Is weak temperature dependence of dephasing possible?

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Weak localization

Noninteracting electron with $p_F \ell \gg \hbar$ passing through scattering media.

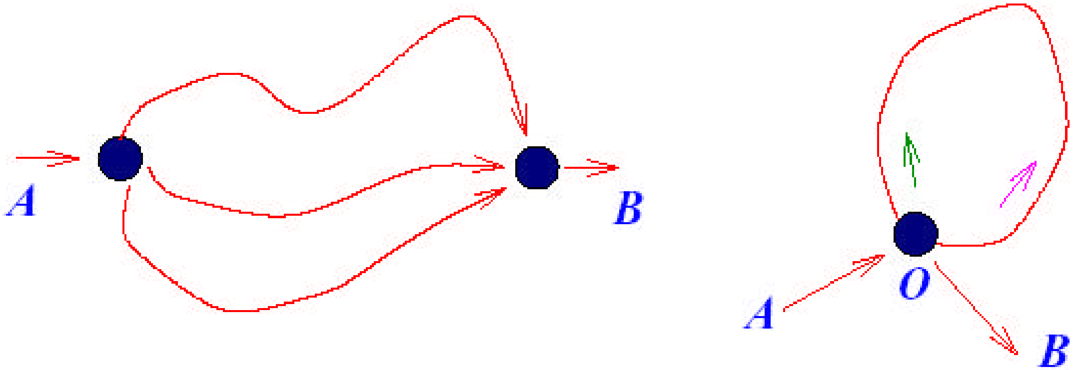


Figure 1: Feynman paths responsible for weak localization

The probability is

$$W = \left| \sum_i A_i \right|^2 = \sum_i |A_i|^2 + \sum_{i \neq j} A_i A_j^* .$$

A_i is the propagation *amplitude* along the path i.

The 1st item – *classical probability*, the 2nd one – *interference* term.

Constructive interference

For the majority of the trajectories the phase gain,

$$\Delta\varphi = \hbar^{-1}\int_A^B \vec{p}\cdot d\vec{l} \gg 1\,,$$

and interference term vanishes.

Special case - trajectories with self-crossings. For these parts, the phase gains are *the same*, and

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2A_1A_2^* = 4|A_1|^2\,.$$

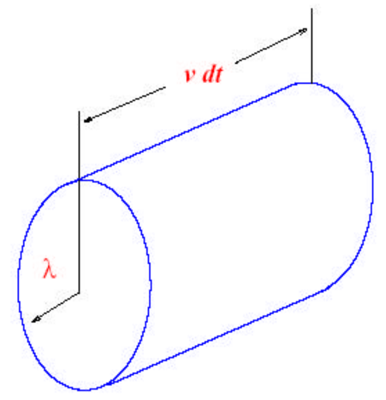
Thus quantum effects double the result. As a result, the total scattering probability at the scatterer at the site O *increases*.

Probability of self-crossing.

The "cross-section" of the site O is λ^2 where $\lambda \sim \hbar/p_F$ is the de Broglie electron wave length.

distance $\sim \sqrt{Dt}$, volume $b\sqrt{Dt}$.

probability $\dfrac{v\lambda^2\,dt}{bDt}$.



relative correction

$$\frac{\Delta\sigma}{\sigma} \sim -\frac{v\lambda^2}{bD}\int_{\tau}^{\tau_\varphi}\frac{dt}{t} = \frac{\Delta\sigma}{\sigma} \sim -\frac{v\lambda^2}{bD}\ln\frac{\tau_\varphi}{\tau}.$$

The lower limit is the elastic time, while τ_φ is the *dephasing time*.

