Competition and Cooperation in Multi-Agent Fisheries

Gian-Italo Bischi
Istituto di Scienze Economiche, University of Urbino, Italy
bischi@econ.uniurb.it

Michael Kopel
Department of Managerial Economics and Industrial Organization,
University of Technology, Vienna, Austria
kopel@mail.ibab.tuwien.ac.at

Ferenc Szidarovszky
Department of System and Industrial Engineering,
University of Tucson, USA
szidar@sie.arizona.edu

This version: May 16, 2002

Preliminary and incomplete draft. Please do not quote!!!

1 Introduction

In this paper oligopoly theory and population dynamics are combined to describe the evolution of a renewable resource subject to harvesting under imperfect competition. A renewable resource, here a fish population, has on the one hand the capacity for reproduction and growth over time, where the population dynamics can be captured by a so-called biological growth or reproduction function. On the other hand, the stock of the resource is diminished by the harvesting activities of either a sole owner or multiple individuals. From an economist's point of view a variety of interesting questions arise in this context: How do (optimal) harvesting paths look like? In which situation, sole owner or multiple agents case, is it more likely to observe conservation or extinction of the resource? What determines the incentives of the agents to drive the resource towards extinction and what is the influence of the market structure?

In many papers on the dynamics of fisheries it has been assumed that the sea is open access, i.e. the fish stock is harvested by a large number of unregulated, competitive fishermen with no barriers to entry or exit. Due to perfectly competitive markets for harvested fish, the price for fish has been taken to be constant. Here however, we assume that due to some form of regulation, e.g. limited entry, access to the fishery is restricted. As a consequence, our multi-agent multi-market bioeconomic model of commercial fishing has to take two effects into account. First, it has to consider the market externality which is present in such an oligopoly situation. This means that every player influences the market prices via its quantity and, accordingly, the payoffs of all other players. Second, it also has to account for the stock externality, i.e. amounts harvested by one player are not available to other players. As a consequence, all players have higher harvesting costs, since these costs can be assumed to be inversely proportional to the available resource stock.

Such multi-market, multi-agent models have been introduced recently by Szidarovszky and Okuguchi (1998, 2000). They analyze an n-player, n-market oligopoly game, and consider a biological growth function given by the logistic law. They derive a one-dimensional dynamical system in continuous time capturing the evolution of the fish population over time which is subject to the harvesting activities of all players and they study two situations: The non-cooperative case, where all players act in their own interests, by maximizing their own profits without taking account the effects on overall profits, and the cooperative venture case, where each player's goal is to maximize joint profits (in other words they act like a sole owner). For both situations, the authors provide existence and stability results for this simple situation. A crucial assumption in their papers – as well as in most of the existing literature – is that fisherman are assumed to have perfect foresight, i.e. they are able to accurately predict the future fish stock prevailing in the sea. This rather restrictive assumption has been relaxed by Bischi and Kopel (2002). Although they consider a similar setup, their model is formulated in discrete time and they assume that fishermen are only boundedly rational. At the time when the agents determine their optimal level of future harvesting activities, the agents do not exactly know the future fish stock. Instead, all the agents have is an imperfect estimate, which is revised as new information about the fish stock becomes available. Obviously, this adaptation changes the dimension of the model, and now the question of the extension of the basins of existing equilibria becomes critical in order to shed some light on the question of extinction. In fact, due to global bifurcations that are specific to two-dimensional dynamical systems represented by iterated noninvertible maps of the plane, the topological structure of these basins might become quite complicated and a thorough understanding of the mechanism causing such changes and the relation to changes in structural parameters becomes an important issue for policy makers.

This paper is organized as follows. First, we briefly summarize some important properties of the logistic growth model for different values of the intrinsic growth rate, since this has been an important topic in the literature. We then investigate the dynamics of the fish stock which is subject to the harvesting activities of boundedly rational agents. We are considering the effects of harvesting costs and inertia in the expectations formation process on the possibility of conservation. We then compare the results with the case of a sole owner.

2 The biological growth law

Let $X(t)$ denote the fish stock. We assume that in the absence of any harvesting the stock of the fish population in period t is determined by the discrete time logistic equation[1]

$$X(t+1) = F_u(X) = X(t)\left(1+\alpha-\beta X(t)\right). \tag{1}$$

The dynamical behavior of this equation has been studied extensively by May (1976, 1987) and May and Oster (1976). The parameter α is referred to as the intrinsic growth rate. To begin with, we briefly summarize some results on the dynamics of the unharvested fish population. The map in (1) is conjugate to the well known standard logistic map $z' = \mu z(1-z)$ with parameter $\mu = 1+\alpha$ through the linear transformation $X = \frac{(1+\alpha)}{\beta}z$. For any $\alpha > 0$ there are two fixed points

$$X_0^* = 0 \quad \text{and} \quad X_1^* = \frac{\alpha}{\beta}. \tag{2}$$

The first represents a particular biological equilibrium, known as extinction of the species. The second, α/β, is called the "carrying capacity" of the species

---

[1] Other similar natural laws for growth can be considered. For example, the Ricker growth $X(t+1) = X(t)\exp\left[r\left(1-X(t)/K\right)\right]$ where r is the intrinsic growth rate, K is the carrying capacity. This can be easily compared with the logistic growth (1) by considering $r = \alpha$ and $K = \alpha/\beta$.

when no harvesting occurs. The equilibrium point $X_0^* = 0$ is unstable for each $\alpha > 0$, and the positive equilibrium X_1^* is stable for $0 < \alpha < 2$. For $2 < \alpha < 3$, even if X_1^* is unstable, a bounded positive attractor exists around it, characterized by oscillatory dynamics (periodic or chaotic) and trapped inside the absorbing interval $I = \left[\frac{(1+\alpha)^2}{16\beta}\left(3 + 2\alpha - \alpha^2\right), \frac{(1+\alpha)^2}{4\beta}\right]$, the upper boundary of which is the maximum value $c = F_u(\alpha/2\beta)$ and the lower boundary is its image $c_1 = F_u(c)$. For each $0 < \alpha < 3$, the basin of attraction of the positive attractor is bounded by the unstable fixed point $X_0^* = 0$ and its rank-1 preimage $(X_0^*)_{-1} = (1 + \alpha)/\beta$, i.e. the basin is given by the interval

$$\mathcal{B} = \left(0, \frac{1+\alpha}{\beta}\right).$$

Hence, if the intrinsic growth rate is not too large, the fish population might fluctuate, but never becomes extinct as long as the initial fish stock is in the interval $\mathcal{B}$. On the other hand, if the initial fish stock is out of the interval $\mathcal{B}$, the trajectory would take on negative values, which can be regarded as extinction of the fish population in finite time (see e.g. Clarke, 1990, p.13).

3 The model

There are n players and n markets, where $n > 1$. The n countries (or players) harvest fish and each player sells the fish in its home market and in the $n - 1$ foreign markets. The inverse demand functions for the markets $i = 1, 2, ..., n$ are given by $p_i = a_i - b_i(x_{1i} + x_{2i} + ... + x_{ni})$, where $x_{ki}(t)$ denotes the amount of fish harvested by player $k = 1, 2, ..., n$ and sold in market $i = 1, 2, ..., n$ at time period t. Each player's harvesting costs depend on the harvest rate and, additionally, on the total fish stock. This latter assumption captures the fact that it is easier and less expensive to catch fish, if the fish population is large. Let $X(t)$ be the total fish biomass at time t in the common sea and $h_k(t) = x_{k1}(t) + x_{k2}(t) + ... + x_{kn}(t)$ the amount of fish *harvested* by player k at time t. Then the cost function of player k is given by $C_k = c_k + \gamma_k \frac{h_k^2}{X}$, which satisfies the common assumptions that costs are decreasing in the fish stock and increasing in harvest (see Clark 1990, Szidarovszky and Okuguchi, 1998; note that this type of cost function can be derived from a Cobb-Douglas-type "production function" with fishing effort and biomass as the two inputs). Let $s_i(t) = x_{1i}(t) + x_{2i}(t) + ... + x_{ni}(t)$ be the amount of fish *supplied (and sold)* in country i at time period t. We assume that the total fish harvested by the players equals the total fish supplied in the markets, i.e. $H(t) = h_1(t) + h_2(t) + ... + h_n(t) = s_1(t) + s_2(t) + ... + s_n(t)$.

As mentioned in the introduction, in resource economics it is usually assumed that fishermen are completely informed about the growth law which governs the reproduction of the fish stock. In determining their future harvesting activities such that profits are maximized, the players can take the effect of the current fish stock on the harvesting costs into account. However, this assumption is rather restrictive. More realistically, the fishermen have access to a collection of past data about the harvested amounts of fish and some other indicators of the size of the fish population, from which they then try to derive an estimate or a prediction of the future fish stock. With every new piece of information, this estimate will be updated and be used to determine the future harvesting activities. We assume that the players have homogeneous expectations with respect to the fish stock (which might be due to their common experience of working in the same industry), which will be denoted as $X^e(t)$ in what follows. This quantity denotes the common expectation the players hold at time $t-1$ of the fish stock prevailing in the sea at time t. Recalling that the dynamics without commercial fishing is given by (1) and taking into account that the total harvesting quantity on the basis of the expected fish stock is $H(X^e(t))$, the evolution of the fish stock subject to harvesting is governed by

$$X(t+1) = X(t)\,(1 + \alpha - \beta X(t)) - H(X^e(t)).$$

Obviously, there are many ways to model how agents derive an estimate from past data. We will assume a simple learning rule called *adaptive expectations*, which states that the new estimate is a weighted average of the previous estimate and current data about the actual fish stock, where the weight on the past estimate (or belief) is a measure of the inertia of the agents. We will assume that at the end of the current period, using past observations about harvests and fish stock, fishermen are able to derive the exact stock of the resource. Hence, in our model fishermen are making a prediction about the future fish stock by forming a weighted average between the current fish stock and their previous prediction. The following 2-dimensional nonlinear dynamical system in discrete time then describes the expectation updating and the evolution of the fish stock

$$\begin{aligned} X(t+1) &= X(t)\left(1+\alpha-\beta X(t)\right) - H(X^e(t)) \\ X^e(t+1) &= \lambda X(t) + (1-\lambda)\, X^e(t). \end{aligned} \tag{3}$$

Note that any non-negative steady state of the 2-D model given by (3) (if existing) has to fulfil $X^e = X$. This condition corresponds to the perfect foresight case, where agents are assumed to be able to make exact predictions of the future fish stock. Consequently, the equilibria in the perfect foresight case and for the 2-dimensional model are the same[2]. The point $X_0^* = 0$ is always an equilibrium. Moreover, a positive X is an equilibrium if and only if

$$\alpha - \beta X = \frac{H}{X} = g(X). \tag{4}$$

Accordingly, the steady state fish stocks are given by the positive intersections of the linear function $\alpha - \beta X$ and the curve of function $g(X)$. In what follows we will derive expressions for the total harvesting quantity $H(X^e(t))$ for the cases where the players behave noncooperatively and cooperatively and we will characterize these quantities. Using the properties of the function g, we can give results on the existence and number of long run steady states in the two cases. Furthermore, we will illustrate the global dynamic properties of the 2-dimensional system – the basins of attraction of the equilibria and the changes these basins undergo as parameters are varied.

4 The non-cooperative game

In the non-cooperative case each player tries to determine its harvesting activity such that their own (short run) expected profit is maximized, without taking into account any effect on the total profits. The expected profit of player k in period t is then

$$\pi_k^e(t) = \sum_{i=1}^{n} \left[a_i - b_i(x_{1i} + x_{2i} + ... + x_{ni})\right] x_{ki} - c_k - \gamma_k \frac{h_k^2(t)}{X^e(t)}$$

---

[2] However, the reader should be aware that the stability properties are different. For a comparison of the stability properties of the perfect foresight case and the 2-D model we refer to Bischi and Kopel (2002).

Note that, since we are interested in the harvesting quantities in equilibrium, we assume that players accurately predict the quantities of their rivals. The first order conditions for player k are:

$$\frac{\partial \pi_k^e}{\partial x_{ki}} = a_i - b_i(x_{1i} + x_{2i} + ... + x_{ni}) - b_i x_{ki} - 2\gamma_k \frac{h_k(t)}{X^e(t)} = 0 \quad i = 1, ..., n$$

from which

$$x_{ki} = \frac{a_i}{b_i} - (x_{1i} + x_{2i} + ... + x_{ni}) - 2\frac{\gamma_k}{b_i}\frac{h_k}{X^e} \quad i = 1, ..., n$$

follows. Solving this (linear) system of equations would give the optimal quantities of fish, x_{ki}^*, harvested by player k and sold in country i as a function of the (current) expectation X^e. Instead, we focus on the total amount of harvest by player k since it is the total amount of fish harvested which affects the dynamics of the fish stock. To derive this quantity, we add the equations above for all $i = 1, ..., n$ to obtain

$$h_k = A - (h_1 + h_2 + .. + h_n) - 2\frac{B\gamma_k}{X^e} h_k \tag{5}$$

where $A = \sum_{i=1}^n (a_i/b_i)$ and $B = \sum_{i=1}^n (1/b_i)$. From this equation, we can get the reaction function for player k,

$$h_k = \frac{A}{2(1 + \frac{B\gamma_k}{X^e})} - \frac{1}{2(1 + \frac{B\gamma_k}{X^e})}(\sum_{j \neq k} h_j). \tag{6}$$

This relation and the fact that total harvest $H = \sum_{j=1}^n h_j = \sum_{j \neq k} h_j + h_k$ imply that

$$h_k = \frac{A}{1 + \frac{2B\gamma_k}{X^e}} - \frac{H}{1 + \frac{2B\gamma_k}{X^e}}.$$

Adding over $k = 1, 2, .., n$ then gives

$$H = A \sum_{k=1}^n \frac{1}{1 + \frac{2B\gamma_k}{X^e}} - H \sum_{k=1}^n \frac{1}{1 + \frac{2B\gamma_k}{X^e}}$$

By defining

$$f(X^e) = \sum_{k=1}^n \frac{1}{1 + \frac{2B\gamma_k}{X^e}}.$$

we obtain the optimal total harvesting quantity of all players in equilibrium:

$$H(X^e) = A\frac{f(X^e)}{1+f(X^e)}. \tag{7}$$

From this expression, we can derive the relative harvest

$$g(X^e) = \frac{H(X^e)}{X^e}. \tag{8}$$

Straightforward, although tedious, calculations show that total harvest H in (7) is strictly increasing and strictly concave in the expected fish stock, i.e. $\partial H/\partial X^e > 0$ and $\partial^2 H/\partial X^{e2} < 0$. Furthermore, the relative harvest in (8) is strictly decreasing and strictly convex in X^e, i.e. $\partial g/\partial X^e < 0$ and $\partial^2 g/\partial X^{e2} > 0$ (see Szidarovszky and Okuguchi, 1998).

