Monetary policy and the distribution of wealth in a OLG economy with heterogeneous agents, money and bequests

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Abstract

Macroeconomic theory generally focuses on the aggregate consequences of monetary policy, without considering its distributional effects. We develop a model in which it is income and wealth distribution themselves to make monetary policy non-superneutral at the individual level. In other words monetary policy may have distributional consequences. We demonstrate that, if agents were homogeneous, that is in a representative agent economy, monetary policy would be superneutral both at the individual and at the aggregate level. If agents differ from one another as far as income and wealth are concerned, there exists a mean field effect that makes money non-superneutral at the individual level. Moreover, if capital markets were incomplete, money may be non-superneutral also at the aggregate level.

1. Introduction

Macroeconomic theory generally focuses on the aggregate consequences of monetary policy, without considering its distributional effects. We develop a model in which it is income and wealth distribution themselves to make monetary policy non-superneutral at the individual level. More precisely, in section 3, we demonstrate that, if agents were homogeneous, that is in a representative agent economy,

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monetary policy would be superneutral: output, consumption and wealth (of the representative agent) are independent of money growth. If agents differ from one another as far as income and wealth are concerned, there exists a mean field effect (see section 4): individual wealth depends also on the average wealth. The mean field effect captures non-strategic interaction between the individual agent and the rest of the population, proxied by the average agent, and makes money non-superneutral at the individual level. In fact the average levels of output, consumption and wealth are independent of money growth but the individual levels of the same variables are indeed affected by changes in the rate of growth of money. In other words, by increasing the rate of money growth, agents who are relatively poor in income/endowment become wealthier whereas relatively rich agents become less wealthy. However, the relative ranking is not reversed. As a consequence, while the first moments of the distributions of output, consumption and wealth do not depend on money (i.e. the distributions are mean preserving), higher moments are influenced by the rate of money growth. If the variance is thought of as a rough measure of inequality, then inequality is decreasing with money growth.

Finally, in section 5, we micro-fund income heterogeneity as the result of an occupational choice made by agents when young. In that context we demonstrate that money may be non-superneutral also at the aggregate level (see subsection 5.3).

Summing up therefore income and wealth distribution acts as a *financial accelerator*. On one hand actually monetary policy has clear asymmetric effects since relatively poor become wealthier, whereas relatively rich become less wealthy. Moreover, on the other hand, it is the distributions of income and wealth themselves that make money non-superneutral when capital markets are incomplete.

2. An OLG economy with money and bequests

For the sake of simplicity, we assume that population is constant and consists of N young and N old people (of the previous generation) per period. The i-th individual can dispose of output y_{it} when young, nothing when old. Output is perishable and therefore cannot be stored to be consumed in the future. For simplicity, preferences are uniform across individuals and the young do not receive utility from consumption. Assuming intergenerational altruism, the well behaved utility function is $U = U(c_{it+1}, b_{it+1})$ where c_{it+1} is consumption of the agent when old and b_{it+1} is bequest of the old to the young (wealth of the young). In a monetary economy, in order to consume when old, the young at time t sells its output to the old of the previous generation at the price P_t in exchange for money M_{it}:

$$M_{it} = P_t y_{it}$$

or

$$\frac{M_{it}}{P_t} = y_{it} \tag{2.1}$$

Aggregating across individuals we get:

$$\frac{M_t}{P_t} = Y_t \tag{2.2}$$

where $M_t \equiv \sum_{i=1}^{N} M_{it}$ is the aggregate demand for money and $Y_t \equiv \sum_{i=1}^{N} y_{it}$ is aggregate output.

Money is a means of payment and a store of value which can be carried on from one period to the next in order to buy goods. When old, the agent spends the money received when young M_{it} plus a money transfer proportional to the *average money holding* $h_t \equiv \frac{H_t}{N}$, where H_t is the aggregate money supply. Assuming that there is equilibrium on the money market in t, i.e. aggregate supply H_t is equal to aggregate demand M_t, we can define the individual money transfer as:

$$T_{it+1} = \mu h_t = \frac{\mu M_t}{N} \tag{2.3}$$

$0 < \mu < 1$. The transfer is uniform across individuals while money balances are not necessarily the same for each and every agent.

The old spend money to buy consumption goods and leave a bequest to the young:

$$M_{it+1} = M_{it} + T_{it+1} = M_{it} + \mu h_t = P_{t+1}\left(c_{it+1} + b_{_{it+1}}\right) \tag{2.4}$$

Dividing by P_{t+1} and substituting (2.1) into (2.4) we obtain the lifetime budget constraint:

$$RMB_i \equiv \frac{M_{it} + T_{it+1}}{P_{t+1}} = \frac{M_{it}}{P_t}\frac{P_t}{P_{t+1}} + \frac{\mu h_t}{P_{t+1}} = \theta_{t+1}\left(y_{it} + \mu\frac{h_t}{P_t}\right) = c_{it+1} + b_{_{it+1}} \tag{2.5}$$

where RMB_i stands for real money balances of the old, $\theta_{t+1} \equiv \frac{P_t}{P_{t+1}}$ is the real rate of return of money, $\theta_{t+1} \equiv \frac{1}{1+\pi_{t+1}}$ and π_{t+1} is inflation in t+1.According to (2.5) real money balances are spent either on consumption goods or bequest.

Equilibrium on the money market is brought about by $M_t = H_t$ so that $\frac{M_t}{P_t} = \frac{H_t}{P_t} = Y_t$. Dividing by N, we get $\frac{h_t}{P_t} = \bar{y}_t$where $\bar{y}_t$ is average output[1]. Therefore, the real money balances of the old can be written as $\theta_{t+1}(y_{it} + \mu\bar{y}_t)$ and the lifetime budget constraint becomes:

$$\theta_{t+1}(y_{it} + \mu\bar{y}_t) = c_{it+1} + b_{it+1} \tag{2.6}$$

Let's assume preferences are represented by a Cobb-Douglas utility function:

$$U = c_{it+1}^{(\gamma)} b_{it+1}^{(1-\gamma)} \tag{2.7}$$

with $0 < \gamma < 1$. Maximizing (2.7) subject to (2.6) yields:

$$c_{it+1} = \gamma\theta_{t+1}(y_{it} + \mu\bar{y}_t) \tag{2.8}$$

$$b_{it+1} = (1-\gamma)\theta_{t+1}(y_{it} + \mu\bar{y}_t) \tag{2.9}$$

Thanks to the Cobb-Douglas utility function, both consumption and bequest are proportional to RMB_i.

Aggregate transfers (to the old) in t+1 is $H_{t+1} - H_t = \sum_{i=1}^{N} T_{it+1} = N\mu\frac{H_t}{N} = \mu H_t$. Hence the supply of money in t+1 is $H_{t+1} = H_t(1+\mu)$. Thanks to equilibrium on the money market in t $H_{t+1} = M_t(1+\mu)$.

Equilibrium on the money market in t+1 is brought about by $M_{t+1} = H_{t+1}$ or $P_{t+1}Y_{t+1} = M_t(1+\mu)$.Dividing by P_t , recalling (2.2) and rearranging we get:

$$\frac{P_{t+1}}{P_t}Y_{t+1} = \frac{M_t}{P_t}(1+\mu) = Y_t(1+\mu)$$

[1]Thus, the money transfer is proportional to average nominal output: $T_{it+1} = \mu h_t = P_t\bar{y}_t$.

or, dividing by N,

$$\frac{P_{t+1}}{P_t}\bar{y}_{t+1} = \bar{y}_t\,(1+\mu)$$

and finally

$$\frac{P_{t+1}}{P_t} \equiv 1+\pi_{t+1} = \frac{1+\mu}{1+g_{t+1}} \tag{2.10}$$

where g_{t+1} is the rate of growth of aggregate (and average) income: $g_{t+1} \equiv \frac{Y_{t+1}}{Y_t} = \frac{\bar{y}_{t+1}}{\bar{y}_t}$. If (2.10) holds, equilibrium in the goods market is assured.[2]

Using (2.10), (2.9) becomes:

$$b_{it+1} = \frac{1-\gamma}{1+\mu}\frac{\bar{y}_{t+1}}{\bar{y}_t}\left(y_{it}+\mu\bar{y}_t\right) \tag{2.11}$$

As an example, let's assume that output y_{it} is the sum of an exogenous variable ω_i and bequest b_{it} (wealth of the young)

$$y_{it} = \omega_i + b_{it} \tag{2.12}$$

ω_i is non-inherited wealth. For the moment, it can be thought of as an exogenous endowment. Later on, we will specify it as income earned by workers and entrepreneurs.