Introducing the *conductance* as $G = \sigma b$ we get

$$\Delta G \sim -\frac{e^2}{\hbar}\ln\frac{L\varphi}{\ell}, \quad L\varphi = (D\tau_\varphi)^{1/2}.$$

In a magnetic field $\vec{p} \to \vec{p} + (e/c)\vec{A}\,, \quad \vec{H} = curl\,\vec{A}$

additional phase $$\Delta\varphi_H = \frac{2e}{c\hbar}\oint \vec{A}\cdot d\vec{l} = \frac{2e}{c\hbar}\oint \left(curl\,\vec{A}\right)\cdot d\vec{S} = 4\pi\frac{\Phi}{\Phi_0}$$

Φ is the magnetic flux $\Phi_0 = 2\pi\hbar c/e$

The role of magnetic field is important at

$$t_H \le \tau_\varphi \quad \to \quad H \ge H_0 \sim \Phi_0/(D\tau_\varphi) \approx \hbar c/L_\varphi^2\,.$$

Quantum effects manifest themselves in extremely weak magnetic fields.

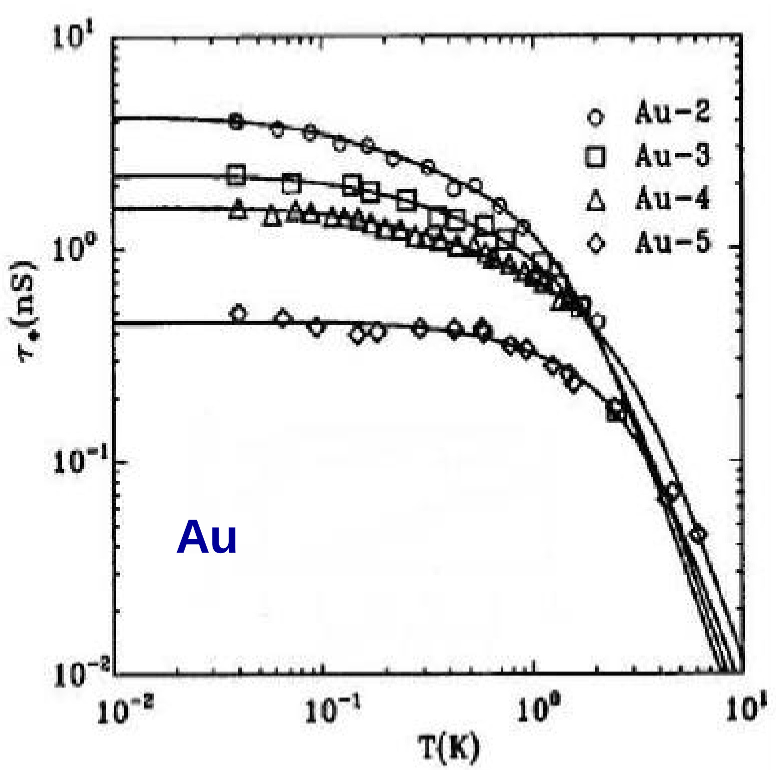
Conventional results

In the simplest situation $\tau_\varphi = \tau_{\text{in}}$. In *clean* materials τ_{in} behaves as

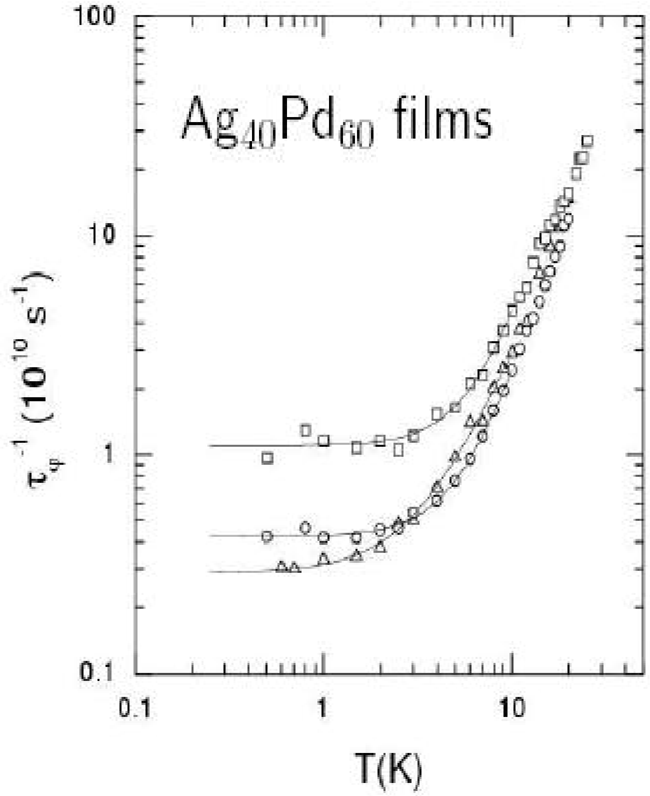
$$\tau_{\text{ph}}^{-1} \propto T^3\,, \quad \tau_{ee}^{-1} \propto T^2\,.$$

