Transition from Hexagons to Spatiotemporal Chaos

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The transitions from periodic stationary patterns to spatiotemporal chaotic regimes is not yet well understood. We address this problem in a nonlinear optical system: a ring cavity filled with a nonlinear self-focusing Kerr medium pumped by an external field. In the mean field approximation the transverse dynamics of the slowly varying electric field amplitude E is governed by a driven-damped nonlinear Schrödinger equation [1]:

$$\partial_t E=-(1+i\theta)E+i\ {}^2E+E_0+i2|E|^2E,$$

where E_0 is the pump field and θ is the cavity detuning.

The homogeneous steady state solution becomes unstable if the pump intensity $I_0=|E_0|^2$ is larger than a critical value leading to the formation of hexagonal patterns. Above threshold the hexagons are not stationary but oscillate. The transition is sub-critical and stationary hexagonal patterns exist for lower values of the pump.

A linear stability analysis of a stationary hexagonal pattern of wavelength λ_1 (fixed point) shows that, increasing the pump intensity, it undergo a Hopf bifurcation at a finite wave length $\lambda_2=\sqrt{3}\lambda_1$. Then, a stable hexagonal super-lattice of wavelength λ_2 can be seen oscillating. A Floquet analysis for increasing values of the pump shows that the oscillating hexagons (limit cycle) undergo a Neimark-Sacker bifurcation introducing a new frequency. Again, the instability takes place at a finite wavelength $\lambda_3=\sqrt{3}\lambda_2=3\lambda_1$ and a stable super-lattice with this wavelength is also observed. For larger pumps, the oscillating hexagons with two frequencies (torus) become unstable leading to a regime of temporal chaos where the hexagonal structure is basically preserved but each peak present chaotic oscillations. Finally for very large pump intensities even the spatial regularity is lost leading to a spatiotemporal chaotic regime.

The dynamics of a single peak is similar to the caviton dynamics in Landau plasmas [2]

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References

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[2] A.C. Newell, D.A. Rand and D. Russell, Physica D **33**, 281 (1998).