The distribution of endowments across agents is the *primary* distribution, while the distribution of income is *secondary* , i.e. derived from the former by adding the bequest. As it will become clear in a moment, also the distribution of bequest is secondary, i.e. derived from the distribution of endowment.

Averaging (2.12) one gets

$$\bar{y}_t = \bar{\omega} + \bar{b}_t \tag{2.13}$$

[2] Aggregating the budget constraints, it turns out that the sum of aggregate consumption and aggregate bequest must be equal to the aggregate real money balances of the old, i.e. $\theta_{t+1}\left(Y_t+\mu N h_t\right) = C_{t+1}+B_{t+1}$ where $C_{t+1} \equiv \sum_i c_{it+1}, B_{t+1} \equiv \sum_i b_{it+1}$. But $h_t = \bar{y}_t$ and $Nh_t = N\bar{y}_t = Y_t$.Therefore: $\theta_{t+1}\left(1+\mu\right)Y_t = C_{t+1}+B_{t+1}$.If (2.10) holds true, then $Y_{t+1} = \theta_{t+1}\left(1+\mu\right)Y_t$. Substituting this expression into the previous one yields $Y_{t+1} = C_{t+1}+B_{t+1}$ which is the equilibrium condition on the goods market.

where $\bar{\omega}$ is average endowment and $\bar{b}_t$ average wealth. We can carry on the dynamic analysis in terms of output or wealth. Substituting (2.12) into (2.11) we obtain the law of motion of output:

$$y_{it+1} = \omega_i + \frac{1-\gamma}{1+\mu} \frac{\overline{y}_{t+1}}{\overline{y}_t} \left[y_{it} + \mu \overline{y}_t \right] \tag{2.14}$$

Averaging (2.14) we obtain

$$\overline{y}_{t+1} = \overline{\omega} + (1-\gamma) \, \overline{y}_{t+1}$$

which simplifies to

$$\overline{y}_{t+1} = \frac{\overline{\omega}}{\gamma}$$

which is constant over time. Therefore we can write

$$\overline{y} = \frac{\overline{\omega}}{\gamma} \tag{2.15}$$

The law of motion of wealth is:

$$b_{it+1} = \frac{1-\gamma}{1+\mu} \frac{\bar{\omega} + \bar{b}_{t+1}}{\bar{\omega} + \bar{b}_t} \left(\omega_i + b_{it} + \mu\bar{\omega} + \mu\bar{b}_t \right) \tag{2.16}$$

There is a *mean field effect* at work, here: individual wealth in t+1 depends not only on individual wealth in t but also on average wealth in t and t+1.The mean field effect captures non-strategic interaction between the individual agent and the rest of the population proxied by the average agent (see Aoki 1996 and the references thereafter).

Averaging (2.16) we get that average wealth is constant over time

$$\bar{b} = \frac{1-\gamma}{\gamma} \bar{\omega} \tag{2.17}$$

Finally it is easy to prove that average consumption is

$$\bar{c} = \bar{\omega} \tag{2.18}$$

3. A representative agent economy

Let's pause now to consider the special case of a representative agent economy. If we adopt the representative agent assumption, (2.1) simplifies to

$$\frac{m_t}{P_t} = y_t \tag{3.1}$$

where m_t and y_t are the demand for money and output of the representative agent. The aggregate demand for money, therefore, is: $M_t \equiv Nm_t$.Assuming equilibrium on the money market in t, i.e. $M_t = H_t$,the equation of the individual transfer becomes

$$T_{it+1} = \frac{\mu H_t}{N} = \frac{\mu N m_t}{N} = \mu m_t$$

and (2.4) boils down to

$$M_{it} + T_{it+1} = m_t (1+\mu) = P_{t+1} (c_{t+1} + b_{t+1}) \tag{3.2}$$

where c_{t+1} and b_{t+1} are consumption and bequest of the representative old agent. Dividing by P_{t+1} and substituting (3.1) into (3.2) we obtain the lifetime budget constraint of the representative agent:

$$\frac{M_{it}}{P_t} (1+\mu) \frac{P_t}{P_{t+1}} = \theta_{t+1} (1+\mu) y_t = c_{t+1} + b_{t+1} \tag{3.3}$$

Maximizing the Cobb-Douglas utility function (2.7) subject to (3.3) yields:

$$c_{t+1} = \gamma \theta_{t+1} (1+\mu) y_t \tag{3.4}$$

$$b_{t+1} = (1-\gamma) \theta_{t+1} (1+\mu) y_t \tag{3.5}$$

We recall now that

$$\frac{1}{\theta_{t+1}} \equiv \frac{P_{t+1}}{P_t} \equiv 1 + \pi_{t+1} = \frac{1+\mu}{1+g_{t+1}}$$

where $1+g_{t+1}=\frac{\bar{y}_{t+1}}{\bar{y}_t}$. Substituting this expression into (3.5) we obtain:

$$b_{it+1}=(1-\gamma)\frac{\bar{y}_{t+1}}{\bar{y}_t}y_t \tag{3.6}$$

At this point, we are able to prove the following

Proposition 3.1. *Output, consumption and wealth (of the representative agent) are independent of money growth, i.e. money is superneutral.*

In the representative agent case, in fact, output and wealth are uniform across individuals and there is no difference between individual and average output or wealth, i.e. $y_t=\bar{y}_t$. As a consequence, (3.6) boils down to:

$$b_{t+1}=(1-\gamma)\,y_{t+1} \tag{3.7}$$

Equation (3.7) implies that the ratio of wealth to income must be constant and equal to $1-\gamma$.This must be true in each period:

$$\frac{b_{t+1}}{y_{t+1}}=\frac{b_t}{y_t}=1-\gamma$$

As an example, assume that

$$y_t=\omega+b_t \tag{3.8}$$

Substituting (3.8) into (3.7) we obtain:

$$b=\frac{1-\gamma}{\gamma}\omega \tag{3.9}$$

Inflation does not affect the accumulation of wealth: in fact, the real rate of return of money θ_{t+1} – which is the reciprocal of the growth factor of the price level – does not show up in (3.9). Consumption and output are respectively:

$$c=\omega$$

$$y=\frac{1}{\gamma}\omega$$

Output is equal to a multiple of the endowment. In each period, the young sells to the old his entire output $y = \frac{1}{\gamma}\omega$ – which consists of endowment and wealth – at the price $P_t = \frac{\gamma}{\omega} M_t$. He does not consume and keeps its money balances M_t "idle" until the next period. The old agent buys output $\frac{1}{\gamma}\omega$, consumes the endowment ω and leaves a bequest $\frac{1-\gamma}{\gamma}\omega$ to his son equal to the bequest received from his father. Since output is constant, money is superneutral and the rate of change of money supply affects only inflation:

$$\pi = \mu$$

4. Heterogeneous agents

In the case of heterogeneous agents, things are not that simple and certainly more interesting. We can summarize our results in

Proposition 4.1. *Money is superneutral on average but is not superneutral at the individual level. In fact the average levels of output, consumption and wealth are independent of money growth but the individual levels of the same variables are indeed affected by changes in the rate of growth of money. As a consequence, while the first moments of the distributions of output, consumption and wealth do not depend on money (i.e. the distributions are mean preserving), higher moments are influenced by the rate of money growth.*

In order to prove this proposition, let's go back to equations 2.15, 2.17 and 2.18[3].

