Interplay of Spin-Orbit Coupling and Interactions in 2D Electron Gas

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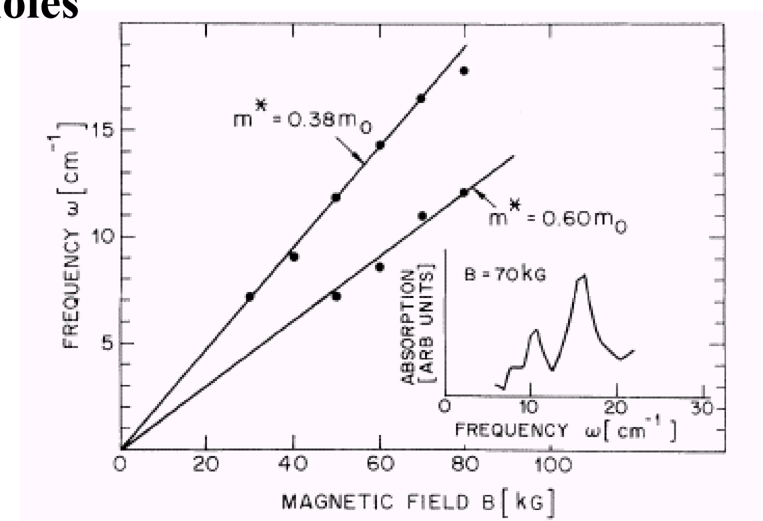
Motivation: Two Experimental Papers

Splitting of cyclotron resonance

[H.L. Stormer *et.al.*, PRL 51, 126 (1983)]

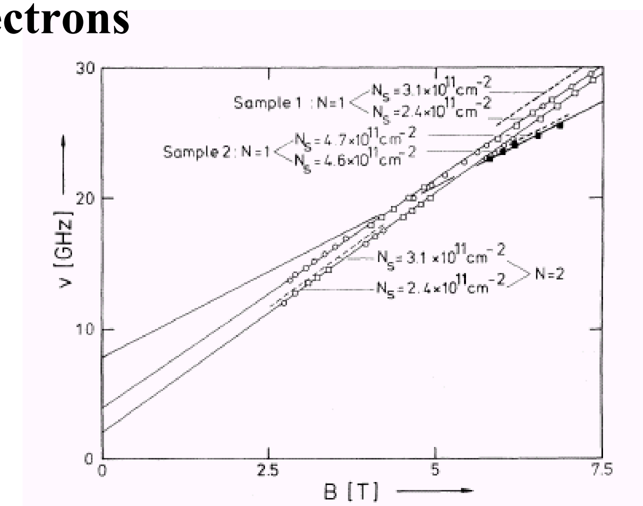
GaAs/$Al_xGa_{1-x}As$ heterostructures – holes



Electron spin resonance

[D. Stein, K.von Klitzing, and G. Weimann, PRL 51, 130 (1983)]

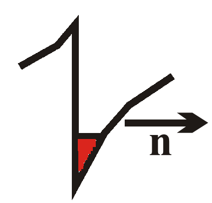
GaAs/$Al_xGa_{1-x}As$ heterostructures – electrons



[Yu.A. Bychkov and E.I. Rashba, JETP Lett. 39, 78 (1984)]:

$$H = \frac{\hbar^2}{2m}\mathbf{k}^2 + \alpha\left[\boldsymbol{\sigma}\times\mathbf{k}\right]\mathbf{n}$$

normal to the hetero-boundary



$$\left[\boldsymbol{\sigma}\times\mathbf{k}\right](\nabla V)$$

Spin-Orbit Hamiltonian: Energy Spectrum

Confinement asymmetry

[Yu.A. Bychkov and E.I. Rashba, JETP Lett. 39, 78 (1984)]

$$H_{SO} = \alpha\,[\boldsymbol{\sigma}\times\mathbf{k}]\,\mathbf{n}$$

Inversion asymmetry of the host crystal

[G. Dresselhaus, Phys. Rev. 100, 580 (1955)]

$$H_{SO} = \beta\left(k_x\sigma_x - k_y\sigma_y\right)$$

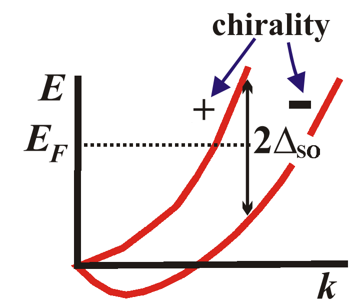
Energy Spectrum:

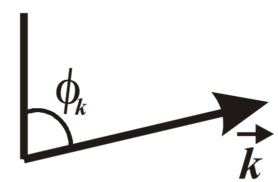
• without magnetic field:

$$E(k) = \frac{\hbar^2 k^2}{2m} \pm \alpha k$$

$$\Delta_{SO} = \alpha k_F$$

$$\chi_{\mathbf{k}}^{\pm} = \frac{e^{i\mathbf{k}\boldsymbol{\rho}}}{\sqrt{2}}\begin{pmatrix} e^{i\phi_{\mathbf{k}}} \\ \pm 1 \end{pmatrix}$$





• in a perpendicular magnetic field:

$$E_{n,\pm} = \hbar\omega_c\left[n \pm \sqrt{\delta^2 + \gamma^2 n}\right]$$

cyclotron frequency

$$\delta = \frac{1}{2}\left[1 - \frac{\Delta_Z}{2\hbar\omega_c}\right]$$

Zeeman energy

$$\gamma = \sqrt{2m\alpha^2/\omega_c}$$

Observation of SO-splitting in 2D Electron Gas

RAPID COMMUNICATIONS

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PHYSICAL REVIEW B VOLUME 38, NUMBER 14 15 NOVEMBER 1988-I

Observation of the zero-field spin splitting of the ground electron subband in GaSb-InAs-GaSb quantum wells

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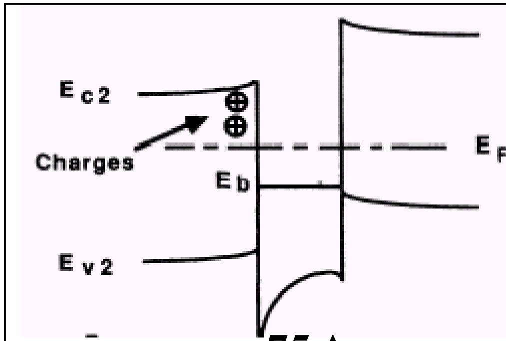
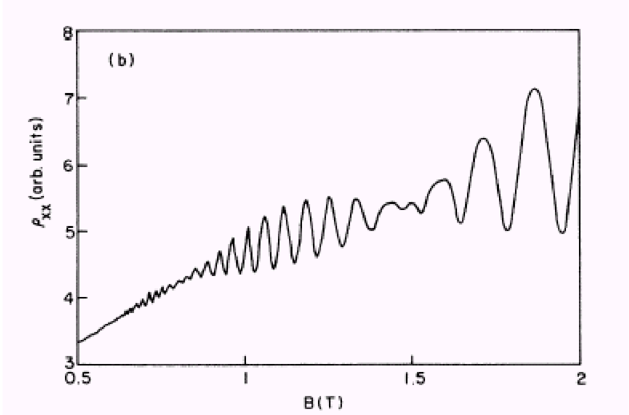
H. Munekata and F. F. Fang
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

P. J. Stiles
Physics Department, Brown University, Providence, Rhode Island 02912



75 A
GaSb - InAs - GaSb
narrow gap
heterostructure



From the experiment

$2\Delta_{SO} \approx 3.5$ meV

Beats of Shubnikov-de Haas Oscillations

$$\rho_{xx} \propto \exp\left(-\frac{\pi}{\omega_c \tau}\right) \cos\left(\frac{2\pi E_F}{\hbar\omega_C}\right)$$

without SO

$$\rho_{xx} \propto \cos\left(2\pi \frac{E_F + \Delta_{SO}}{\hbar\omega_C}\right) + \cos\left(2\pi \frac{E_F - \Delta_{SO}}{\hbar\omega_C}\right)$$

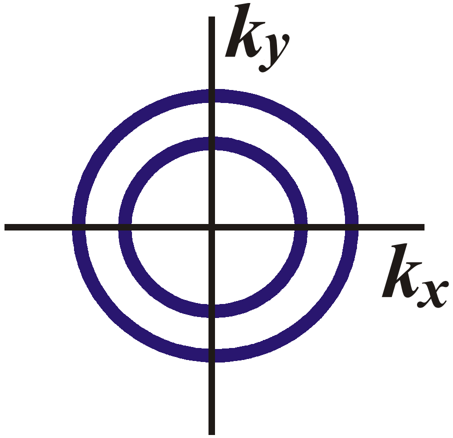
$$\propto \exp\left(-\frac{\pi}{\omega_c \tau}\right) \cos\left(\frac{2\pi E_F}{\hbar\omega_C}\right) \cos\left(\frac{2\pi \Delta_{SO}}{\hbar\omega_C}\right)$$

with SO

envelope

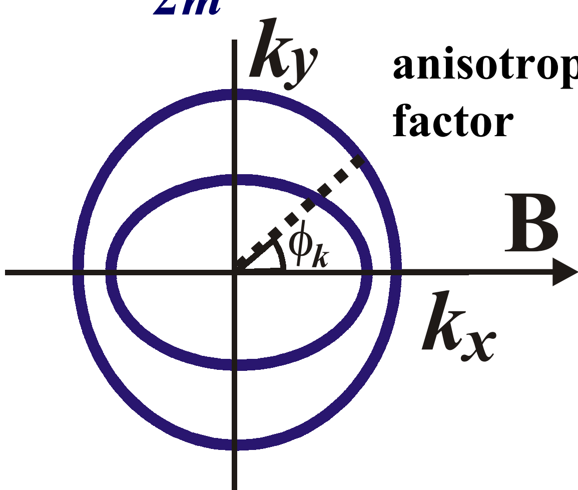
SO-Induced Anisotropy of the In-Plane Magnetoresistance

either $\Delta_Z = 0$ or $\Delta_{so} = 0$



$\Delta_Z \neq 0$ and $\Delta_{so} \neq 0$

$$E(\mathbf{k}) = \frac{\hbar^2 k^2}{2m} \pm \sqrt{\Delta_{so}^2 + \Delta_Z^2 + 2\Delta_{so}\Delta_Z \sin\phi_k}$$



anisotropy factor $\rightarrow \dfrac{\rho_{\parallel}(B) - \rho_{\perp}(B)}{\rho_{\parallel}(B) + \rho_{\perp}(B)}$