In disordered materials $\longrightarrow$ quasi-elastic $e-e$ $\longrightarrow$ $\tau_\varphi^{-1} \propto T\,.$

Some experiments → saturations of dephasing time



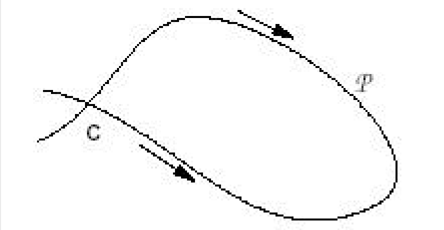
Mohanty et al., 1997



Lin et al., 2000

Dephasing due to “slow” degrees of freedom

Qualitative considerations



a slowly varying potential field $U(\vec{r}, t)$ action $S = \int ds \sqrt{2m(\mathcal{E} - U)}$

variation ΔS

$$\Delta S = -\int \frac{ds}{v} U(s,t) = -\int dt\, U(\vec{s}_t, t).$$

phase difference $\Delta\varphi = [(\Delta\varphi)_+ - (\Delta\varphi)_-] \propto \int_0^{t_0} dt\, [U(s_t, t) - U(s_{t_0 - t}, t_0 - t)]$

no spatial correlation between the scattering centers, $\overline{U(s_t, t) U(s_{t'}, t')} \propto \delta(s_t - s_{t'})$

single-point correlation function

$$\overline{U(s,t)U(s,t')} \equiv \overline{U^2} f(t - t')\,,\ \overline{U^2} \equiv \overline{U^2(s,t)}\,,\ f(0) = 1$$

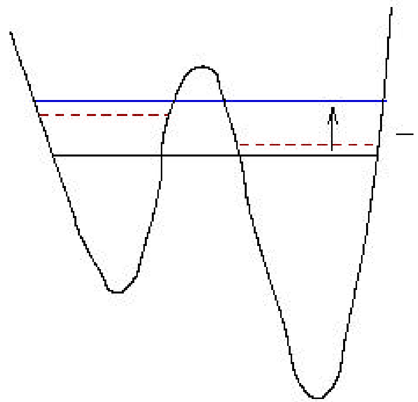
variance

$$\overline{\Delta\varphi^2} \propto \sum_s \int_0^{t_0} \frac{dt}{\tau_s}\,[1 - f_s(2t - t_0)]$$

The main reason for dephasing to slow down is a large correlation time comparing to the typical traversal time t_0. Then for typical times the correlation function is close to 1, and the phase variance turns out to be small.

Model for a dynamic defect

two-level tunneling states (TLS)



$$\mathcal{H}_\mathrm{d} = (\Delta\,\sigma_3 - \Lambda\,\sigma_3)/2$$

Δ is the diagonal level splitting

Λ is the tunneling amplitude

Previous studies of TLS-induced dephasing:

Y. Imry, H. Fukyama, and P. Schwab, Europhys. Letters, **47**, 608 (1999).

A. Zawadowski, Jan von Delft and D. C. Ralph, Phys. Rev. Lett. **83**, 2632 (1999).

Kagn-Hun Ahn, P. Mohanty, Phys. Rev. B **63**, 195301 (2001)

New features:

Two mechanisms of dephasing

Phase jumps versus phase wandering (diffusion)

Role of the average procedure over different TLSs

Two mechanisms of dephasing

1. Direct transitions between the two TLS's states accompanied by electron-hole pair creation/annihilation

If the energy transfer E is large , the phase relaxation time τ_φ is equal to the typical inelastic relaxation time $\tau_1(E,\Delta)$.