Notice now that if average output is constant over time, the rate of growth of average output is *zero*, i.e. $\frac{\bar{y}_{t+1}}{\bar{y}_t} = 1$,and the inflation rate is constant and equal to the rate of growth of money: $\pi = \mu$. Therefore $\theta_{t+1} = \frac{1}{1+\mu}$, (2.14) simplifies

[3]From the equilibrium condition on the goods markete one gets $\bar{c} = \bar{y} - \bar{b}$.These equations happen to coincide with the ones derived for the representative agent case (see the previous section) but in the heterogeneous agents case they refer to the average agent which does not coincide by construction with the representative agent.

to[4]:

$$y_{it+1} = \omega_i + \frac{1-\gamma}{1+\mu}\left(y_{it} + \mu\bar{y}\right) \tag{4.1}$$

and (2.16) boils down to:

$$b_{it+1} = \frac{1-\gamma}{1+\mu}\left(\omega_i + b_{it} + \mu\bar{\omega} + \mu\bar{b}\right) \tag{4.2}$$

which is a linear difference equation incorporating a *linear mean field effect.* The mean field effect was already present in (2.16) but now it is simpler: individual wealth in t+1 depends *linearly* on average wealth. Changes in average wealth play the role of a *positive macroeconomic externality* on individual wealth: the higher average wealth, the higher average output, the higher the money transfer in t+1[5] and the higher individual wealth in t+1, coeteris paribus.

The steady state of (4.2) is

$$b_i^* = \frac{1-\gamma}{\mu+\gamma}\left(\omega_i + \mu\bar{\omega} + \mu\bar{b}\right) \tag{4.3}$$

At this level of the analysis, the mean field effect is present also in the steady state: the individual steady state of wealth in fact depends *linearly* on (steady state) average wealth.

Averaging b_i^* from (4.3) and rearranging, however, one gets[6]

$$\bar{b}^* = \frac{1-\gamma}{\gamma}\bar{\omega}$$

which can be plugged into (4.2) to obtain

$$b_{it+1} = \frac{1-\gamma}{1+\mu}\left(\omega_i + b_{it} + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{4.4}$$

This new law of motion makes individual wealth in t+1 depend *linearly* on average endowment. The phase diagram of (4.4) is represented in figure 4.1.

[4]Remember that $\bar{y}_t = \overline{y}$, at any t.

[5]In fact, according to (2.3) $T_{it+1} = \mu h_t$ but $\frac{h_t}{P_t} = \bar{y}_t = \overline{y} = \bar{\omega} + \bar{b}$, hence $\frac{T_{it+1}}{P_{t+1}} = \frac{P_t}{P_{t+1}}\left(\mu\bar{\omega} + \mu\bar{b}\right) = \frac{1}{1+\mu}\left(\mu\bar{\omega} + \mu\bar{b}\right)$. In words, the real value of the money transfer in t+1 for each individual is increasing with average wealth.

[6]Of course this expression is the same as (2.17).

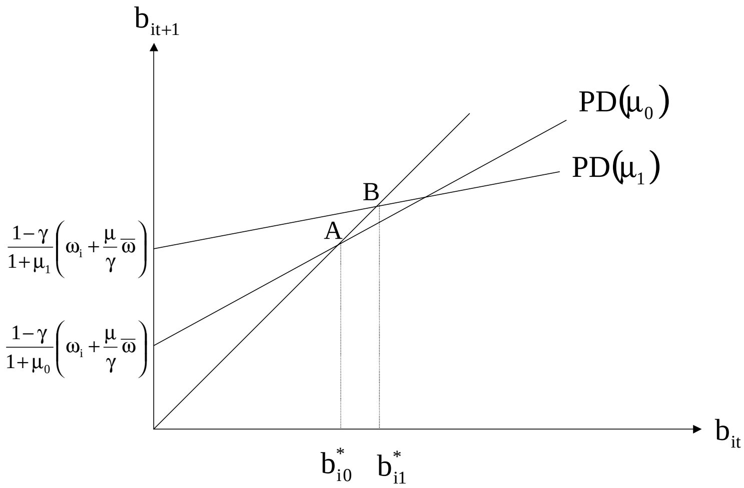

Figure 4.1

The steady state of (4.4) is

$$b_i^* = \frac{1-\gamma}{\mu+\gamma}\left(\omega_i + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{4.5}$$

Applying the same modelling strategy to output and consumption, we obtain the steady state values:

$$y_i^* = \frac{1}{\mu+\gamma}\left[\omega_i\left(1+\mu\right) + \left(1-\gamma\right)\frac{\mu}{\gamma}\bar{\omega}\right] \tag{4.6}$$

and

$$c_i^* = \gamma\frac{1+\mu}{\mu+\gamma}\omega_i + \frac{1-\gamma}{\mu+\gamma}\mu\bar{\omega} \tag{4.7}$$

Comparing (2.17)(2.15)(2.18) with (4.5) (4.6) and (4.7) it is clear that in the steady state individual wealth, output and consumption are indeed affected by the rate of growth of money (which coincides with inflation) and money is not superneutral while *on average* consumption, output and wealth are independent of money growth, i.e. money is superneutral.

Let's pause now to characterize the relationship between money growth and the steady state of the i-th agent's wealth (output and consumption). In order to do so, notice first of all that an increase in the rate of money growth makes the slope of the phase diagram smaller but it may lead to an increase of the intercept with contrasting effects on steady state wealth.

In order to understand the nature and consequences of these effects, it is necessary to interpret (4.4) as follows $b_{it+1} = (1-\gamma) \times RMB_i$, where $RMB_i = \frac{M_{it} + T_{it+1}}{P_{t+1}} = \theta_{t+1}\frac{M_{it} + T_{it+1}}{P_t} = \theta_{t+1}\left(y_{it} + \mu\bar{y}\right)$ are the real money balances of the old. Recalling that $\theta_{t+1} = \frac{1}{1+\mu}$, $y_{it} = \omega_i + b_{it}$ and $\bar{y} = \frac{\bar{\omega}}{\gamma}$, RMB_i can be written as follows:

$$RMB_i = \frac{1}{1+\mu}\left(\omega_i + b_{it} + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{4.8}$$

RMB_i is the product of the real return on money $\theta_{t+1} = \frac{1}{1+\mu}$ times the value of individual money balances in t+1 deflated by P_t (see the expression in brackets) which in turn is the sum of the real money balances carried over from the previous period: $\frac{M_{it}}{P_t} = y_{it} = \omega_i + b_{it}$ and of the money transfer received in t+1 deflated by P_t: $\frac{T_{it+1}}{P_t} = \mu\frac{h_t}{P_t} = \mu\bar{y} = \frac{\mu}{\gamma}\bar{\omega}$.

An increase in money growth has contrasting effects on money holdings in real terms. It

- boosts inflation which reduces the real rate of return on money $\theta_{t+1} = \frac{1}{1+\mu}$ and real money balances, coeteris paribus: this is the *inflation tax effect* of money growth on real money balances;

- implies an increase in the money transfer to the old $\frac{T_{it+1}}{P_t} = \frac{\mu}{\gamma}\bar{\omega}$: this is the *money transfer effect.*

Which effect is prevailing? In order to answer this question, we must compute the derivative of RMB_i with respect to money growth from (4.8):

$$\begin{aligned}\frac{\partial RMB_i}{\partial \mu} &= \frac{\partial \theta_{t+1}}{\partial \mu} \times \left(\omega_i + b_{it} + \frac{\mu}{\gamma}\bar{\omega}\right) + \theta_{t+1}\frac{\partial\left(\omega_i + b_{it} + \frac{\mu}{\gamma}\bar{\omega}\right)}{\partial \mu} = \\ &= -\frac{\omega_i + b_{it} + \frac{\mu}{\gamma}\bar{\omega}}{(1+\mu)^2} + \frac{1}{1+\mu}\frac{\bar{\omega}}{\gamma}\end{aligned}$$

The first (negative) term captures the inflation tax effect, while the second (positive) term reflects the money transfer effect. After some algebraic manipulation we end up with

$$\frac{\partial RMB_i}{\partial \mu} = -\frac{1}{(1+\mu)^2}\left(\omega_i + b_{it} - \frac{\bar{\omega}}{\gamma}\right) = -\frac{1}{(1+\mu)^2}\left(y_{it} - \bar{y}\right) \tag{4.9}$$

According to (4.9) if the agent has relatively little output $(y_{it} < \bar{y})$, an increase in money growth brings about higher real money balances, i.e. the money transfer effect prevails over the inflation tax effect. If the opposite is true $(y_{it} > \bar{y})$, an increase in money growth yields lower real money balances, i.e. the inflation tax effect prevails over the money transfer effect.