Given that the total harvesting quantity is (7), the dynamical system (3) can be written as

$$\begin{aligned} X(t+1) &= X(t)\left(1+\alpha-\beta X(t)\right) - \frac{Af(X^e(t))}{1+f(X^e(t))} \\ X^e(t+1) &= \lambda X(t) + (1-\lambda)\, X^e(t) \end{aligned} \tag{9}$$

where the parameters γ_k, α, β, A and B are positive and $0 \leq \lambda \leq 1$. The equilibrium condition (4) is now given by

$$\alpha - \beta X = g(X) = \frac{Af(X)}{(1+f(X))X}.$$

Let $C = \sum_{i=1}^{n}(1/\gamma_k)$, then $g(0) = AC/2B$ and $g'(0) = -A(\sum_{i=1}^{n}(1/\gamma_k^2) + C^2)/4B^2$. Given the properties of the function g, we can now state the following results on the existence and number of steady states[3]:

(i) Assume first that $g(0) > \alpha$ and $g'(0) < -\beta$. Since $\lim\limits_{X^e\to\infty} g'(X^e) = 0$ and g' strictly increases, there is a unique $\overline{X}$ such that $g'(\overline{X}) = -\beta$.
(iA) If $g(\overline{X}) > \alpha - \beta\overline{X}$, then no positive equilibrium exists.
(iB) If $g(\overline{X}) = \alpha - \beta\overline{X}$, then there is a unique positive equilibrium.
(iC) If $g(\overline{X}) < \alpha - \beta\overline{X}$, then there are two positive equilibria.

---

[3] Note that the equilibrium analysis presented here holds for all relative harvest functions with the same qualitative properties as the function g.

(ii) Assume next that $g(0) > \alpha$ and $g'(0) \geq -\beta$. Then no positive equilibrium exists.

(iii) Assume that $g(0) = \alpha$.
(iiiA) If $g'(0) < -\beta$, then there is a unique positive equilibrium.
(iiiB) If $g'(0) \geq -\beta$, then there is no positive equilibrium.

(iv) Assume finally that $g(0) < \alpha$. Then there is a unique positive equilibrium.

Notice that (iiiB) is the borderline case of (ii), (iiiA) is a borderline case of (iv), and (ii) and (iA) can be treated in the same way. So we have four basic cases (see figs. 1a-d). In case 1 no positive equilibrium exists. Case 2 is characterized by the existence of a unique positive equilibrium. In case 3 two positive equilibria X_1^* and X_2^* exist, where $X_1^* < X_2^*$. In case 4, again only 1 unique equilibrium exists.

4.1 A special case: The symmetric game

In addition to players having the same expectations due to a common background or experience, we may also assume that they use the same fishing technology which is currently available. In a situation where $\gamma_k \equiv \gamma$ for all k, the game becomes symmetric, since all players face the same costs and supply the same markets. The total harvest in equilibrium in the symmetric non-cooperative game is

$$H^S = \frac{nAX^e}{(n+1)X^e + 2B\gamma}. \tag{10}$$

Obviously, since the game is symmetric

$$h_k = h^S = \frac{H^S}{n} = \frac{AX^e}{(n+1)X^e + 2B\gamma}.$$

Note that this implies also that $x_{ki} = x_i = \frac{a_i}{b_i} - s_i - 2\frac{\gamma}{b_i}\frac{h^S}{X^e}$, i.e. all players supply the same amount of fish for market i.

Using these expressions, we can show that, when fixed costs c_k are small, the profit of each player is positive for all t. Since in the symmetric game the players are identical, we can focus on the 'representative player'. It suffices to show that total profits are positive, since each player receives the same share. From the equation given in the previous paragraph, by summing over all players k, we get $s_i = nx_i = n\frac{a_i}{b_i} - ns_i - 2\frac{\gamma}{b_i}\frac{H^S}{X^e}$. This yields $s_i = \frac{na_i}{(n+1)b_i} - \frac{2\gamma H^S}{(n+1)b_i X^e}$. We will use this expression to show that the total profit is positive. The expected total profit is

$$\pi^e = \textstyle\sum_{k=1}^{n} \pi_k^e(t) = \sum_{k=1}^{n} \left\{ \sum_{i=1}^{n} \left[a_i - b_i(x_{1i} + x_{2i} + ... + x_{ni})\right] x_{ki} - c_k - \gamma_k \frac{h_k^2(t)}{X^e(t)} \right\} = \tag{11}$$

$$= \textstyle\sum_{i=1}^{n} \left[a_i(x_{1i} + x_{2i} + ... + x_{ni}) - b_i(x_{1i} + x_{2i} + ... + x_{ni})^2\right] - \sum_{k=1}^{n} c_k - \frac{1}{X^e(t)} \sum_{k=1}^{n} \gamma_k h_k^2(t)$$

which for the symmetric non-cooperative game reduces to (in what follows we neglect the expression for the fixed costs)

$$\begin{aligned} \pi^e &= \sum_{i=1}^{n} \left[a_i - b_i s_i\right] s_i - \frac{n\gamma(h^S)^2}{X^e} = \\ &= \sum_{i=1}^{n} \left[a_i - b_i s_i\right] s_i - \frac{\gamma(H^S)^2}{nX^e} \end{aligned}$$

Taking into account that $a_i - b_i s_i = \frac{a_i}{(n+1)} + \frac{2\gamma H^S}{(n+1)X^e}$, this yields

$$\begin{aligned} \pi^e &= \frac{1}{n+1} \textstyle\sum_{i=1}^{n} a_i s_i + \frac{2\gamma H^S}{(n+1)X^e} \sum_{i=1}^{n} s_i - \frac{\gamma(H^S)^2}{nX^e(t)} = \\ &\frac{1}{n+1} \textstyle\sum_{i=1}^{n} a_i s_i + \frac{2\gamma(H^S)^2}{(n+1)X^e} - \frac{\gamma(H^S)^2}{nX^e(t)} = \\ &\frac{1}{n+1} \textstyle\sum_{i=1}^{n} a_i s_i + \frac{\gamma(H^S)^2}{X^e(t)} \frac{(n-1)}{n(n+1)} > 0 \end{aligned}$$

Hence we have shown that the total profit in the noncooperative case is positive if fixed costs are zero. If fixed costs are positive, but sufficiently small, then π^e remains positive. Note that this is true for all expected fish stocks (i.e. also in the transient phase towards equilibrium) and not only in equilibrium.

From the equilibrium condition it is easy to see that the positive steady states in the symmetric game can be given as solutions of the quadratic equation

$$\beta(n+1)X^2 + (2\beta B\gamma - (n+1)\alpha)X + nA - 2B\gamma\alpha = 0$$

The existence conditions which characterize the four different cases described above for the general (non-symmetric) game (see fig. 1) can be given in detail when the game is symmetric. Since $C = n/\gamma$, we get $g^S(0) = nA/2B\gamma$, $g^{S\prime}(0) = -nA(n+1)/4B^2\gamma^2$. Additionally, from $g^{S\prime}(\overline{X}) = -\beta$, we get $\overline{X} = \sqrt{nA/(n+1)\beta} - 2B\gamma/(n+1)$. As shown above, a comparison of the expressions $g^S(\overline{X}) = \sqrt{\beta nA/(n+1)}$ and $\alpha - \beta\overline{X}$ provides insights on the existence and number of equilibria. For example, case 3 with two positive equilibria can be characterized by the conditions

$$\begin{aligned} \alpha &< \frac{nA}{2B\gamma} \\ \beta &< \frac{n(n+1)A}{4B^2\gamma^2} \\ \alpha &> 2\sqrt{\frac{\beta nA}{n+1}} - \frac{2\beta\gamma B}{n+1} \end{aligned}$$

which are restrictions on the range of biological parameters α and β.

4.2 Entry of a new player in the non-cooperative case

Let us now assume that a new player enters the game. Hence the number of players and markets increases from n to $n+1$. What can be said about the corresponding total harvests in the non-cooperative game? Under which conditions will an increase in the number of players lead to an increase in the total harvest?

To answer these questions, we denote $A_n = \sum_{i=1}^{n}(a_i/b_i)$, $B_n = \sum_{i=1}^{n}(1/b_i)$, and $f_n(X^e) = \sum_{k=1}^{n} 1/(1 + \frac{2B_n\gamma_k}{X^e})$. Under which conditions does

$$H_{n+1} = A_{n+1}\frac{f_{n+1}(X^e)}{1 + f_{n+1}(X^e)} > A_n\frac{f_n(X^e)}{1 + f_n(X^e)} = H_n$$

hold? Using the relation $A_{n+1} = A_n + a_{n+1}/b_{n+1}$, it is easy to see that this inequality is equivalent to

$$a_{n+1} > \frac{b_{n+1}A_n[f_n(X^e) - f_{n+1}(X^e)]}{[1 + f_n(X^e)]f_{n+1}(X^e)}$$

Hence, two cases have to be considered:

- Case 1: If $f_n(X^e) - f_{n+1}(X^e) \leq 0$, then this inequality holds always. Hence, $H_{n+1} > H_n$ holds always.
- Case 2: If $f_n(X^e) - f_{n+1}(X^e) > 0$, then this inequality is fulfilled if a_{n+1} is sufficiently large (observe that the right-hand side of the inequality is independent of a_{n+1}).
  The condition $f_n(X^e) - f_{n+1}(X^e) > 0$ holds if and only if

  $$\sum_{k=1}^{n} \frac{1}{1 + 2B_n\gamma_k/X^e} > \sum_{k=1}^{n} \frac{1}{1 + 2B_{n+1}\gamma_k/X^e} + \frac{1}{1 + 2B_{n+1}\gamma_{n+1}/X^e} \quad (12)$$

  Notice that only the last term on the right hand side depends on γ_{n+1}. Furthermore, since $B_{n+1} = \sum_{i=1}^{n+1} \frac{1}{b_i} > \sum_{i=1}^{n} \frac{1}{b_i} = B_n$, for $k = 1, 2, ...n$, we have

  $$\frac{1}{1 + 2B_n\gamma_k/X^e} > \frac{1}{1 + 2B_{n+1}\gamma_k/X^e}.$$

  Accordingly, adding over all k yields

  $$LHS = \sum_{k=1}^{n} \frac{1}{1 + 2B_n\gamma_k/X^e} > \sum_{k=1}^{n} \frac{1}{1 + 2B_{n+1}\gamma_k/X^e} = RHS$$

  Clearly, if $LHS - RHS \geq 1$, then (12) holds and $f_n(X^e) - f_{n+1}(X^e) > 0$ is always fulfilled. On the other hand, if $LHS - RHS < 1$, then γ_{n+1} has to be sufficiently large for (12) to hold.

For the symmetric game a sufficient condition for $H^S_{n+1} > H^S_n$ is that $\frac{1}{b_{n+1}} \leq \frac{B_n}{n}$. In other words, if the sensitivity of demand with respect to price changes of market $n + 1$ is less than or equal to the average price sensitivity of demand over all the remaining markets, then total harvest after entry of the new player increases.

5 Cooperative venture case

Let us now assume that the players form a cooperative venture. In this case each player determines its harvesting activity such that the *joint profit of all players* is maximized. That is, each players harvesting activity x_{ki} maximizes the total sum of expected profits (see (11)). The first-order conditions for each player k are

$$\frac{\partial \pi^e}{\partial x_{ki}} = a_i - 2b_i(x_{1i} + x_{2i} + ... + x_{ni}) - 2\gamma_k \frac{h_k(t)}{X^e(t)} = 0 \quad i = 1, ..., n$$

We rewrite these conditions as

$$\frac{a_i}{b_i} - 2(x_{1i} + x_{2i} + ... + x_{ni}) - 2\gamma_k \frac{h_k(t)}{b_i X^e(t)} = 0 \quad i = 1, ..., n \tag{13}$$

Adding over all i yields

$$A - 2H - 2\gamma_k \frac{B h_k(t)}{X^e(t)} = 0$$

where we used the same definitions of A and B as before. Rewriting this condition as

$$\frac{A - 2H}{2B\gamma_k} = \frac{h_k}{X^e}$$

and summing over all players gives the expression for the total harvest if players form a cooperative venture

$$H^V = \frac{ACX^e}{2(CX^e + B)} \tag{14}$$

where as before $C = \sum_{k=1}^{n} \frac{1}{\gamma_k}$. The relative harvest in this case is given by

$$g^V(X^e) = \frac{H^V}{X^e} = \frac{AC}{2(CX^e + B)}$$

It is easy to see that H^V is strictly increasing and strictly concave and that g^V is strictly decreasing and strictly convex in X^e (see Szidarovszky and Okuguchi, 2000). Moreover, for given X^e and n, the total harvest in the cooperative case (14) is smaller than in the non-cooperative case (7): $H^V =$

$\dfrac{ACX^e}{2(CX^e+B)} < H = A\dfrac{f(X^e(t))}{1+f(X^e(t))}$, where $f(X^e) = \sum_{k=1}^n \dfrac{1}{1+\frac{2B\gamma_k}{X^e}}$.
Proof: The inequality given above is equivalent to

$$f(X^e(t)) > \frac{CX^e}{CX^e+2B}.$$

Let $z_k = \dfrac{X^e}{2B\gamma_k}$. Then $f(X^e) = \sum_{k=1}^n \dfrac{1}{1+\frac{1}{z_k}} = \sum_{k=1}^n \dfrac{z_k}{1+z_k}$. The right hand side of the inequality can be also expressed in terms of z_k as $\dfrac{CX^e}{CX^e+2B} = \dfrac{1}{1+2B/CX^e} = \dfrac{1}{1+1/\sum_{k=1}^n z_k} = \dfrac{\sum_{k=1}^n z_k}{\sum_{k=1}^n z_k + 1}$. Since, for $k = 1, 2, ..., n$ we have $\dfrac{z_k}{1+z_k} > \dfrac{z_k}{1+\sum_{k=1}^n z_k}$, adding these inequalities for all k proves the claim.

Note that this result is valid only for a given expected fish stock. Since the positive stable long-run equilibria fish stocks for the cooperative game and the non-cooperative game might be different, nothing is said how these steady states and the corresponding total harvests in these steady states relate to each other.

In the cooperative venture case with adaptive expectations, the evolution of the fish stock subject to harvesting is governed by

$$\begin{aligned} X(t+1) &= X(t)\left(1+\alpha-\beta X(t)\right) - \frac{ACX^e}{2(CX^e+B)} \qquad (15)\\ X^e(t+1) &= (1-\lambda)X^e(t) + \lambda X(t) \end{aligned}$$

The equilibrium condition (4) is now given by

$$\alpha - \beta X = g^V(X) = \frac{AC}{2(CX+B)}.$$

Since the function g^V has the same qualitative properties as the function g in the noncooperative case, conditions (i)-(iv) presented above can be used to characterize the existence and number of steady states in the cooperative venture case (see figures 1a-d, where a qualitative illustration is given). We now have $g^V(0) = AC/2B = g(0)$ but $g^{V\prime}(0) = -AC^2/2B^2 < g'(0)$. Furthermore, since $H^V(X) < H(X)$, we also have $g^V < g$. Therefore, case 1 may remain the same, or even cases 2 or 3 will occur. That is, if there is no positive equilibrium for the non-cooperative case, full cooperation may result in the emergence of one or two (positive) equilibria, X_1^{*V} and X_2^{*V} (fig. 1a). In case 2, full cooperation will result in the appearance of two positive equilibria, X_1^{*V} and X_2^{*V}, where the original positive equilibrium (for the non-cooperative case) is always between the two new equilibria (for the full cooperation game); see fig. 1b. In case 3, with full cooperation two equilibria still exist. The change occurs with respect to the location of the equilibria: the smaller equilibrium decreases and the larger one increases, i.e. $X_1^{*V} < X_1^*$ and $X_2^{*V} > X_2^*$ (fig. 1c). Considering case 4 we notice that with full cooperation the unique positive equilibrium increases with respect to the noncooperative game, i.e. $X^{*V} > X^*$ (fig. 1d).

