Non Attracting Invariant Sets: What They Can Do and How We Can Find Them

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When studying the time evolution of dissipative dynamical systems one is usually interested in the asymptotic behaviors that can be observed. These asymptotic behaviors correspond to the evolution on the attractors of the system. However, not all invariant sets are attracting. Furthermore, non attracting invariant sets play a relevant role in shaping the observable dynamics. For example, unstable periodic orbits are dense inside chaotic attractors and most of the quantities that are used to characterize these attractors, such as Lyapunov exponents, can be calculated in terms of these periodic orbits. Non attracting invariant sets can also be chaotic, and are responsible, among other things, for the long chaotic transients that can be observed quite frequently in high dimensional systems. Finding attractors numerically is not difficult, since there is a measurable set of initial conditions that approach them. Finding non attracting sets is much more challenging. The typical method for finding periodic orbits is Newton's method. This method has been applied succesfully in the case of spatially extended systems. The main drawback in such a case is the determination of a good enough initial guess that guarantees the convergence of the procedure. The numerical computation of non-attracting chaotic sets imposes further challenges. In this talk we will describe a method for finding non attracting chaotic sets that can be used to obtain good initial guesses for the calculation of periodic orbits in spatially extended systems. We will complement the talk with a brief introduction to the theory of dynamical systems and on the role of non-attracting invariant sets on the observable dynamics.

Pattern Transitions And Two-Dimensional Flows In The Gray-Scott Model

The Gray-Scott model is a two-variable reaction-diffusion system that displays a large variety of patterns. This variety of patterns occurs, in particular, for parameter values for which the spatially homogeneous system only has one fixed point that attracts all initial conditions. In this talk we will show how some of the patterns and transitions among them can be understood in terms of the collection of planar vector fields that can be constructed via perturbations of the spatially homogeneous evolution equations by treating the diffusion terms as parameters. We will discuss why this point of view might be helpful for the construction of models from experimental observations.