Overview of Transport Models in Cuprates

Victor Yakovenko

University of Maryland

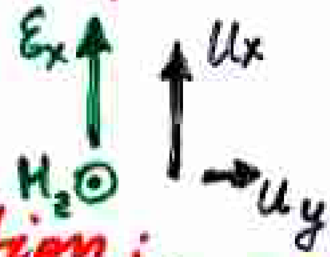
College Park, MD, USA

A mystery of the (ac, normal-state) Hall conductivity

1) Introduction to the Drude theory
2) 2-tau models of the Hall effect
3) Tau^2 fit for θ_H
4) The Abrahams-Varma theory of θ_H
5) The vortex flow model of the normal-state Hall effect

Drude theory of electron transport

(u_x, u_y) is the electron velocity,
$\gamma = 1/\tau$ is a relaxation time.



Electron's equations of motion:

$(\omega_H \tau \ll$

$$\frac{d}{dt} u_x + \gamma u_x = \frac{e}{m}\mathcal{E}_x \Rightarrow u_x = \frac{e/m}{-i\omega + \gamma}\mathcal{E}_x$$

$$\frac{d}{dt} u_y + \gamma u_y = \underbrace{\left(\frac{eH}{mc}\right)}_{=\omega_H} u_x \Rightarrow u_y = \frac{\omega_H}{-i\omega + \gamma} u_x$$

$$\boxed{\mathcal{E}_x} \xrightarrow{\text{Electric force}} \boxed{u_x} \xrightarrow{\text{Lorentz force}} \boxed{u_y}$$

$$\mathcal{E}_x \frac{e/m}{-i\omega + \gamma} = u_x; \quad u_x \frac{\omega_H}{-i\omega + \gamma} = u_y$$

Electric conductivities:

$$\sigma_{xx} = \frac{j_x}{\mathcal{E}_x} = \frac{e n u_x}{\mathcal{E}_x} = \frac{ne^2/m}{-i\omega + \gamma} = \frac{\omega_p^2}{-i\omega + \gamma}$$

$$\tan\theta_H = \frac{u_y}{u_x} = \frac{\omega_H}{-i\omega + \gamma} \approx \theta_H$$

$$\sigma_{xy} = \frac{j_y}{\mathcal{E}_x} = \frac{e n u_y}{\mathcal{E}_x} = \frac{u_y}{u_x} \cdot \frac{e n u_x}{\mathcal{E}_x} = \theta_H \sigma_{xx} = \frac{\omega_p^2 \omega_H}{(-i\omega + \gamma)^2}$$

$$R_H = \frac{1}{H}\frac{\sigma_{xy}}{\sigma_{xx}^2} = \frac{1}{nec} = \mathrm{const}(\tau, \omega) = \frac{\theta_H}{H \sigma_{xx}}$$

dc transport in cuprates

- Experiment shows that

$$\sigma_{xx} \sim \frac{1}{T} \text{ and } \theta_H \sim \frac{1}{T^2} \quad \text{(Ong 1991)}$$

- Drude theory contradicts the experiment

$$\sigma_{xx} = \omega_p^2 \, \tau(T) \text{ and } \theta_H = \omega_H \, \tau(T)$$

- P.W. Anderson's (1991) proposal – two relaxation times:

$$\sigma_{xx} = \omega_p^2 \, \tau_{tr} \text{ and } \theta_H = \omega_H \, \tau_H$$

$$\tau_{tr} = \frac{1}{\gamma_{tr}} \sim \frac{1}{T} \text{ and } \tau_H = \frac{1}{\gamma_H} \sim \frac{1}{T^2}$$

- H.D. Drew's (1996) generalization to finite ω (ac transport):

$$\sigma_{xx} = \frac{\omega_p^2}{-i\omega + \gamma_{tr}} \text{ and } \theta_H = \frac{\omega_H}{-i\omega + \gamma_H}$$

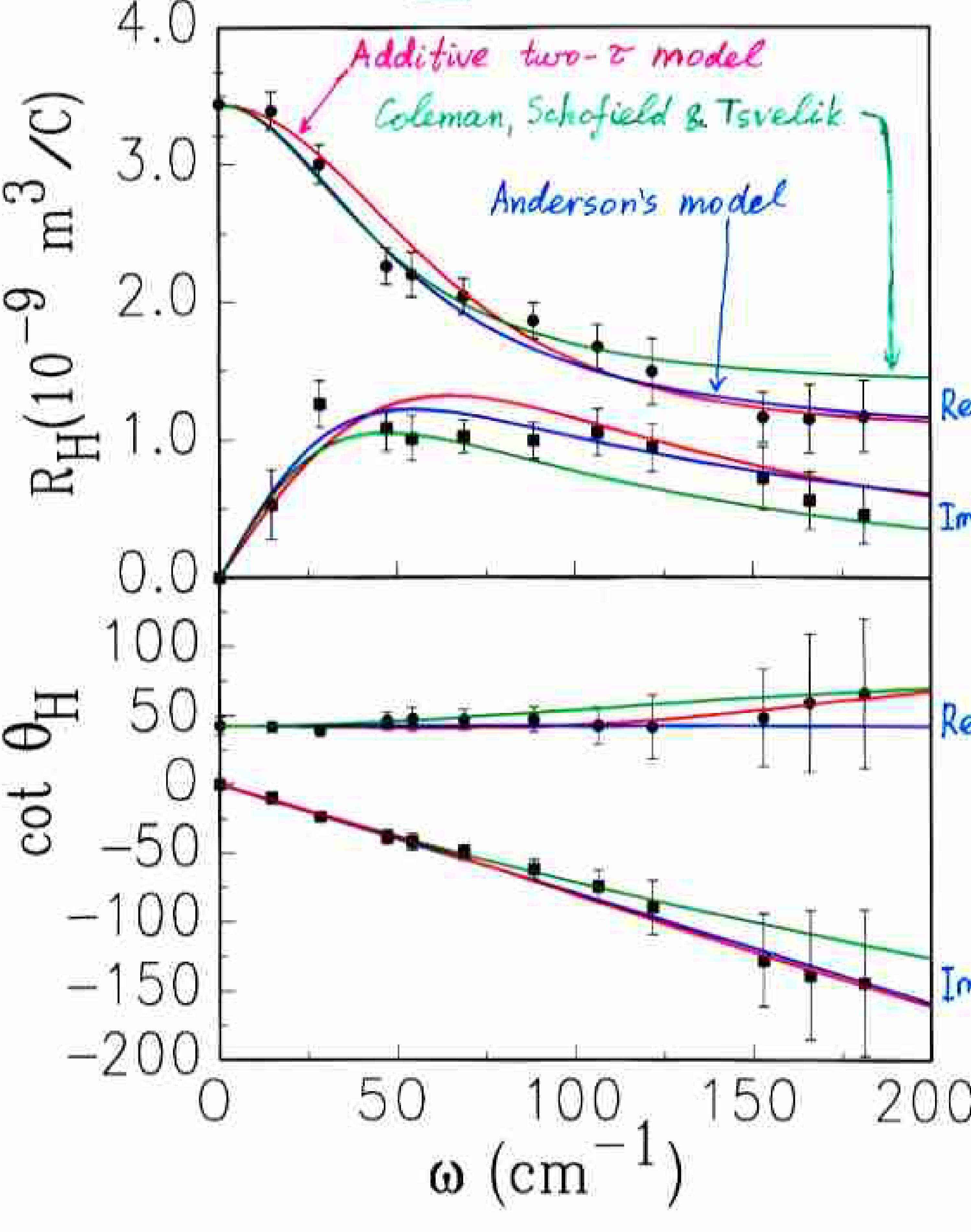
$$\theta_H^{-1} = \frac{-i\omega}{\omega_H} + \frac{\gamma_H(T)}{\omega_H} \quad \text{(same as Drude)}$$

$$R_H = \frac{\theta_H}{H \, \sigma_{xx}} = \frac{1}{nec} \cdot \frac{-i\omega + \gamma_{tr}}{-i\omega + \gamma_H} \neq \text{const}(\omega, T)$$