The criterion of "large" E $$E\tau_1 >> \hbar.$$

For smaller E

Estimate the dephasing time:

effective number of defects – $\bar{N} \sim t/\tau_1$;

phase factor – $e^{\pm Et/2\hbar} \to \cos(Et/2\hbar)$ if $T \gtrsim E$;

correlation function – $f(t) = \cos(Et/\hbar)$;

typical phase shift – $\bar{N}^{1/2}Et/\hbar \to \tau_\varphi \sim \hbar^{2/3}\tau_1^{1/3}/E^{2/3}$.

phase diffusion or wandering

$$\tau_\varphi^{(1)} = \tau_1 \max\left\{1, (\hbar/E\tau_1)^{2/3}\right\}$$

2. Apparently elastic scattering of electrons by a "breathing" scattering potential associated with the dynamic defect.

Estimate the dephasing time:

the correlation function $f(t)$ for statistically independent defects is

$$f(t) = e^{-2\gamma|t|}$$

where γ is the the defect transition rate.

In a similar way, we get

$$\tau_\varphi^{(3)} = \max\left\{\tau_3, (\tau_3/\gamma)^{1/2}\right\} .$$

Conclusion: Phase wandering at $\gamma\tau_3 \ll 1$.

Quantitative results

$$\delta\sigma = -\frac{e^2}{2\pi^2\hbar}\ln\frac{\tau_\varphi}{\tau}$$

where τ_φ is defined according to the equation

$$\begin{aligned}
\ln\frac{\tau_\varphi}{\tau} &\equiv \int_\tau^\infty \frac{dt}{t} e^{-\Gamma_1(t,E,\Lambda)-\Gamma_3(t,E,\Lambda)}, \\
\Gamma_1(t,E,\Lambda) &= \frac{1}{\tau_1}\left[t - \frac{\sin(t\,E/\hbar)}{E\tau/\hbar}\right], \\
\Gamma_3(t,E,\Lambda) &= \frac{1}{\tau_3}\left[t - \frac{\hbar}{2\gamma\tau}(1 - e^{-2\gamma t/\hbar})\right].
\end{aligned}$$

Average over different dynamic defects

We need the distribution function $\mathcal{P}(E,\gamma)$

Δ is determined by the defect's neighborhood,

Λ is determined by the distance between two metastable states.

Thus Δ and Λ are assumed to be uncorrelated.

Two typical model distributions:

"glass-model", GM (Anderson et al., Phillips).

$\mathcal{P}_\Lambda \propto \Lambda^{-1}$ exponentially-broad distribution of relaxation rates.

Amorphous materials, glasses

"tunneling-states-model" (TM) (Kozub&Rudin)

more appropriate for crystalline materials.

tunneling integrals Λ are almost the same for all dynamical defects.

Δ is determined by long-range interactions,
distributed smoothly within some band.

Then

$$\mathcal{P}_{TM}(E,\Lambda) = \frac{\Theta(E^* - E)}{E^*} \frac{E}{\sqrt{E^2 - \Lambda_0^2}} \delta(\Lambda - \Lambda_0) \, .$$

In the following we will assume that the dynamical defects are characterized by $\Lambda_0 \ll T$.

one has to replace $\Gamma_{1,3}$ by the averages

$$\bar{\Gamma}_i(\eta) = \int dE\, d\Lambda\, \mathcal{P}(E,\Lambda)\, \Gamma_i(\eta, E, \Lambda)\,.$$

Detailed calculations are rather tedious, and the concrete interplay between the mechanisms depends on the temperature.

The main conclusion is that there is a temperature region $T \geq T_\alpha, T_\beta$ in which the σ_3-channel dominates.

In this region,

$$\frac{1}{\tau_\varphi} = \frac{1}{\tau_\Lambda}\left[\left(\frac{T}{T_\alpha}\right)^{1/3} + \zeta\right], \quad \zeta \approx 1\,.$$

Here $\tau_\Lambda = \tau_3(E^*/\Lambda_0)$ nd $T_\alpha = \hbar/\chi\tau_\Lambda$ depend on the properties of the defect distribution and strength of the electron-defect interaction.