VOLUME 88, NUMBER 7 PHYSICAL REVIEW LETTERS 18 FEBRUARY 2002

Weak Anisotropy and Disorder Dependence of the In-Plane Magnetoresistance in High-Mobility (100) Si-Inversion Layers

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[3] *Institut für Halbleiterphysik, Johannes Kepler Universtät, Linz A4040, Austria*

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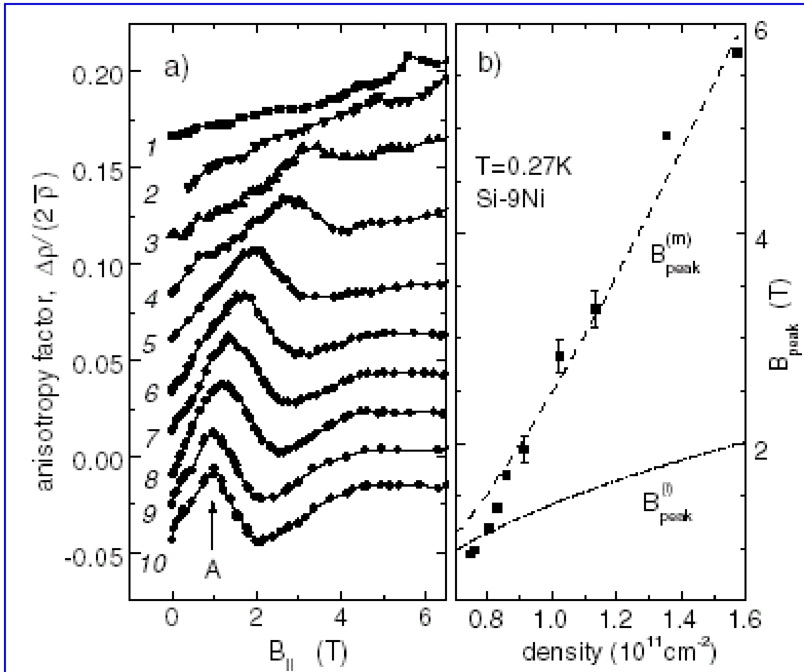

Experimentally Observed Effects that are due to Finite Δ_{so} : "hidden" period in conductance fluctuations

VOLUME 80, NUMBER 5 PHYSICAL REVIEW LETTERS 2 FEBRUARY 1998

Ensemble-Average Spectrum of Aharonov-Bohm Conductance Oscillations: Evidence for Spin-Orbit-Induced Berry's Phase

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G. Borghs
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1050

VOLUME 88, NUMBER 14 PHYSICAL REVIEW LETTERS 8 APRIL 2002

Aharonov-Bohm Oscillations with Spin: Evidence for Berry's Phase

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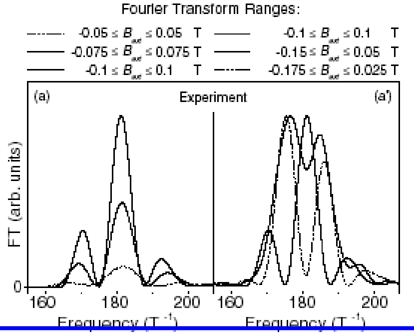
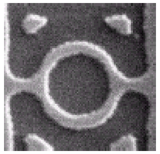
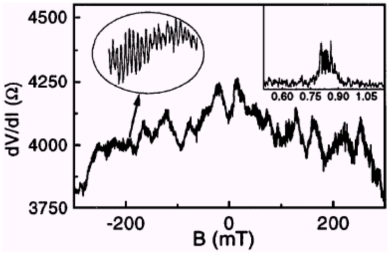
146801-1

- **GaAs/AlGaAs herero-structure – 2D hole system**
- **diameter 1μm**
- **$n \sim 2.4 \times 10^{15}$ m^{-2}**
- **$l \sim 2 \div 3\mu$m**



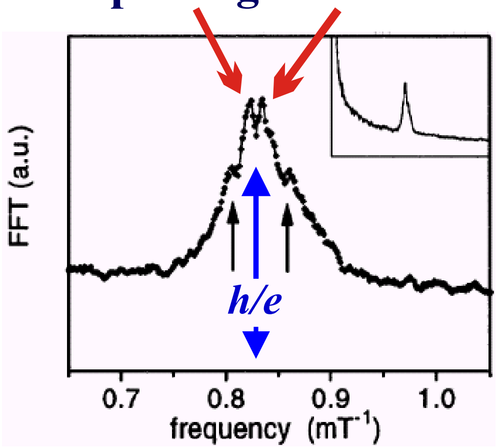
- **diameter 0.9 ÷ 2.1μm**
- **AlSb/InAs/AlSb heterostructure**
- **$n \sim 1.0 \times 10^{16}$ m^{-2}**
- **$l \sim 1\mu$m**



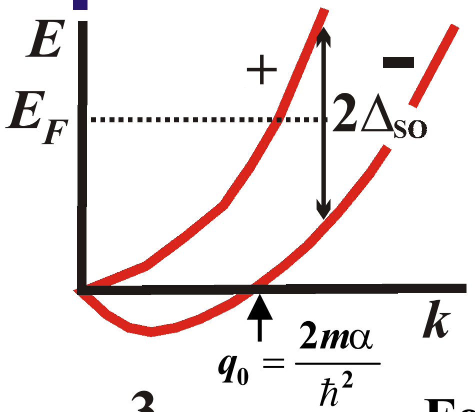


Aharonov-Bohm effect

Splitting due to SO

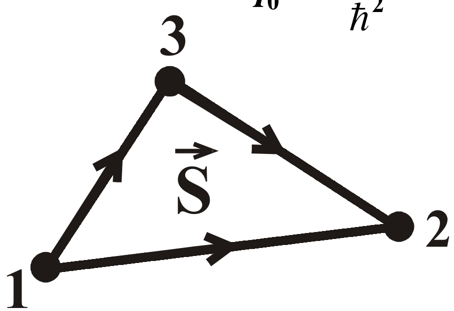


SO-induced Magnetic Field



$$\hat{G}(\mathbf{r}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{e^{i\mathbf{kr}}}{E - \frac{\hbar^2 k^2}{2m} - \alpha(\mathbf{k}\times\boldsymbol{\sigma})\mathbf{n}} = $$

$$= e^{-\left[\frac{iq_0}{2}\mathbf{n}(\boldsymbol{\sigma}\times\mathbf{r})\right]} G_0(r)$$



For localized electrons, $E < 0$

$$\hat{A} = \hat{G}(\mathbf{r}_{12}) + Q\,\hat{G}(\mathbf{r}_{13})\,\hat{G}(\mathbf{r}_{32}) \qquad e^{\hat{B}} e^{\hat{C}} = e^{\hat{B}+\hat{C}+\frac{1}{2}[\hat{B},\hat{C}]+\dots}$$

scattering amplitude

$$\hat{G}(\mathbf{r}_{13})\,\hat{G}(\mathbf{r}_{32}) = G_0(r_{13})\,G_0(r_{32})\cdot\exp\left\{-\frac{iq_0}{2}\mathbf{n}(\boldsymbol{\sigma}\times\mathbf{r}_{12}) - \frac{q_0^2}{8}\left[\mathbf{n}(\boldsymbol{\sigma}\times\mathbf{r}_{13}), \mathbf{n}(\boldsymbol{\sigma}\times\mathbf{r}_{32})\right]\right\}$$

$$[\ldots] = 4i(\mathbf{S}\times\mathbf{n})(\boldsymbol{\sigma}\times\mathbf{n})$$

vector area

$$\hat{A} = e^{-\frac{iq_0}{2}\mathbf{n}(\boldsymbol{\sigma}\times\mathbf{r}_{12})}\left[G_0(r_{12}) + Q\,G_0(r_{13})\,G_0(r_{32})\,e^{-\frac{2\pi i}{\Phi_0}(\mathbf{B}_{SO}\cdot\mathbf{S})}\right]$$

where $\mathbf{B}_{SO} = \frac{q_0^2\Phi_0}{4\pi}(\boldsymbol{\sigma}\cdot\mathbf{n})\,\mathbf{n}$

Suppression of Mesoscopic Fluctuations in Quantum Dots

VOLUME 86, NUMBER 10 PHYSICAL REVIEW LETTERS 5 MARCH 2001

Spin Degeneracy and Conductance Fluctuations in Open Quantum Dots

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C. M. Marcus
Department of Physics, Stanford University, Stanford, California 94305
and Department of Physics, Harvard University, Cambridge, Massachusetts 02138
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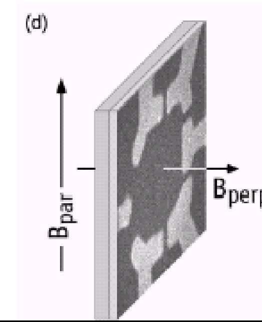
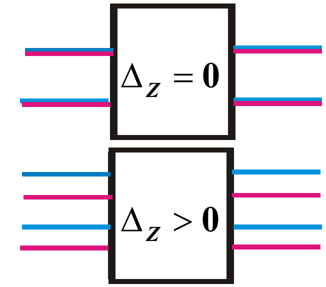