(not as Drude)

A:\R_H.cot.omega.ps

PRB 57, 3089 (1998)



Transport models

1) Single-τ models

- Marginal Fermi liquid

  Varma et al., PRL 1989

  $$\rho_{xx} \propto \frac{1}{\tau} \propto \mathrm{Im}\,\Sigma \propto \max(T, \omega)$$

- Luttinger liquid (Anderson, PRB 1997

  $$\sigma_{xx}(\omega) = \frac{A}{\left(\frac{1}{\tau} - i\omega\right)^{\alpha}} \; ; \quad \frac{\mathrm{Im}\,\sigma_{xx}(\omega)}{\mathrm{Re}\,\sigma_{xx}(\omega)} \approx \text{const at } \omega \gg \frac{1}{\tau}$$

↑ Do not give predictions about $\sigma_{xy}(T, \omega)$

- Holon-spinon fluid

  D.K. Lee & P.A. Lee, J. Phys. Cond. Mat. 1997

  $$\omega_H \propto \frac{1}{T} \; ; \quad \theta_H = \frac{\omega_H(T)}{\frac{1}{\tau} - i\omega}$$

- Skew scattering $\propto \frac{1}{T}$

  Kotliar, Sengupta, Varma, PRB 1996

↑ Unsatisfactory because give $R_H(\omega) =$ $=$ const

2) Global two-τ models

– Multiplicative model

Anderson; Ong, PRL, 1991

$$\sigma_{xx} \propto \tau_{tr} \propto \frac{1}{T}$$

$$\sigma_{xy} \propto \tau_{tr}\,\tau_H \propto \frac{1}{T^3}\text{, where } \tau_H \propto \frac{1}{T^2}$$

For $\omega \neq 0$: $\dfrac{1}{\tilde{\tau}_{tr,H}} = \dfrac{1}{\tau_{tr,H}} - i\omega$

Drew et al. PRL 1996

– Charge-conjugation model

Coleman, Schofield, Tsvelik, PRL 1996

$\hat{C}\,\hat{\psi} = \pm\,\hat{\psi}$ (normally $\hat{C}\,\hat{\psi} = \hat{\psi}^+$)

relaxation rates $\Gamma_f \propto T$ & $\Gamma_s \propto T^2$

$$\rho_{xy} \propto \Gamma_f\,\Gamma_s \propto T^3;\ \rho_{xx} \propto (\Gamma_f + \Gamma_s) \propto T + T^2$$

$$\theta_H = \frac{\rho_{xy}}{\rho_{xx}} = \frac{(\Gamma_f - i\omega)(\Gamma_s - i\omega)}{\Gamma_f + \Gamma_s - 2i\omega}$$

3) Variation of τ over the Fermi surface.

- Additive two-τ model

Carrington, Mackenzie, Lin, Cooper, PRL 1992

Kendziora, Mandrus, Mihaly, Forro, PRB 1992

$\sigma_{xx}(T)$, $\sigma_{xy}(T)$

Zheleznyak, Yakovenko, Drew, PRB 1998, 1999

$\sigma_{xx}(\omega)$, $\sigma_{xy}(\omega)$; $\Delta\sigma_{xx}(H,T)$

- Hussey, cond-mat/0110187 (2001)

- Antiferromagnetic fluctuations

Hlubina, Rice, PRB 1995

Stojković, Pines, 1996 - 1997

- Cold spots

Ioffe, Millis, PRB 1998

$$\frac{1}{\tau} = \frac{1}{\tau_0} + \Gamma k_t^2, \quad \frac{1}{\tau_0} \propto T^2, \quad \Gamma = \text{const}$$

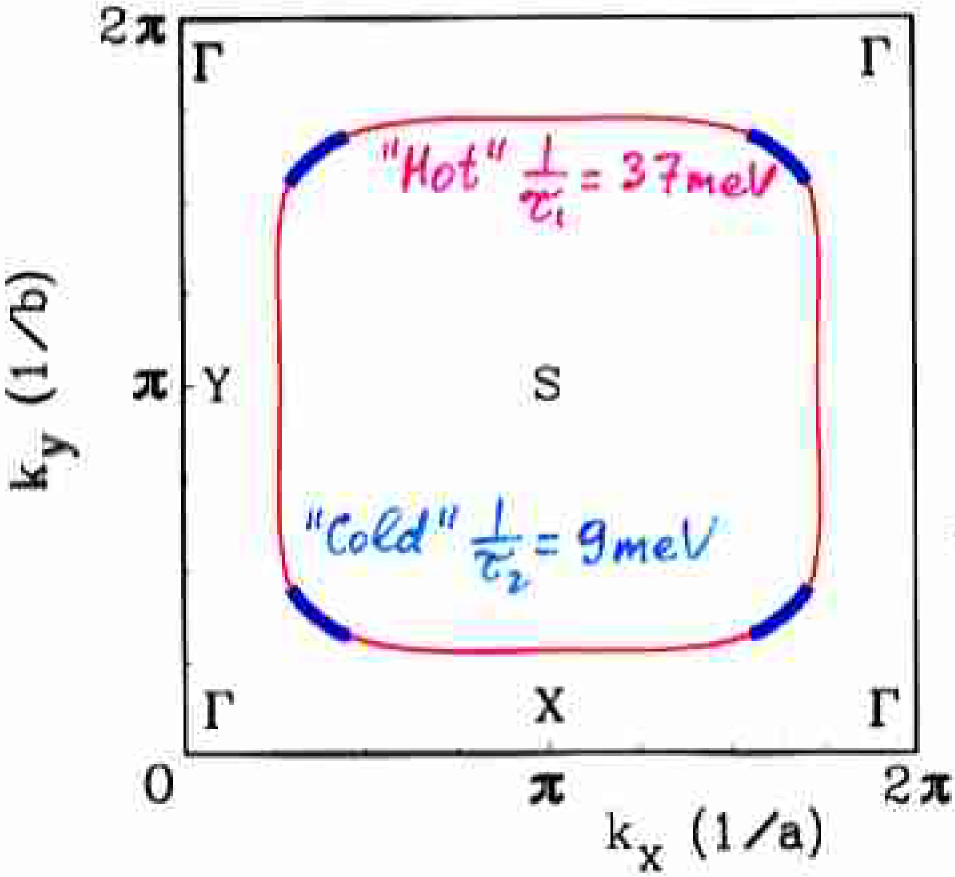
van der Marel, PRB 1999

$$\frac{1}{\tau} = \frac{1}{\tau_0} + \Gamma \sin^2(2\theta)$$

$$\sigma_{xx} \propto \frac{1}{\sqrt{\frac{1}{\tau_0} - i\omega}} \, \frac{1}{\sqrt{\frac{1}{\tau_0} + \Gamma - i\omega}} \propto \begin{cases} \frac{1}{\sqrt{-i\omega}}, & \frac{1}{\tau_0} \ll \omega \ll \Gamma \\ \frac{1}{-i\omega}, & \Gamma \ll \omega \end{cases}$$