We may summarize the intuition of these results by the following statements. Comparing the noncooperative and the full cooperation cases, we observe that for the same expected fish stock, agents acting cooperatively harvest less than if they act in a noncooperative way. This aggregate behavior eventually leads to a higher fish stock remaining in the sea in the long run. Cooperation leads to conservation. These insights are in line with earlier results for game-theoretic models of fisheries (e.g. Clarke 1990, Levhari and Mirman 1982) and are typical of the well known prisoner's dilemma situation[4].

Finally, note that, in contrast to the non-cooperative case, it is possible to use the equilibrium condition to derive explicit expressions for the equilibria. In addition to the steady state $X_0^* = 0$, we get in the cooperative case further steady states as the solutions of the quadratic equation

$$2C\beta X^2 + (2B\beta - 2\alpha C)X + AC - 2B\alpha = 0.$$

---

[4] One might imagine the following fishermen's dilemma (see Clark 1990). Two fishermen can employ two different strategies, severe exploitation (competitive attitude) or moderate exploitation (cooperative attitude). For suitably chosen payoff the well known conclusions of a symmetric 2x2 game is that cooperation (a bi-lateral agreement for less fishing effort) implies conservation. A higher fish stock implies higher harvesting results for each, resulting in higher profits.

5.1 A special case: The symmetric game

If we consider the symmetric case again, that is $\gamma_k \equiv \gamma$ for all k, then $C = n/\gamma$, and it is easy to see that

$$H^{VS} = \frac{AnX^e}{2(nX^e + \gamma B)} \tag{16}$$

We have for the harvesting quantity of a single player

$$h_k = h^{VS} = \frac{H^{VS}}{n} = \frac{AX^e}{2(nX^e + B\gamma)}.$$

Again it is rather straight forward to show that total profit is positive. Since players are identical, using the relation given above, we get from (13) that $s_i = \frac{a_i}{2b_i} - \frac{\gamma H^{VS}}{nb_iX^e}$. Furthermore, this yields $a_i - b_is_i = \frac{a_i}{2} + \frac{\gamma H^{VS}}{nX^e}$. Hence, the total profit is

$$\begin{aligned}\pi^e &= \sum_{i=1}^{n} [a_i - b_is_i]\, s_i - \frac{\gamma(H^{VS})^2}{nX^e} \\ &= \frac{1}{2}\sum_{i=1}^{n} a_is_i + \frac{\gamma H^{VS}}{nX^e}\sum_{i=1}^{n} s_i - \frac{\gamma(H^{VS})^2}{nX^e} = \\ &= \frac{1}{2}\sum_{i=1}^{n} a_is_i > 0\end{aligned}$$

An interesting results follows, if we assume that positive steady states exist in the *symmetric* game with and without full cooperation and compare the expressions for the total harvesting quantities in these steady states. Let $\Delta = X_2^{*VS} - X_2^{*S}$ denote the difference between the positive stable steady states in the cooperative venture case (X_2^{*VS}) and the non-cooperative case (X_2^{*S}) for the symmetric game, where $X_2^{*VS} > X_2^{*S}$. Then $H^{VS}(X_2^{*VS}) = \frac{AnX_2^{*VS}}{2(nX_2^{*VS} + \gamma B)} > H^S(X_2^{*S}) = \frac{nAX_2^{*S}}{(n+1)X_2^{*S} + 2B\gamma}$ if and only if $\Delta > (n-1)X_2^{*VS}X_2^{*S}/2\gamma B$. In other words, the total harvest in equilibrium in the cooperative case is larger than in the non-cooperative case if and only if some relative difference between the positive stable steady states X_2^{*VS} and

X_2^{*S} is sufficiently large. Hence, in this case cooperation does not only lead to higher conservation, but the larger stock eventually leads also to a higher total harvest (due to increased growth of the resource).

Moreover, we can again provide details on the existence conditions of equilibria and compare them with the conditions for the noncooperative game. In the symmetric full cooperation game we have $g^{VS}(0) = An/2B\gamma$ and $g^{VS\prime}(0) = -An^2/2B^2\gamma^2$.

From $g^{VS\prime}(\overline{X}) = -\beta$, we get $\overline{X} = \sqrt{A/2\beta} - B\gamma/n$. A comparison of the expressions $g^{VS}(\overline{X}) = \sqrt{\beta A/2}$ and $\alpha - \beta\overline{X}$ again provides insights on the existence and number of equilibria. For example, case 3 in the full cooperation case with two positive equilibria can be characterized by the conditions

$$\begin{aligned}\alpha &< \frac{nA}{2B\gamma}\\ \beta &< \frac{n^2A}{2B^2\gamma^2}\\ \alpha &> \sqrt{2A\beta} - \frac{\beta\gamma B}{n}.\end{aligned}$$

The first condition coincides with the condition in the noncooperative game, but the second condition is less restrictive.

5.2 Entry of a new player in the cooperative case

We are now studying the effect of entry of a new player opening a new market. A_n, B_n and $f_n(X^e)$ are defined as before. Let $C_n = \sum_{k=1}^{n} \frac{1}{\gamma_k}$. Again, we are interested in conditions such that harvest after entry occurred is larger, i.e.

$$H_{n+1}^V = \frac{A_{n+1}C_{n+1}X^e}{2(C_{n+1}X^e + B_{n+1})} > \frac{A_nC_nX^e}{2(C_nX^e + B_n)} = H_n^V$$

Using again the relation $A_{n+1} = A_n + a_{n+1}/b_{n+1}$, it is easy to see that this inequality is equivalent to

$$a_{n+1} > b_{n+1}\frac{A_n(C_nB_{n+1} - C_{n+1}B_n)}{C_{n+1}(C_nX^e + B_n)}$$

Two cases have to be considered:

- Case 1: If $C_nB_{n+1} - C_{n+1}B_n \leq 0$, then this inequality always holds and entry increases total harvest.
- Case 2: If $C_nB_{n+1} - C_{n+1}B_n > 0$, then this inequality holds if a_{n+1} is sufficiently large. Observe that $C_nB_{n+1} = C_n(B_n + \frac{1}{b_{n+1}}) > C_{n+1}B_n = (C_n + \frac{1}{\gamma_{n+1}})B_n$ if and only if $\gamma_{n+1} > B_nb_{n+1}/C_n$, i.e. if γ_{n+1} is sufficiently large.

If we consider the symmetric game, a sufficient condition for $H_{n+1}^{VS} > H_n^{VS}$ is again $\frac{1}{b_{n+1}} \leq \frac{B_n}{n}$. As in the non-cooperative game, if the price sensitivity of demand in the new market is less than or equal to the average price sensitivity of demand over all the remaining markets, then total harvest increases after entry of the new player.

6 Local and global stability of equilibria

After having extensively discussed the existence of equilibria in the non-cooperative and full cooperation game and the special cases of symmetric games, we are now turning our attention to an analysis of the stability properties of the long run steady states. The two-dimensional discrete dynamical system in (3) can be represented as an iterated point mapping

$$T : \begin{cases} X' = X\left(1 + \alpha - \beta X\right) - H\left(X^e\right) \\ X^{e'} = \lambda X + \left(1 - \lambda\right) X^e \end{cases} \tag{17}$$

Each time the map T is applied, a point of the plane (X, X^e) is moved to another point, which represents the state of the system at the next time step. A trajectory of the system with adaptive expectations

$$\tau(X(0), X^e(0)) = \left\{(X(t), X^e(t)) = T^t(X(0), X^e(0)), t \geq 0\right\}$$

is generated by T starting from an initial condition $(X(0), X^e(0))$. The projection of a point along the trajectory on the horizontal axis gives the time evolution of the fish stock, the projection on the vertical axis gives the time evolution of expectations. Time periods at which the trajectory is close to the diagonal $X^e = X$ correspond to periods at which expectations are quite accurate, whereas points far from the diagonal represent over/under estimates of the real fish stock (depending on the fact if the point is above or below the 45-degree line).

In the *non-cooperative game*, the total harvest H is given in (7). To derive stability conditions for the equilibria, we have to analyze the eigenvalues of the Jacobian matrix

$$DT(X,X^e)=\left[\begin{array}{cc} 1+\alpha-2\beta X & -A_n\frac{Df(X^e)}{[1+f(X^e)]^2} \\ \lambda & 1-\lambda \end{array}\right],$$

computed at the fixed point considered, where $f(X^e)=\sum_{k=1}^{n}X^e/(X^e+2B\gamma_k)$. For example, let us consider the equilibrium $O=(0,0)$. We have

$$DT(0,0)=\left[\begin{array}{cc} 1+\alpha & -\frac{AC}{2B} \\ \lambda & 1-\lambda \end{array}\right],$$

and the characteristic equation is

$$z^2-Tr(O)z+Det(O)=0$$

where $Tr(O)=2+\alpha-\lambda$ and $Det(O)=(1-\lambda)(1+\alpha)+\lambda AC/(2B)$. The conditions for stability are

$$\begin{aligned} 1-Tr(O)+Det(O)&>0 && (=0 \text{ means that } z=1 \text{ is an eigenvalue})\\ 1+Tr(O)+Det(O)&>0 && (=0 \text{ means that } z=-1 \text{ is an eigenvalue})\\ 1-Det&>0 && (=0 \text{ means that both the eigenvalues have } |z|=1)\end{aligned}$$

The first condition gives $AC/(2B)>\alpha$. It is easy to see that this condition coincides with the stability condition of the model where agents have perfect foresight; see Bischi and Kopel (2002). The second condition becomes $4+(2-\lambda)\alpha+\lambda AC/(2B)-2\lambda>0$, which is always satisfied. The third condition gives the extra stability condition

$$AC/(2B)<(\lambda-\alpha(1-\lambda))/\lambda.$$

So, we can conclude that the range of local asymptotic stability of the extinction equilibrium, defined by

$$\alpha<AC/(2B)<1-\alpha\frac{1-\lambda}{\lambda}$$

is non-empty only if $\alpha<\lambda$. It is again easy to show that the range of stability of the extinction equilibrium is smaller under adaptive expectations than under perfect foresight (Bischi and Kopel 2002). We cannot conclude, however, that extinction is less probable. In fact, the conditions above only concern asymptotic stability, whereas extinction might even occur in finite time, due to trajectories which exit the positive quadrant. For example, whenever the fixed point $(0,0)$ is an unstable focus (complex eigenvalues with modulus greater than one), trajectories would eventually assume negative values. In such a case, we can say that extinction in finite time occurs, even if the extinction equilibrium is unstable (see Clark 1990 and Bischi and Kopel, 2002 for a more detailed discussion on this point).

For the study of the basins of the long run steady states it is important to notice that the map T which describes the adaptive process in the noncooperative case, is a noninvertible (or many-to-one) map[5]. Given a point $\left(X', X^{e'}\right)$ several distinct points may exist which are mapped into $\left(X', X^{e'}\right)$. Put differently, several preimages can be obtained by solving (17) with respect to (X, X^e), so that the inverse relation $(X, X^e) = T^{-1}\left(X', X^{e'}\right)$ is a multivalued function. In fact, from

$$\begin{cases} X' = X\left(1+\alpha-\beta X\right) - H\left(X^e\right) \\ X^{e'} = \lambda X + \left(1-\lambda\right) X^e \end{cases}$$

we obtain $X = \frac{1}{\lambda}\left(X^{e'} - \left(1-\lambda\right) X^e\right)$, where X^e is a real solution of the equation

$$\frac{\beta\left(1-\lambda\right)}{\lambda^2}\left(X^e\right)^2 + \frac{1-\lambda}{\lambda}\left(\frac{2\beta}{\lambda}X^{e'} - \left(1+\alpha\right)\right)X^e - X' + \frac{1+\alpha}{\lambda}X^{e'} - \frac{\beta}{\lambda}X^{e'2} = H\left(X^e\right)$$

The positive solutions of this equation are located at the intersections (if any) of a parabola and the increasing and concave function H. Hence, we can have two, one or no positive solutions (indeed, there are several negative solutions, but we can neglect these preimages).

As the map T is continuously differentiable, it is easy to obtain the equation of LC_{-1}, since it is included in the set of points at which the determinant of the Jacobian vanishes:

$$\det DT\left(X, X^e\right) = \left(1-\lambda\right)\left(1+\alpha-2\beta X\right) + \lambda DH_n(X^e) = 0. \qquad (18)$$

---

[5] This property also holds for the one-dimensional map which gives the dynamics under PF, i.e. $X^e = X$ for all time periods t. It is easy to see that this map has an unimodal graph (see Bischi and Kopel, 2002). The map is a $Z_0 - Z_2$ noninvertible map, i.e. a point of its codomain may have two preimages or no preimage, like the logistic map.

The graph of the curve LC_{-1} is shown in fig. 2. Applying the map T to points of LC_{-1} yields the critical curve of rank-1, $LC = T\left(LC_{-1}\right)$. The curve LC can now be used to identify regions of the plane whose points have different number of preimages, just as the critical points of a one-dimensional map can be used to locate regions with different preimages (see e.g. the quadratic map). These critical curves separate the phase plane into regions Z_k whose points have k preimages, or, equivalently, where k distinct inverses of T are defined (see e.g. Mira et al., 1996, or Agliari et al., 2002). It is interesting to note that for $\lambda < 1$ the locus LC_{-1} of merging preimages (or critical curve of rank-0) has equation

$$X = \frac{1}{2\beta}\left[1 + \alpha + \frac{\lambda}{1-\lambda}DH(X^e)\right]$$

whereas if $\lambda = 1$ (i.e. in the case of naive expectations), then $\det DT$ never vanishes, since $\det DT = DH(X^e) > 0$ for each X^e. So, for $\lambda = 1$ no critical curves exist. The presence of critical curves may have important consequences on the structure of the basins' boundaries. The fact that several preimages may exist give rise to the possibility of having non-connected or multiply connected basins. The creation and the structure of the basins can be explained by using the concept of critical curves, since the occurrence of global bifurcations, i.e. contacts between critical curves and the boundaries of the basin, give rise to important and significant changes (for recent applications in economics, see Bischi et al., 2000, Bischi and Kopel 2001, 2002, Bischi et al. 2002).

