Risk measures and financial regulation

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Abstract

For a portfolio of financial instruments whose fluctuations obey Gaussian statistics the surfaces corresponding to a given level of risk are (hyper)ellipsoids in the space of portfolio weights. Real-life instruments are non-Gaussian and often display heavy tailed behaviour. Constructing the distribution of the weighted sums of such variables and characterising the risk included in such a portfolio are highly nontrivial tasks. It can, however, be assumed that the level surfaces of risk will be closed and convex in the space of portfolio weights under any reasonable risk measure. Convexity is also an elementary property of the coherent risk measures [1] which have been strongly advocated recently by several groups, and expresses the simple fact that diversification decreases risk.

The level surfaces of risk also constitute a major concern for international financial regulation. In particular, the current rules dictated by the "standard model" for the calculation of the capital requirements of bond, FX, and equity portfolios, respectively, as described in the Capital Adequacy Directive (CAD) of the European Union, can be viewed as definitions of surfaces corresponding to constant regulatory capital or, by implication, to constant risk. These "regulatory surfaces" are polyhedra, and represent crude approximations to the real constant risk surfaces.

In this paper we explicitly construct these polyhedra, and briefly analyse how their properties reflect the presumable regulatory intentions. The main results of our study can be summarised as follows: The capital requirements of the specific risk of bonds and also those of the FX portfolio define two different regulatory polyhedra which are convex, as they should be. Depending on the composition, however, the level surfaces of risk for an equity portfolio can become concave and, paradoxically, this is due precisely to the special rule (absent in the original CAD, but introduced in some of its national implementations) which was designed to penalise excessive concentration of equity portfolios. Admittedly, for the equity portfolio the effect is rather small, of the order of only a few percent, so the inconsistency is of a theoretical rather than practical nature. This is not the case for the general risk of bonds where the regulatory surface can be deeply concave. As a result, one can easily construct model portfolios for which a smaller exposure attracts twice as large a capital than a larger one. In addition, for some portfolios the capital requirement exhibits wild fluctuations due to the transition of the portfolio components between different maturity zones.

Due to the improper choice of regulatory risk measures, financial institutions can also become confronted with extremely hard optimisation problems, familiar from other chapters of the theory of complex systems. The 1998 amendment of CAD permits institutions to use internal models instead of the standard one, but stipulates stringent rules concerning these own models, and a factor 3 to 4 to multiply the capital requirement resulting from them. In view of this, several banks decided to keep to the standard model for reporting purposes, and set up an internal model only for the sake of their own risk management. Now if a bank wants to optimise its bond, FX or equity portfolio according to some internal criteria (like minimal variance, minimal VaR, etc.), then the rules of CAD start to act as additional non-linear constraints, which leads to the appearance of a huge number (exponential in the size of the portfolio) of nearly degenerate solutions that are, in addition, extremely sensitive to any misspecification of the parameters of the problem.

Reference

[1] P. Artzner, F. Delbaen, J.-M. Eber, D. Heath: Thinking coherently, *RISK* **10** (11) (1997); P. Artzner, F. Delbaen, J.-M. Eber, D. Heath: Coherent measures of risk, *Math. Fin.* **9**(3), 203-228, (1999)