Notice that when $b_{it} = 0$, following an increase in money growth, the money transfer effect prevails if $\omega_i < \frac{\bar{\omega}}{\gamma}$, while the inflation tax effect prevails if $\omega_i > \frac{\bar{\omega}}{\gamma}$.

Let's go back to the phase diagram of 4.4. As we have already acknowledged, an increase of μ makes the slope $\frac{1-\gamma}{1+\mu}$ smaller, due to the inflation tax effect. As to the intercept $\frac{1-\gamma}{1+\mu}\left(\omega_i + \frac{\mu}{\gamma}\bar{\omega}\right)$, it is easy to realize that an increase of μ makes the intercept greater if $\omega_i < \frac{\bar{\omega}}{\gamma}$ while it leads to a lower intercept if $\omega_i > \frac{\bar{\omega}}{\gamma}$[7]. In other words, the intercept increases if the money transfer effect prevails over the inflation tax effect, it decreases if the opposite is true.

As to the effect of money growth on the steady state (4.5), it is clear that if the agent can dispose of a relatively big output $\left(\omega_i > \frac{\bar{\omega}}{\gamma}\right)$, an increase of μ yields a smaller steady state, due to the joint effects of a smaller slope and a smaller

[7] In fact $\frac{\partial}{\partial \mu}\left(\frac{1-\gamma}{1+\mu}\left(\omega_i + \frac{\mu}{\gamma}\bar{\omega}\right)\right) = -\frac{1-\gamma}{(1+\mu)^2}\left(\omega_i\gamma - \bar{\omega}\right)$

intercept of the phase diagram. If the agent can dispose of relatively little output $\left(\omega_i < \frac{\bar{\omega}}{\gamma}\right)$, an increase of μ may yield either a greater or a smaller steady state, due to the contrasting effects of a smaller slope and a greater intercept of the phase diagram. In figure 4.1, we have represented the former case: when the rate of money growth is μ_0, steady state wealth of the i-th agent is b^*_{i0} (point A). An increase of the rate of money growth to μ_1 makes the phase diagram shift upward and rotate so that steady state wealth goes up to b^*_{i1} (point B).

So much for the single agent. Let's now look at the effects of money growth on the distribution of wealth. In figure 4.2 we have represented equation (4.5) on the (b^*_i, ω_i) plane.

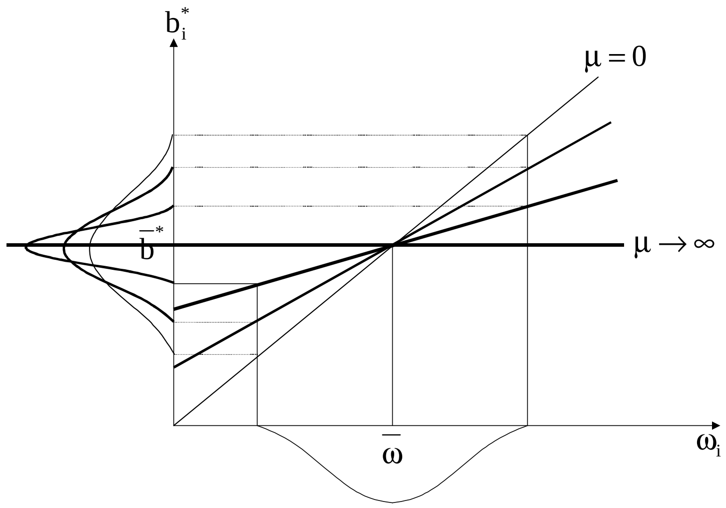

Figure 4.2

Both the intercept $\left(\frac{1-\gamma}{\mu+\gamma}\frac{\mu}{\gamma}\bar{\omega}\right)$ and the slope $\left(\frac{1-\gamma}{\mu+\gamma}\right)$ of the *wealth-endowment line* are positive but the former is increasing while the latter is decreasing with respect to μ. If money were constant ($\mu = 0$), steady state wealth would be represented by the solid line of equation $b^*_i = \frac{1-\gamma}{\gamma}\omega_i$. By increasing μ the line shifts up and rotates around point A, the *average agent point* of coordinates $\left(\bar{\omega}, \bar{b}^*\right)$(see for instance the dotted line). If $\mu \to \infty$ steady state wealth would be represented by the solid bold line of equation $b_i^* = \frac{1-\gamma}{\gamma}\bar{\omega} = \bar{b}^*$.

In other words, by increasing μ, agents who are relatively poor in endowment (i.e. characterized by $\omega_i < \bar{\omega}$) become wealthier (i.e. their b_i^* increases) while relatively rich agents (characterized by $\omega_i > \bar{\omega}$) become less wealthy. However, the relative ranking is not reversed: agents who are relatively poor in endowment ($\omega_i < \bar{\omega}$) remain less wealthy than the average $\left(b_i^* < \bar{b}^*\right)$and tend to the average only asymptotically.

The primary distribution of ω_i can be represented ideally on the x-axis. Let the support of ω_i be $(\min \omega_i, \max \omega_i)$. From the (primary) distribution of the endowment, for each μ the wealth-endowment line generates the (secondary) distribution of wealth, which can be represented ideally on the y-axis. In other words, the wealth-endowment line maps the distribution of the endowment into the distribution of wealth. On the same diagram, we can represent also the secondary distribution of output: the intercept of the straight line of equation $b_i^* = y_i^* - \omega_i$ passing through each point of coordinates (b_i^*, ω_i) belonging to the wealth-endowment line is the steady state individual output. The vertical distance between y_i^* and b_i^* measures consumption c_i^*.

While the primary distribution is independent of money growth (by construction), the secondary distribution of wealth is affected by μ[8]. It is easy to see that increasing μ, the support of the distribution of wealth shrinks and so does the variance. The widest range of $b_i^*$is associated to the constant money scenario $(\min b_i^* \mid \mu = 0, \max b_i^* \mid \mu = 0)$. In the limit, as $\mu \to \infty$ the support of the distribution collapses to a point (corresponding to average wealth) and the variance of the distribution tends to zero[9].

The first moment of the distribution of endowment and wealth (the coordinates of the average agent point) are independent of money growth, while the variance of the secondary distribution is decreasing with money growth. In fact, computing the variance of steady state wealth from (4.5) one gets:

$$V\left(b_i^*\right) = \left(\frac{1-\gamma}{\mu+\gamma}\right)^2 V\left(\omega_i\right)$$

[8] The same is true for the distributions of output and consumption, as it is clear from (4.6) and (4.7).

[9] It is easy to see that in this case one obtains the the most narrow range also of output and consumption.

The variance of wealth falls in the range[10]:

$\min V\left(b_i^*\right) = V\left(b_i^*\right) \mid \mu \to \infty = 0$

$\max V\left(b_i^*\right) = V\left(b_i^*\right) \mid \mu = 0 = \left(\dfrac{1-\gamma}{\gamma}\right)^2 V\left(\omega_i\right)$

As to output, the variance of output is:

$$V\left(y_i^*\right) = \left(\frac{1+\mu}{\mu+\gamma}\right)^2 V\left(\omega_i\right)$$

and falls in the range:

$\min V\left(y_i^*\right) = V\left(y_i^*\right) \mid \mu \to \infty = V\left(\omega_i\right)$

$\max V\left(y_i^*\right) = V\left(y_i^*\right) \mid \mu = 0 = \dfrac{1}{\gamma^2} V\left(\omega_i\right)$

Finally the variance of consumption is:

$$V\left(c_i^*\right) = \left(\gamma\frac{1+\mu}{\mu+\gamma}\right)^2 V\left(\omega_i\right)$$

and falls in the range:

$\min V\left(c_i^*\right) = V\left(c_i^*\right) \mid \mu \to \infty = \gamma^2 V\left(\omega_i\right)$

$\max V\left(c_i^*\right) = V\left(c_i^*\right) \mid \mu = 0 = V\left(\omega_i\right)$

If the variance is thought of as a rough measure of inequality, then *inequality is decreasing with money growth.*

Another insightful way to look at the effect of money growth on wealth distribution exploits the relation between steady state wealth and money growth. In figure 4.3 we have represented equation (4.5) on the (b_i^*, μ) plane.