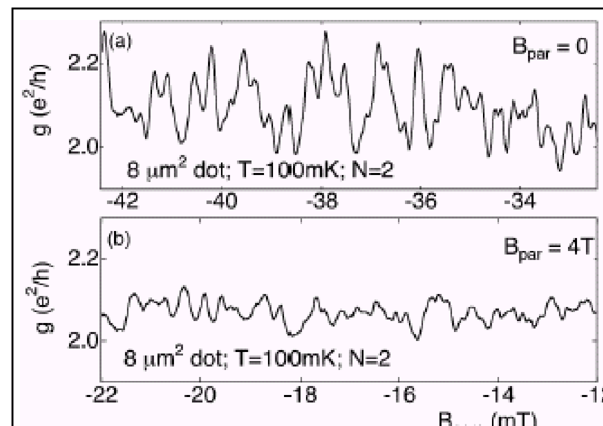


- GaAs/AlGaAs hetero-structure
- $\mu \approx 14\, m^2/V\,s$ $l \sim 1.5 \mu m$
- dots: $1\,\mu m^2$ and $8\,\mu m^2$
- $N_e =$ 2000 and 16000

The variance of fluctuations at high parallel field is reduced compared to the low-field value:

- **due to Zeeman splitting – reduction factor – 2.**
- **experimentally – reduction factor – from 2 to 5.5**
- **larger reduction factor is due to SO**

	8 μm²				1 μm²			
	N=1 100mK	N=3 100mK	N=1 200mK	N=1 300mK	N=1 100mK	N=3 100mK	N=1 300mK	N=3 300mK
Reduction Factor	5.5	4.2	5.3	4.0	1.9	2.2	2.6	2.7
Char. Field	0.9T	1.2T	1.4T	1.6T	1.1T	1.9T	3.9T	4.7T



Weak Antilocalization

VOLUME 48, NUMBER 15 PHYSICAL REVIEW LETTERS 12 APRIL 1982

Influence of Spin-Orbit Coupling on Weak Localization

Gerd Bergman

Institut für Festkörperforschung der Kernforschungsanlage Jülich, D-517 Jülich, West Germany

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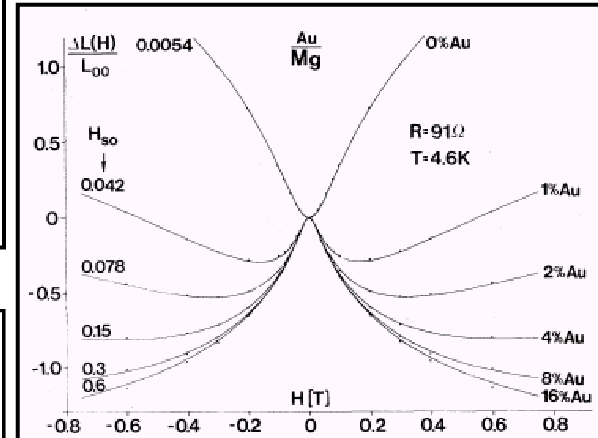
magnetoconductance



- **magnetoconductance of *Mg* films with different coverages of *Au***
- **spin-relaxation mechanism – scattering by *Au* impurities**

S. Hikami, A.I Larkin, and Y. Nagaoka, Prog. Theor. Phys. 63, 707 (1980)

VOLUME 68, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JANUARY 1992

Observation of Spin Precession in GaAs Inversion Layers Using Antilocalization

P. D. Dresselhaus, C. M. A. Papavassiliou,[a] and R. G. Wheeler

Department of Applied Physics, Yale University, New Haven, Connecticut 06520-2157

R. N. Sacks

United Technologies Research Center, East Hartford, Connecticut 06108

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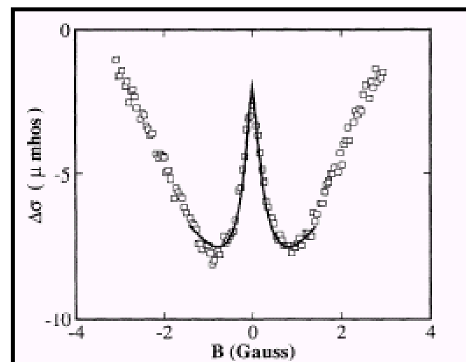
magnetoconductance



$GaAs/Al_{0.3}Ga_{0.7}As$ heterostructure

$n \sim 6.4 \times 10^{15}\ \mathrm{m}^{-2}$

$\mu \sim 10\ \mathrm{m}^2/\mathrm{V\,sec}$

- **SO determines spin-relaxation mechanism**
- **SO enters in the combination $\Delta_{so}\tau$**

Weak Antilocalization

PHYSICAL REVIEW B   VOLUME 53, NUMBER 7   15 FEBRUARY 1996-I

Weak antilocalization and spin precession in quantum wells

W. Knap, C. Skierbiszewski,* A. Zduniak,† E. Litwin-Staszewska,*D. Bertho, F. Kobbi, and J. L. Robert

G. E. Pikus F. G. Pikus S. V. Iordanskii V. Mosser K. Zekentes Yu. B. Lyanda-Geller

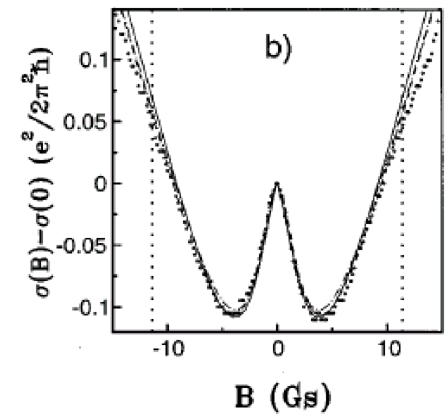
3912

AlGaAs/InGaAs/GaAs quantum well

540 A / 130 A / 8000 A

$n \sim 1 \times 10^{16}\,\mathrm{m}^{-2}$

$\mu \sim 4\ \mathrm{m}^2/\mathrm{V\,sec}$



APS March Meeting 2002

[D19.002] Gate-Controlled Spin-Orbit Coupling in a GaAs Two-Dimensional Electron Gas

J. B. Miller, D. M. Zumbuhl, C. M. Marcus (Harvard University),

K. Campman, A. C. Gossard (UC Santa Barbara)

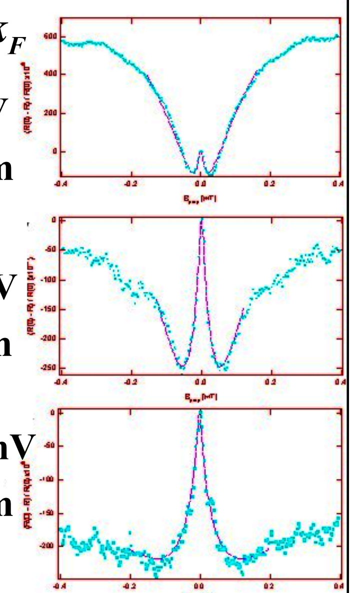
crossover in magnetotransport from weak localization to weak antilocalization in a gated Hall bar as a function of gate voltage…

$\Delta_{SO} = \alpha\, k_F$

V_g=-70 mV
$l_{so} = 22\,\mu\mathrm{m}$

V_g=+10 mV
$l_{so} = 17\,\mu\mathrm{m}$

V_g=+110 mV
$l_{so} = 10\,\mu\mathrm{m}$

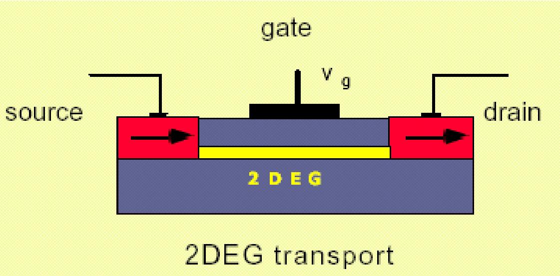


Applications: Spin Transistor

Electronic analog of the electro-optic modulator

Supriyo Datta and Biswajit Das
School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907

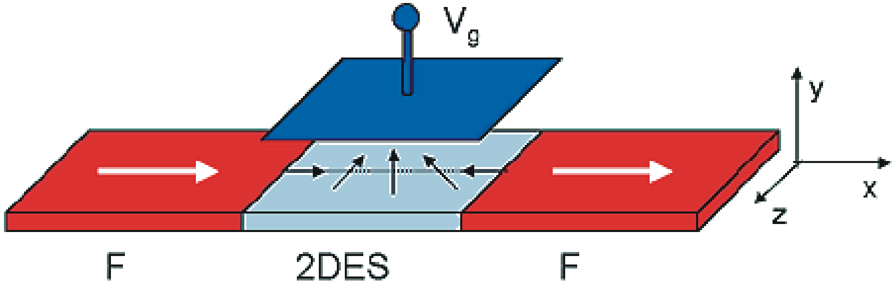
Appl. Phys. Lett 56, 665 (1990)



$$H = \frac{\hbar^2 k_x^2}{2m} + \alpha \sigma_z k_x$$

$$k_{x,+} - k_{x,-} = \frac{2m\alpha}{\hbar^2}$$

$$\Delta\theta = \left(k_{x,+} - k_{x,-}\right) L = \frac{2m\alpha L}{\hbar^2}$$



- **Spin precession of the injected electrons – due to SO splitting**
- **Gate control of SO** (α)
- **High mobility structures,** $L < l$