At very low temperatures τ_φ^{-1} vanishes.

Estimates

We use numbers, obtained from experiments on zero-bias anomalies in point contacts. Based on these estimates and taking

$P_d \approx 10^{34}$ erg^{-1}cm^{-3}, $\sigma_{\rm in} \approx 10^{-15}$ cm^2, $v_F \approx 10^8$ cm/s, and $\Lambda_0 \approx 10$ mK

we obtain $\tau_\Lambda \approx 10^{-9}$ s.

According to the estimates, at temperatures larger than $T_\Lambda \approx \Lambda_0 \approx 10$ mK one expects temperature-independent contribution of resonant processes.

For the relaxation channel, one obtains $T_\alpha \approx T_\beta \approx 10$ mK. Consequently, at $T \gtrsim T_\alpha \approx T_\Lambda \approx 10$ mK one expects that dephasing rate obeys the above equation with $\tau_\Lambda \approx 10^{-9}$ s.

Conclusions

- The dynamical defects can be responsible for the saturation of the temperature dependence of electron dephasing at low temperatures.
- This saturation is however terminated at $= T \to 0$ at temperatures of the order of the defects tunneling matrix element Λ_0.
- Two factors responsible for the dephasing:
  1. direct inelastic scattering of electrons by the defects,
  2. breaking of time reversal symmetry by time-dependent scattering potential.

  The first channel can indeed lead to the saturation behavior while the second one still contains a temperature dependence although a weak one.

Appendix: First-principle calculation

Spinless electrons which scatter against tunneling defects.

The Hamiltonian

$$\tilde{\mathcal{H}} = \tilde{\mathcal{H}}_\mathrm{d} + \sum_{\vec{p}} \epsilon_p c^+_{\vec{p}} c_{\vec{p}} + \tilde{\mathcal{H}}_\mathrm{int} \tag{3}$$

where

$$\tilde{\mathcal{H}}_\mathrm{d} = (\Delta\,\sigma_3 - \Lambda\,\sigma_1)/2 \tag{4}$$

$$\tilde{\mathcal{H}}_\mathrm{int} = \frac{1}{2} \sum_{\vec{p}\vec{p}_1, n} \left(\hat{1}\tilde{V}^+_{\vec{p}\vec{p}_1} + \sigma_3 \tilde{V}^-_{\vec{p}\vec{p}_1} \right) c^+_{\vec{p}} c_{\vec{p}_1}\, e^{i(\vec{p}-\vec{p}_1)\cdot\vec{r}_n/\hbar}, \tag{5}$$

$V^\pm = V_l \pm V_r$ represent components of a short-range defect potential in the "left" and "right" positions. Estimates for $\tilde{V}^\pm$ were given by several authors, e. g., by J. Black and by Y. Imry.

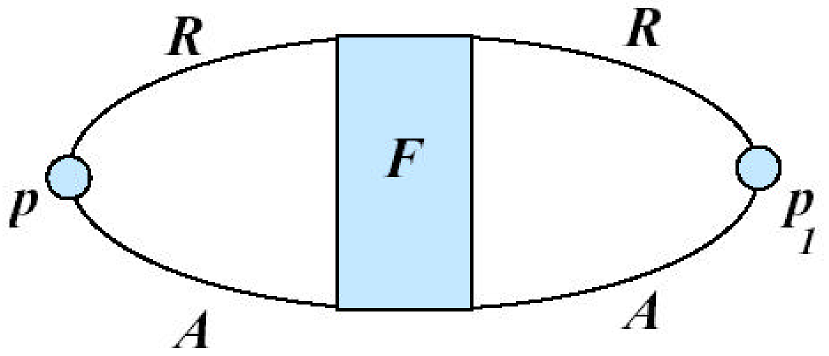
After the transform which makes $\tilde{\mathcal{H}}_{\mathrm{d}}$ diagonal we arrive at the Hamiltonian

