the network, one may approximate progressively more complex non-linear phenomena, but at the cost of an increasingly larger parameter set.

The approach of this study is to use relatively simple neural networks, between two and four neurons, in order to show that even relatively simple neural network specifications do well for approximating non-linear relations implied by forward-looking expectations in stochastic dynamic general equilibrium models.

Each agent solves the optimization problem for γ^i for each agent in order to minimize the sum of squares of the following error metric:

$$
\underset{\{\gamma^i\}}{Min} \sum_{t=1}^{T} [\varsigma_t^i]^2 \qquad (28)
$$

$$
\varsigma_t^i = \beta \Psi_t^i(k_t^i, e_t^i, K_t; \gamma^i) - \beta E\left[\left(c_{t+1}^i\right)^{-\sigma_i}(1+r_t^l)\right] \text{ for } b_t^i \geq 0 \qquad (29)
$$

$$
\varsigma_t^i = \beta \Psi_t^i(k_t^i, e_t^i, K_t; \gamma^i) - \beta E\left[\left(c_{t+1}^i\right)^{-\sigma_i}(1+r_t^b)\right] \text{ for } b_t^i < 0 \qquad (30)
$$

The error function is minimized, subject to the following constraints:

$$
\begin{aligned}
K_t &> 0 \qquad (31)\\
c_t^i &> 0 \qquad (32)\\
b_t &= \sum_i |a_t^i| - K_t \qquad (33)\\
b_t^i &\geq -\frac{\overline{b} e_t^i}{1+r^b} \qquad (34)
\end{aligned}
$$

where b_t represents aggregate borrowing at time t. Individuals are net borrowers if their asset holding are less than zero.

Marcet and Singleton (1998) discuss the computational procedure for parameterized expectations solutions to problems with constraints on asset holdings. The approach they suggest is to first solve the model as if all agents are unconstrained by the limits on their portfolio transactions. Secondly, if an agent borrows too much, the relevant asset position is set at its limits. They note that once these steps are completed, "the possibility of being constrained in the future affects agents' decisions today", even if the debt limits are not currently binding [Marcet and Singleton (1998), p. 15].

Since the parameterized expectation solution is a relatively complex non-linear function, the optimization problem is solved with a repeated hybrid approach. First a global search method, genetic algorithm, similar to the one developed by Duffy and McNelis (2001), is used to find the initial parameter set $\{\gamma^i\}$, then a local optimization, the BFGS method, based on the quasi-Newton algorithm, is used to "fine tune" the genetic algorithm solution.

De Falco (1998) applied the genetic algorithm to nonlinear neural network estimation, and found that his results "proved the effectiveness" of such algorithms for neural network estimation.. The main drawback of the genetic algorithm is that it is slow. For even a reasonable size or dimension of the coefficient vector, the various combinations and permutations of the coefficients which the genetic search may find "optimal" or close to optimal, at various generations, may become very large. This is another example of the well-known "curse of dimensionality" in non-linear optimization. Thus, one needs to let the genetic algorithm "run" over a large number of generations—perhaps several hundred—in order to arrive at results which resemble unique and global minimum points.

Quagliarella and Vicini (1998) point out that hybridization may lead to better solutions than those obtainable using the two methods individually. They argue that it is not necessary to carry out the quasi-Newton optimization until convergence, if one is going to repeat the process several times.. The utility of the quasi-Newton BFGS algorithm is its ability to improve the "individuals it treats", so "its beneficial effects can be obtained just performing a few iterations each time" [Quagliarella and Vicini (1998): 307].

4 Calibration

Table II lists the parameter configuration we use in the baseline simulations of the model.

Table II: Parameter Specification

Discount Rate	$\beta = .99486$
Production and Depreciation	$A = 1, \alpha = .36, \delta = .025$
Borrowing Limits	$\overline{b}= -1$
Risk Aversion	$\sigma^i = 1.5,\ i = 1, 2.$
Share process	$\rho_s= .91, \sigma_s= .022$
Endowment process	$c_e = .001536, \rho_e = .62, \sigma_a = .001$
Spread parameters	$\{\omega_i\}= [.03, 1, 1, 0], [.03, 1, 1, 2]$

The parameter specification is similar to previous studies. The production and depreciation parameters come from Jermann (1997), while the other parameters are identical to those used by den Haan and Spear (1998).

