Renormalization Group Analysis of Transition to Chaos in Coupled MapLattices

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The lectures contain a review of results concerning some generalizations of the Feigenbaum renormalization-group approach to spatially-extended lattices or media composed of point elements represented by period-doubling systems. Concepts of universality and scaling are outlined. A variety of dynamical regimes (pattern formation, auto-wave propagation, turbulence) is illustrated numerically. It is demonstrated that they are possible on different scaling levels, i.e. similar dynamics takes place with different scales of time, space, and dynamical variable. The review contains three parts: (i) Coupled period-doubling maps, lattices, and networks. Renormalization group, universality and scaling. (ii). From coupled map lattices to continuous media. Spatio-temporal scaling of patterns at the onset of chaos. (iii) Some experimental and numerical studies relating to realistic physical systems.

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