Zheleznyak, Yakovenko, Drew, Mazin
PRB 57, 3089 (1998)

Additive two-τ model



$\frac{1}{\tau_1} \sim T$

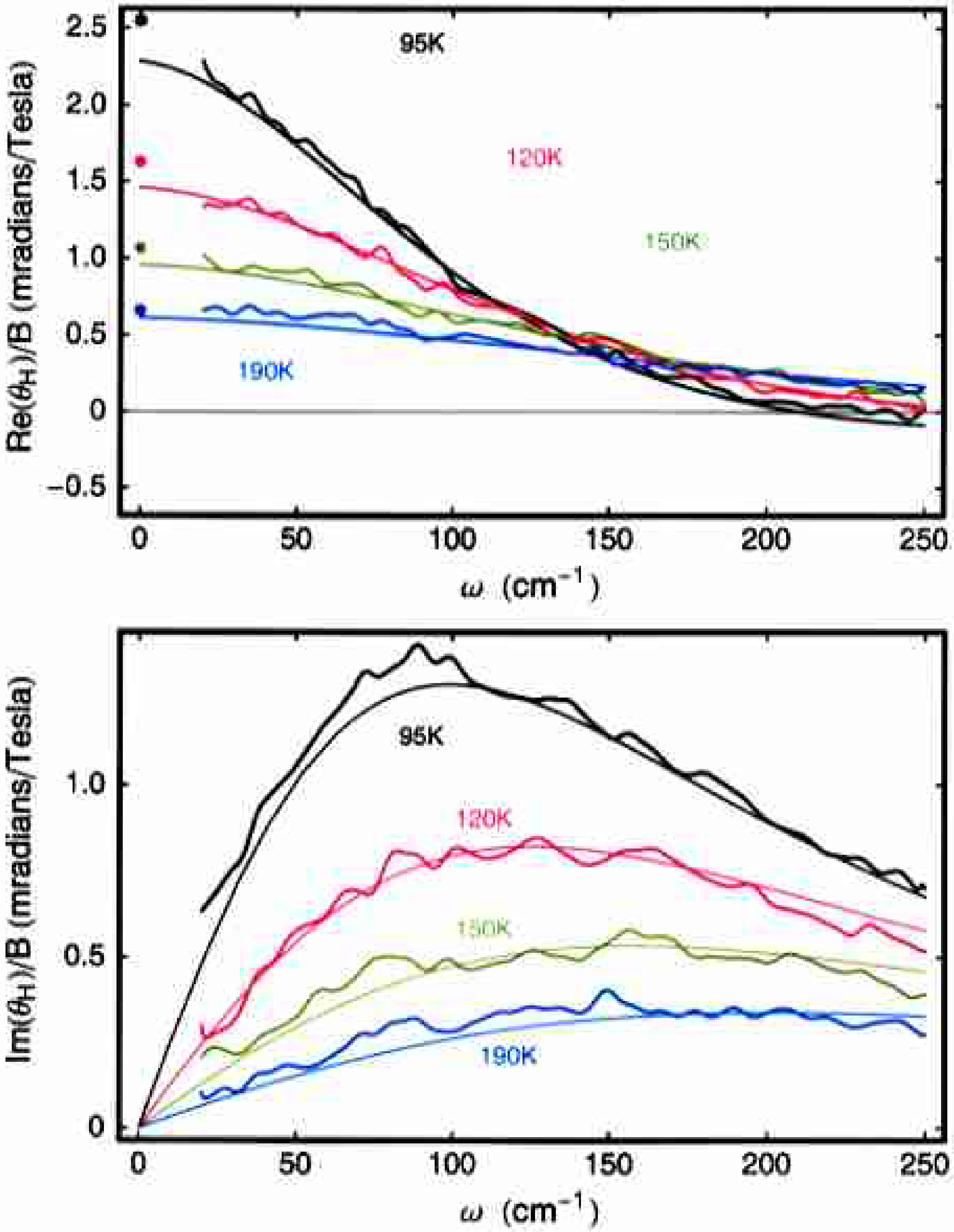
$\frac{1}{\tau_2} \sim T^2$

$$\sigma_{xx}(\omega) = A_1\tilde{\tau}_1(\omega) + A_2\tilde{\tau}_2(\omega)$$
$$\sigma_{xy}(\omega) = B_1\tilde{\tau}_1^2(\omega) + B_2\tilde{\tau}_2^2(\omega)$$
$$1/\tilde{\tau}_{1,2}(\omega) = 1/\tau_{1,2} - i\omega, \quad \tau_2/\tau_1 = 4$$

where

$$A_{1,2} \propto \int_{1,2} v(k_t)\, dk_t, \quad A_1 : A_2 = 9 : 1$$

$$B_{1,2} \propto \int_{1,2} \mathbf{v}(k_t) \times d\mathbf{v}(k_t), \quad B_1 : B_2 = 7 : 3$$



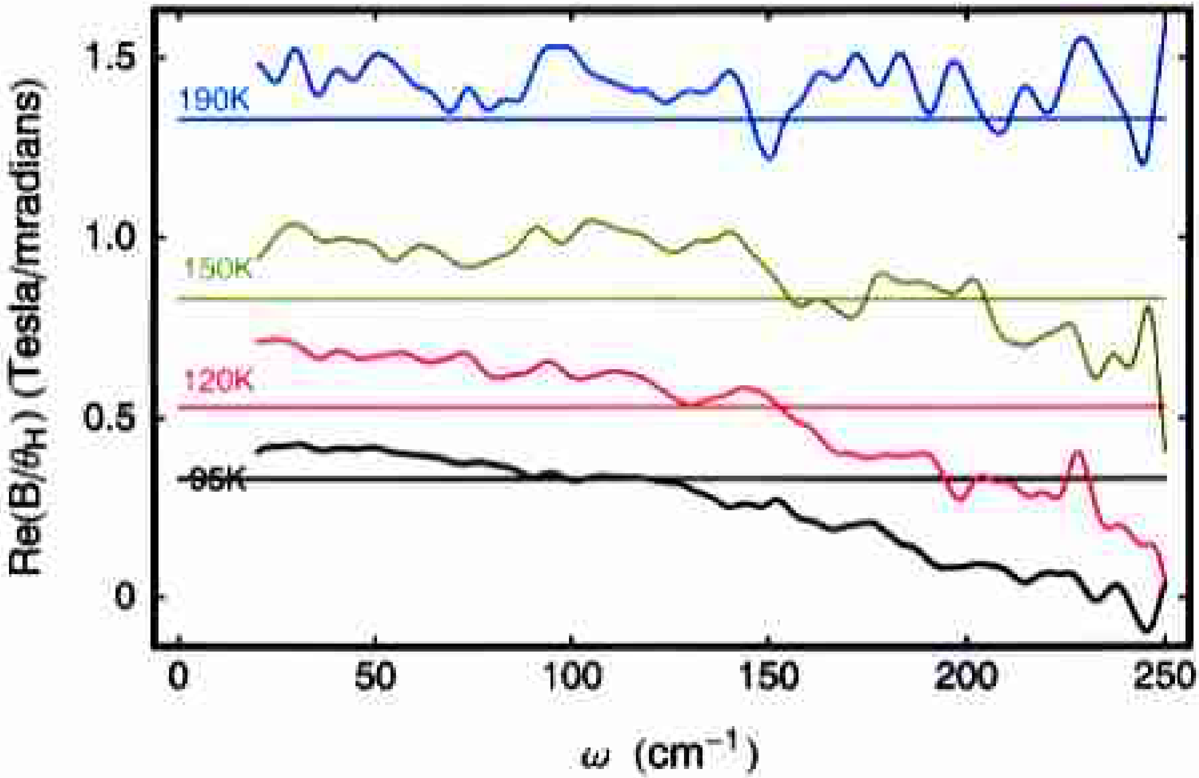
Holon drag model $\frac{a\,\omega H}{\Gamma H - i\,\omega} + \frac{\omega H\,\Omega P}{(\Gamma H - i\,\omega)^2}$