[10] Notice that b_i and ω_i are positively correlated and the correlation is perfect (thanks to (4.5)). Therefore $V(y_i^*) = V(\omega_i) + V(b_i) + 2Cov(\omega_i, b_i)$ and $Cov(\omega_i, b_i) = \sqrt{V(\omega_i)V(b_i)}$ But $V(b_i) = c^2 V(\omega_i)$ where $c = \dfrac{1-\gamma}{\mu+\gamma}$. Therefore $Cov(\omega_i, b_i) = \sqrt{V(\omega_i)V(b_i)} = cV(\omega_i)$. As a consequence $V(y_i) = V(\omega_i) + c^2 V(\omega_i) + 2cV(\omega_i) = V(\omega_i)\left(1+c\right)^2$.

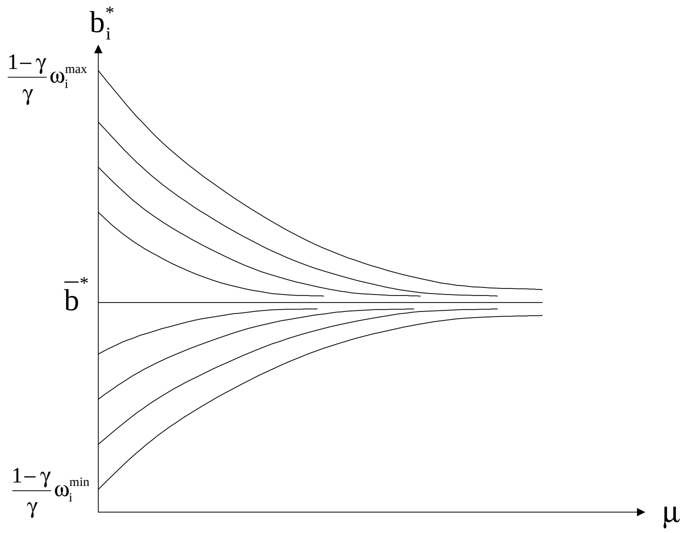

Figure 4.3

The shape and location of the *wealth-money curve* depends on individual endowment. In fact, taking the derivative of b_i^* with respect to μ one gets:

$$\frac{\partial b_i^*}{\partial \mu} = -\frac{1-\gamma}{(\mu+\gamma)^2}(\omega_i - \bar{\omega})$$

If the i-th agent is relatively poor $(\omega_i < \bar{\omega})$, the wealth-money curve is an increasing concave function of money growth, asymptotically tending to $b_{i\infty}^* = \bar{b}^*$. Symmetrically, if the agent is relatively rich $(\omega_i > \bar{\omega})$, the wealth-money curve is a decreasing convex function of money growth, asymptotically tending to $b_{i\infty}^* = \bar{b}^*$. In words: a relatively poor (rich) individual becomes less and less poor (rich) – i.e. his wealth keeps increasing (decreasing) if money growth becomes higher and higher,but he remains relatively poor (rich) even if money grows at an infinite rate[11].

In figure 4.3 we have represented different wealth-money curves associated to different endowments. One can get a picture of the distribution of wealth

[11] By an entirely similar argument, it is easy to conclude that the same is true for output and consumption: if the agent is relatively poor (rich), an increase in money growth increases (decreases) both output and consumption.

by sectioning the sheaf of curves at a given growth rate of money. It is clear that increasing μ the first moment of the distribution remains constant but the second moment goes down. The limiting distribution (associated to $\mu \to \infty$) is characterized by minimum variance (as a matter of fact, the variance tends to zero).

The transition from A to B in figure 4.1, which leads to a higher steady state wealth as a consequence of an increase in money growth, requires not only that the money transfer effect prevails over the inflation tax effect – which occurs if $\omega_i < \dfrac{\bar{\omega}}{\gamma}$) – with a corresponding increase of the intercept of the phase diagram, but also that the increase of the intercept more than offset the decrease of the slope of the phase diagram, – which occurs if $\omega_i < \bar{\omega}$).

5. Occupational choice, bequests and the origin of heterogeneity

5.1. Skilled and unskilled workers

Since their birth, agents are endowed with a level of individual ability/efficiency e_i. When young, they make an occupational choice, which consists in being a worker of the skilled or unskilled type. The young population, therefore, consists of skilled and unskilled workers. Both types of workers supply workhours to the "production sector" which produces output and sells it to the old of the previous generation against money. This generates the income of the young, i.e. the wage, which plays the role of the endowment in previous section. The wage of the unskilled worker is fixed at w. The wage of the skilled worker is proportional to his ability. In order to simplify the argument and without loss of generality we assume that it is equal to his ability. Therefore, unskilled workers have the same wage, skilled workers' wage is differentiated in order to recognize different abilities. The old receive also the money transfer. Finally, as in the previous case, the young receives also a bequest b_{it}. Real output therefore is $y^u_{it} = w + b_{it}$ and $y^s_{it} = e_i + b_{it}$ for the unskilled and the skilled worker respectively.

Preferences are as before: $U = (c_{it+1})\,\gamma\,(b_{it+1})^{1-\gamma}$. Therefore the young do not consume. They exchange their output (income and bequest) for money: $P_t y_{it} = M_{it}$. The old receive money transfers ($T_{it+1} = \mu h_t$) and spend their money to consume and leave a bequest. LBC is (see section 2) : $\theta_{t+1}\left(y_{it} + \mu\frac{h_t}{P_t}\right) = c_{it+1} + b_{it+1}$where $\theta_{t+1}\left(y_{it} + \mu\frac{h_t}{P_t}\right)$ are real money balances of the old.

Since, given the preferences, indirect utility is

$$U=\left(\gamma\theta_{t+1}\left(y_{it}+\mu\frac{h_t}{P_t}\right)\right)\gamma\left[\left(1-\gamma\right)\theta_{t+1}\left(y_{it}+\mu\frac{h_t}{P_t}\right)\right]^{1-\gamma}$$

or

$$U=\left(\gamma\right)\gamma\left(1-\gamma\right)^{1-\gamma}\theta_{t+1}\left(y_{it}+\mu\frac{h_t}{P_t}\right)$$

and the real rate of return on money and money transfers are uniform across the population, the occupational choice depends on the relative magnitude of income obtained when young as skilled or unskilled worker.

The individual becomes skilled worker if $e_{it}+b_{it}\geq w+b_{it}$ or

$e_{it}\geq w\equiv\hat{e}$, i.e. if his ability is high enough to yield a wage as skilled worker higher than the wage of the unskilled worker. $\hat{e}$ is the minimum efficiency a worker must have in order to work as skilled and get a skilled worker wage equal to his efficiency. It turns out, quite simply, that the minimum efficiency of the skilled is equal to wage of the unskilled workers. Let's assume that efficiency is distributed as a uniform random variable with support (0,1). It is clear that $w\equiv\hat{e}$ is also the share of unskilled workers in the population[12]. This share is constant and independent of money growth.