In the *full cooperation game*, the dynamic evolution of the fish stock and the expectations is described by (3), where H is replaced by

$$H^V\left(X^e\right) = \frac{ACX^e}{2\left(CX^e + B\right)}.$$

The qualitative properties of the resulting 2-dimensional dynamical system are quite similar to the properties of (17). However, important differences in the quantitative effects are caused by the lower total harvesting quantity, the different conditions for the existence of equilibria and the different location of the positive equilibria with respect to the noncooperative game. Also the shape of LC_{-1}, whose equation is $(1-\lambda)\left(1+\alpha-2\beta X\right)+\lambda DH_n^V(X^e) = 0$, is quite similar, and the vertical asymptote in the $\left(X, X^e\right)$ –plane has the same equation $X = \left(1+\alpha\right)/\beta$. However, as $DH^V\left(X^e\right) < DH\left(X^e\right)$, the curve

LC_{-1}^{V} is to the left of (and below of) LC_{-1}. The main differences between the two cases, non-cooperative and full cooperation game, as far as stability and stability extent is concerned (i.e. extension and structure of the basins, which give information about the robustness with respect to exogenous perturbations both in fish stock and in fish stock estimates) can be better appreciated by carrying out some numerical experiments, where these numerical studies are guided by the analytic/geometric results described above.

6.1 Numerical Explorations

For the intrinsic growth rate and the biological parameter β we choose

$$\alpha = 3, \beta = 1. \tag{19}$$

In this situation, the dynamics of the unharvested population evolving according to (1) would exhibit chaotic oscillations around the carrying capacity $\alpha/\beta = 3$.

We now consider the non-cooperative game with two players and two markets, $n = 2$, where $a_1 = 5, a_2 = 4.5, b_1 = 2, b_2 = 2$. This set of parameters yields the aggregate parameters $A = 4.75$, $B = 1$. In the numerical explorations we will focus on the impact of changes in the cost parameters γ_1 and γ_2 on the extent of the basins and their structure. Furthermore, we will also investigate the effect of variations in λ, which measures the inertia of the fishermen to revise their expectations as new information becomes available. The influence of changes in the cost parameters is of significant interest, since the costs of harvesting can be changed by such methods as restricting the length of the fishing season, setting total catch limitations, and regulating the type of fishing gear used. The influence of changes in the expectations formation process is interesting from a behavioral point of view. If fishermen put a higher weight on the most recently observed fish stock, does this lead to more conservative behavior or not? We start our explorations with high costs for both players. Intuitively, high harvesting costs should prevent over-fishing and lead to conservation. Indeed, this intuition is confirmed by the numerical results shown in fig. 3a. Since $AC/(2B) < \alpha$ there is a unique positive equilibrium at $(2.384, 2.384)$ and the basin of this equilibrium (shown in light color) is rather large. The equilibrium is far away from the basin boundaries, which can be taken as an indication that in such a situation even exogenous shocks will not result in disaster; the model is robust with respect to noise.

The grey region represents the set of initial conditions which result in extinction of the species (where we do not distinguish extinction in the long run from extinction in finite time). The fixed point $(0, 0)$ is in this situation a saddle point, and its stable set constitutes the boundary which separates the two regions. The situation changes for decreasing cost parameters, see figs. 3b-d. First, notice that two positive equilibria exist, where (X_1^*, X_1^*) is a saddle point, and its stable set now constitutes the boundary which separates the basin of the stable equilibrium (X_2^*, X_2^*) and the set of points which lead to extinction. The size of the basin $\mathcal{B}$ becomes smaller and smaller until it reduces to a set which is so small that "stability" has lost any practical meaning (see fig. 3d). To investigate the role of the speed of adjustment, we used the same parameters as in fig. 3d, but increased the value of λ from 0.5 to 0.9 in fig. 3e. The basin of the stable equilibrium is enlarged. Obviously, less inertia in revising expectations results in an enlargement of the basin of the stable equilibrium. We can conclude that there are two possible ways to achieve "more stability" in terms of the extent of the basin of the equilibrium. A policy maker may increase the costs of harvesting the resource and may prevent over-fishing the resource. This result is quite reasonable. If it is more expensive for fishermen to harvest the resource, the total harvesting activity is reduced. As a result, conservation of the resource is achieved from a larger set of initial combinations of the actual and expected fish stock. A different route to higher stability is to make the fishermen believe that the use of the most recent observation of the fish stock gives a better prediction of future fish stocks than relying previous observations.

Surprising and unexpected effects are obtained when slightly different sets of parameters are considered. Let us put

$$a_1 = a_2 = 5.5 \ ; \ \ b_1 = b_2 = 1 \ \ ; \lambda = 0.3.$$

Figs. 4a-e show the basins of the positive stable equilibrium for decreasing values of the cost parameters γ_1 and γ_2. Here we can observe global bifurcations which drastically change the topological structure of the basin $\mathcal{B}$ from a simply connected set (figs. 4a and b) to a multiply connected set (connected with holes inside) in fig. 4c, to a non-connected set (union of disjoint portions) in figs. 4d and 4e. These significant changes of the basins are due to contacts between the basin boundary and the critical curve LC (see e.g. Mira et al., 1996, Abraham et al., 1997, Bischi, Gardini and Kopel, 2000, Bischi and Kopel, 2001, Agliari et al., 2002). Hence, they are known as contact bifurcations. Of course, basin structures like those shown in figs 4c and d have important consequences for practical considerations. In contrast to the previous situations, a slight dislocation of a point in the phase space can now have serious consequences with respect to the long run fate of the fish stock. Such a *final-state sensitivity* (see Grebogi et al. 1983; Brock and Hommes 1997), where the complexity does refer to any attracting set but concerns the choice of the initial condition, is of particular relevance in fishery economics. Small differences or noise can lead to vastly different outcomes, namely conservation or extinction. It is important to note that such a complexity of the basin (multiply connected or non-connected) can only arise in discrete dynamical systems generated by the iteration of a noninvertible map.[6]

We now consider the full cooperation game. Recall that in this case, the agents choose their individual harvesting quantities such that the joint profits of all players is maximized. Hence, we can refer to this situation as if players are conducting a cooperative venture. In order to obtain a comparison with the non-cooperative game, we consider the same parameters as those used in fig. 4e and examine the extent of the basin of the stable positive equilibrium in the full cooperation case (fig. 5). Obviously, the basin is simply connected and larger than in the non-cooperative case. The complex topological structure, which causes a loss of robustness and predictability, cannot be observed here for this set of parameters. Simply connected basins are obtained even with lower values of the cost parameters.

Our results confirm (and generalize) the conclusion about the role of cooperation which can be found in the literature (see Clarke 1990, Levhari and Mirman 1982, and in particular, Szidarovszki and Okuguchi 2000). As far as the stability extent of the positive stable equilibrium is concerned, cooperation leads to more stability. In addition to these differences, we can also observe that the stable equilibrium in the non-cooperative game is closer to the basin boundary than in the full cooperation game for similar values of the cost parameters (cf. figs. 4b and 5). This raises another issue, namely how "stability" can be measured in terms of the extent of the basins of attraction. In fig. 4b, although the basin might be considered as "large", the equilibrium

---

[6] A higher value of the parameter λ (e.g. $\lambda = 0.8$) would again achieve stability as far as the extension of the basin is concerned. In this case increasing values of λ also imply that any complexity in the topological structure of the basins is lost. This is due to the fact that as $\lambda \rightarrow 1$ the Jacobian determinant never vanishes, so no critical curves exist. It is also interesting that under the assumption of PF with the same parameters as in the present scenario no complexities arise.

is quite close to the basin boundary. Such a situation might not be considered as being very stable despite this large basin, since displacements of the trajectory which are due to small perturbations or small errors in the prediction of the fish stock might cause the fish stock to go extinct. Small mistakes can make all the difference between conservation or extinction. The situation is different in the full cooperation game, where the equilibrium is further away from the boundary. Here, small mistakes do not matter that much, and we would consider such a situation as being more stable (for a more rigorous treatment of these issues we refer the interested reader to McDonald et al. 1985)

7 Conclusions

In this paper we introduced a multi-market multi-agent game-theoretic model, where a renewable resource is subject to the harvesting activities of n agents. The fishermen harvest the resource and offer it for sale on n different markets. In contrast to the existing literature, we assume that fishermen are only boundedly rational, i.e. they do not know the reproduction function of the resource. Instead they use an adaptive expectations scheme to update their beliefs about the fish stock.

Given this framework, we have calculated the total harvesting quantities in the cooperative and the non-cooperative case and provided existence conditions for the equilibria of our model. In the cooperative case players choose their actions in order to maximize joint profits instead of maximizing their individual payoffs. It has been shown that for given expectations the total harvesting quantity is smaller if players form a cooperative venture.

We also studied the global stability of the equilibria in our model, i.e. the extent of the set of points which result in conservation and the structure of this set. As a conclusion, our investigation confirms the intuition that cooperation leads to conservative behavior. We have observed that for the same set of parameters the basin of attraction of the equilibrium in the cooperative game with adaptive expectations is larger than the basin of the equilibrium in the non-cooperative game. Furthermore, the structure of the basins is less complex. Although we could see non-connected and multiply connected basins for the non-cooperative case, if players are assumed to cooperate the basins of the corresponding equilibrium is connected and simple. Our results show that earlier results continue to hold in an extended expectations-feedback- framework, where multiple agents compete on different markets.

References

[1] Agiza, H.N., Bischi, G.I. and Kopel, M., Multistability in a dynamic Cournot game with three oligopolists, *Mathematics and Computers in Simulation 51*, 63-90, (1999).

[2] Arnold, V., Varchenko, A. and Goussein-Zade, S., *Singularites des Applications Differentiables*, Editions MIR, Moscow, (1986).

[3] A. Agliari, G.I. Bischi and L. Gardini, 2002. Some methods for thr Global Analysis of Dynamic Games represented by Noninvertible Maps, Chapter 3 in *Oligopoly Dynamics: Tools & Models*, T. Puu and I. Sushko (eds.), Springer Verlag, pp. 31-83.

[4] Abraham, R., Gardini, L., Mira, C., 1997. *Chaos in discrete dynamical systems (a visual introduction in two dimensions)* Springer-Verlag.

[5] Bischi, G.I. and Kopel, M., 2001, "Equilibrium Selection in a Nonlinear Duopoly Game with Adaptive Expectations" *Journal of Economic Behavior and Organization*, vol. 46/1, 73-100.

[6] G.I. Bischi and M. Kopel, 2002. The Role of Competition, Expectations and Harvesting Costs in Commercial Fishing Chapter 4 in *Oligopoly Dynamics: Tools & Models*, T. Puu and I. Sushko editors, Springer Verlag, pp. 85-109.

[7] G.I. Bischi and R. Marimon Global Stability of Inflation Target Policies with Adaptive Agents, Macroeconomic Dynamics vol. 5 n.2, pp. 148-179 (2001).

[8] Bischi, G.I., Gardini, L., Kopel, M., 2000. Analysis of Global Bifurcations in a Market Share Attraction Model, *Journal of Economic Dynamics and Control*, 24, 855-879.

[9] G.I. Bischi, H. Dawid and M. Kopel, Spillover Effects and the Evolution of Firm Clusters, *Journal of Economic Behavior and Organization* (in press).

[10] Bischi, G.I., H. Dawid and M. Kopel, Gaining the Competitive Edge Using Internal and External Spillovers: A Dynamic Analysis, *Journal of Economic Dynamics and Control*, (in press).

[11] Brock W., Hommes C. (1997): A Rational Route to Randomness. Econometrica 65, 1059–1095.

[12] J.M. Conrad, C.W. Clark, Natural Resource Economics. Notes and problems. Cambridge University Press, 1987.

[13] J.M. Conrad, Bioeconomic Models of the Fishery, in: Handbook of Environmental Management, D.W. Bromley, Ch. 18, 405-432.

[14] J.M. Conrad, Resource Economics, Cambridge University Press, 1999.

[15] H. Dawid, M. Kopel, On the Economically Optimal Exploitation of a Renewable Resource: The Case of a Convex Environment and a Convex Return Function, Journal of Economic Theory, 76, 272-297, 1997.

[16] P.K. Dutta, R.K. Sundaram, The Tragedy of the Commons? Economic Theory, 3, 413-426, 1993.

[17] Grebogi C., McDonald S.W., Ott E., Yorke J.A. (1983): Final state sensitivity: an obstruction to predictability, Physics Letters 99A (9), 415-418.

[18] D. Levhari, L.J. Mirman, The Great Fish War: An Example Using a Dynamic Cournot-Nash Solution, in: Essays in the Economics of Renewable Resources, L.J. Mirman and D.F. Spulber (Eds.), 1982, North-Holland, 243-258.

[19] K.-G. Mäler (2000), Development, ecological resources and their management: a study of complex dynamic systems, European Economic Review 44, 645-665.

[20] R.M. May, Chaos and the Dynamics of Biological Populations, Proc. R. Soc. London, A 413, 1987, 27-44.

[21] R.M. May, Simple Mathematical Models With Very Complicated Dynamics, Nature, 261, 1976, 459-467.

[22] R.M. May, G.F. Oster, Bifurcations and Dynamic Complexity in Simple Ecological Models, The American Naturalist, 110, 1976, 573-599.

[23] McDonald W., Grebogi C., Ott E., Yorke J A. (1985): Fractal basin boundaries, Physica 17D, 125-153.

[24] M. Mesterton-Gibbons (1993), Game-theoretic resource modeling, Natural Resource Modeling 7, 93-147.

[25] C. Mira, D. Fournier-Prunaret, L. Gardini, H. Kawakami and J.C. Cathala, "Basin bifurcations of two-dimensional noninvertible maps: fractalization of basins", International Journal of Bifurcations and Chaos, 4, 343-381, (1994).

[26] C. Mira, L. Gardini, A. Barugola and J.C. Cathala, *Chaotic Dynamics in Two-Dimensional Noninvertible Maps*, World Scientific, (1996).

[27] P.A. Neher, Natural Resource Economics. Conservation and Exploitation, Cambridge University Press, 1990.

[28] K. Okuguchi, Long-run fish stock and imperfectly competitive international commercial fishing, Keio Economic Studies, 35, 1998, 9-17

[29] D.W. Pearce, R.K. Turner, Economics of Natural Resources and the Environment, John Hopkins University Press, 1990.

[30] J.B. Rosser, Jr. "Complex ecologic-economic dynamics and environmental policy" *Ecological Economics* (2001) pp. 23-37.

[31] R.Sethi and E. Somanathan (1996), The evolution of social norms in common property resource use, American Economic Review 86, 766-788.

[32] F. Szidarovszky, K. Okuguchi, An Oligopoly model of commercial fishing, Seoul Journal of Economics, 1998, Vol. 11, 321-330.

[33] F. Szidarovszky and K. Okuguchi (2000): A Dynamic Model of International Fishing, Seoul Journal of Economics 13, 471-476.

[34] J.C.J.M. Van den Bergh and J.M. Gowdy (2000), Evolutionary theories in environemental and resource economics: approaches and applications, Environmental and Resource Economics 17, 37-57.Abraham, R., Gardini, L. and Mira, C., *Chaos in discrete dynamical systems (a visual introduction in two dimensions)*, Springer, (1997).