The sample size for the model is 2000. The number of neurons set for each agent is three.

5 Simulation Results

This section first takes up the results for the den Haan/Spear framework, followed by the extended models: capital with no adjustment costs, capital with adjustment costs, and capital with learning and low adjustment costs.

5.1 den Haan/Spear Model

The evolution of the borrowing rate on loans in the model without capital appears in Figure 2.

Fiugre 2: Borrowing Rate in den Haan-Spear Model

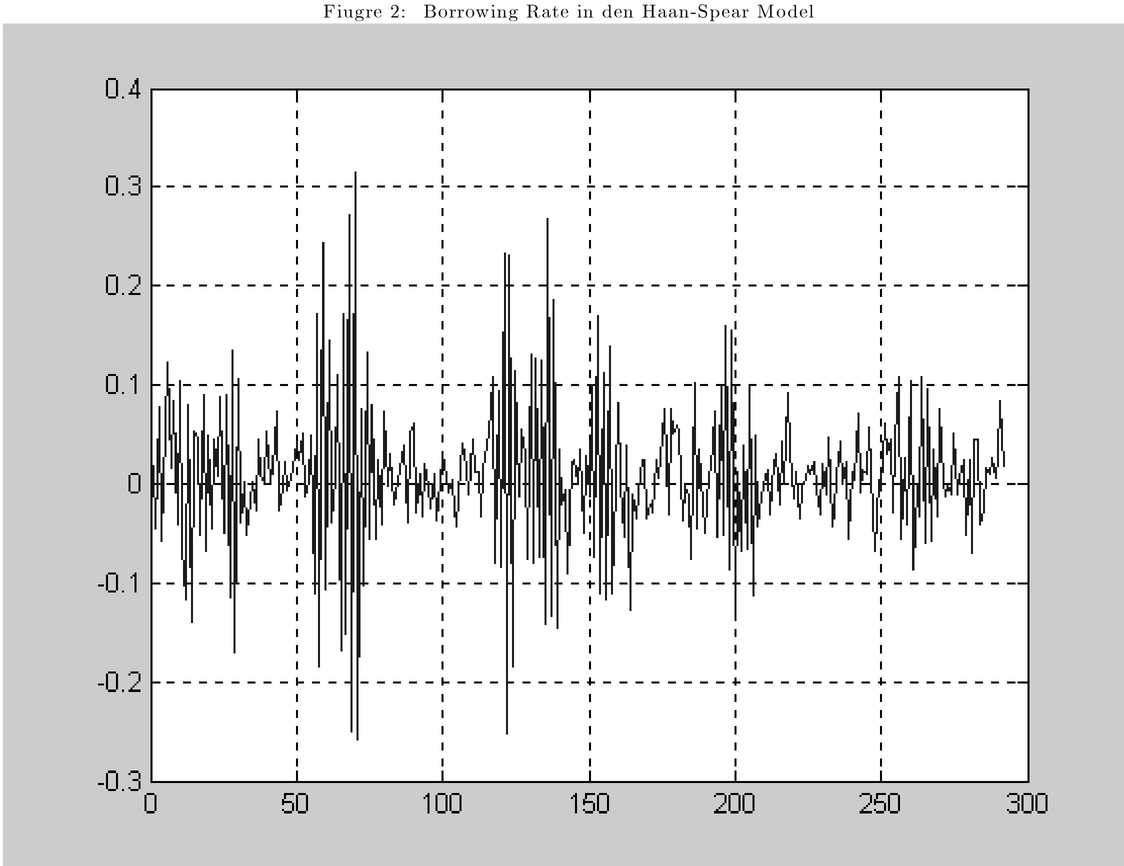