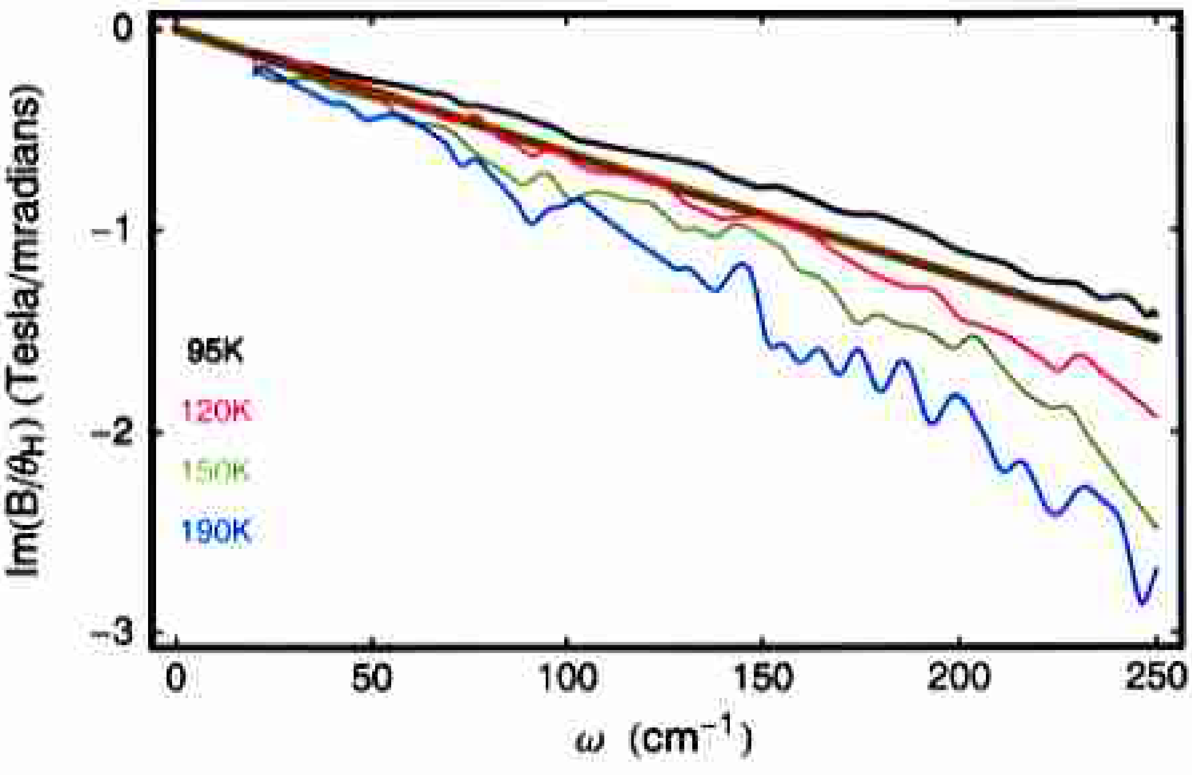
$a = .1;$

$\omega H = 2.3\ \mathrm{cm}^{-1};$

$\Omega P = 220\ \mathrm{cm}^{-1};$

$\Gamma H = \frac{1}{.55}\ T\ (K) + \frac{25}{250}\ \omega\ (\mathrm{cm});$



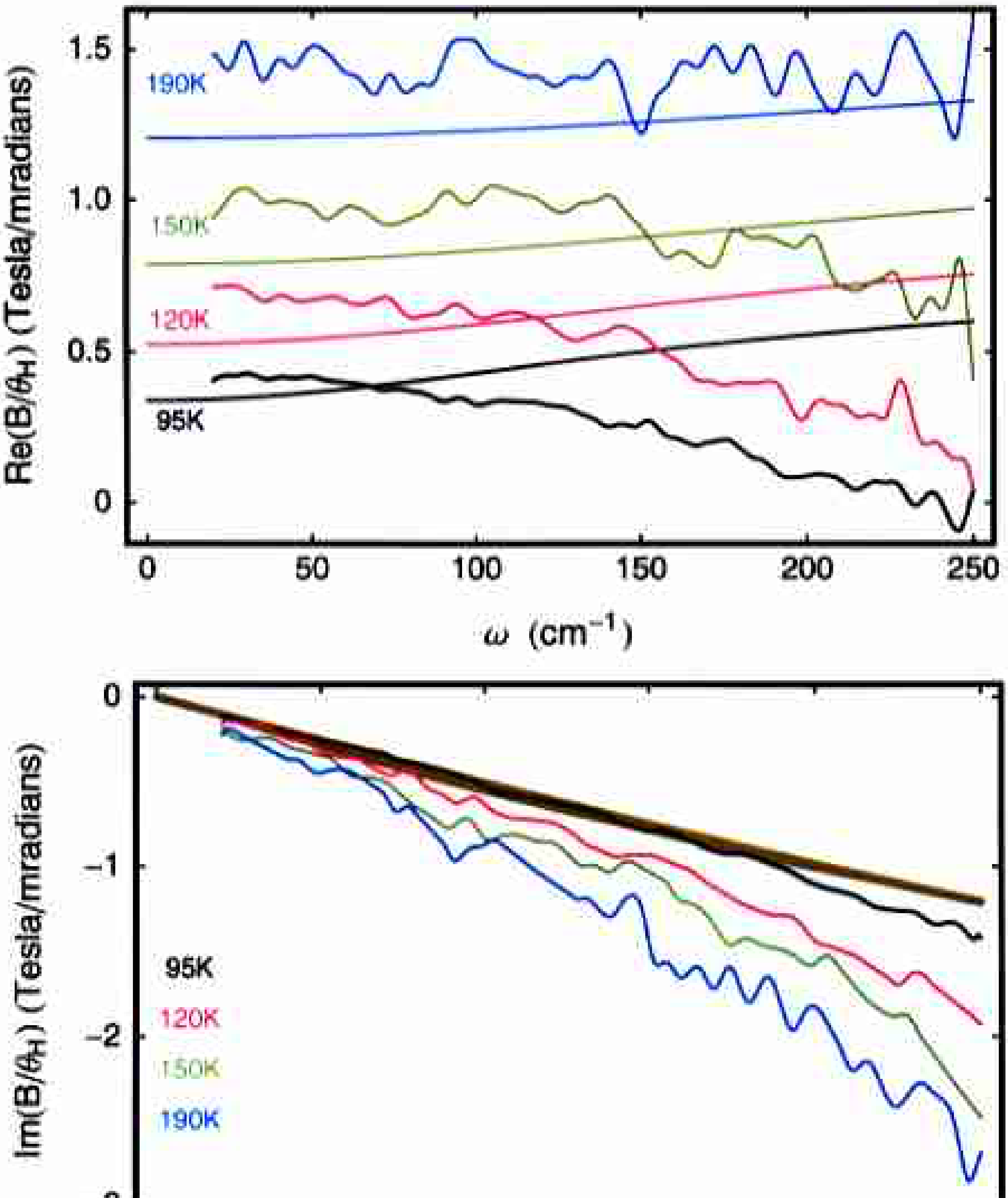


Anderson's model: $\sigma xx = c\,\frac{\omega p^2}{4\pi}\,\tau tr$ $\qquad$ $\tau tr = \left(185\,\frac{T\,(K)}{95\,K} - i\,\omega\,(cm^{-1})\right)^{-1}$