Gate Control of Spin-Orbit Splitting

VOLUME 78, NUMBER 7 PHYSICAL REVIEW LETTERS 17 FEBRUARY 1997

Gate Control of Spin-Orbit Interaction in an Inverted $In_{0.53}Ga_{0.47}As/In_{0.52}Al_{0.48}As$ Heterostructure

Junsaku Nitta, Tatsushi Akazaki, and Hideaki Takayanagi
NTT Basic Research Laboratories, 3-1 Wakamiya, Morinosato, Atsugi-shi, Kanagawa 243-01, Japan

Takatomo Enoki
NTT System Electronics Laboratories, 3-1 Wakamiya, Morinosato, Atsugi-shi, Kanagawa 243-01, Japan

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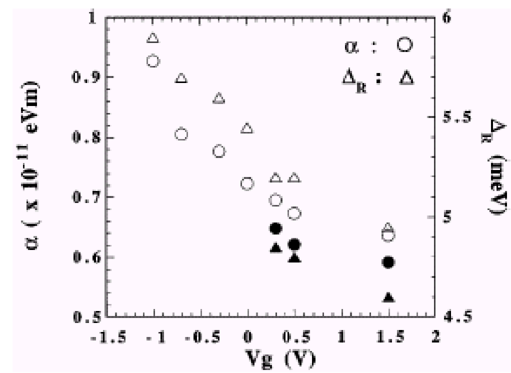
InGaAs/InAlAs heterostructure

$n \sim 1.9 \times 10^{16}\,\mathrm{m^{-2}}$

$\mu \approx 3.6\,\mathrm{m^2}/V\,\mathrm{s}$



PHYSICAL REVIEW B VOLUME 55, NUMBER 4 15 JANUARY 1997-II

Experimental and theoretical approach to spin splitting in modulation-doped $In_xGa_{1-x}As/InP$ quantum wells for $B \rightarrow 0$

G. Engels, J. Lange, Th. Schäpers, and H. Lüth
Institut für Schicht- und Ionentechnik, Forschungszentrum Jülich GmbH, 52425 Jülich, Germany

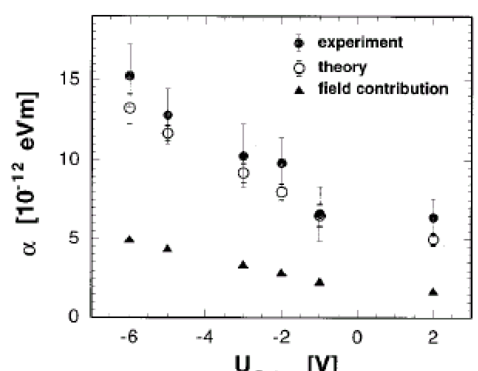
R1958

InP/InGaAs/InP quantum well

$n \sim 1.59 \times 10^{16}\,\mathrm{m^{-2}}$

$\mu \approx 20\,\mathrm{m^2}/V\,\mathrm{s}$

From the beats of SdH oscillations



From the beats of SdH oscillations

$\Delta_{so} = 5\,\mathrm{meV}$

$L_{\Delta\theta=\pi} = 0.5\,\mu m$

$l = 1\,\mu m$

Spin-Orbit Splitting in AlGaAs/GaAs heterostructures

PHYSICAL REVIEW B VOLUME 55, NUMBER 11 15 MARCH 1997-I

Zero-magnetic-field spin splittings in $Al_xGa_{1-x}As/GaAs$ heterojunctions

P. Ramvall, B. Kowalski, and P. Omling
Solid State Physics, Lund University, Box 118, S-221 00 Lund, Sweden

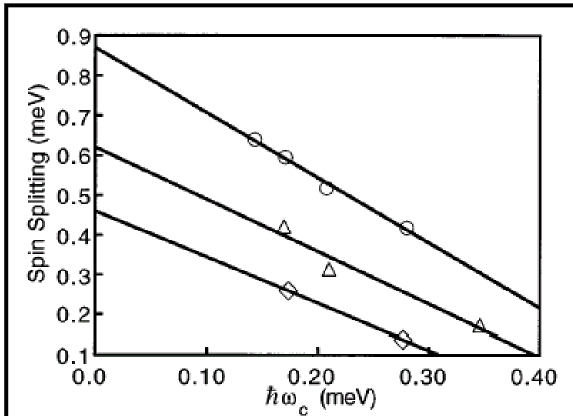
7160

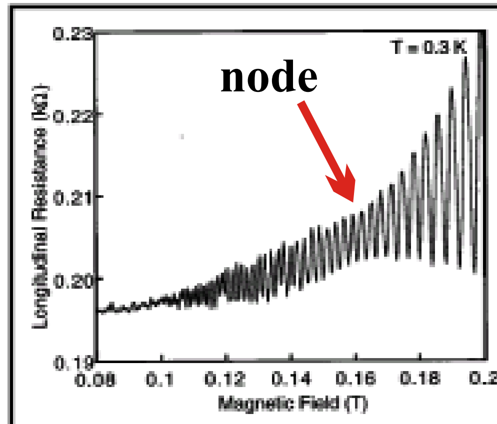
AlGaAs/GaAs heterostructure

$n \sim 0.37 \times 10^{16}\,\mathrm{m}^{-2}$

$\mu \approx 38\,\mathrm{m}^2 / V\,\mathrm{s}$

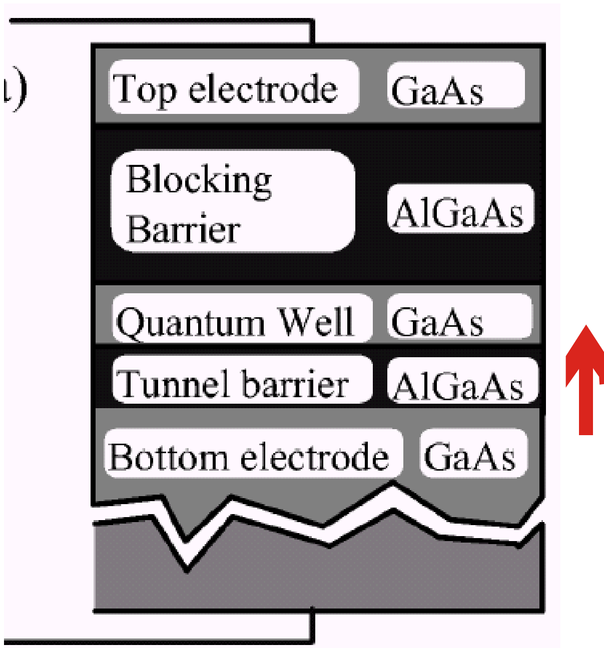
- **SO splitting is determined from linear extrapolation** $\omega_c \to 0$
- **Results are ambiguous**



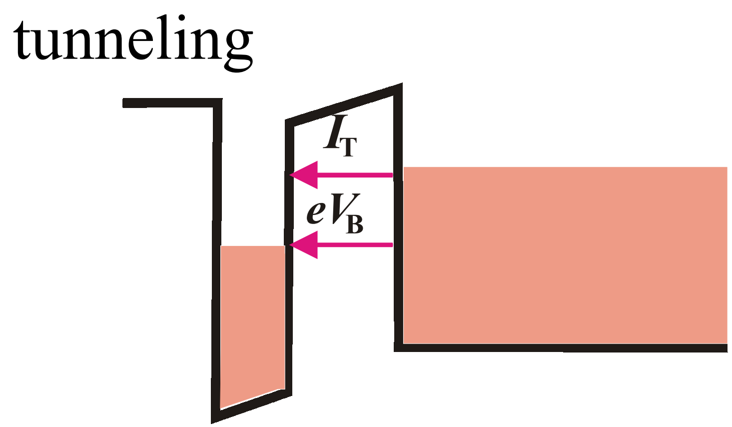


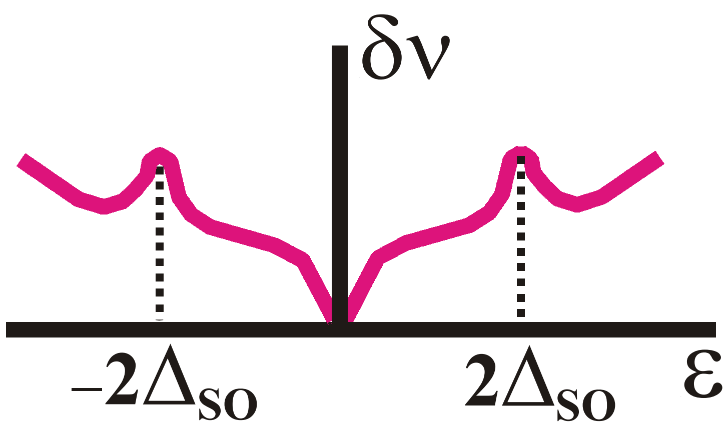
Treatment before measuring	Electron concentration (10^{11} cm^{-2})	Zero-field spin splitting (meV)
No irradiation, no illumination	3.7	Unmeasurable
No irradiation, illumination	3.9	0.46
Irradiation, no illumination	3.7	0.61
Irradiation, illumination	4.5	0.87

Tunneling into 2D Electron Gas



- Tunneling conductance $\propto$ 2D (tunneling) density of states, ν
- Very fine feature can be resolved, $\Delta\varepsilon < 0.01$ meV





$$I_T(V_B) \sim \int_0^{eV_B} d\varepsilon\, \nu(\varepsilon)$$

$$G(V_B) = \frac{dI_T}{dV_B} \sim \nu(eV_B)$$

Zero Bias Anomaly (2D): Theory

VOLUME 44, NUMBER 19 PHYSICAL REVIEW LETTERS 12 MAY 1980

Interaction Effects in Disordered Fermi Systems in Two Dimensions

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and

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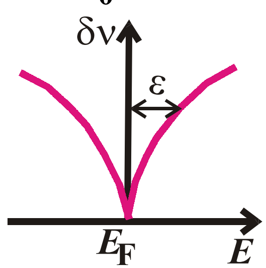
Bell Laboratories, Murray Hill, New Jersey 07974