Figure 3: Conditional Variance of Differenced Returns in den Haan-Spear Model

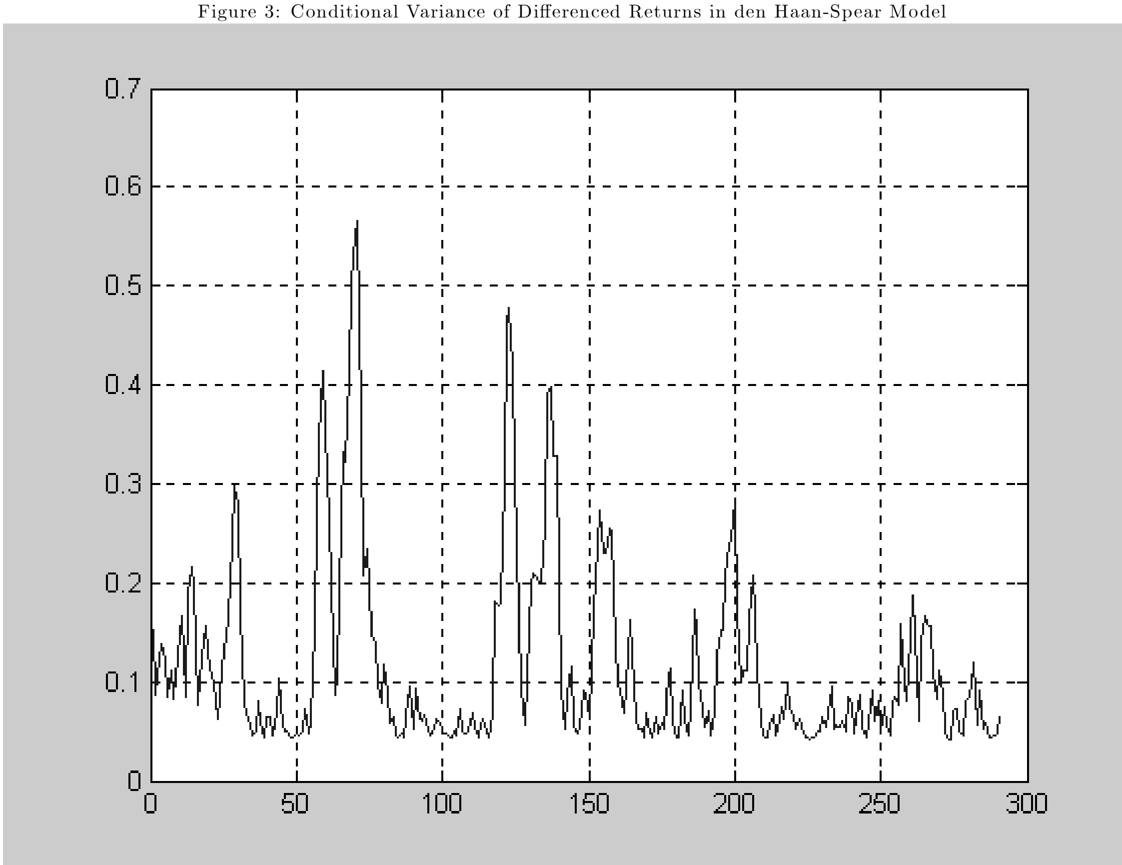


Table 2 summarizes the statistical properties of the asset returns in the den Haan - Spear model.

Statistical Properties of Real Returns in den Haan - Spear Model

Mean	.008
Standard Deviation	.0796
Skewness	.1877
Kurtosis	5.0932

GARCH Model for First-Differenced Real Returns in den Haan - Spear Model

$$\mathrm{r}_t = r_{t-1} + \epsilon_t$$
$$\epsilon_t = N(0, \sigma_t^2)$$
$$\sigma_t^2 = \beta_0^2 + \beta_1^2 \widehat{\epsilon}_{t-1}^2 + \beta_2^2 \sigma_{t-1}^2$$