$\sigma xy = c\,\frac{\omega H\,\omega p^2}{4\pi}\,\tau tr\,\tau H$ $\qquad$ $\tau H = \left(54\,\left(\frac{T\,(K)}{95}\right)^2 - i\,\omega\,(cm^{-1})\right)^{-1}$

$\omega p = 9.2\times10^3\ cm^{-1}$

$\omega H = 1.3\ cm^{-1}$

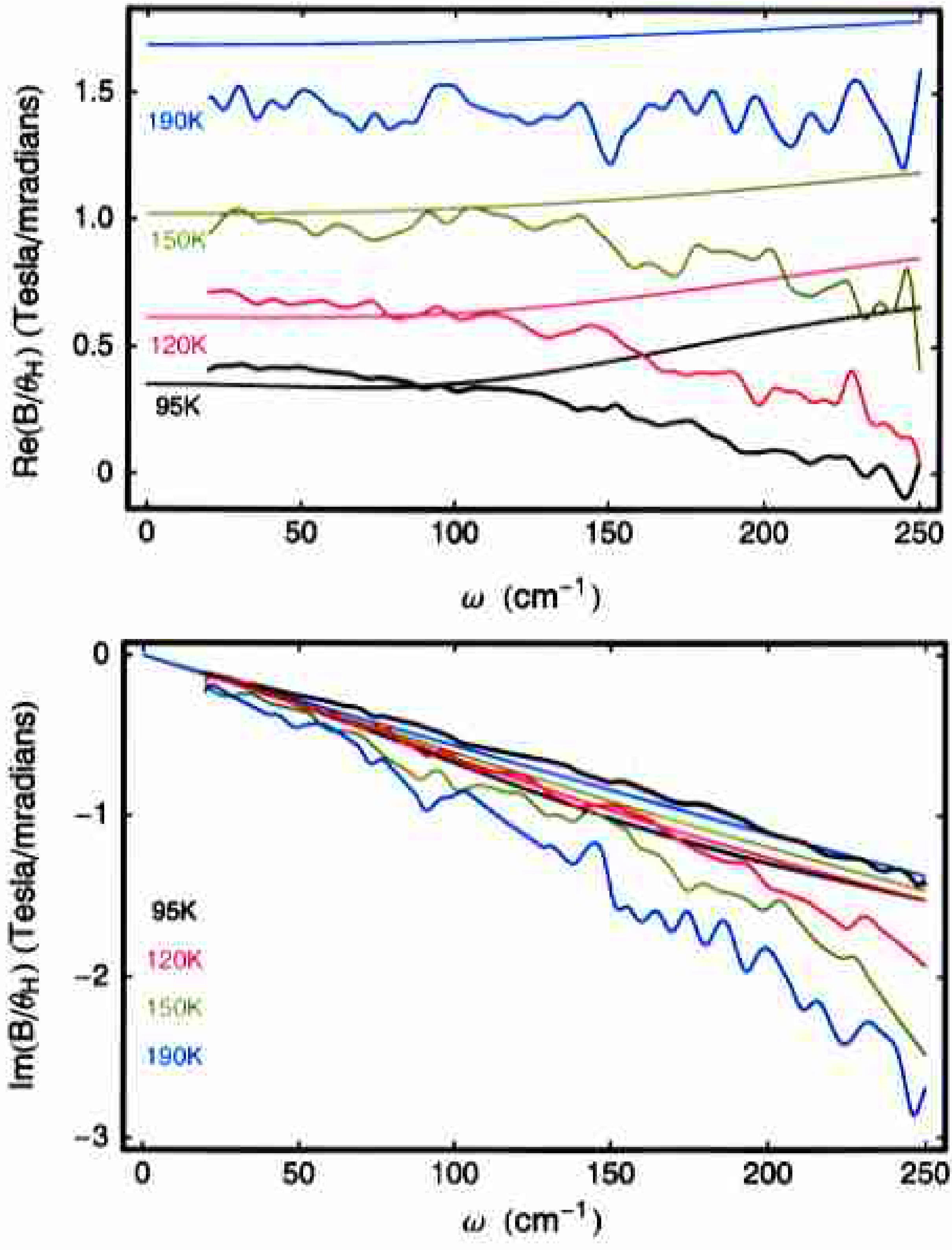


Charge conjugation model: $\sigma xx = c\,\frac{\omega p^2}{2\pi}\,\frac{1}{\Gamma f + \Gamma s}$ $\quad \Gamma f := \frac{T}{95\,K}\,322\ \mathrm{cm}^{-1} - i\,\omega\ (\mathrm{cm}^{-1})$

$\sigma xy = c\,\frac{\omega H\,\omega p^2}{4\pi}\,\frac{1}{\Gamma f\,\Gamma s}$ $\quad \Gamma s = \left(\frac{T}{95\,K}\right)^2 49\ \mathrm{cm}^{-1} - i\,\omega\ (\mathrm{cm}^{-1})$

$\omega p = 9.2\times10^3\ \mathrm{cm}^{-1}$

$\omega H = 2\ \mathrm{cm}^{-1}$



Yakovenko's two τ

$$\sigma xx = \frac{\omega p^2}{4\pi}\,(a\,\tau 1 + (1 - a)\,\tau 2)$$

$$\sigma xy = \frac{\omega H\,\omega p^2}{4\pi}\,(b\,(\tau 1)^2 + (1 - b)\,(\tau 2)^2)$$

$$\tau 1 = (2\pi c)^{-1}\left(\eta\,\frac{T\,kB}{c\,hF} - i\,\omega\right)^{-1}$$

$$\tau 2 = (2\pi c)^{-1}\left(\frac{T\left(\frac{T\,kB}{hF}\right)}{c\,W} - i\,\omega\right)^{-1}$$

$a = 0.9$
$b = 0.71$
$\eta = 4.5$
$W = 82.5\ K$
$\omega p = 2\pi c\,10000\ cm^{-1}$
$\omega H = 2\pi c\,1.7\ cm^{-1}$

Grayson, Rigal, Schmadel, Drew, Kung

PRL 89, 037003 (2002)