1288

<u>Diffusive regime ($\varepsilon\tau \ll 1$):</u>

Interaction-induced corrections to the tunneling density of states:

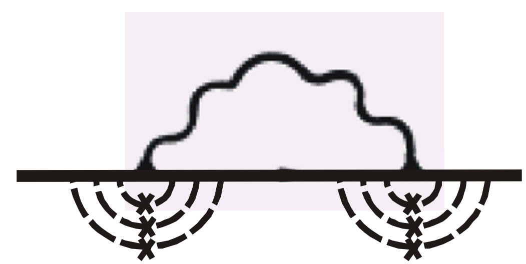
$$\frac{\delta\nu(\varepsilon)}{\nu_0} = \frac{1}{4\pi\varepsilon_F\tau}\ln(|\varepsilon|\tau)$$



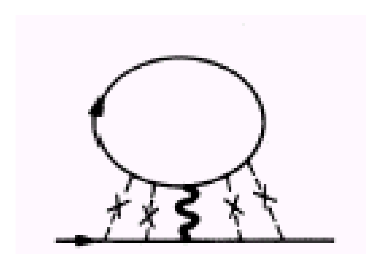
small in parameter

$$\frac{<V_{scr}(k)>_\varphi}{V_{scr}(0)} \sim \frac{1}{k_F a_B} \sim r_s << 1$$

Exchange correction



Hartree correction



Zero Bias Anomaly (2D): Experiment

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Density-of-States Anomalies in a Disordered Conductor: A Tunneling Study

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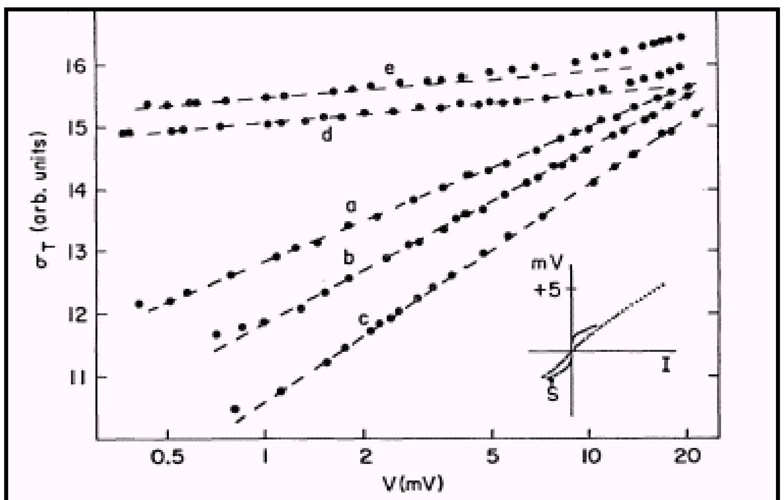
841

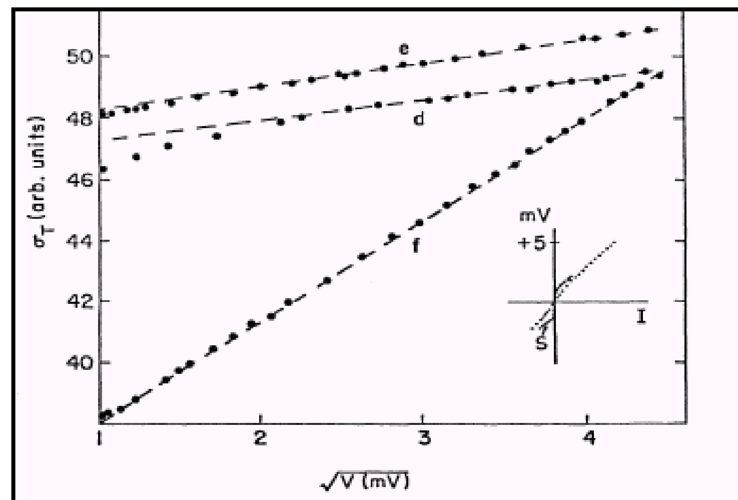
InO_x films of various thicknesses

$n \sim 0.95 \times 10^{26}\,\mathrm{m}^{-3}$



$d < 460\,\mathrm{A}$

$\sigma_T \sim \delta\nu \sim \ln(V)$ **2D behaviour**

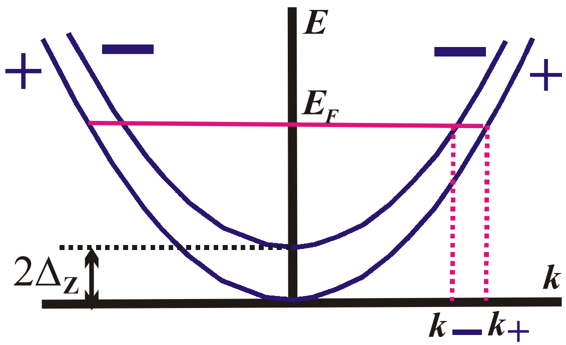


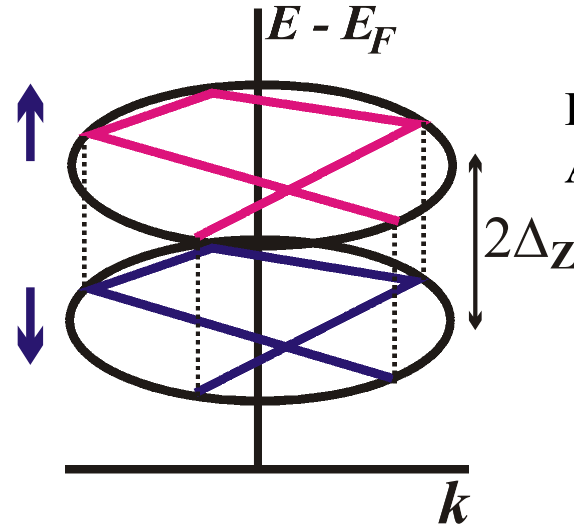
$d > 460\,\mathrm{A}$

$\sigma_T \sim \delta\nu \sim \sqrt{V}$ **3D behaviour**

Zeeman Satellites of a Zero-Bias Anomaly

Two Zeeman-splitted Fermi surfaces



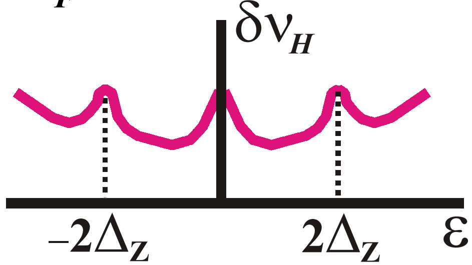


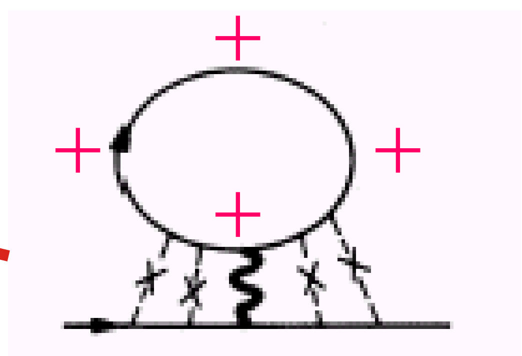
B.L. Altshuler and A.G. Aronov (1985)

Satellites of a zero-bias anomaly at $\varepsilon = \pm 2\Delta_Z$

Hartree Correction

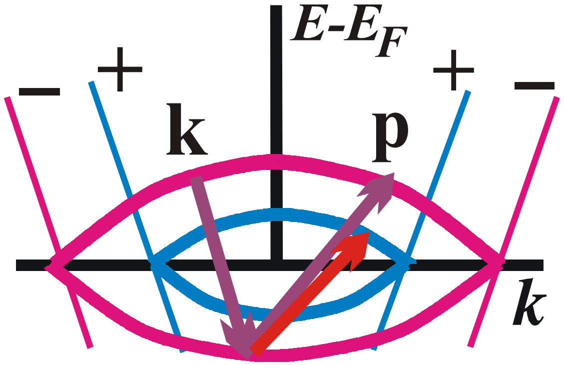
$$\frac{\delta\nu_Z(\varepsilon)}{\nu_0} = -\frac{\nu_0 < V_{scr}(\mathbf{k}) >_\varphi}{8\pi\,\varepsilon_F \tau} \ln\left(\tau^2 \left|\varepsilon^2 - 4\Delta_Z^2\right|\right)$$





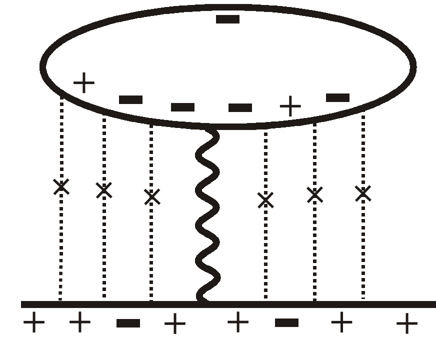
Zero-Bias Anomaly in the Presence of SO

- short range disorder $\rightarrow$ large momentum transfer



$$W_{\mathbf{kp}}^{\mu,\mu} = W_{\mathbf{kp}}\left[\frac{1+\cos\phi_{\mathbf{kp}}}{2}\right]$$

$$W_{\mathbf{kp}}^{\mu,-\mu} = W_{\mathbf{kp}}\left[\frac{1-\cos\phi_{\mathbf{kp}}}{2}\right]$$