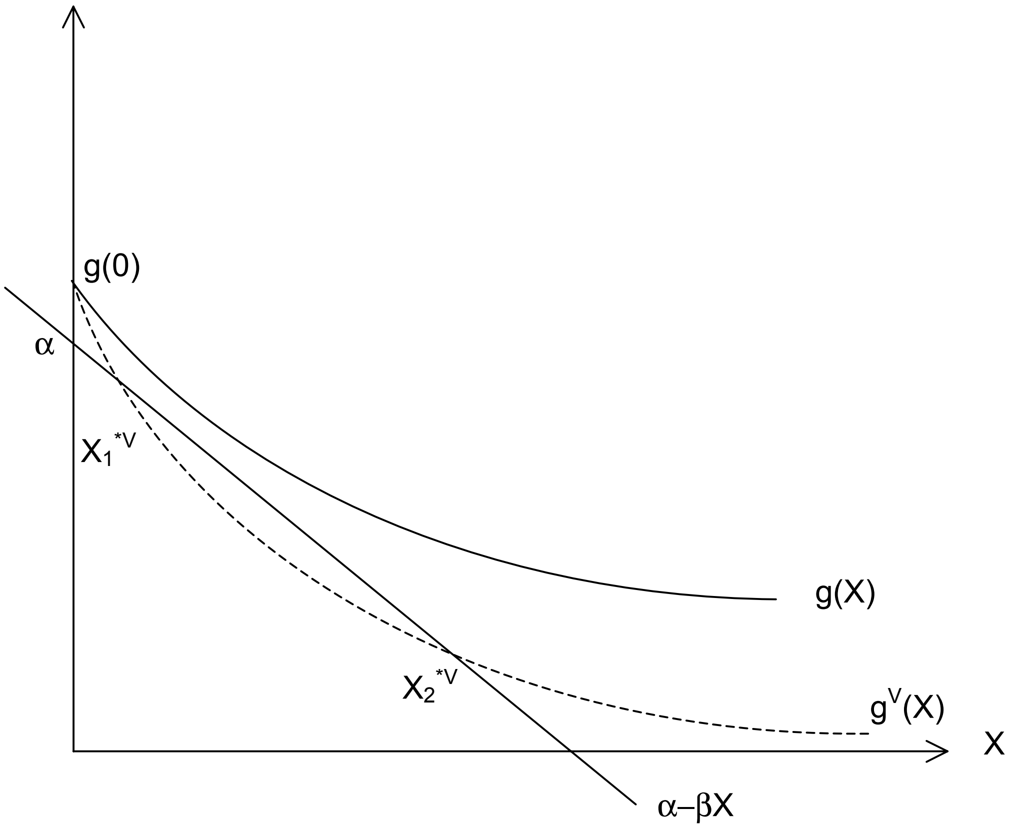


Figure 1a: Players act noncooperatively and no equilibrium exists (case 1). If players start cooperating, then two positive equilibria can occur (case 3).

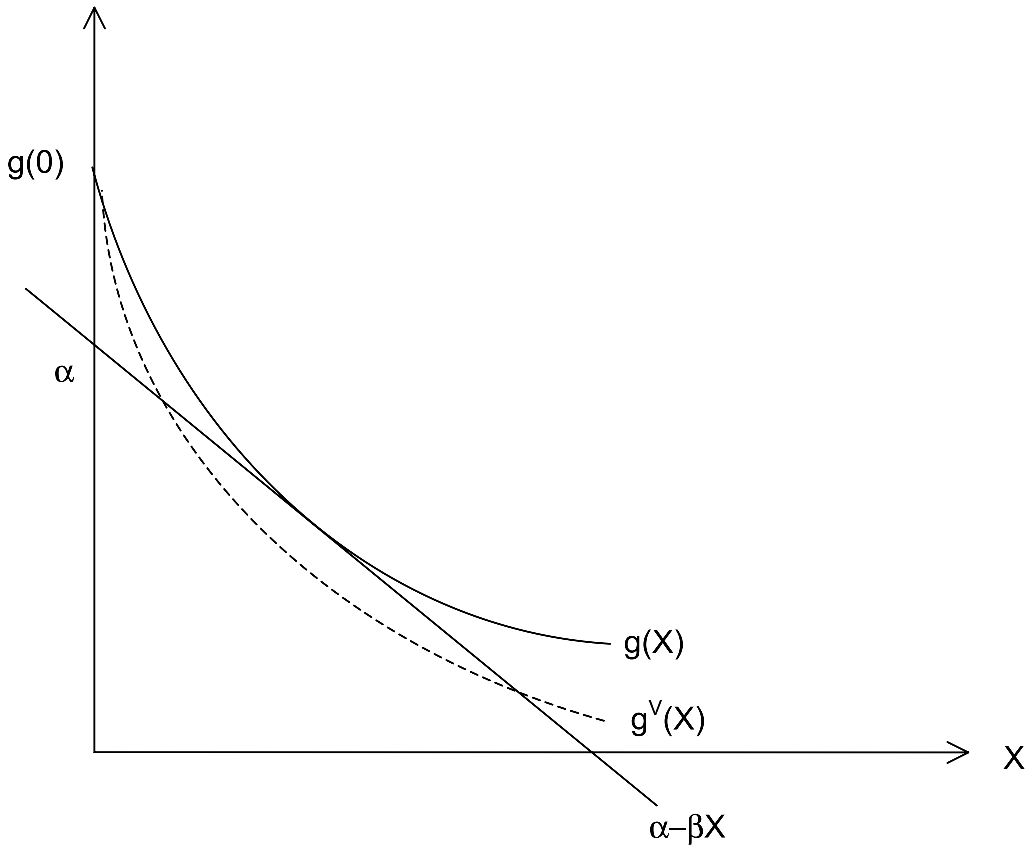


Figure 1b: Players act noncooperatively and a unique equilibrium exists (case 2). If players start cooperating, then two positive equilibria emerge (case 3).

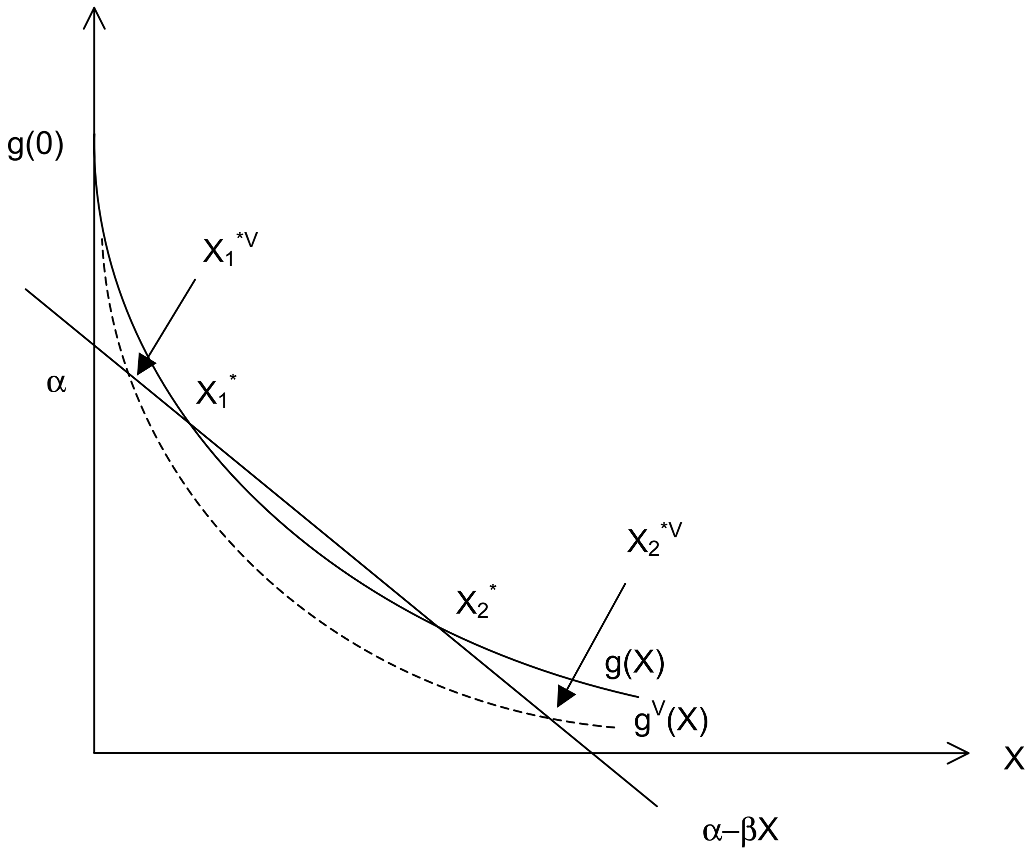


Figure 1c: Players act noncooperatively and a two positive equilibria exist (case 3). If players start cooperating, then the equilibria are shifted.

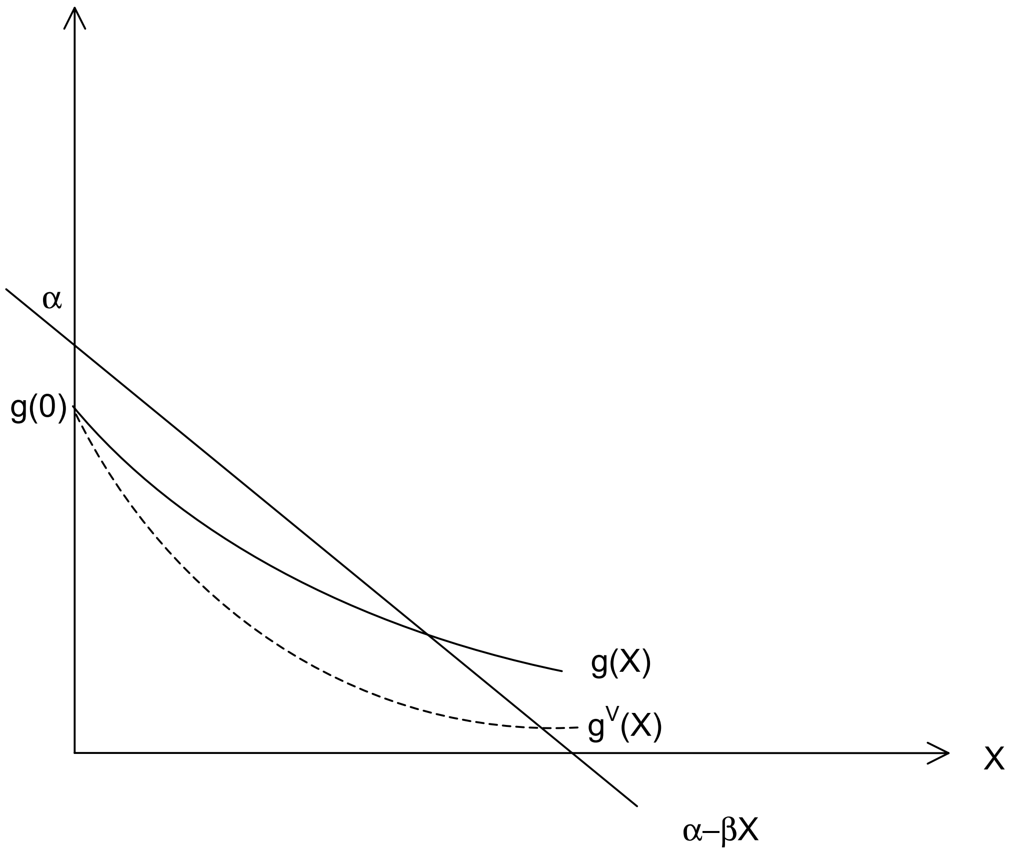
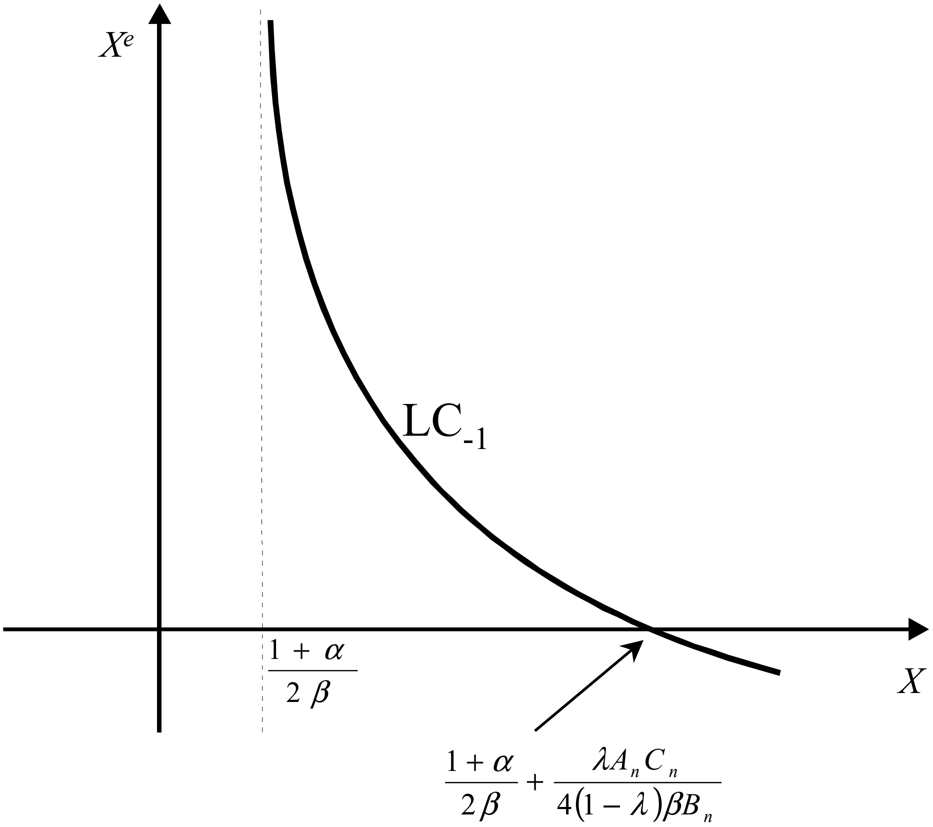
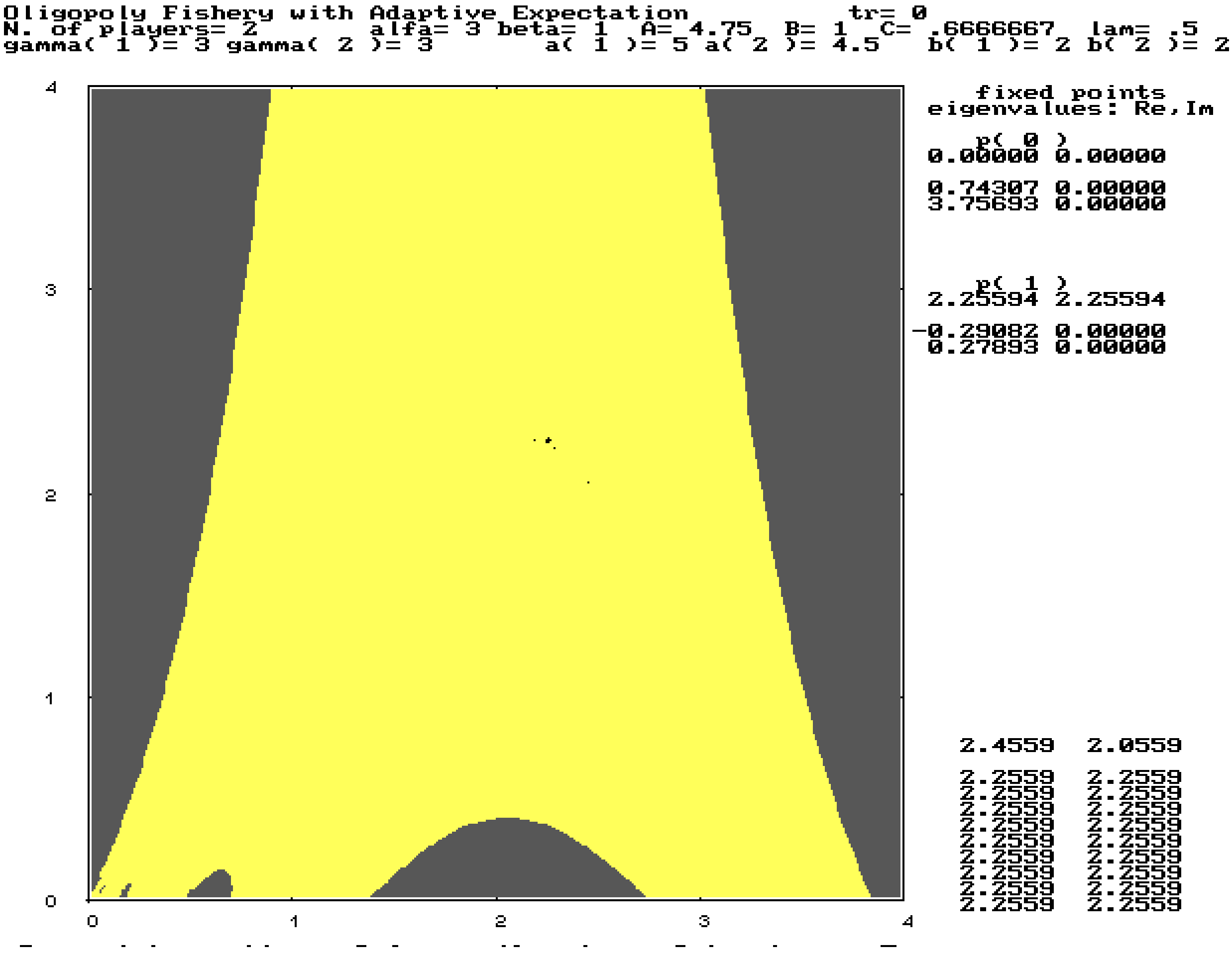