$$\begin{aligned} \mathcal{H} &= \frac{1}{2}\sum_{n} E_n \,\boldsymbol{\sigma}_3 + \sum_{\vec{p}} \epsilon_p c^{+}_{\vec{p}} c_{\vec{p}} + \frac{1}{2}\sum_{\vec{p}\vec{p}_1, n} \Big\{ \hat{\mathbf{1}} V^{+}_{\vec{p}\vec{p}_1} \\ &\quad + \left(\frac{\Lambda_n}{E_n}\boldsymbol{\sigma}_1 + \frac{\Delta_n}{E_n}\boldsymbol{\sigma}_3 \right) V^{-}_{\vec{p}\vec{p}_1} \Big\} c^{+}_{\vec{p}} c_{\vec{p}_1} \, e^{i(\vec{p}-\vec{p}')\cdot\vec{r}_n/\hbar} . \end{aligned} \tag{6}$$

$\Rightarrow$ two processes of electron-defect interaction described by the items proportional to $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_3$, respectively.

They correspond to the two mechanisms discussed above.

Quantum contribution to conductance



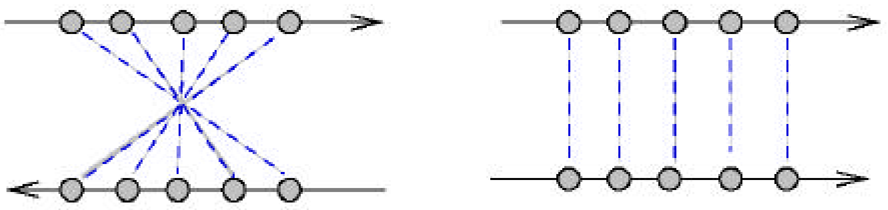
$$\delta\sigma = \frac{e^2}{2m^2}\int (dp)\,(dq)p^2 \int \left(-\frac{dn}{d\varepsilon}\right) d\varepsilon \int \frac{d\omega}{2\pi i} N(\omega)$$
$$\times G_R(\varepsilon,\vec{p})G_A(\varepsilon,\vec{p})\,F(\varepsilon,\omega,\vec{p},\vec{q}-\vec{p})$$
$$\times G_R(\varepsilon+\omega,\vec{q}-\vec{p})G_A(\varepsilon+\omega,\vec{q}-\vec{p})\,. \qquad (7)$$

Here $(dp) \equiv d^2p/(2\pi\hbar)^2$,

$n(\varepsilon)$ is the Fermi function,

$N(\omega)$ is the Planck function,

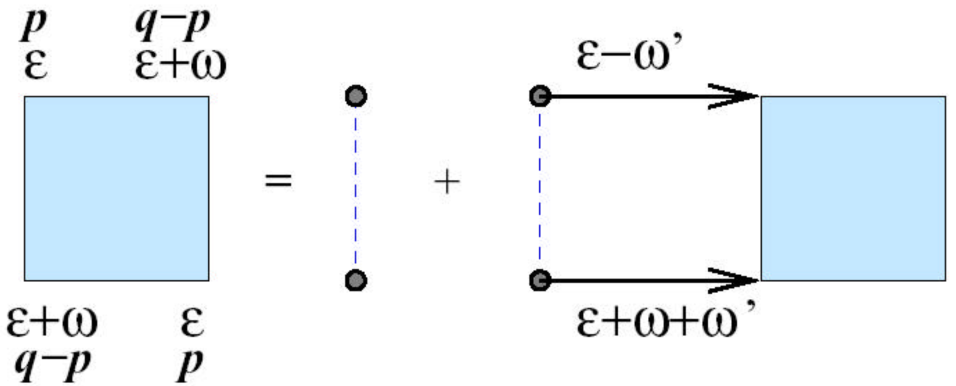
$F(\varepsilon, \omega, \vec{p}, \vec{p}_1)$ is a sum of the maximally-crossed diagrams – Cooperon.



Dashed lines - "propagators" of dynamic defects.

The Cooperon is a sum of a ladder in the particle-particle channel.