New ac experiment

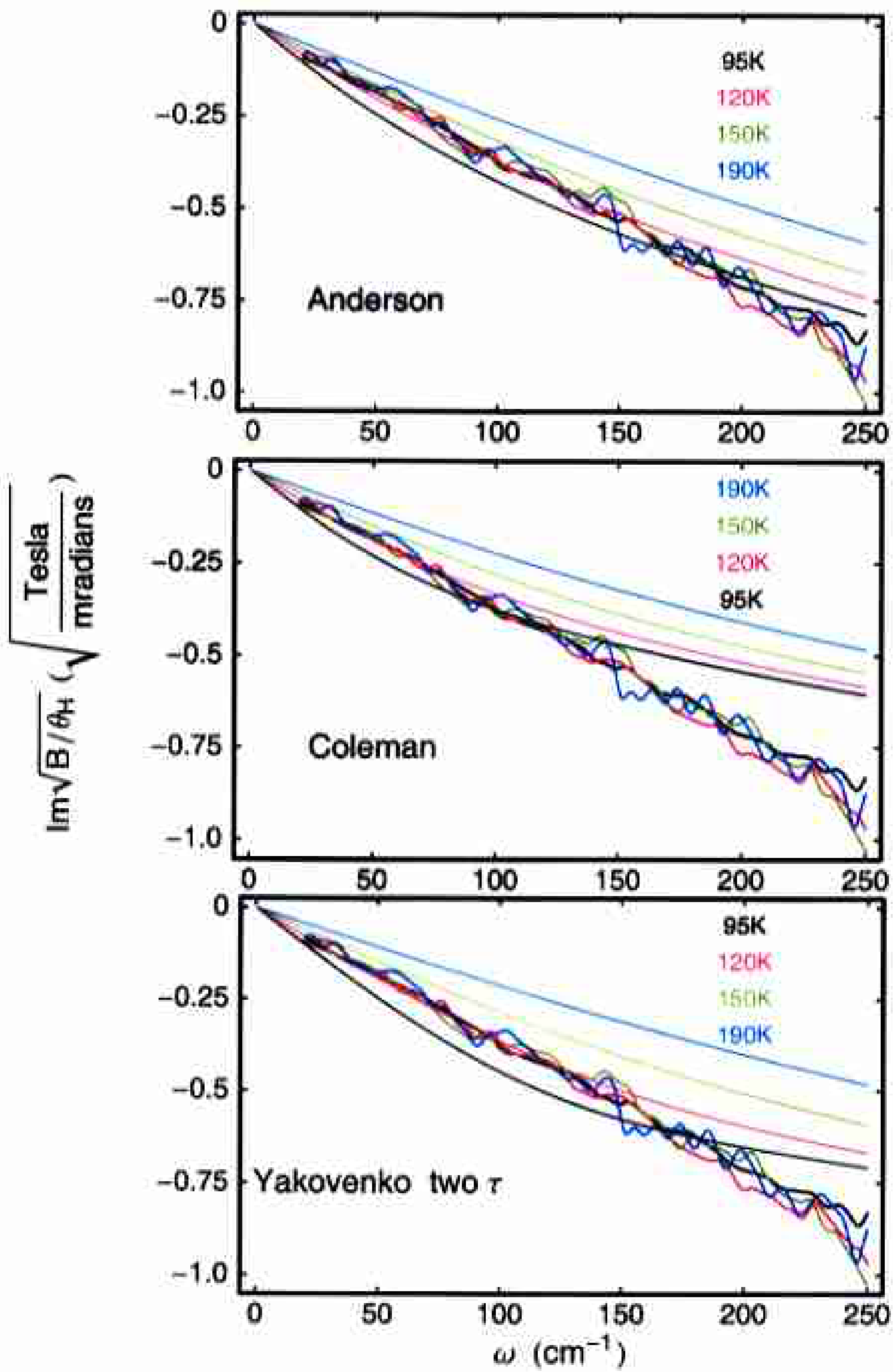
$$\theta_H(\omega, T) = \frac{\omega_H \, \omega_0}{\left[-i\omega + \gamma(T)\right]^2}$$

$$\Downarrow$$

$$\sqrt{\theta_H^{-1}} \sim -i\omega + \gamma(T)$$

$$\theta_H^{-1} = \frac{\left[-i\omega + \gamma(T)\right]^2}{\omega_H \, \omega_0} \sim \gamma^2(T) - \omega^2 - 2i\omega\gamma(T)$$

For $\gamma(T) \sim T$, $\theta_H^{(dc)} \sim \frac{1}{\gamma^2} \sim \tau^2 \sim \frac{1}{T^2}$



Anderson
Coleman
Yakovenko two τ
95K
120K
150K
190K
$\mathrm{Im}\sqrt{B/\theta_H}\,\left(\sqrt{\frac{\mathrm{Tesla}}{\mathrm{mradians}}}\right)$
ω (cm^{-1})
0
−0.25
−0.5
−0.75
−1.0
0
50
100
150
200
250

Varma and Abrahams theory
(PRL 2001, erratum 2002)

- Two relaxation mechanisms

$\gamma_M = \frac{1}{\tau_M} \sim T$ - as in the marginal Fermi liquid, uniform over the Fermi surface

$\gamma_i = \frac{1}{\tau_i} \sim f(\theta)$ - the small-angle impurity scattering, strongly varies over the Fermi surface

- Assuming that $\gamma_M \gg \gamma_i$, they found

$$\theta_H(T) = \omega_H \tau_M(T) + \omega_H \tau_M^2(T) \left\langle \frac{1}{\tau_i} \right\rangle,$$

claimed that the second term is $\sim 10^2$ times greater than the first one.

- Actually, with the assumptions of the derivation (circular Fermi surface, etc), the second term is **zero**.

- In general, the second term is not zero, but **small** when $\gamma_M \gg \gamma_i$. In the opposite limit $\gamma_M \ll \gamma_i$, $\theta_H \approx \omega_H \langle \tau_i \rangle \approx \mathrm{const}(T)$.

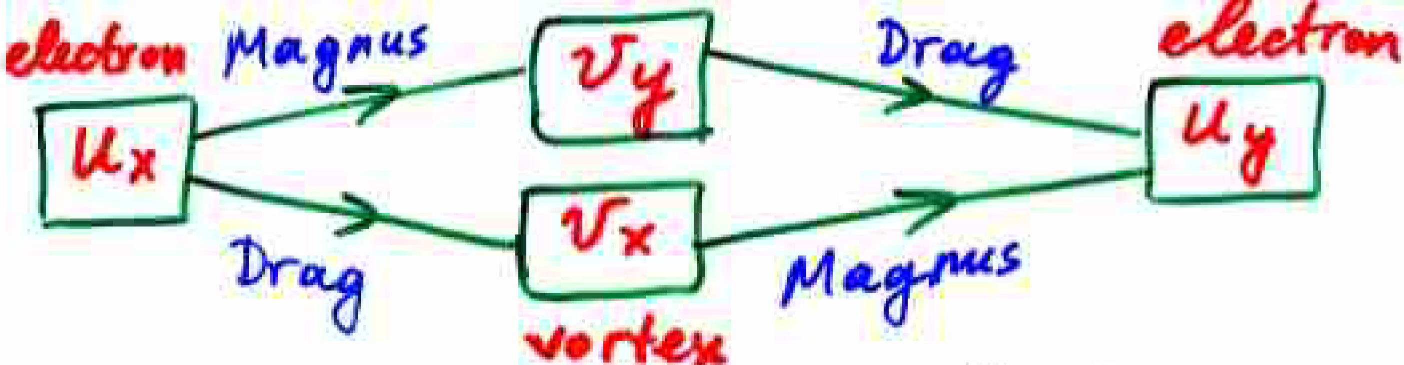
- Thus, the Varma-Abrahams theory cannot reproduce the Grayson-Drew fit.

~~???~~ This conclusion is confirmed by <u>Carter and Schofield</u> (2002), who also studied magnetoresistance.