Figure 1d: Players act noncooperatively and a unique positive equilibrium exists (case 4). If players start cooperating, then this equilibrium is shifted to the right.



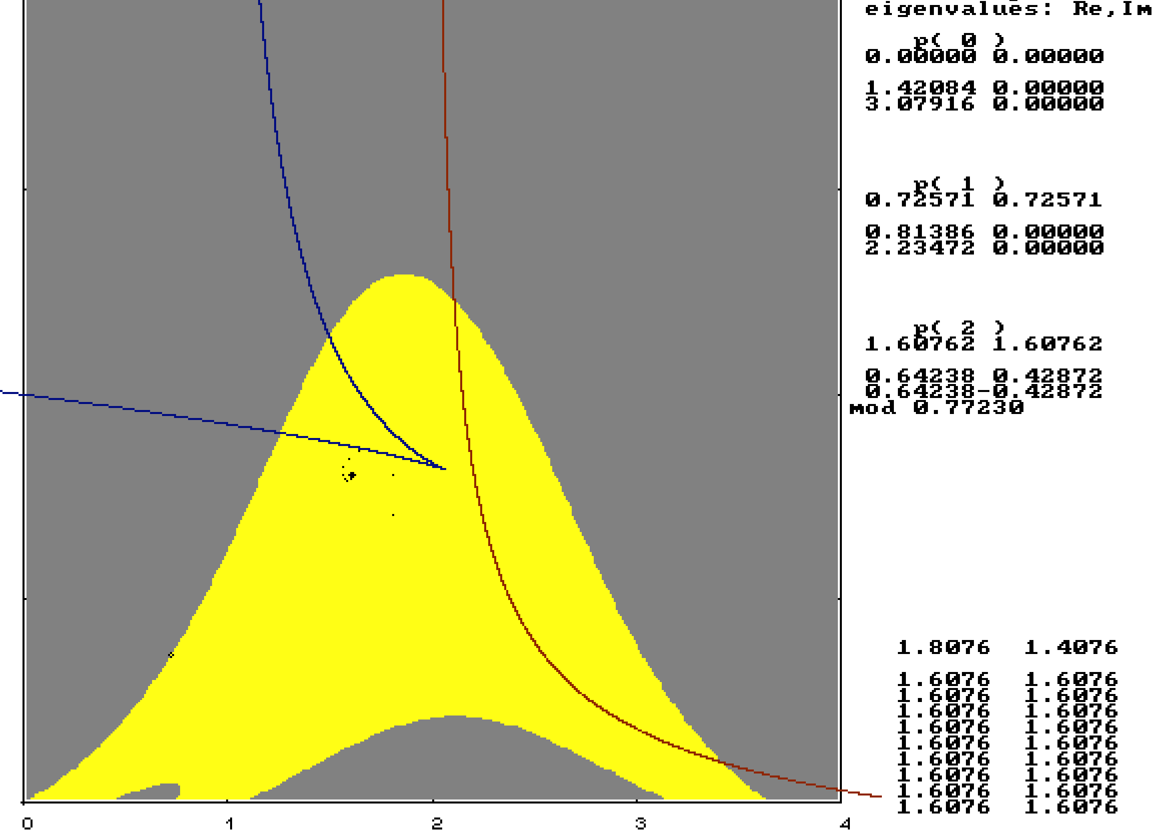
X^e
LC$_{-1}$
$\frac{1+\alpha}{2\beta}$
X
$\frac{1+\alpha}{2\beta}+\frac{\lambda A_n C_n}{4(1-\lambda)\beta B_n}$

Fig. 2

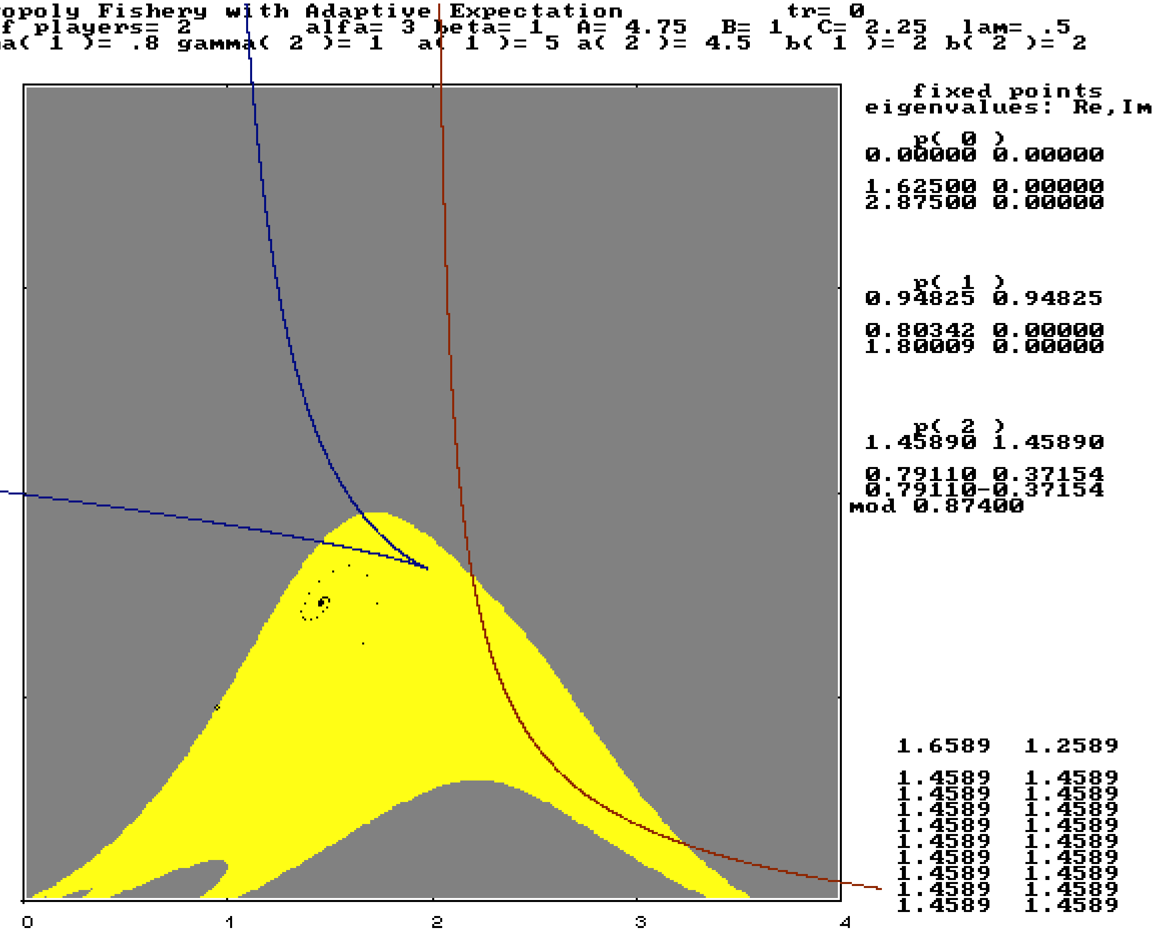


```
Oligopoly Fishery with Adaptive Expectation          tr= 0
N. of players= 2        alfa= 3 beta= 1  A= 4.75  B= 1  C= .6666667  lam= .5
gamma( 1 )= 3 gamma( 2 )= 3        a( 1 )= 5 a( 2 )= 4.5    b( 1 )= 2 b( 2 )= 2

   fixed points
eigenvalues: Re,Im

   p( 0 )
0.00000 0.00000

0.74307 0.00000
3.75693 0.00000


   p( 1 )
2.25594 2.25594

-0.29082 0.00000
0.27893 0.00000


 2.4559   2.0559

 2.2559   2.2559
 2.2559   2.2559
 2.2559   2.2559
 2.2559   2.2559
 2.2559   2.2559
 2.2559   2.2559
 2.2559   2.2559
 2.2559   2.2559
 2.2559   2.2559
```

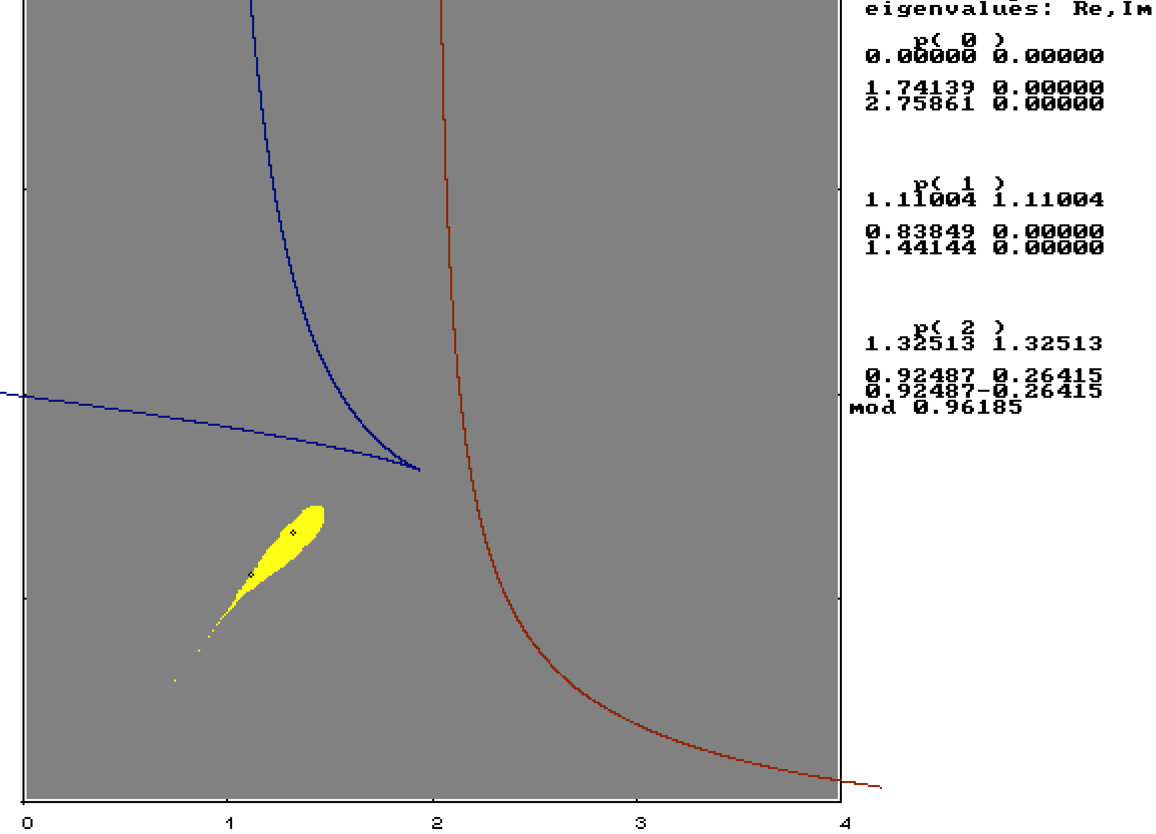
Fig. 3a



```
eigenvalues:  Re,Im
      p( 0 )
0.00000 0.00000
1.42084 0.00000
3.07916 0.00000

      p( 1 )
0.72571 0.72571
0.81386 0.00000
2.23472 0.00000

      p( 2 )
1.60762 1.60762
0.64238 0.42872
0.64238-0.42872
mod 0.77230

  1.8076   1.4076

  1.6076   1.6076
  1.6076   1.6076
  1.6076   1.6076
  1.6076   1.6076
  1.6076   1.6076
  1.6076   1.6076
  1.6076   1.6076
  1.6076   1.6076
  1.6076   1.6076
```