The income of the skilled worker falls in the interval $(\hat{e},1)$ where 1 is the maximum efficiency of the skilled (by the assumption above). But $w\equiv\hat{e}$ so that average income of the skilled is

$$\bar{e}^s=\frac{1+\hat{e}}{2}=\frac{1+w}{2} \tag{5.1}$$

Therefore, average income is

$$\bar{\omega}=w\hat{e}+\bar{e}^s\left(1-\hat{e}\right)=\frac{w^2+1}{2} \tag{5.2}$$

Let's define now the laws of motion. Recalling, as shown above, that $\frac{h_t}{P_t}=\bar{y}_t=\bar{y}$ where $\bar{y}$ is average output of the whole economy, and $\bar{y}=\bar{\omega}+\bar{b}$ where $\bar{\omega}$ is average income and $\bar{b}$ average wealth, each and every unskilled worker has the following law of motion of wealth:

$$b^u_{t+1}=\frac{1-\gamma}{1+\mu}\left(w+b^u_t+\mu\bar{\omega}+\mu\bar{b}\right) \tag{5.3}$$

[12] By assumption therefore $w<1$.

The steady state of (5.3) is

$$b^u = \frac{1-\gamma}{\mu+\gamma}\left(w + \mu\bar{\omega} + \mu\bar{b}\right) \tag{5.4}$$

The skilled worker of efficiency $e_i^s > w$ has the following law of motion of wealth:

$$b_{it+1}^s = \frac{1-\gamma}{1+\mu}\left(e_i^s + b_{it}^s + \mu\bar{\omega} + \mu\bar{b}\right) \tag{5.5}$$

whose steady state is

$$b_i^s = \frac{1-\gamma}{\mu+\gamma}\left(e_i^s + \mu\bar{\omega} + \mu\bar{b}\right) \tag{5.6}$$

Averaging (5.5) we obtain the law of motion of the average wealth of the skilled

$$\bar{b}_{t+1}^s = \frac{1-\gamma}{1+\mu}\left(\bar{e}^s + \bar{b}_t^s + \mu\bar{\omega} + \mu\bar{b}\right) \tag{5.7}$$

whose steady state is:

$$\bar{b}^s = \frac{1-\gamma}{\mu+\gamma}\left(\bar{e}^s + \mu\bar{\omega} + \mu\bar{b}\right) \tag{5.8}$$

Plugging (2.17) into (5.3) and (5.5) we get:

$$b_{t+1}^u = \frac{1-\gamma}{1+\mu}\left(w + b_t^u + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{5.9}$$

and

$$b_{it+1}^s = \frac{1-\gamma}{1+\mu}\left(e_i^s + b_{it}^s + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{5.10}$$

and the steady states are

$$b^{u*} = \frac{1-\gamma}{\mu+\gamma}\left(w + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{5.11}$$

$$b_i^{s*} = \frac{1-\gamma}{\mu+\gamma}\left(e_i^s + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{5.12}$$

$$\bar{b}^{s*} = \frac{1-\gamma}{\mu+\gamma}\left(\bar{e}^s + \frac{\mu}{\gamma}\bar{\omega}\right) \qquad (5.13)$$

As to the impact of money growth on bequest, from the previous section we know that an increase in the rate of money growth is beneficial for (because it increases the wealth of) the relatively poor ($\omega_i < \bar{\omega}$) and detrimental for the relatively wealthy ($\omega_i > \bar{\omega}$).

The average income of the economy (5.2) is a quadratic function of w. It is clear that the wage of the unskilled (45 degree line) is always lower than average income: therefore an increase in money growth always boosts their wealth. For each w (for instance w_0) the range of the wages of the skilled workers is the distance between w and 1. There are some skilled workers – those with an efficiency which falls in the range $\bar{\omega} > e_i^s > w$ – who gain from an acceleration in monetary expansion, others – those with an efficiency which falls in the range $1 > e_i^s > \bar{\omega}$ – who loose. On average, however, money growth does not affect wealth (output and consumption). This is the simplest illustration of Proposition 4.1 in a context where we explicitly consider the occupational choice: in the steady state, money is superneutral on average but is not superneutral at the individual level. As a consequence, while the first moments of the distributions of output, consumption and wealth do not depend on money, higher moments are influenced by the rate of money growth.

5.2. Workers and entrepreneurs

Agents are endowed with an investment project which can be used to produce capital and whose outcome e_i is related to the level of individual ability/efficiency. When young, they make an occupational choice. They supply capital (if entrepreneurs) or labor (if workers) to the production sector. In the former case, they have to purchase and employ an input whose fixed cost is x. Therefore x is the input requirement of entrepreneurial production [13]. In order to do so, they can use internal funds b_{it}. If internal funds are more than enough to finance the fixed cost, the entrepreneur is self-financed, if the opposite is true, a financing gap occurs equal to $x - b_{it}$.

The production sector produces the good, sells it to the old of the previous generation against money and generates income for the young: profit of the young

[13] A different interpretation of the same environment is an extension of the previous subsection. The occupational choice is between skilled and unskilled labour but in order to be skilled the worker must invest in human capital at the fixed cost x.

entrepreneur $\omega_i^e \equiv e_{it} - x$ and wage of the young worker $\omega_i^w \equiv w$.

Preferences are as before (see equation 2.7).Therefore the occupational choice depends on the relative magnitude of income obtained as worker or entrepreneur. Let's assume for the moment and for the sake of discussion that an individual could fill the financing gap at no cost. In this case the entrepreneur gets

$$y_{it}^e = e_i + b_{it} - x$$

independently from his financial condition while the worker gets

$$y_{it}^w = w + b_{it}$$

The individual becomes entrepreneur if $e_i + b_{it} - x \geq w + b_{it}$ or

$\omega_i^e \equiv e_{it} - x \geq \omega_i^w \equiv w$. In other words, the agent becomes entrepreneur if $e_{it} \geq w + x \equiv \hat{e}$, i.e. if his ability is high enough to yield a revenue as entrepreneur higher than the sum of the wage and the input requirement. $\hat{e}$ is the minimum efficiency an agent must have in order to be an entrepreneur and get a profit which is increasing with his efficiency. If efficiency is distributed as a uniform random variable with support (0,1), $w + x \equiv \hat{e}$ is also the share of workers in the population[14]. This share is constant and independent of money growth.

The income of the entrepreneur falls in the interval $(w, 1 - x)$ where $1 - x$ is the maximum profit (by the assumption above). Therefore, the average income of the entrepreneur is

$$\bar{e}^e = \frac{1 - x + w}{2} \tag{5.14}$$

and average income is

$$\bar{\omega} = w\hat{e} + \bar{e}^e (1 - \hat{e}) = \frac{1}{2} \left(w^2 + 2wx + 1 - 2x + x^2 \right) \tag{5.15}$$

In figure 5.1 we have represented the income of the worker and of the entrepreneur as a function of efficiency.

[14]By assumption therefore $w + x < 1$.

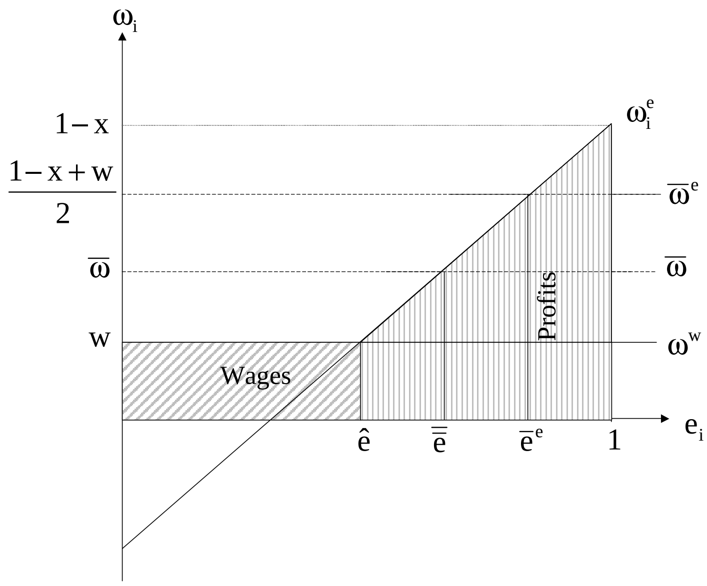

Figure 5.1

Let's define now the laws of motion. Each and every worker has the law of motion of wealth (5.3), whose steady state[15] is the same as equation 5.11

$$b^{w*} = \frac{1-\gamma}{\mu+\gamma}\left(w + \frac{\mu}{\gamma}\bar{\omega}\right)$$

while the entrepreneur of efficiency $e_i^e > w + x$ has the following law of motion of wealth:

$$b_{it+1}^e = \frac{1-\gamma}{1+\mu}\left(e_i^e - x + b_{it}^e + \mu\bar{\omega} + \mu\bar{b}\right) \tag{5.16}$$

whose steady state is

$$b_i^{e*} = \frac{1-\gamma}{\mu+\gamma}\left(e_i^e - x + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{5.17}$$