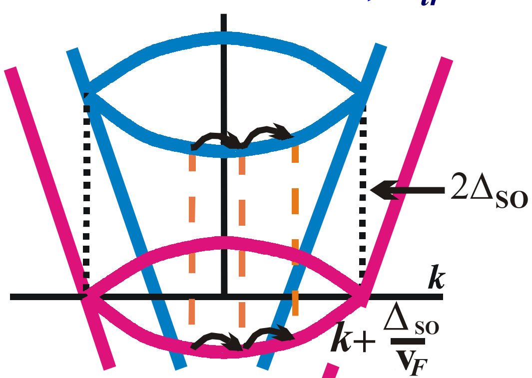
no conservation of the branch index $\longrightarrow$ no SO-induced singularity at $\varepsilon = \pm 2\Delta_{SO}$

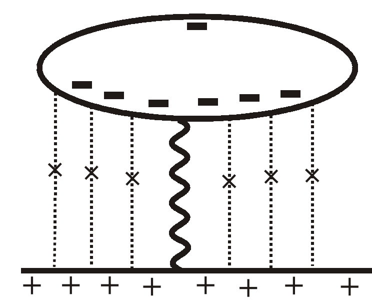
- smooth disorder, $\tau_{tr} >> \tau$

$\longrightarrow$ small momentum transfer, $\phi_{\mathbf{kp}} \ll 1$

$\longrightarrow$ approximate conservation of the branch index

$\longrightarrow$ SO-induced singularity at $\varepsilon = \pm 2\Delta_{SO}$





A.M. Rudin, I.L. Aleiner, and L.I. Glazman PRL 78, 709 (1997)

A.P. Dmitriev and V.Yu. Kachorovskii, PRB 63, 113301 (2001)

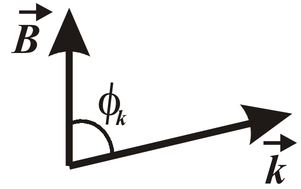
Calculation of $\delta\nu(\varepsilon)$: Hamiltonian

- $$\hat{H} = \frac{\hbar^2 k^2}{2m} + \alpha\left(\mathbf{k}\times\hat{\boldsymbol{\sigma}}\right)\mathbf{n} + \Delta_Z\hat{\sigma}_x$$

- $$E_\mu(k) = E_F + \hbar \mathbf{v}_F\left(k - k_F\right) + \mu\,\Delta(\mathbf{k})$$

$$\Delta(\mathbf{k}) = \sqrt{\Delta_{SO}^2 + \Delta_Z^2 + 2\Delta_{SO}\Delta_Z \sin\phi_k}$$

$$\mu = \pm 1$$



- $$\hat{H} = \sum_\mu E_\mu(\mathbf{k})\hat{\Lambda}_\mu(\mathbf{k})$$

$$\hat{\Lambda}_\mu(\mathbf{k}) = \frac{1}{2}\begin{pmatrix} 1 & \mu\exp(-i\varphi_\mathbf{k}) \\ -\mu\exp(i\varphi_\mathbf{k}) & 1 \end{pmatrix}$$

$$\tan\varphi_\mathbf{k} = \tan\phi_\mathbf{k} + \frac{\Delta_Z}{\Delta_{SO}\cos\phi_\mathbf{k}}$$

- $$\hat{G}_E^{(0)} = \sum_\mu \frac{\hat{\Lambda}_\mu(\mathbf{k})}{E - E_\mu(\mathbf{k})}$$

Calculation of $\delta\nu(\varepsilon)$: diffusive regime

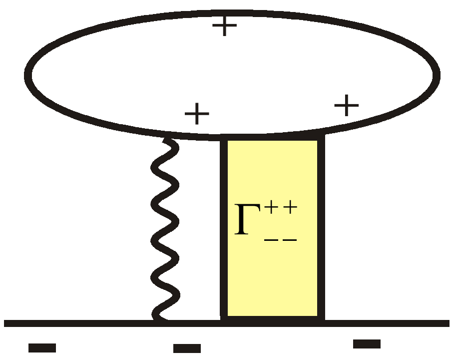
Hartree correction:

interaction — Vertex function

$$\frac{\delta\nu(\varepsilon)}{\nu_0} = \frac{1}{2\pi}\frac{\partial}{\partial\varepsilon}\mathrm{Re}\int_{\varepsilon}^{\infty} d\omega \int \frac{d\mathbf{q}}{(2\pi)^2}\int \frac{d\mathbf{p}}{(2\pi)^2}\int \frac{d\mathbf{p}'}{(2\pi)^2} V(\mathbf{p}-\mathbf{p}')\Gamma_{--}^{++}(\mathbf{p},\mathbf{p}',\mathbf{q},\omega)$$

$$\times\left(1-\lambda_{\mathbf{p+q},\mathbf{p}'+\mathbf{q}}\right)\left(1-\lambda_{\mathbf{p},\mathbf{p}'}\right)G_1^R(\varepsilon+\omega,\mathbf{p+q})\,G_1^R(\varepsilon+\omega,\mathbf{p}'+\mathbf{q})\,G_{-1}^A(\varepsilon,\mathbf{p})\,G_{-1}^A(\varepsilon,\mathbf{p}')$$

Disorder-induced mixing of different branches — Green functions

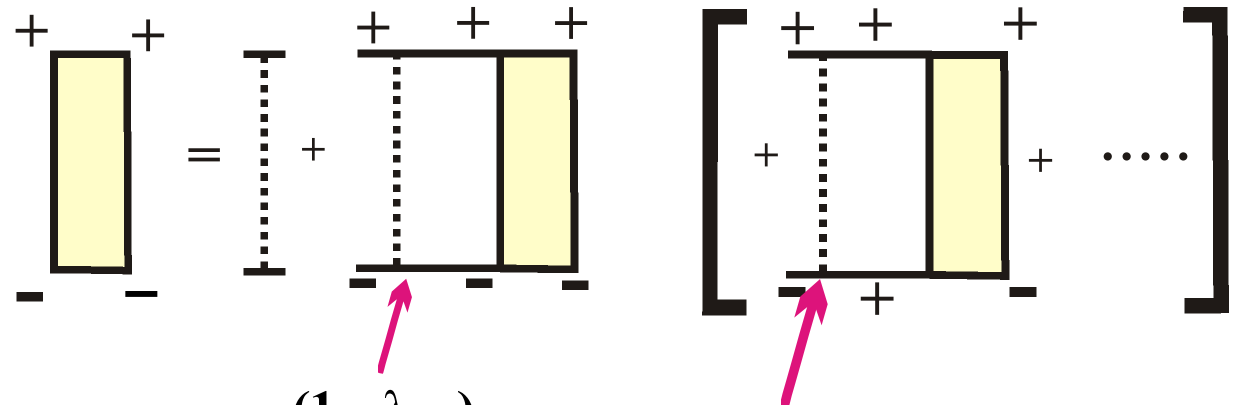


- $\lambda_{\mathbf{k},\mathbf{p}} = \mathrm{Tr}\left(\hat{\Lambda}_{\mu}(\mathbf{k})\hat{\Lambda}_{-\mu}(\mathbf{p})\right) \sim \left(\phi_{\mathbf{k}} - \phi_{\mathbf{p}}\right)^2 << 1$ smooth potential

- $G_{\mu}^{R,A} = \dfrac{1}{\varepsilon - E_{\mu}(k) \pm i/2\tau}$ τ - scattering time

Calculation of $\delta\nu(\varepsilon)$: Vertex Function

- $\Gamma^{++}_{--}(p,p',q,\omega) =$



$(1-\lambda_{k,p})$ additional small factor $\lambda_{k,p} << 1$

$[\cdots]$ are small in parameter $\dfrac{1}{\Delta_{SO}\tau_{\text{int}}} << 1$

- $$\Gamma^{++}_{--} \propto \frac{1}{-i\,\big(\omega - 2\Delta(\mathrm{p})\big)\tau + Dq^2\tau + \tau/\tau_{\text{int}}(\mathrm{p})}$$

SO splitting

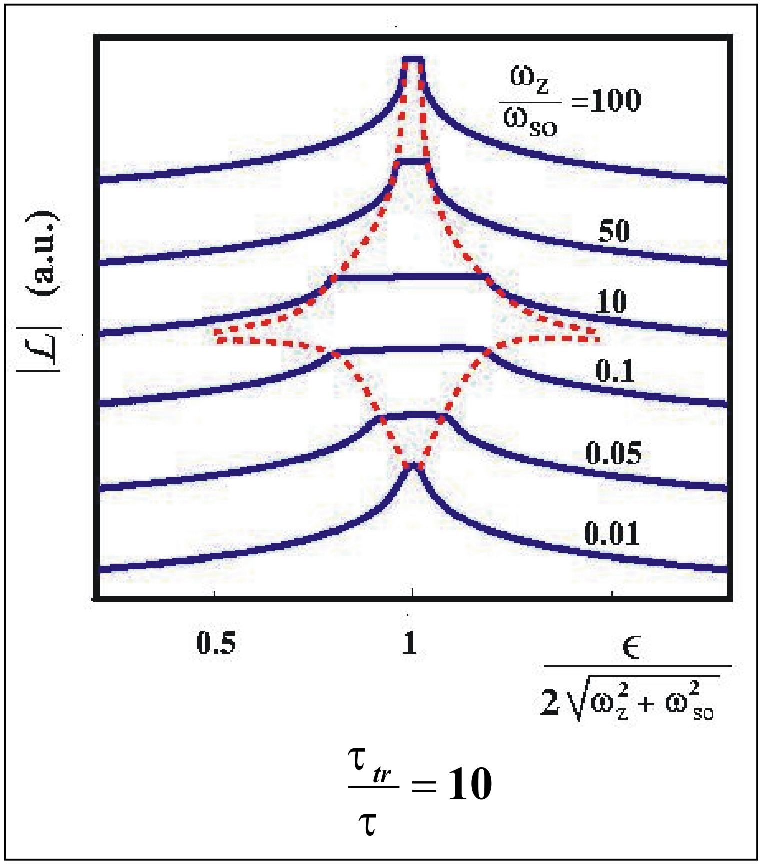
Diffusion coefficient: $D = \dfrac{v_F^2\tau_{tr}}{2}$