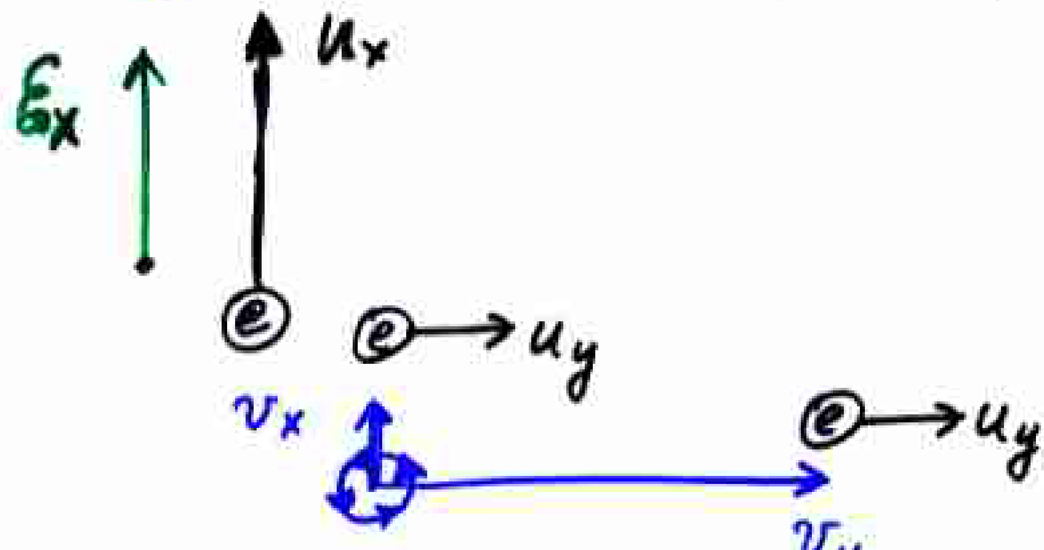
Vortex-flow model of the normal-state Hall effect

(Yakovenko, Drew 2002)

Qualitatively similar to Ong's picture of the Nernst effect.



(v_x, v_y) is the vortex velocity



Let us treat the "normal" state as a superconducting state with proliferated, depinned vortices and antivortices (similar to Kosterlitz-Thouless

Electron equation of motion

$$\left(\frac{d}{dt}+\gamma_e\right)u_y = \cancel{\omega_H u_x} \quad \text{- Lorentz force}$$

$$+\,\omega_H(v_x - \cancel{u_x}) \quad \text{- Magnus force}$$

$$+\,\gamma'_{ev}(v_y - u_y) \quad \text{- Electron-vortex drag}$$

Vortex equations of motion

$$\left(\frac{d}{dt}+\gamma_v\right)v_y = \Omega\, u_x \quad \text{- Magnus force}$$

$$\left(\frac{d}{dt}+\gamma_v\right)v_x = \gamma_{ev}(u_x - v_x) \quad \text{- Electron-vortex drag}$$

Relaxation rates:

γ_e - electron to lattice,

γ_v - vortex to lattice,

$\gamma_{ev}^{(')}$ - electron-vortex drag.

Result:

$$\theta_H = 2\,\frac{\omega_H\,\gamma_{ev}}{(-i\omega+\gamma_e+\gamma'_{ev})(-i\omega+\gamma_v+\gamma_{ev})}$$

$$\frac{\gamma'_{ev}}{\gamma_{ev}} = \frac{NM}{nm}\ (\ll 1\,?)$$ - vortex / - electron concentration × mas

$$\theta_H \simeq 2\,\frac{\omega_H\,\gamma_{ev}}{[-i\omega+\gamma_e(T)][-i\omega+\gamma_v(T)+\gamma_{ev}]}$$

We assume $\gamma_e, \gamma_v \sim T$, $\gamma_{ev} = \mathrm{const}(T)$.

More general case: $\rho_n + \rho_s = 1$

$$\left(\frac{d}{dt} + \gamma_e\right) u_y = \omega_H u_x + \rho_s \omega_H (v_x - u_x) =$$

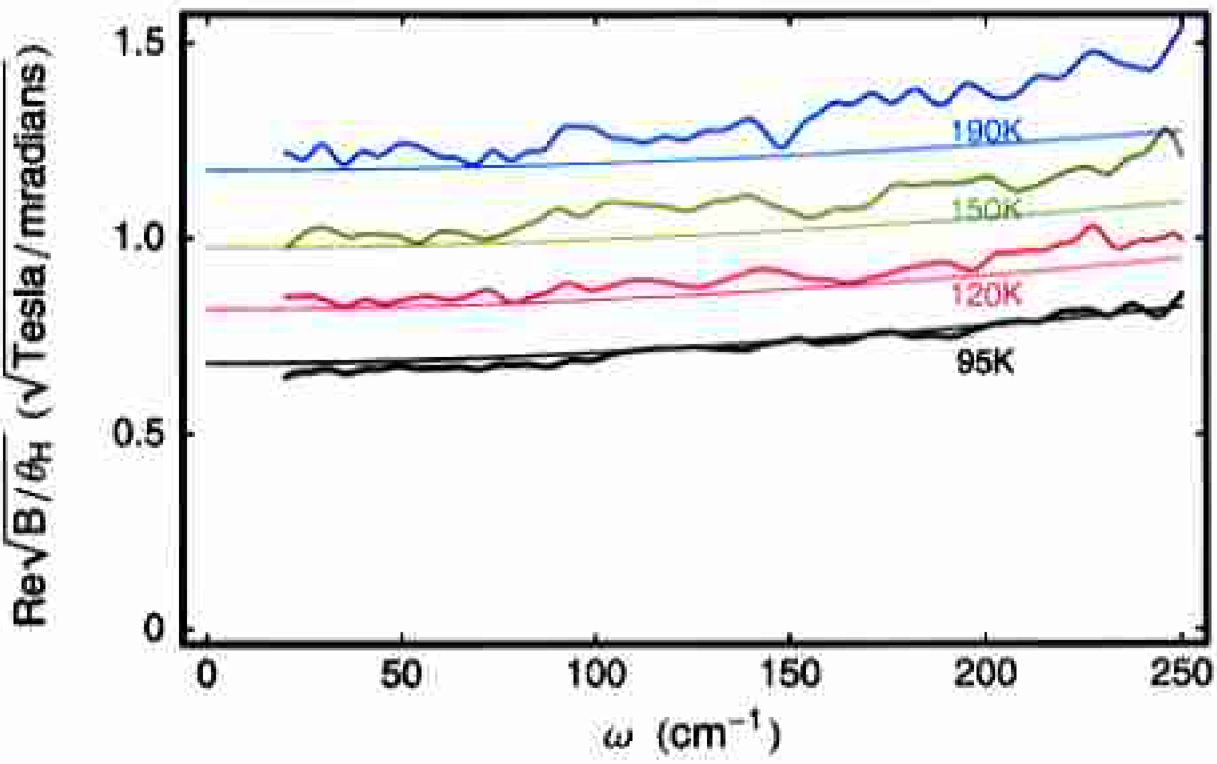
$$= \rho_n \omega_H u_x + \rho_s \omega_H v_x .$$

