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SUMMER SCHOOL ON ASTROPARTICLE PHYSICS AND COSMOLOGY

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LARGE EXTRA DIMENSIONS

Lecture 1 - Kaluza-Klein Theories

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Please note: These are preliminary notes intended for internal distribution only.

Large Extra Dimensions
(by G. Gabadadze)

Lecture # 1: Kaluza-Klein Theories

Reasons to study extra dimensions:

- Unification of gravity and gauge interactions of particle physics
- The hierarchy problem
- The cosmological constant problem

⋮

Conventions: $\hbar = c = 1$;

Metric $(- + + + + + \cdots)$

Free scalar field in $(4+1)$D.

$$\mathcal{L} = -\frac{1}{2}\,\partial_A \phi\, \partial^A \phi \qquad A, B = 0, 1, 2, 3, \underline{\underline{5}}.$$

$$\phi = \phi(t, \vec{x}, y) \equiv \phi(x_\mu, y) \; : \; \mu = 0, 1, 2, 3.$$

The y-direction is compactified:

$$y \Longleftrightarrow y + 2\pi L$$

Therefore:

$$\phi(x, y) = \phi(x, y + 2\pi L)$$

$$\phi(x, y) = \sum_{n=-\infty}^{+\infty} \phi_n(x)\, e^{i n y / L}$$

Note: $\phi_n^* = \phi_{(-n)}$

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu \phi\, \partial^\mu \phi + \partial_y \phi\, \partial_y \phi\right)$$

$$= -\frac{1}{2} \sum_{n,m=-\infty}^{+\infty} \left(\partial_\mu \phi_n\, \partial^\mu \phi_m - \frac{nm}{L^2} \phi_n \phi_m\right) e^{i(n+m)y/L}$$

<u>Action:</u> $$S = \int d^4x \int_0^{2\pi L} dy \;\mathcal{L}$$

Integrate w.r.t. y:

$$S = -\frac{2\pi L}{2} \int d^4x \sum_{n=-\infty}^{+\infty} \left(\partial_\mu \phi_n\, \partial_\mu \phi_n^* + \frac{n^2}{L^2} \phi_n \phi_n^*\right)$$

introducing notations:

$$\varphi_n \equiv \sqrt{2\pi L}\;\phi_n$$

$$S = \int d^4x \left[-\frac{1}{2} \left(\partial_\mu \varphi_0 \, \partial^\mu \varphi_0^* \right) \right]$$

$$- \int d^4x \sum_{K=1}^{\infty} \left(\partial_\mu \phi_K^* \, \partial^\mu \phi_K + \frac{K^2}{L^2} \phi_K^* \phi_K \right)$$

The resulting spectrum:

- A massless mode φ_0 (zero mode)
- Massive modes, $m_K^2 = K^2/L^2$;
  there are an ifinite number of them.

These are called the Kaluza-Klein modes

At low energies, i.e., $E \ll 1/L$ only the zero mode is important.

Abelian gauge field in 5D

$$\mathcal{L} = -\frac{1}{4g_5^2} F_{AB}F^{AB}$$

$$[A_B] = [m]$$

$$[g_5^2] = [m^{-1}]$$

$$F_{AB}^2 = F_{\mu\nu}^2 + 2\left(\partial_\mu A_5 - \partial_5 A_\mu\right)^2$$

$$A_\mu(x,y) = \sum_{n=-\infty}^{+\infty} A_\mu^{(n)}(x)\, e^{iny/L} \qquad A_5 = \sum_{n=-\infty}^{+\infty} A_5^{(n)} e^{iny/L}$$

Calculate the action:

$$S = \int d^4x \int_0^{2\pi L} dy\, \mathcal{L} \equiv \int d^4x\, \mathcal{L}_{(4)}$$

$$\mathcal{L}_{(4)} \equiv \int_0^{2\pi L} dy\, \mathcal{L}$$

(1.5

$\mathcal{L}_{(4)}$ can be reduced to: (using gauge transform.)