Averaging (5.16) we obtain the law of motion of the average wealth of the entrepreneur

$$\bar{b}_{t+1}^e = \frac{1-\gamma}{1+\mu}\left(\bar{e}^e - x + \bar{b}_t^e + \mu\bar{\omega} + \mu\bar{b}\right) \tag{5.18}$$

[15] Remember that $\overline{b} = \frac{1-\gamma}{\gamma}\overline{\omega}$.

whose steady state is:

$$\bar{b}^{e*} = \frac{1-\gamma}{\mu+\gamma}\left(\bar{e}^{e} - x + \frac{\mu}{\gamma}\bar{\omega}\right) \tag{5.19}$$

As to the impact of money growth on bequest, we know that an increase in the rate of money growth is beneficial for the relatively poor and detrimental for the relatively wealthy. It is clear that the wage is always lower than average income: therefore an increase in money growth always boosts the wealth of workers. As to entrepreneurs, some of them – those with an efficiency which falls in the range $\bar{\bar{e}} = \bar{\omega} + x > e_i^e > w + x = \hat{e}$ – gain from an acceleration in monetary expansion, the others – whose efficiency falls in the range $1 + x > e_i^s > \bar{\bar{e}}$ – loose. On average, however, money growth does not affect wealth (output and consumption). Notice that also in this context in the steady state, money is superneutral on average but is not superneutral at the individual level. As a consequence, while the first moments of the distributions of output, consumption and wealth do not depend on money, higher moments are influenced by the rate of money growth. In particular, monetary policy has clear asymmetric effects: small entrepreneurs bear the brunt of a deceleration of monetary expansion while big entrepreneurs gain from it. However, average entrepreneurial income is necessarily greater than average economy-wide income. Therefore on average, entrepreneur gain from a deceleration of money growth.

5.3. Financing constraints

In this subsection we consider a variant of the environment described in the previous one. Suppose that capital markets are imperfect in the sense that there is no credit market to carry on investment. In this case the agent with an efficiency higher than $\hat{e}$ is a potential entrepreneur who can actually carry on his project if and only if he is self-financed, i.e. if $b_{it} - x > 0$. A financially constrained entrepreneur, i.e. an agent whose efficiency is higher than $\hat{e}$ but whose internal funds are insufficient to pay for the fixed cost ($b_{it} - x < 0$) must necessarily revert to the condition of worker. In this context, therefore, an entrepreneur must be not only relatively efficient but also relatively wealthy. A relatively efficient agent who cannot afford incurring the fixed cost x falls behind in the social ladder and is lumped together with the inefficient agents in the working class[16].

[16]In the interpretation given in the previous note, this is the case of an agent who cannot afford paying for the investment in human capital.

We can envisage two different scenarios. The first happens when $w \geq x\frac{\mu+\gamma}{1-\gamma} - \frac{\mu}{\gamma}\overline{\omega}$ and is depicted in figure 5.2. In this case the economy consists only of self-financed individuals and the absence of a credit market does not prevent the implementation of all the investment projects. As a consequence, all the results of the previous section are confirmed.

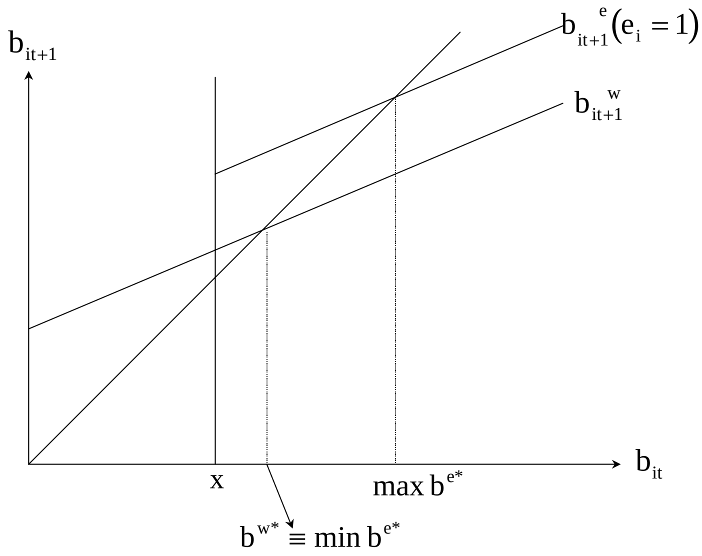

Figure 5.2

A different and more interesting scenario happens, symmetrically, when

$$w < x\frac{\mu+\gamma}{1-\gamma} - \frac{\mu}{\gamma}\overline{\omega} \tag{5.20}$$

and is depicted in figure 5.3.

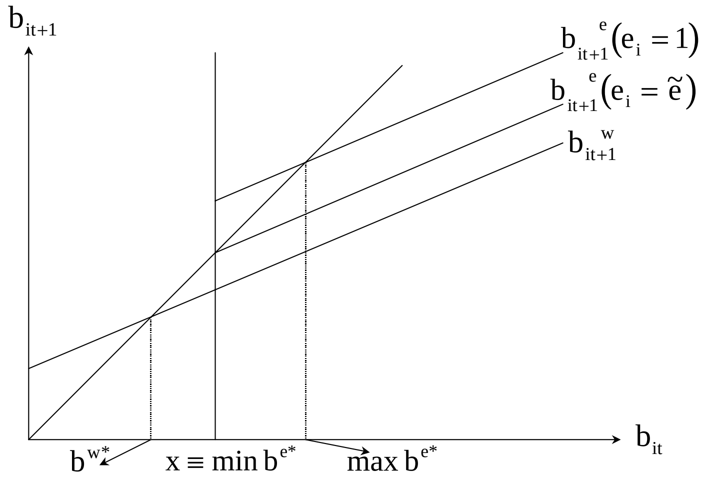

Figure 5.3

In this case there are some potential entrepreneurs who must give up the investment project, because of financial constraints, and be workers. As a consequence not all the agents efficient enough to become entrepreneurs actually become entrepreneurs in the steady state. Potential entrepreneurs with an efficiency level such that $\frac{1-\gamma}{1+\mu}\left(e_i^e - x + \frac{\mu}{\gamma}\bar{\omega}\right) < x$, i.e.

$$e_i^e < \frac{1+\mu}{1-\gamma}x - \frac{\mu}{\gamma}\bar{\omega} = \tilde{e} \tag{5.21}$$

will never catch up with the self financed entrepreneurs. In the steady state the wealth of entrepreneurs falls in the range $(x, \max b_i^e)$. x is the steady state of the entrepreneur with efficiency $e_i^e = \tilde{e}$. $\max b_i^e$ is the steady state of the entrepreneur with maximum efficiency $e_i^e = 1$ whose law of motion is

$$b_{it+1}^e = \frac{1-\gamma}{1+\mu}\left(1 - x + b_{it}^e + \frac{\mu}{\gamma}\bar{\omega}\right)$$

Therefore

$$\max b_i^e = \frac{1-\gamma}{\mu+\gamma}\left(1 - x + \frac{\mu}{\gamma}\bar{\omega}\right)$$