$\tau_{\text{int}}(\mathrm{p})$ - inter-subband scattering time

τ_{tr} - transport relaxation time

Calculation of $\delta\nu(\varepsilon)$: Results

$$\frac{\delta\nu(\varepsilon)}{\nu_0} = -\left(\frac{1}{16\pi E_F \tau_{tr}}\right) L(\varepsilon)$$



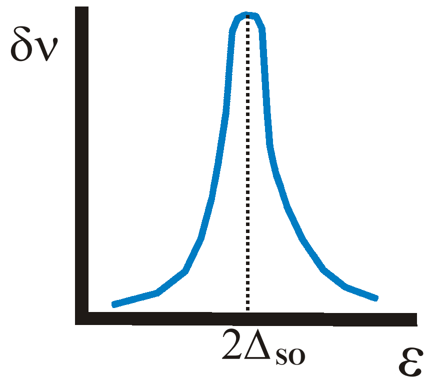
$$L(\varepsilon) = \int_0^{2\pi} \frac{d\phi}{2\pi} \ln\left[\left(\varepsilon - 2\Delta(\phi)\right)^2 \tau^2 + \frac{\tau^2}{\tau_{\mathrm{int}}^2(\phi)}\right]$$

$$\tau_{\mathrm{int}}(\phi) = 2\tau_{tr} \frac{\left(\Delta_Z^2 + \Delta_{SO}^2 + 2\Delta_Z \Delta_{SO} \sin\phi\right)^2}{\Delta_{SO}^2 \left(\Delta_{SO} + \Delta_Z \sin\phi\right)^2}$$

$$\Delta(\phi) = \sqrt{\Delta_{SO}^2 + \Delta_Z^2 + 2\Delta_{SO}\Delta_Z \sin\phi}$$

Limiting cases

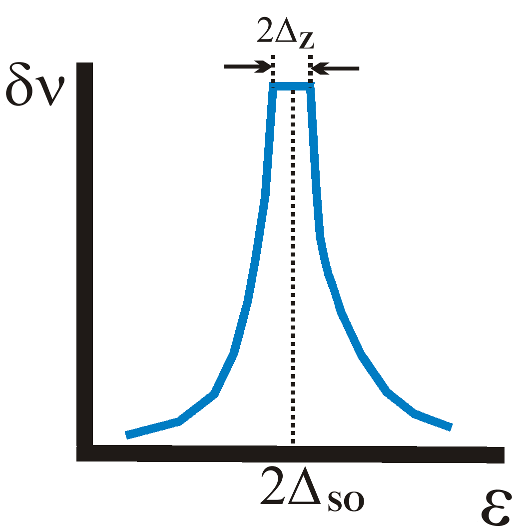
- $\Delta_Z = 0$



$$\frac{\delta\nu(\varepsilon)}{\nu_0} = -\left(\frac{1}{16\pi E_F \tau_{tr}}\right) \ln\left[(\varepsilon - 2\Delta_{SO})^2 \tau^2 + \frac{\tau^2}{2\tau_{tr}^2}\right]$$

- $\tau_{tr}^{-1} << \Delta_Z << \Delta_{SO}$

$$\frac{\delta\nu(\varepsilon)}{\nu_0} = -\left(\frac{1}{8\pi E_F \tau_{tr}}\right) \ln\left|\left|\frac{\varepsilon}{2} - \Delta_{SO}\right|\tau + \sqrt{\left(\frac{\varepsilon}{2} - \Delta_{SO}\right)^2 \tau^2 - \Delta_Z^2 \tau^2}\right|$$



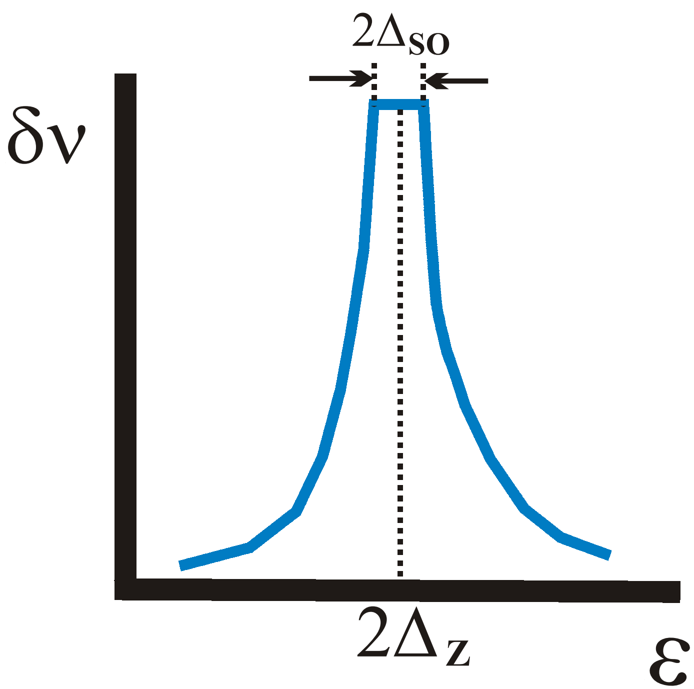
Plateau of width $2\Delta_Z$ at the top of the SO satellite

$$\int_0^{2\pi} \frac{d\varphi}{2\pi} \ln(a_\varepsilon + b\cos\varphi)^2 = 2\ln\frac{\left||a_\varepsilon| + \sqrt{a_\varepsilon^2 - b^2}\right|}{2}$$

if $b > a_\varepsilon$ **, then** $\left||a_\varepsilon| + i\sqrt{b^2 - a_\varepsilon^2}\right| = |b|$

Limit of small SO coupling $(\Delta_{SO} << \Delta_Z)$

$$\frac{\delta\nu(\varepsilon)}{\nu_0} = -\left(\frac{1}{8\pi E_F \tau_{tr}}\right) \ln\left| \left|\frac{\varepsilon}{2} - \Delta_Z\right|\tau + \sqrt{\left(\frac{\varepsilon}{2} - \Delta_Z\right)^2 \tau^2 - \Delta_{SO}^2 \tau^2} \right|$$



$$\tau_{\text{int}}(\phi) = 2\tau_{tr} \frac{\left(\Delta_Z^2 + \Delta_{SO}^2 + 2\Delta_Z \Delta_{SO} \sin\phi\right)^2}{\Delta_{SO}^2 \left(\Delta_{SO} + \Delta_Z \sin\phi\right)^2}$$

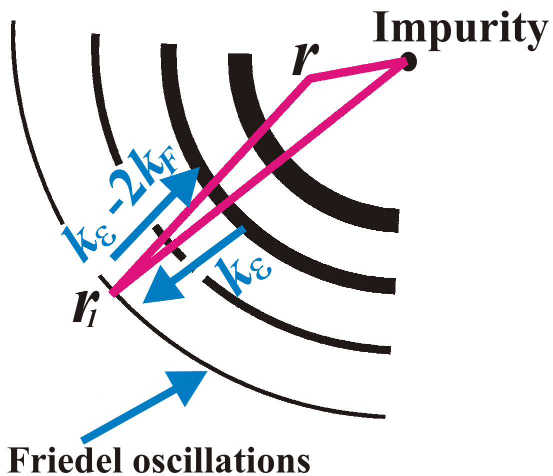
for $\Delta_{SO} << \Delta_Z$ $\quad \tau_{\text{int}} = \tau_{tr} \left(\frac{\Delta_Z}{\Delta_{SO}}\right)^2$

applicability: $\tau << \tau_{\text{int}} = \tau_{tr} \left(\frac{\Delta_Z}{\Delta_{SO}}\right)^2$

Plateau of width $2\Delta_{SO}$ emerges at the top of the Zeeman satellites

Limit of small SO coupling $(\Delta_{SO} \ll \Delta_Z)$:quasiballistic regime

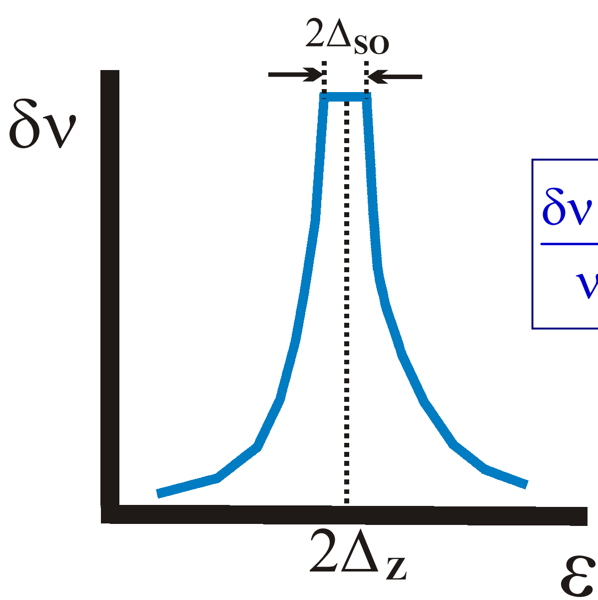
A.M. Rudin, I.L. Aleiner, and L.I. Glazman, PRB 55, 9322 (1997)



$$\frac{\delta\nu(\varepsilon)}{\nu_0} = -\left(\frac{1}{16\pi E_F \tau}\right) L(\varepsilon)$$

$$L(\varepsilon) = \int_0^{2\pi} \frac{d\phi}{2\pi} \ln\left[\frac{(\varepsilon - 2\Delta(\phi))^2}{E_F^2}\right]$$

$$\Delta(\phi) = \sqrt{\Delta_{SO}^2 + \Delta_Z^2 + 2\Delta_{SO}\Delta_Z \sin\phi}$$