$$\mathcal{L}_{(4)} = -\frac{1}{4g_4^2}\Big\{ F^{(0)}_{\mu\nu} F^{(0)\mu\nu}$$

$$+ 2\sum_{k=1}^{\infty}\Big[F^{(k)}_{\mu\nu} F^{*(k)\mu\nu} + \frac{2k^2}{L^2} A^{(k)}_{\mu} A^{*(k)\mu}\Big]$$

$$+ 2\left(\partial_\mu A_5^{(0)}\right)^2 \Big\}$$

- Massless gauge field with the gauge coupling constant

$$\boxed{g_4^2 \equiv \frac{g_5^2}{2\pi L}}$$

- Massive Kaluza-Klein gauge bosons (KK)

$$m_K^2 = K^2/L^2$$

- Massless field $A_5^{(0)}$

- Higgs mechanism takes place at each massive level.

Counting physical degrees of freedom (pdf)

5 dimensional massless gauge field $N_{pdf} = 3$

→ massless level: 4 dimensional massless gauge field ($N_{pdf} = 2$), plus 1 real scalar: total $= 3$

→ massive level: 4 dimensional massive gauge field $N_{pdf} = 3$. (Massless gauge field „eats" one massless scalar and becomes massive, the Higgs mechanism).

Gravity:

Unification of gravity and electromagnetism (Kaluza)

$$G_{AB} = \left(\begin{array}{c|c} G_{\mu\nu} & G_{5\mu} \\ \hline G_{\nu 5} & G_{55} \end{array} \right)$$

$G_{\mu\nu}$: is like a 4 dim graviton

$G_{\mu 5}$ $(G_{\nu 5})$: like 4 dim. vector bosons

G_{55} : 4 dim scalar.

$$S = \frac{M_*^3}{2} \int d^4x \, dy \sqrt{G} \, R_5$$

$$G_{AB}(x,y) = \sum_{n=-\infty}^{+\infty} G_{AB}^{(n)} e^{iny/L}$$

Let us concentrate on the zero modes only (neglecting for a moment massive modes)

$$G_{AB}^{(0)} \equiv g_{AB} \equiv e^{\phi/\sqrt{3}} \left(\begin{array}{c|c} g_{\mu\nu} + e^{-\sqrt{3}\phi} A_\mu A_\nu & e^{-\sqrt{3}\phi} A_\mu \\ \hline e^{-\sqrt{3}\phi} A_\nu & e^{-\sqrt{3}\phi} \end{array} \right)$$

$$S \xrightarrow[\text{only}]{\text{zero mode}} \frac{M_*^3 (2\pi L)}{2} \int d^4x \sqrt{g} \left(R_4(g) - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - \frac{1}{4} e^{-\sqrt{3}\phi} F_{\mu\nu} F^{\mu\nu} \right)$$

- Conventional 4 dim Action:

$$\left(M_P^2/2\right) \int d^4x \sqrt{g}\, R_4$$

Therefore: $\boxed{M_P^2 = M_*^3 \, (2\pi L)}$

$$M_P^2 = \frac{1}{8\pi G_N} \Rightarrow \boxed{G_N = \frac{1}{16\pi^2 M_*^3 L}}$$

- We obtain massless gauge fields from higher dimensional components of gravity.

- pdf's: massless level:

4 dim graviton	2 pdf
4 dim. gauge boson	2 pdf
real scalar	1 pdf
total	5 pdf

correspond to 5 pdf's of 5 dimensional massless graviton.

- Massive levels:

4 dimensional massive gravitons

$$m_k^2 = k^2/L^2$$

with 5 pdf's.

Higgs mechanism: (at each massive level)

One massless graviton (2 pdf's) "eats"

1 massless gauge boson (2 pdf's) plus

One real scalar (1 pdf). Total $\underline{\underline{5 \text{ pdf's.}}}$

- KK modes are charged under $A_\mu^{(e)}$.

$$q_n \sim \frac{n}{L M_p} = \frac{m_n}{M_p} \qquad q_1 \sim e \Rightarrow$$

$\Rightarrow m_1 \sim M_p$. $\Rightarrow$ All charged particles are very heavy $\left(1/L \sim M_p \sim 10^{19}\ \text{GeV} \right)$.