	Esimates /T-Statistics	
β_0	.0016	2.953
β_1	.1084	1.376
β_2	.8682	4.700

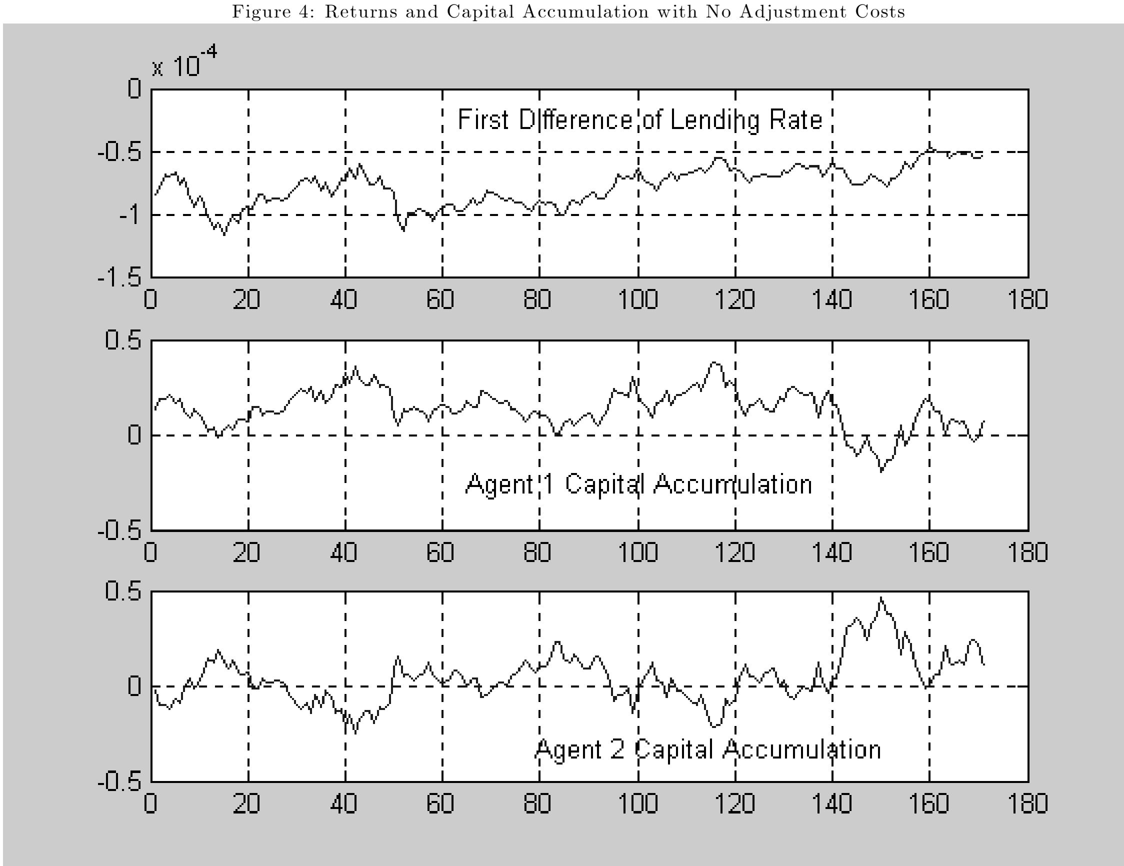
Table 2

While the den Haan-Spear model replicates well the excess kutosis of long-term real returns, it does not deliver negative skewness. While the GARCH coefficient is close to the actual garch coefficient, the arch coefficient is insignificant.

5.2 Capital Accumulation with No Adjustment Costs

Figure 4 pictures the first difference of the return on loans as well as the accumulation of capital of the two agents, when capital accumulation is introduced into the model with no adjustment costs.

Figure 4: Returns and Capital Accumulation with No Adjustment Costs



As should be clear from this figure, there is little or no volatility persistence in the first difference of real returns. Once capital accumulation is introduced, the volatility is "transferred" from real returns to capital accumulation itself, since there are no costs for adjusting these costs.

The statistical properties of the first difference of the real returns appear in Table 3.