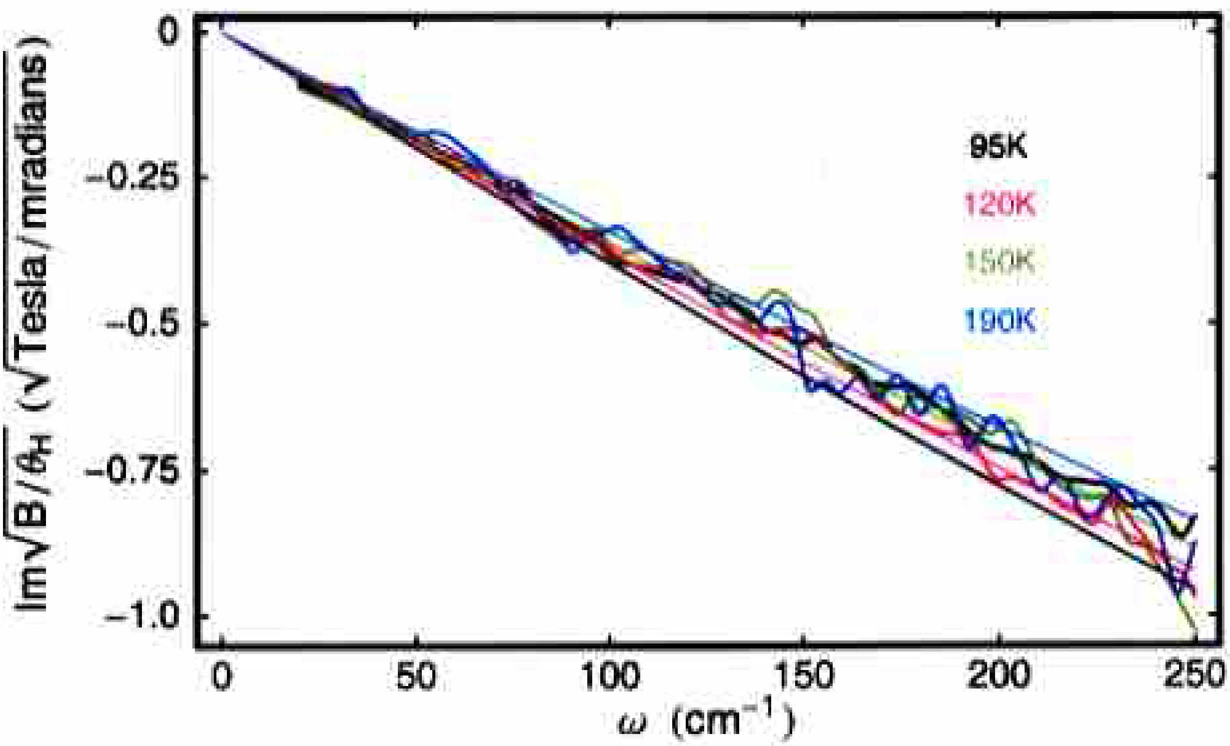
Then

$$\theta_H = \rho_n \frac{\omega_H}{-i\omega + \gamma_e} +$$

$$+ \rho_s \frac{\omega_H \gamma_{ev}}{(-i\omega + \gamma_e)(-i\omega + \gamma_v + \gamma_{ev})}$$

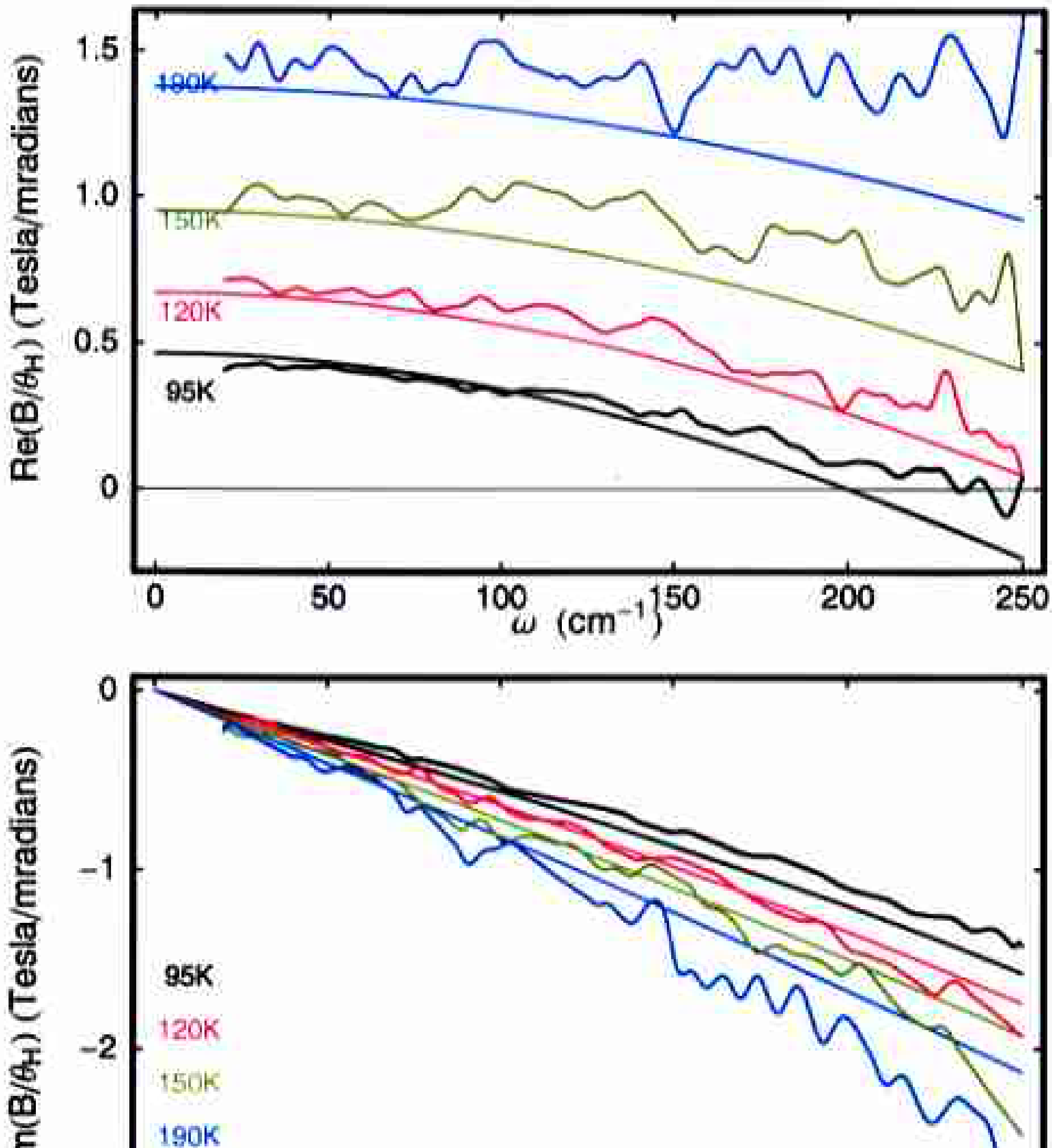
This equation produces a reasonable fit of the data.





Vortex model $\frac{a\omega H}{(\Gamma H - i\,\omega)} + \frac{\omega H\, CC}{(\Gamma H - i\,\omega)\,(\Gamma H - i\,\omega + CC)}$

$\omega H = 6.5\ \mathrm{cm}^{-1}$;
$CC = 50\ \mathrm{cm}^{-1}$;
$a = .08$;
$\Gamma H = \frac{T}{K}\,1.4$;



Vortex model $\frac{a\omega H}{(\Gamma H - i\,\omega)} + \frac{\omega H\,CC}{(\Gamma H - i\,\omega)\,(\Gamma H - i\,\omega + CC)}$

$\omega H = 6.5\ \mathrm{cm}^{-1}$;
$CC = 50\ \mathrm{cm}^{-1}$;
$a = .08$;
$\Gamma H = \frac{T}{K}\,1.4$;

Conclusions

1) New experimental data (Grayson et al. PRL 2002) show that

$$\theta_H = \frac{\omega_H \, \omega_0}{[-i\omega + \gamma(T)]^2}$$

2) Old transport theories fail to produce this dependence

3) Vortex flow model of the normal-state transport gives

$$\theta_H = \rho_n \frac{\omega_H}{-i\omega + \gamma_e} +$$

$$+ \rho_s \frac{\omega_H \gamma_{ev}}{(-i\omega + \gamma_e)(-i\omega + \gamma_v + \gamma_{ev})}$$

This formula reasonably (but not perfectly) fits the data.