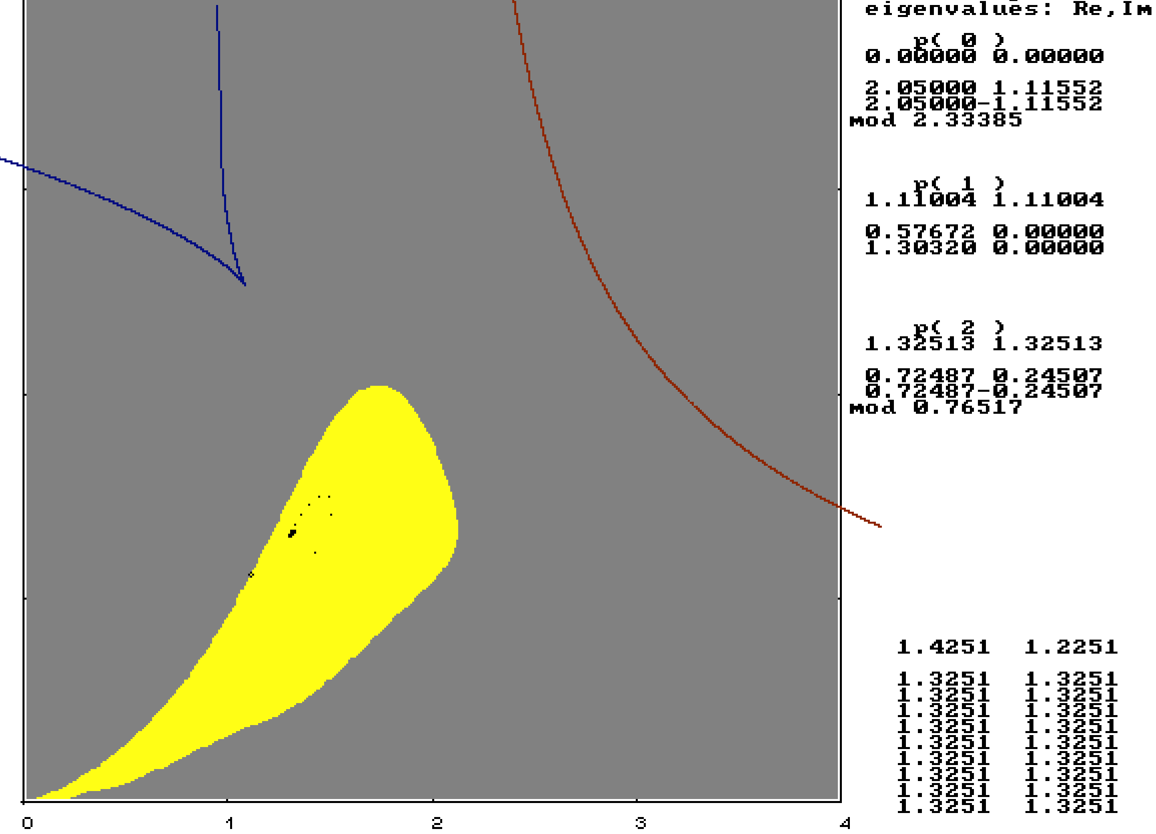


```
opoly Fishery with Adaptive Expectation          tr= 0
f players= 2        alfa= 3 beta= 1  A= 4.75  B= 1  C= 2.25  lam= .5
a( 1 )= .8 gamma( 2 )= 1  a( 1 )= 5 a( 2 )= 4.5  b( 1 )= 2 b( 2 )= 2

   fixed points
eigenvalues:  Re,Im
      p( 0 )
0.00000 0.00000
1.62500 0.00000
2.87500 0.00000

      p( 1 )
0.94825 0.94825
0.80342 0.00000
1.80009 0.00000

      p( 2 )
1.45890 1.45890
0.79110 0.37154
0.79110-0.37154
mod 0.87400

  1.6589   1.2589

  1.4589   1.4589
  1.4589   1.4589
  1.4589   1.4589
  1.4589   1.4589
  1.4589   1.4589
  1.4589   1.4589
  1.4589   1.4589
  1.4589   1.4589
  1.4589   1.4589
```



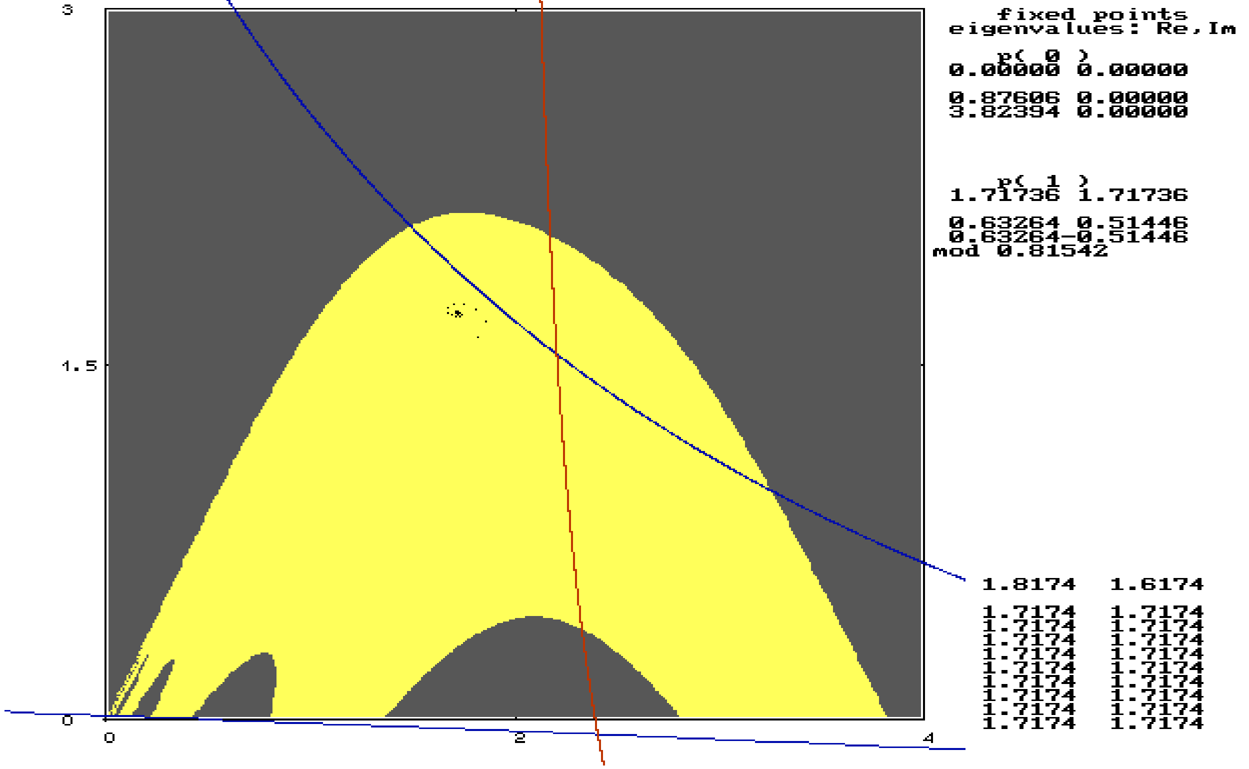
```
eigenvalues: Re,Im
     p( 0 )
0.00000 0.00000

1.74139 0.00000
2.75861 0.00000


     p( 1 )
1.11004 1.11004

0.83849 0.00000
1.44144 0.00000


     p( 2 )
1.32513 1.32513

0.92487 0.26415
0.92487-0.26415
mod 0.96185
```

Fig. 3d

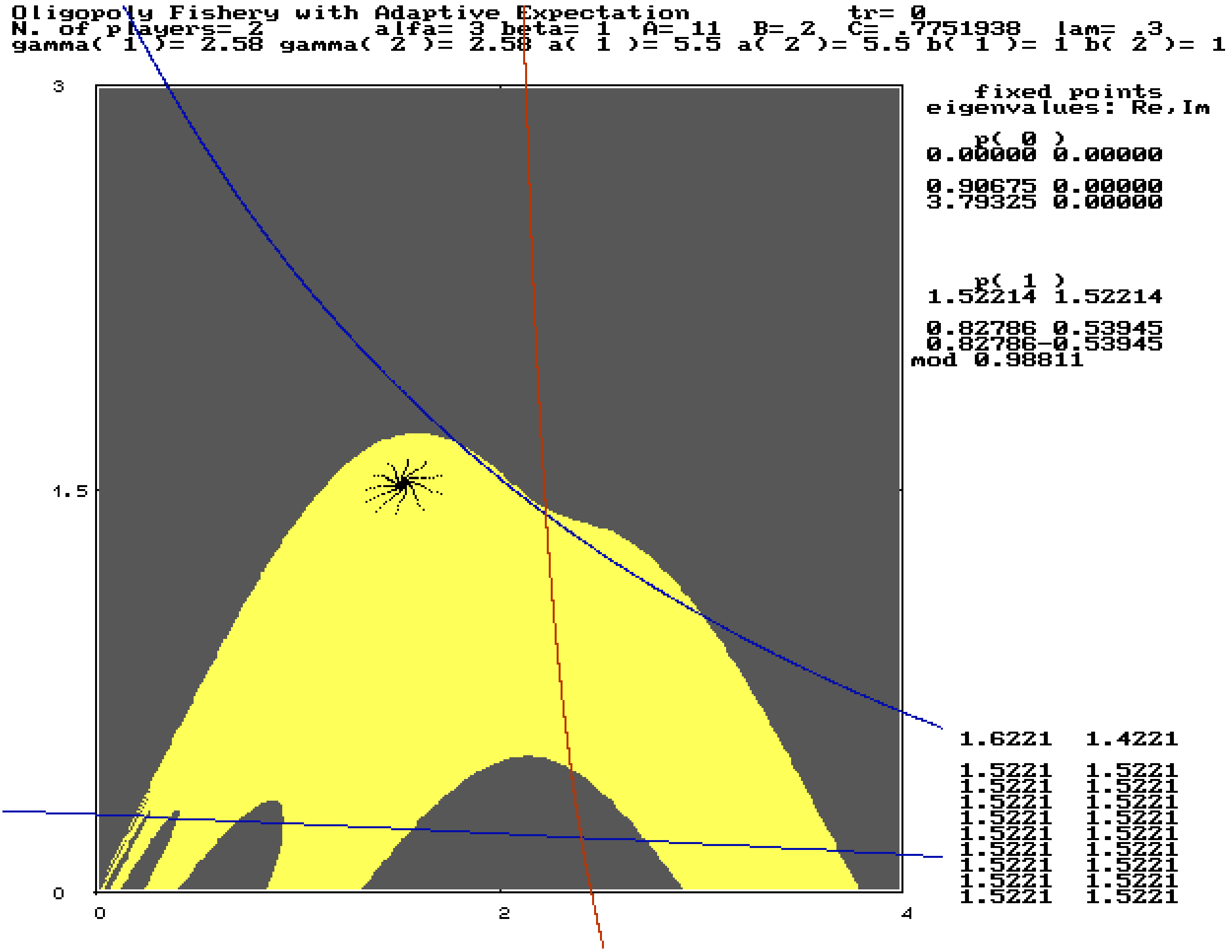


```
  eigenvalues:  Re,Im

     p( 0 )
  0.00000 0.00000

  2.05000 1.11552
  2.05000-1.11552
mod 2.33385


     p( 1 )
  1.11004 1.11004

  0.57672 0.00000
  1.30320 0.00000


     p( 2 )
  1.32513 1.32513

  0.72487 0.24507
  0.72487-0.24507
mod 0.76517



      1.4251   1.2251

      1.3251   1.3251
      1.3251   1.3251
      1.3251   1.3251
      1.3251   1.3251
      1.3251   1.3251
      1.3251   1.3251
      1.3251   1.3251
      1.3251   1.3251
      1.3251   1.3251
```

Fig. 3e

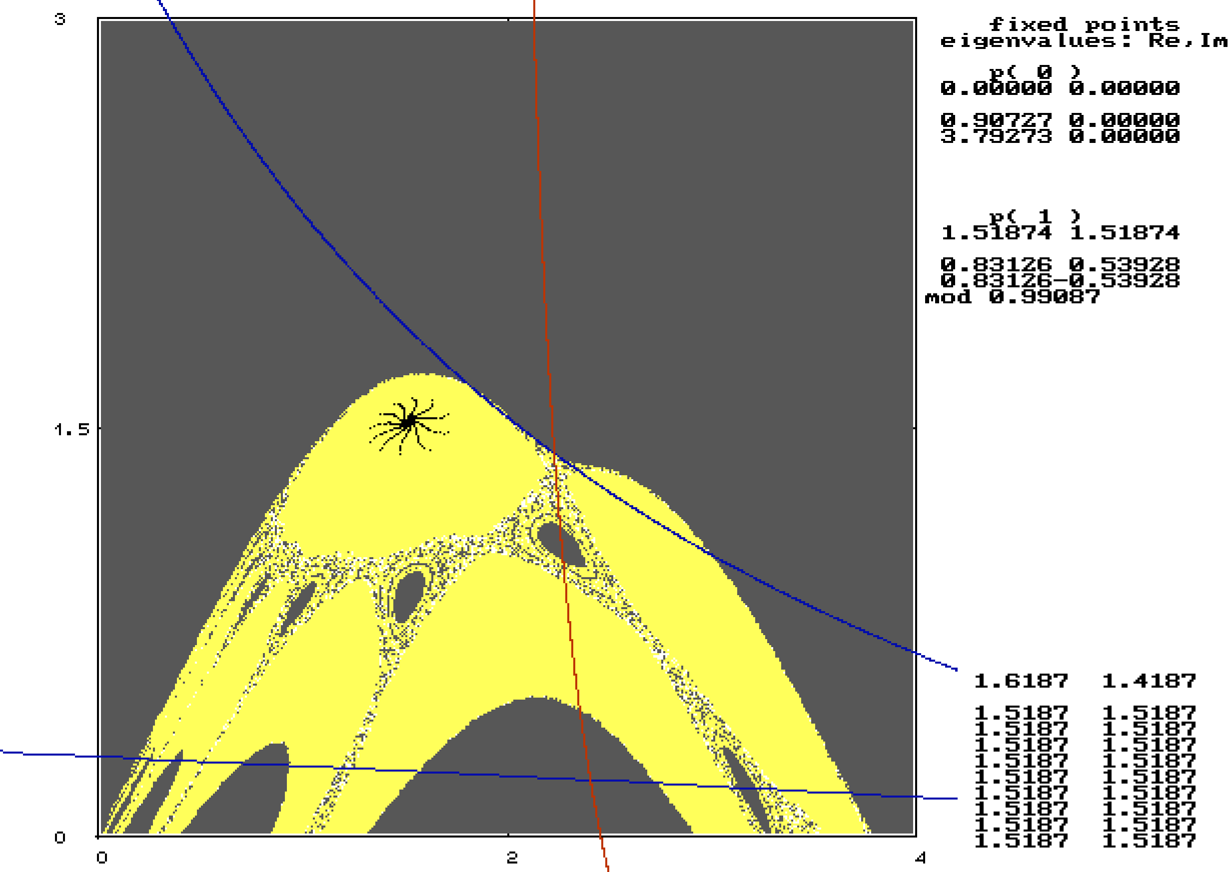


```
fixed points
eigenvalues: Re,Im
p( 0 )
0.00000 0.00000
0.87606 0.00000
3.82394 0.00000

p( 1 )
1.71736 1.71736
0.63264 0.51446
0.63264-0.51446
mod 0.81542

1.8174   1.6174

1.7174   1.7174
1.7174   1.7174
1.7174   1.7174
1.7174   1.7174
1.7174   1.7174
1.7174   1.7174
1.7174   1.7174
1.7174   1.7174
1.7174   1.7174
```



```
Oligopoly Fishery with Adaptive Expectation            tr= 0
N. of players= 2         alfa= 3 beta= 1  A= 11  B= 2  C= .7751938  lam= .3
gamma( 1 )= 2.58 gamma( 2 )= 2.58 a( 1 )= 5.5 a( 2 )= 5.5 b( 1 )= 1 b( 2 )= 1
```