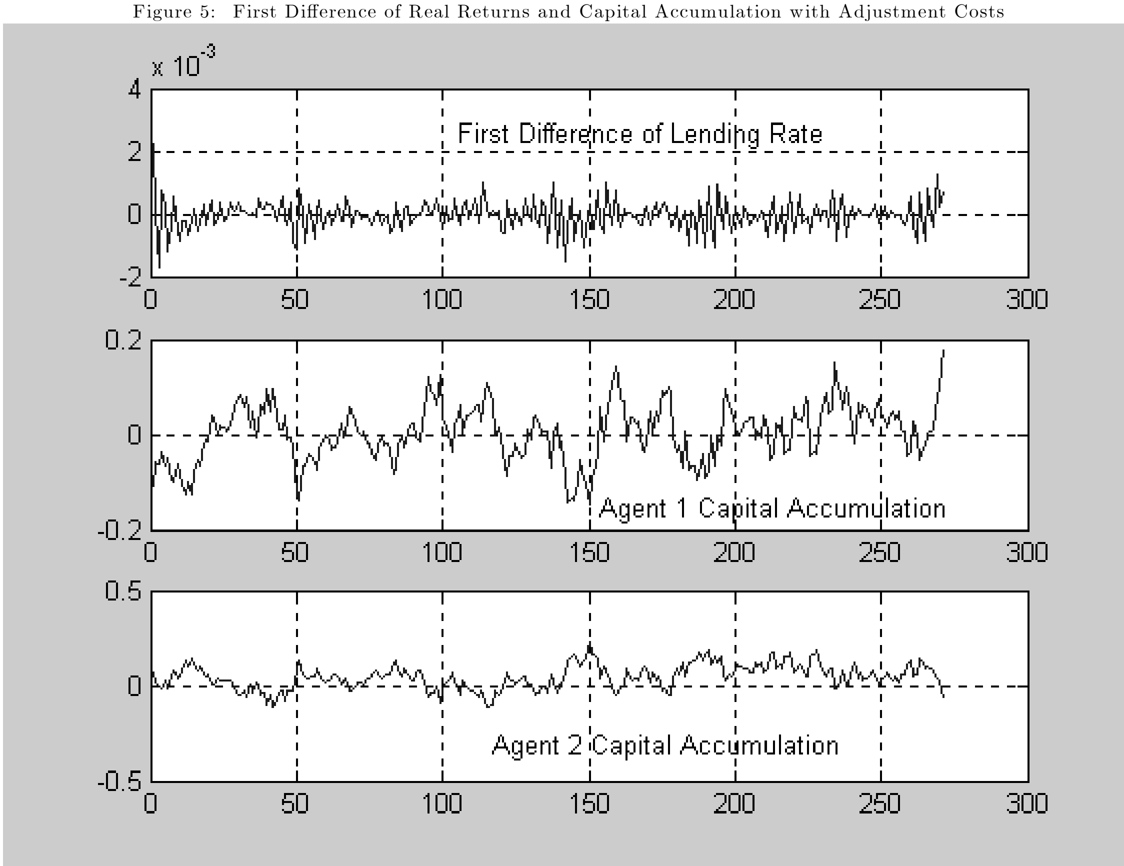
Statistical Properties of Real Returns in Model with Capital and No Adjustment Costs	
Mean	0.0067
Standard Deviation	0.0040
Skewness	0.1633
Kurtosis	1.7640

Table 3

5.3 Capital Accumulation with Adjustment Costs

Figure 5 pictures the first difference of the return on loans as well as the accumulation of capital of the two agents, when capital accumulation is introduced into the model with adjustment costs.

Figure 5: First Difference of Real Returns and Capital Accumulation with Adjustment Costs



The conditional variance appear of real returns appears in Figure 6.