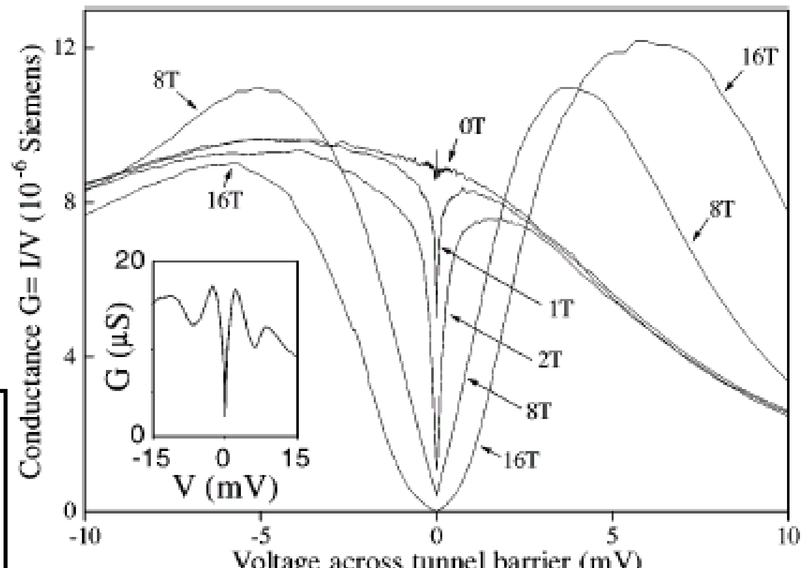
$$\frac{\delta\nu(\varepsilon)}{\nu_0} = -\left(\frac{1}{8\pi E_F \tau}\right) \ln\left[\left(|\varepsilon - 2\Delta_Z| + \sqrt{(\varepsilon - 2\Delta_Z)^2 - 4\Delta_{SO}^2}\right) E_F^{-1}\right]$$

$$\Delta_{SO}\tau \gg 1$$

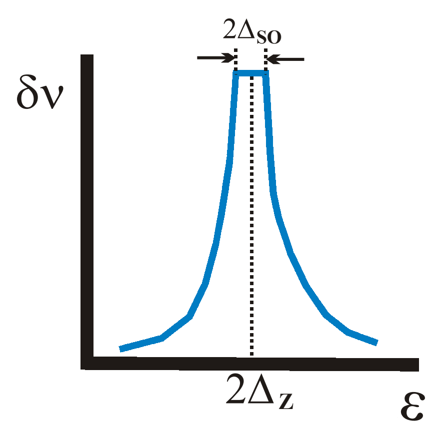
Tunneling into 2D Electron Gas: Accuracy

- Very fine feature can be resolved, $\Delta\varepsilon \sim 0.01$ meV
- Accuracy is limited by the noise
- The accuracy is increased with increasing the bias voltage

H.B. Chan, P.I. Glicofridis, R.C. Ashoori, and M.R. Melloch, PRL 79, 2867 (1997)

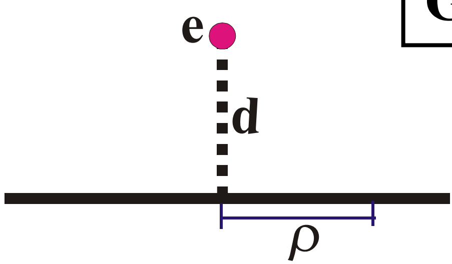


- AlGaAs/GaAs - $\Delta_{so} < 0.5$ meV
- SO splitting, as the plateau of width $2\Delta_{so}$ at the top of the Zeeman satellites, can be *resolved* in tunneling experiments at finite bias ($V_B = 2\Delta_Z / e$)



Oscillations of the Screening Potential with Finite Δ_{so}

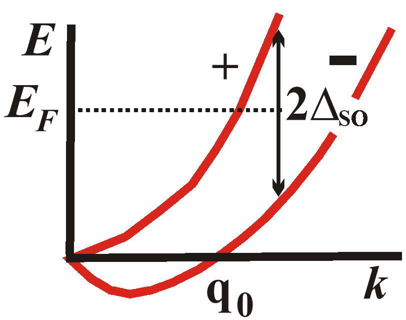
G.-H. Chen and M.R. PRB 59, 5090 (1999)



$$V(\rho) = e\int_0^\infty dq\, e^{-qd}\,\frac{J_0(q\rho)}{\varepsilon(q)}$$

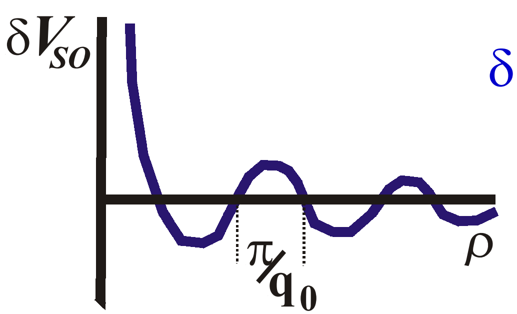
RPA:

$$\frac{\varepsilon(q)}{\varepsilon_0} = 1 - \frac{2\pi e^2}{q}\sum_{\mathbf{k},\mu,\mu'} \mathrm{Tr}\left(\hat{\Lambda}_\mu(\mathbf{k})\hat{\Lambda}_{\mu'}(\mathbf{k}+\mathbf{q})\right)\frac{f_0\left(E_\mu(\mathbf{k})\right) - f_0\left(E_{\mu'}(\mathbf{k}+\mathbf{q})\right)}{E_\mu(\mathbf{k}) - E_{\mu'}(\mathbf{k}+\mathbf{q})}$$



contains $\sqrt{q_0 - q}$ anomaly

Long-period contribution to $V(\rho)$



$$\delta V_{SO}(\rho) = -\left(\frac{2e^2}{\varepsilon_0 a_B}\right)\left(\frac{q_0}{2k_F}\right)^2 \frac{\cos(q_0\rho)}{(q_0\rho)^2}$$

not smeared out by the temperature

Exchange-Induced Enhancement of Δ_{so}

G.-H. Chen and M.R. PRB 60, 4826 (1999)

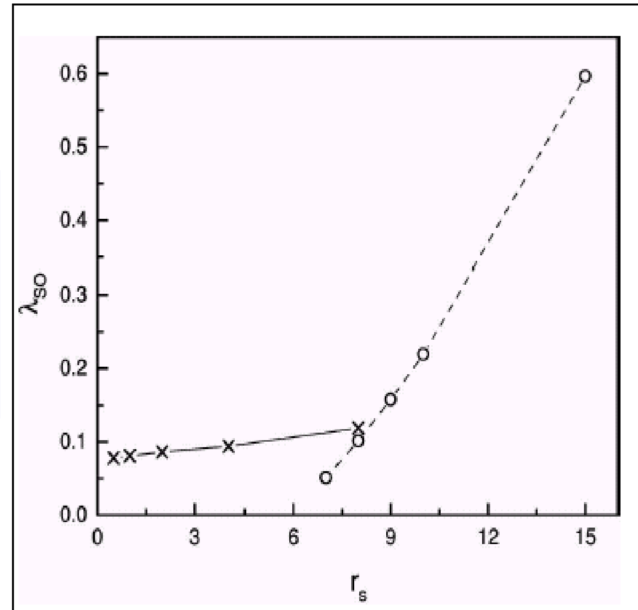
$$\hat{H} = \sum_{\mu=\pm1} E_\mu(\mathbf{k})\hat{\Lambda}_\mu(\mathbf{k}) \qquad \hat{\Lambda}_\mu(\mathbf{k}) = \frac{1}{2}\begin{pmatrix} 1 & \mu\exp(-i\phi_{\mathbf{k}}) \\ -\mu\exp(i\phi_{\mathbf{k}}) & 1 \end{pmatrix}$$

$$\hat{\Sigma}(\mathbf{k}) = -\int \frac{d^2\mathbf{k}'}{(2\pi)^2} V_{eff}(|\mathbf{k}-\mathbf{k}'|) \sum_{\mu=\pm1} \hat{\Lambda}_\mu(\mathbf{k}') f_0\left(E_F - E_\mu(\mathbf{k}')\right) =$$

self-energy

$$= \Sigma_1(\mathbf{k})\hat{\Lambda}_1(\mathbf{k}) + \Sigma_{-1}(\mathbf{k})\hat{\Lambda}_{-1}(\mathbf{k})$$

$$\Sigma_1(\mathbf{k}) - \Sigma_{-1}(\mathbf{k}) = \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \cos(\phi_{\mathbf{k}} - \phi_{\mathbf{k}'}) V_{eff}(|\mathbf{k}-\mathbf{k}'|)$$
$$\times\left[f_0\left(E_F - E_1(\mathbf{k}') - \Sigma_1(\mathbf{k}')\right) - f_0\left(E_F - E_{-1}(\mathbf{k}') - \Sigma_{-1}(\mathbf{k}')\right)\right]$$



$$m^* = \hbar^2 k_F \left(\left.\frac{\partial E(k)}{\partial k}\right|_{k_F} \right)^{-1}$$

$$\Delta^*_{so} = \frac{\Delta_{so}}{1 - \frac{m^*}{m}\lambda_{so}} \qquad \lambda_{so} = \frac{m}{(2\pi\hbar)^2} \int_0^{2\pi} d\phi \; \cos\phi \; V_{eff}\left(2k_F \sin\frac{\phi}{2}\right)$$