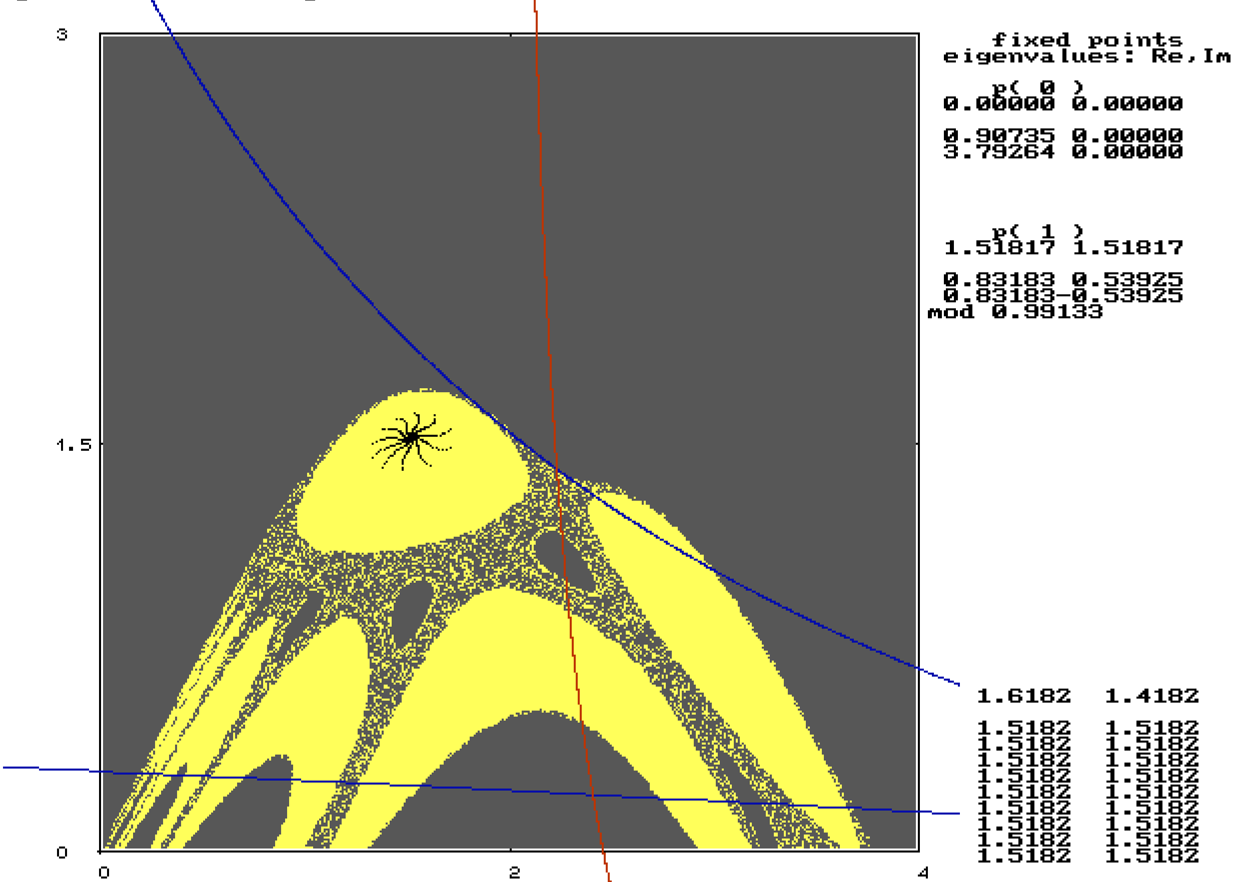
```
fixed points
eigenvalues: Re,Im
p( 0 )
0.00000 0.00000
0.90675 0.00000
3.79325 0.00000

p( 1 )
1.52214 1.52214
0.82786 0.53945
0.82786-0.53945
mod 0.98811

1.6221   1.4221

1.5221   1.5221
1.5221   1.5221
1.5221   1.5221
1.5221   1.5221
1.5221   1.5221
1.5221   1.5221
1.5221   1.5221
1.5221   1.5221
1.5221   1.5221
```



```
  fixed points
eigenvalues: Re,Im

    p( 0 )
0.00000 0.00000

0.90727 0.00000
3.79273 0.00000


    p( 1 )
1.51874 1.51874

0.83126 0.53928
0.83126-0.53928
mod 0.99087
```

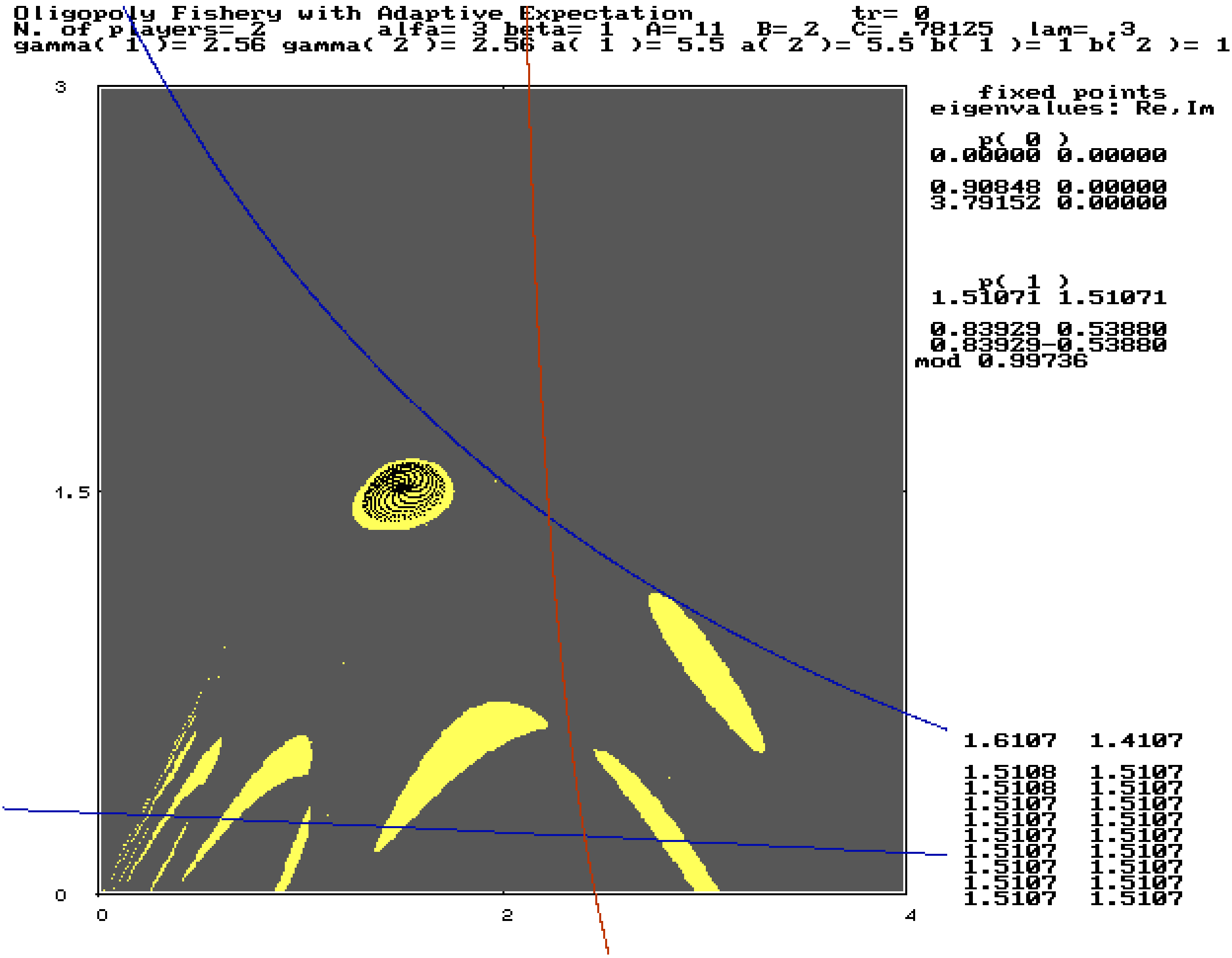
```
1.6187   1.4187

1.5187   1.5187
1.5187   1.5187
1.5187   1.5187
1.5187   1.5187
1.5187   1.5187
1.5187   1.5187
1.5187   1.5187
1.5187   1.5187
1.5187   1.5187
```

```
Oligopoly Fishery with Adaptive Expectation              tr= 0
N. of players= 2        alfa= 3 beta= 1  A= 11  B= 2  C= .7773028  lam= .3
gamma( 1 )= 2.573 gamma( 2 )= 2.57a( 1 )= 5.5 a( 2 )= 5.5 b( 1 )= 1 b( 2 )= 1
```



```
  fixed points
eigenvalues: Re,Im

    p( 0 )
0.00000 0.00000

0.90735 0.00000
3.79264 0.00000


    p( 1 )
1.51817 1.51817

0.83183 0.53925
0.83183-0.53925
mod 0.99133
```

```
1.6182   1.4182

1.5182   1.5182
1.5182   1.5182
1.5182   1.5182
1.5182   1.5182
1.5182   1.5182
1.5182   1.5182
1.5182   1.5182
1.5182   1.5182
1.5182   1.5182
```

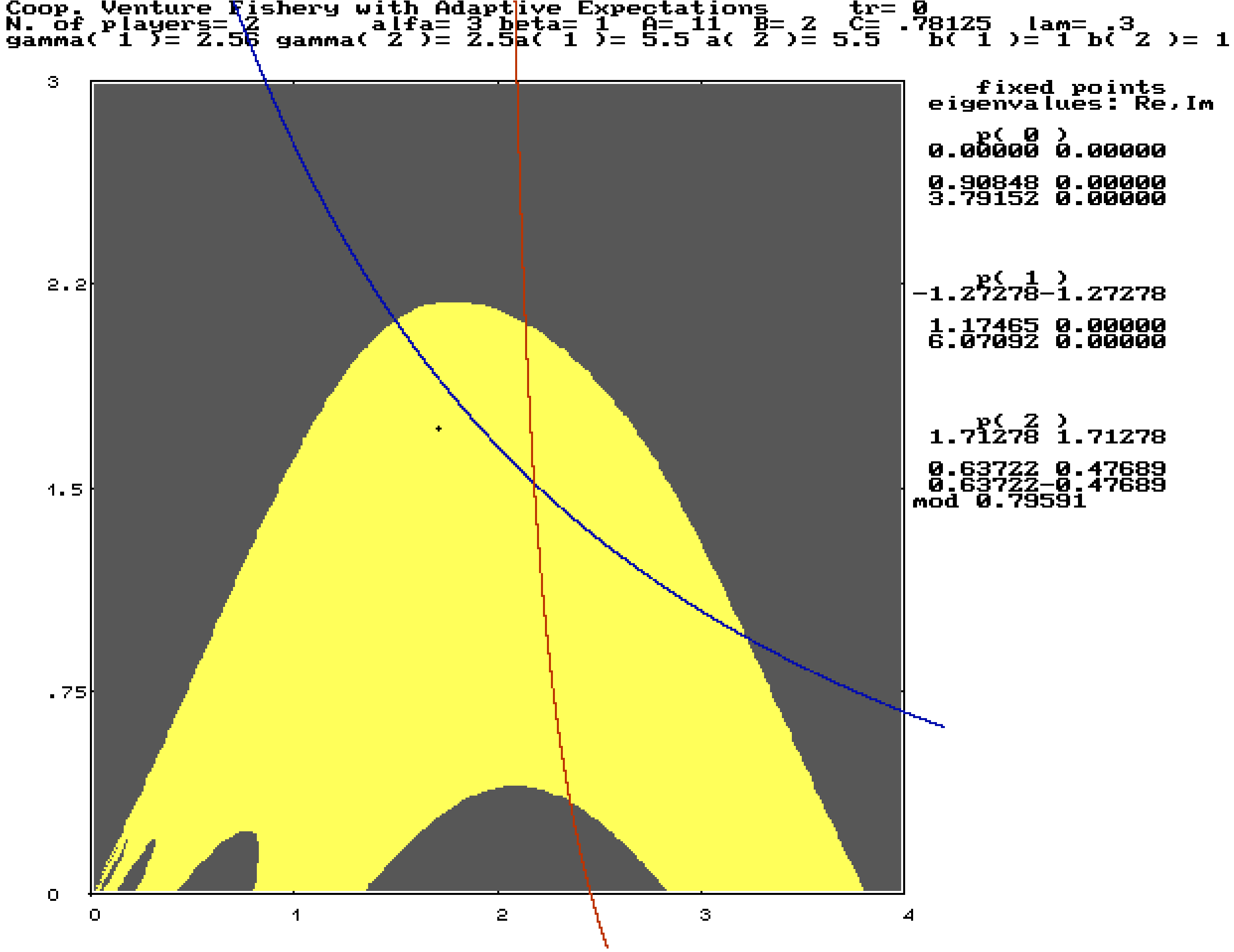


```
Oligopoly Fishery with Adaptive Expectation                tr= 0
N. of players= 2          alfa= 3 beta= 1  A= 11  B= 2  C= .78125  lam= .3
gamma( 1 )= 2.56 gamma( 2 )= 2.56 a( 1 )= 5.5 a( 2 )= 5.5 b( 1 )= 1 b( 2 )= 1
```

```
   fixed points
 eigenvalues: Re,Im

    p( 0 )
 0.00000 0.00000

 0.90848 0.00000
 3.79152 0.00000


    p( 1 )
 1.51071 1.51071

 0.83929 0.53880
 0.83929-0.53880
mod 0.99736
```

```
1.6107   1.4107

1.5108   1.5107
1.5108   1.5107
1.5107   1.5107
1.5107   1.5107
1.5107   1.5107
1.5107   1.5107
1.5107   1.5107
1.5107   1.5107
1.5107   1.5107
```

Fig. 4e



```
Coop. Venture Fishery with Adaptive Expectations      tr= 0
N. of players= 2        alfa= 3 beta= 1  A= 11  B= 2  C= .78125  lam= .3
gamma( 1 )= 2.56 gamma( 2 )= 2.5a( 1 )= 5.5 a( 2 )= 5.5   b( 1 )= 1 b( 2 )= 1
```

```
    fixed points
 eigenvalues: Re,Im

    p( 0 )
 0.00000 0.00000

 0.90848 0.00000
 3.79152 0.00000

    p( 1 )
-1.27278-1.27278

 1.17465 0.00000
 6.07092 0.00000

    p( 2 )
 1.71278 1.71278

 0.63722 0.47689
 0.63722-0.47689
mod 0.79591
```

Fig. 5