Figure 6: Conditional Variance of Real Returns with Adjustment Costs

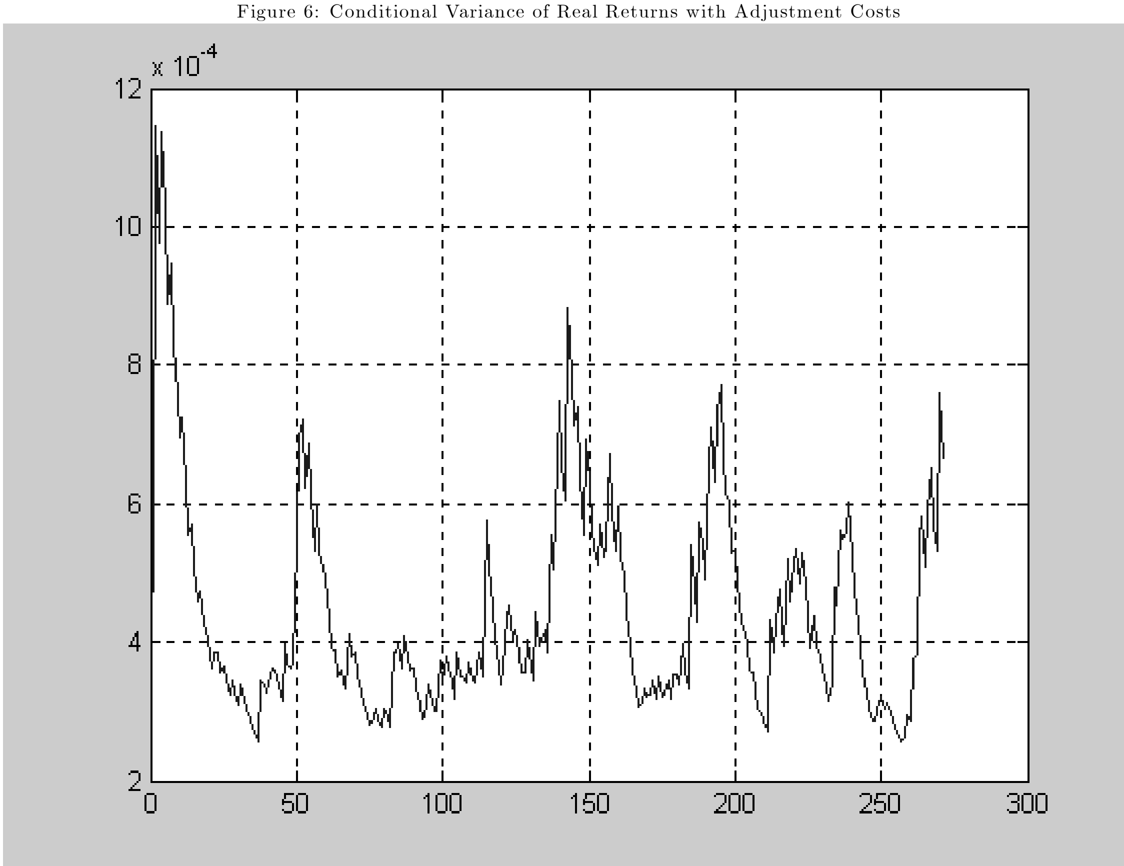


Table 4 summarizes the statistical properties of the asset returns in the model with adjustment costs.

Statistical Properties of Real Returns in Adjustment Costs Model

Mean	.0163
Standard Deviation	.0049
Skewness	-.3608
Kurtosis	1.765

GARCH Model for First-Differenced Real Returns in Adjustment Costs Model

$$\mathrm{r}_t = r_{t-1} + \epsilon_t$$
$$\epsilon_t = N(0, \sigma_t^2)$$
$$\sigma_t^2 = \beta_0^2 + \beta_1^2 \hat{\epsilon}_{t-1}^2 + \beta_2^2 \sigma_{t-1}^2$$

	Esimates /T-Statistics	
β_0	.00000001534466	2.755
β_1	0.217	7.6688
β_2	0.7168	36.7713

Table 4

6 Conclusion

This study shows that the introduction of capital does indeed "smooth out" the volatility persistence found in real returns in models with financial frictions but no captial accumulation. Introducing adjustment costs does indeed cause volatility persistence to reappear in real returns. It also brings out negative skewness in the level of real returns, which pure friction models without capital accumulation fail to reproduce.

The results show that "heterogeneity" delivers many of the same results as "habit persistence". Which assumption is more realisitic, of course, is a matter of judgement. Introducing either one of these assumptions increases the complexity of the model, but in different ways. So appealing to "Occan's razor" does not resolve the issue. However, given the widespread use of heterogeneity of agents for helping to explain other macroeconomic phenomena, it makes sense to stay with this framework for analyzing asset return phenomena as well.

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