It satisfies the following Dyson equation

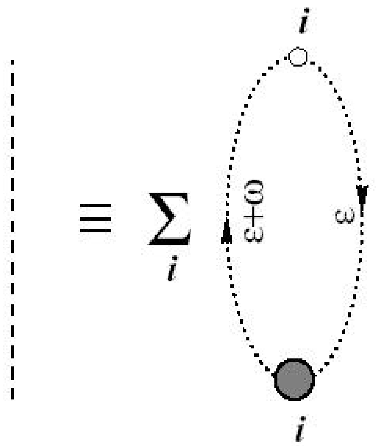


filled square – the Cooperon,

thick lines – the Green's functions averaged over the defect positions, as well as over the states of the thermal bath,

dashed lines – propagators for electron scattering against dynamic defects.

The propagator can be expressed as a loop graph where dotted lines represent Green's functions for a dynamic defect.



Since the interaction Hamiltonian (6) contain the items of three types ($\propto \hat{1}, \sigma_1, \sigma_3$), each propagator consists of a sum of three items.

Propagators are derived by the Abrikosov technique developed for the Kondo effect – a dynamic defect is interpreted as a pseudo-Fermion with the Green's function

$$g_{\pm}(\epsilon) = (\epsilon \mp E/2 - \lambda + i\delta)^{-1}, \tag{8}$$

where λ is an auxiliary "chemical potential".

To remove extra unphysical states, at the initial stage $\lambda \to \infty$.

As a result, the retarded propagator in the σ_1-channel is

$$\mathcal{D}_1^R(\omega) = -\tanh\frac{E}{2T}\left(\frac{1}{\omega - E + i\delta} - \frac{1}{\omega + E + i\delta}\right), \quad \delta \to +0. \quad (9)$$

The propagator for the σ_3-channel is (cf. with Maleev's expression for glasses)

$$\mathcal{D}_3^R(\omega) = \frac{1}{T\cosh^2(E/2T)}\frac{2i\gamma}{\omega + 2i\gamma}. \quad (10)$$

Here

$$\gamma(\Lambda, E) = \left(\frac{\Lambda}{E}\right)^2 \gamma_0(E), \quad \gamma_0(E) = \frac{\chi E}{\tanh E/2T}, \quad (11)$$

where $\chi = 0.01 - 0.3$ is dimensionless constant dependent on the matrix element $V^{(1)}$ where $\gamma_0(E)$ has the meaning of *maximum* hopping rate for the systems with a given interlevel spacing (J. Black).

For the *1*-channel we define the propagator as

$$\mathcal{D}_0^R(\omega) = \frac{\nu}{2T}\left(\frac{1}{\omega + \nu + i\delta} - \frac{1}{\omega - \nu + i\delta}\right), \quad \delta, \nu \to +0. \quad (12)$$

The propagators do not include the electron-defect coupling constant, hence each propagator should be multiplied by $|W^{(i)}|^2$ where

$$W^{(0)} = V^+ \,,\; W^{(1)} = (\Lambda/E)V^- \,,\; W^{(3)} = (\Delta/E)V^- \,.$$

Then, summation over different dynamic defects should be performed.

The resulting equation for $F(\varepsilon, \omega, \vec{p}, \vec{q} - \vec{p}_1)$ obtained by a proper analytical continuation of the Matsubara Green's functions, has the form

$$F(\varepsilon, \omega, \vec{p}, \vec{q} - \vec{p}) = \mathcal{D}(\omega) - \int \frac{(dp_1)\, d\omega'}{2\pi i} F(\varepsilon, \omega', \vec{p}_1, \vec{q} - \vec{p}) \mathcal{D}(\omega - \omega')$$
$$\times G^R(\varepsilon + \omega - \omega', \vec{p}_1')\, G^A(\varepsilon + \omega', \vec{q} - \vec{p}_1)\, [N_0(\omega') - N_0(\omega' - \omega)] \quad (13)$$

where $\quad \mathcal{D}(\omega) \equiv \sum_{is} |W_s^{(i)}|^2 \left[\mathcal{D}_i^R(\omega) - \mathcal{D}_i^A(\omega)\right].$