The average wealth of the entrepreneurs therefore is:

$$\bar{b}^e = \frac{\min b_i^e + \max b_i^e}{2} = \frac{1}{2}\left(\frac{\mu+2\gamma-1}{\mu+\gamma}x + \frac{1-\gamma}{\mu+\gamma}\left(1+\frac{\mu}{\gamma}\bar{\omega}\right)\right)$$

where average income is

$$\bar{\omega} = w\tilde{e} + \overline{\omega}^e\left(1-\tilde{e}\right) = w\tilde{e} + \left(\overline{e}^e - x\right)\left(1-\tilde{e}\right)$$

and

$$\overline{e}^e = \frac{1+\tilde{e}}{2}$$

Therefore

$$\bar{\omega} = \frac{1}{2} - x + \tilde{e}\left(w+x\right) - \frac{\tilde{e}^2}{2} \tag{5.22}$$

From (5.21) it comes out:

$$\bar{\omega} = \frac{1+\mu}{1-\gamma}\frac{\gamma}{\mu}x - \frac{\gamma}{\mu}\tilde{e} \tag{5.23}$$

Let's define $A = \frac{1+\mu}{1-\gamma}\frac{\gamma}{\mu}x$, $B = \frac{\gamma}{\mu}$, $C = \frac{1}{2} - x$ and $D = (w+x)$. (5.22) and (5.23) therefore become:

$$\bar{\omega} = A - B\tilde{e} \tag{5.24}$$

$$\bar{\omega} = C + D\tilde{e} - \frac{\tilde{e}^2}{2} \tag{5.25}$$

Note that, according to (5.25), $\bar{\omega}$ is a concave non monotonic function of $\widetilde{e}$, which presents a maximum in $\widetilde{e} = \widehat{e} = w + x$. It is clear that the *effective* entrepreneurial share of the population must be non-greater than the *potential* one. That is $1 - \widetilde{e} \leq 1 - \widehat{e}$. Therefore we have to focus only on the decreasing part of (5.25), corresponding to $\widetilde{e} \geq \widehat{e}$. According to (5.24), $\bar{\omega}$ is a linear decreasing function of $\widetilde{e}$. (5.24) and (5.25) represent a system in two unknowns: $\bar{\omega}$ and $\widetilde{e}$. The solution of the system is depicted in figure 5.4.

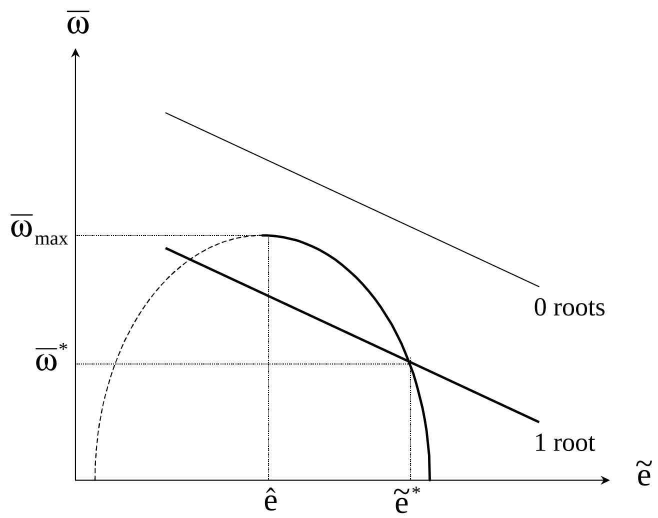

Figure 5.4

It is interesting now to pass analyzing the effects of money growth. For this purpose, first notice that (5.25) is not affected by μ, whereas it is (5.24). Actually we can write

$$\bar{\omega} = A(\mu) - B(\mu)\,\tilde{e} \tag{5.26}$$

where $\frac{\partial A}{\partial \mu} < 0$ and $\frac{\partial B}{\partial \mu} < 0$.

Graphically speaking therefore μ reduces the intercept of the straight line in figure 5.4 and makes it flatter. The effects of an increase in μ on $\bar{\omega}^*$ and on $\widetilde{e}^*$ are therefore ambiguous, namely it may happen either that $\bar{\omega}^*$ increases and $\widetilde{e}^*$ decreases or the other way around. In figure 5.5 we plot the former case and 5.6 we plot the latter.

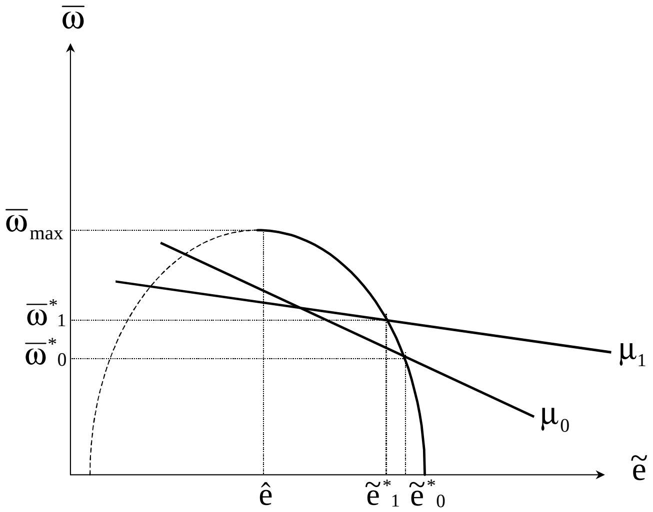

Figure 5.5

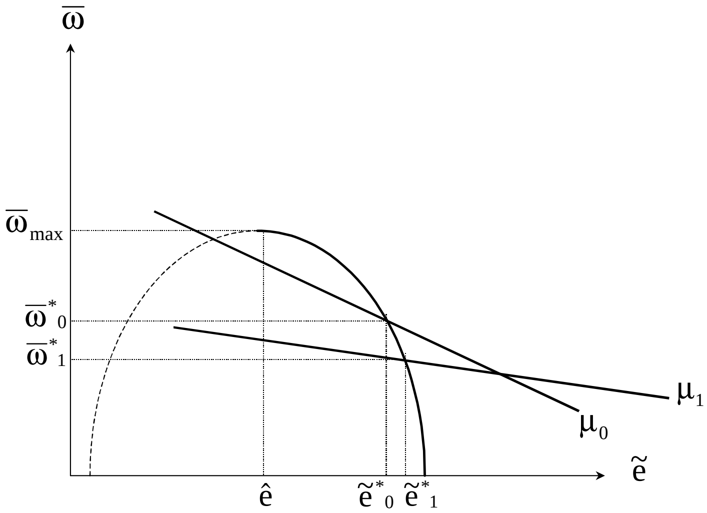

Figure 5.6

What we have sketched is relevant for consideration about individual and aggregate superneutrality of money.

Proposition 5.1. *When capital markets are incomplete, money may be non-superneutral. Actually as far as average income is affected by μ, monetary policy is superneutral neither at individual nor at aggregate level.*

In fact

$$\frac{\partial \overline{b}^*}{\partial \mu} = \frac{1-\gamma}{\gamma}\frac{\partial \overline{\omega}^*}{\partial \mu} \tag{5.27}$$

$$\frac{\partial \overline{c}^*}{\partial \mu} = \frac{\partial \overline{\omega}^*}{\partial \mu} \tag{5.28}$$

$$\frac{\partial \overline{y}^*}{\partial \mu} = \frac{1}{\gamma}\frac{\partial \overline{\omega}^*}{\partial \mu} \tag{5.29}$$

Moreover, notice that (5.20) is also a condition on μ. Actually it means that $\overline{\omega}^*$ has to be sufficiently low, namely lower than a critical level $\underline{\overline{\omega}}$, which turns out to be a function of μ.

$$\overline{\omega}^* < x\frac{\mu+\gamma}{1-\gamma}\frac{\gamma}{\mu} - w\frac{\gamma}{\mu} \equiv \underline{\overline{\omega}} \tag{5.30}$$

It is easy to demonstrate that $\frac{\partial \underline{\overline{\omega}}^*}{\partial \mu} > 0$, if and only if $\gamma < \frac{w}{x+w}$. This means that an increase in inflation may lead from the scenario depicted in figure 5.3 to the scenario depicted in figure 5.2.

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