The above equation describes the dominant contribution provided the sum of the incoming momenta, q, is small:

$$q\ell \ll 1 \,.$$

Here $\ell = v_F \tau$ is the electron mean free path, while τ is the electron life time,

$$\tau^{-1} = \tau_e^{-1} + \tau_1^{-1} + \tau_3^{-1} \,.$$

τ_e^{-1} **- elastic rate**

$$\tau_1^{-1} = 2\pi \rho n_d (\Lambda/E)^2 |V_d^-|^2/\hbar \,,$$

$$\tau_3^{-1} = 2\pi \rho n_d (\Delta/E)^2 |V_d^-|^2/\hbar \,.$$

We transform (13) to the form of the diffusion equation. Using the inequalities

$$p_F\ell/\hbar \gg 1,\ \hbar\omega \ll T$$

and expressing results in terms of a new function

$$\mathcal{F}(\varepsilon,\vec{q},\omega) \equiv \frac{F(\varepsilon,\omega,\vec{p},\vec{q}-\vec{p})}{\omega(1-i\tau\vec{q}\cdot\vec{v})} \tag{14}$$

we obtain the equation

$$(1+Dq^2\tau)\mathcal{F}(\varepsilon,q,\omega) = \frac{\Phi(\omega)}{4\pi\rho}$$
$$-T\int\frac{d\omega'}{(2\pi i)(\omega-2\omega'+i/2\tau)}\mathcal{F}(\varepsilon,q,\omega')\Phi(\omega-\omega'). \tag{15}$$

Here $D = v_F\ell/2$ is the diffusion constant, while $\Phi(\omega) \equiv \mathcal{D}(\omega)/\omega$. In the time representation with respect to ω we obtain

$$1+Dq^2\tau)\mathcal{F}(\varepsilon,q,t) = \frac{\Phi(\varepsilon,t)}{2\tau T\rho}+\int_{-\infty}^{t}\frac{dt'}{\tau}e^{(t'-t)/\tau}\mathcal{F}(\varepsilon,q,t')\Phi(\varepsilon,2t-t'),$$

$$\Phi(\varepsilon,t) \equiv \frac{\tau}{\tau_e}+\frac{\tau}{\tau_1}\cos\frac{Et}{\hbar}+\frac{\tau}{\tau_3}e^{-2\gamma t/\hbar}.$$

The above equation can be solved exactly. The results have the simplest form at $\tau_e \ll \tau_1, \tau_3, \hbar/\gamma$, and what we need is $\mathcal{F}(0,q)$,

$$\mathcal{F}(0,q) = \int_{-\infty}^{0} \frac{dt'}{\tau} \, \Phi(t') \, e^{\vartheta(t')} ,$$

$$\vartheta(t) = Dq^2 t + \left[\frac{t}{\tau_1} - \frac{\sin(Et/\hbar)}{E\tau_1/\hbar}\right] + \left[\frac{t}{\tau_3} - \frac{\hbar}{2\gamma\tau_3}(e^{2\gamma t/\hbar} - 1)\right], \; t < 0 .$$

The final result can be formulated as

$$\delta\sigma = -\frac{e^2}{2\pi^2\hbar} \ln \frac{\tau_\varphi}{\tau}$$

where τ_φ is defined according to the equation

$$\begin{aligned} \ln \frac{\tau_\varphi}{\tau} &\equiv \int_1^\infty \frac{d\eta}{\eta} e^{-\Gamma_1(\eta,E,\Lambda) - \Gamma_3(\eta,E,\Lambda)} , \\ \Gamma_1(\eta,E,\Lambda) &= \frac{\tau}{\tau_1}\left[\eta - \frac{\sin(\eta\, E\tau/\hbar)}{E\tau/\hbar}\right] , \\ \Gamma_3(\eta,E,\Lambda) &= \frac{\tau}{\tau_3}\left[\eta - \frac{\hbar}{2\gamma\tau}(1 - e^{-2\eta\gamma\tau/\hbar})\right] , \end{aligned} \tag{16}$$

where $\eta = t/\tau$. This equation is obtained by the integration over q.